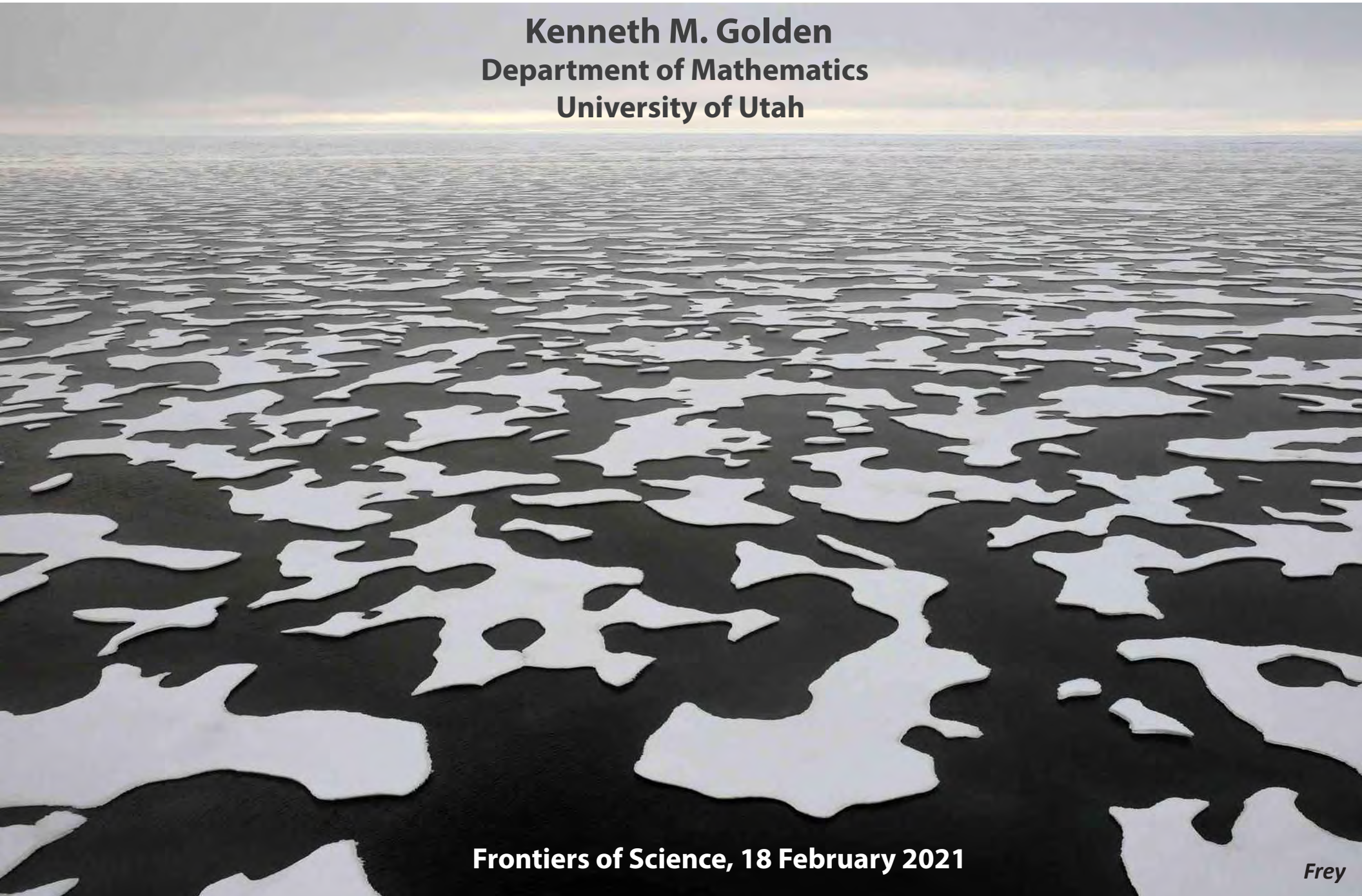


# On Thinning Ice: Modeling Sea Ice in a Warming Climate

**Kenneth M. Golden**  
Department of Mathematics  
University of Utah



**Frontiers of Science, 18 February 2021**

*Frey*



# SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of **climate change**





# polar ice caps critical to global climate in reflecting incoming solar radiation



white snow and ice  
reflect

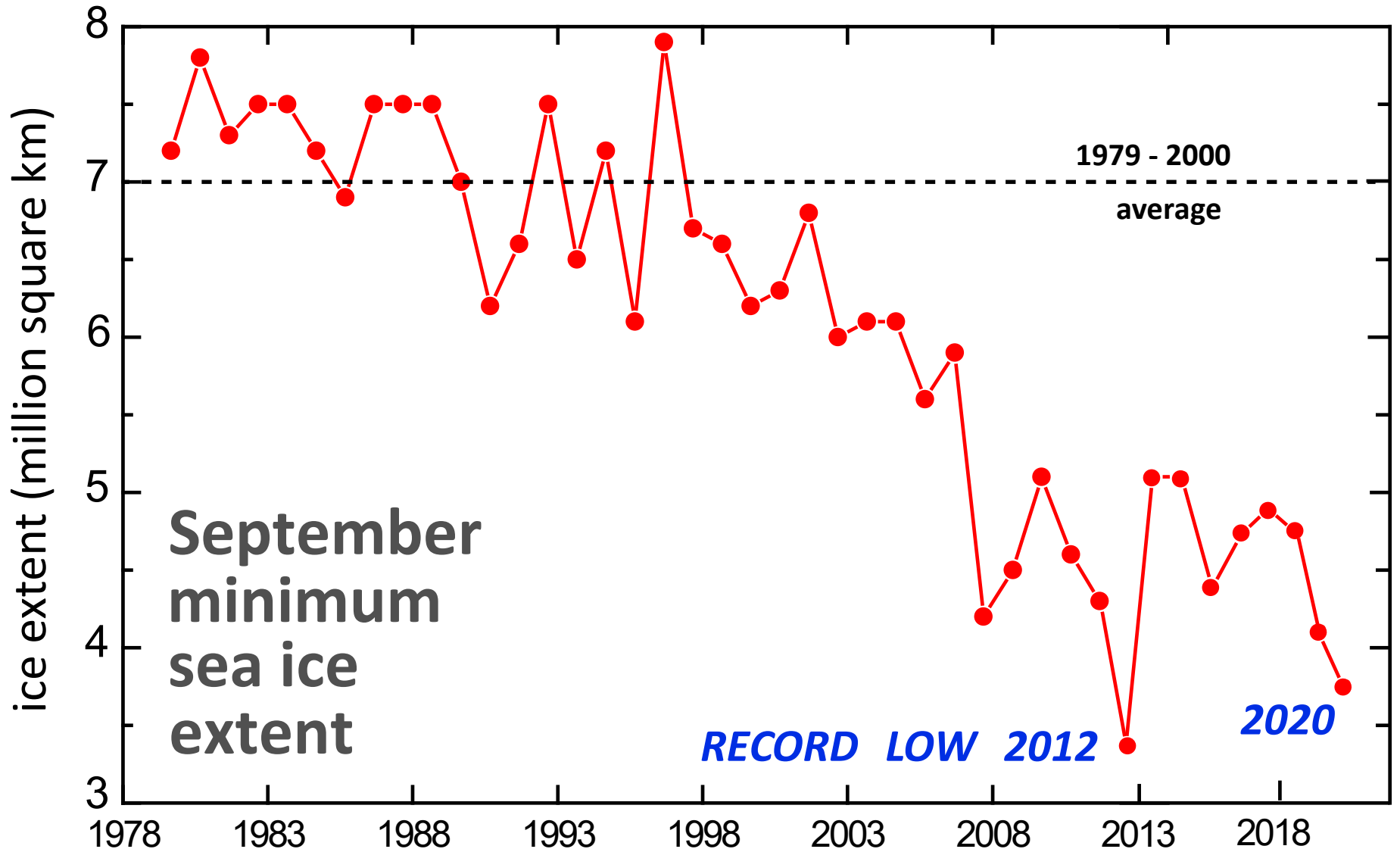


dark water and land  
absorb

$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$



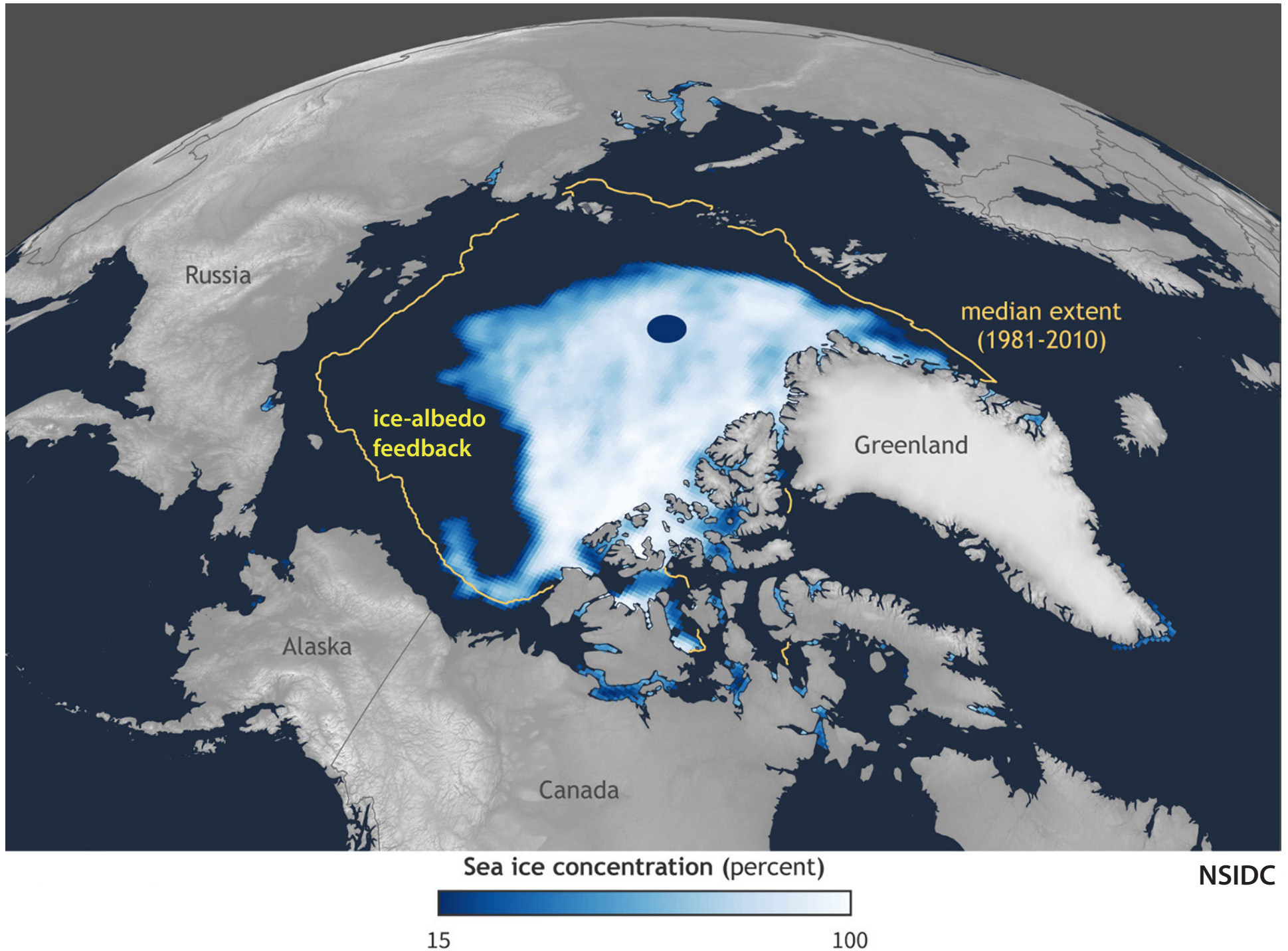
# *the summer Arctic sea ice pack is melting*





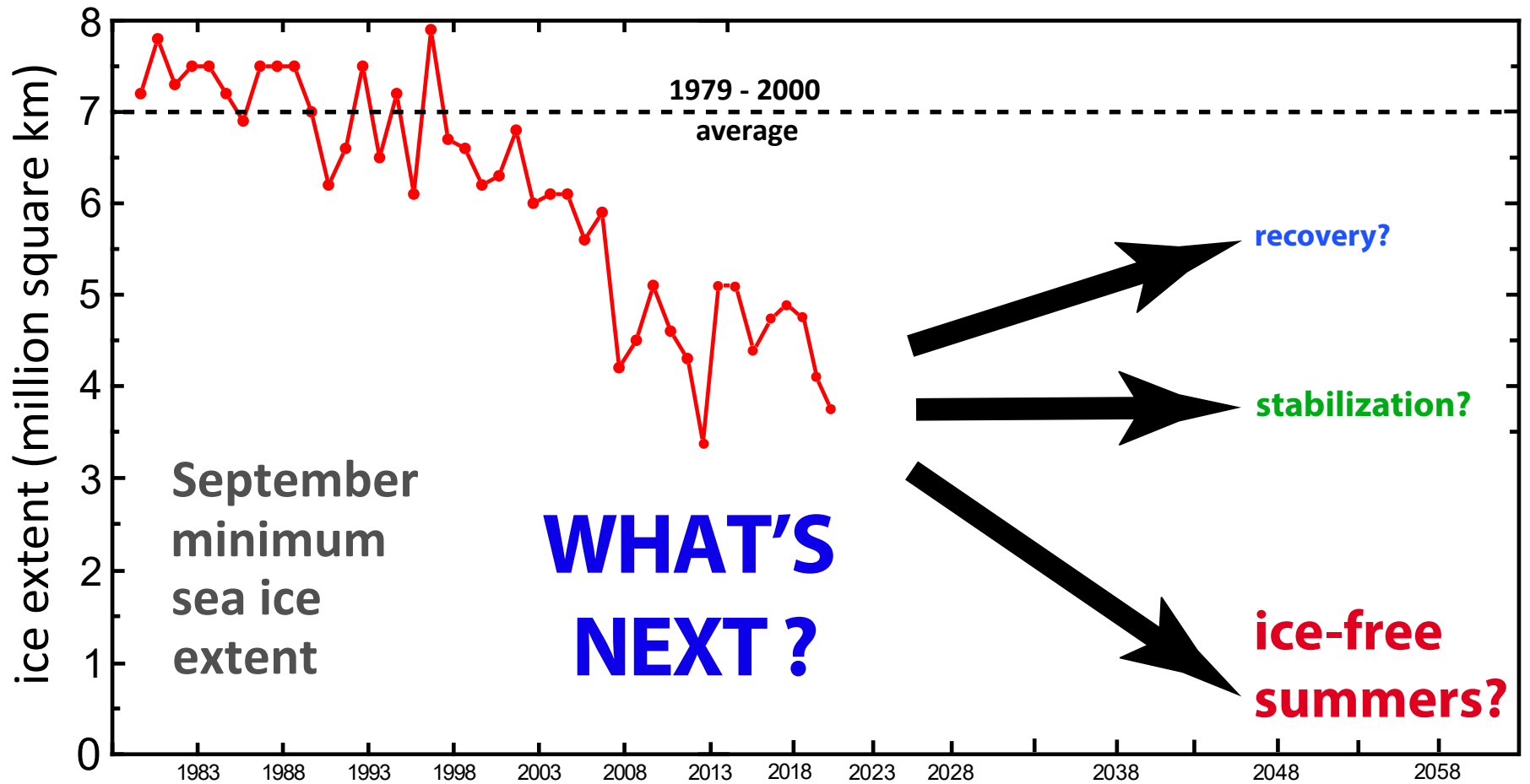
# Arctic sea ice extent

September 15, 2020





# *Predicting what may come next requires lots of math modeling.*

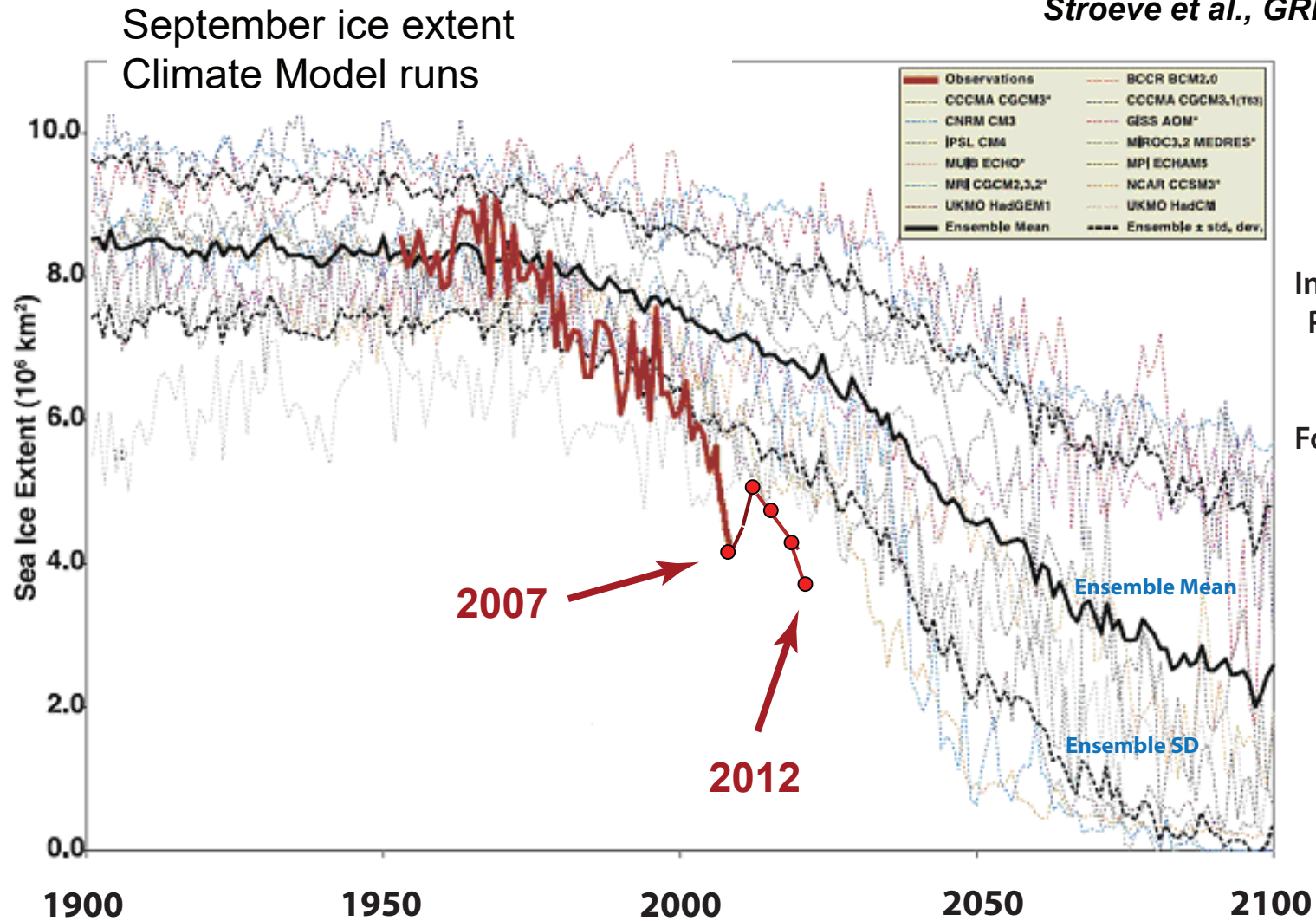




# Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007

Stroeve et al., GRL, 2012



**IPCC AR4  
Models**

Intergovernmental  
Panel on Climate  
Change (IPCC)

Fourth Assessment  
AR4, 2007

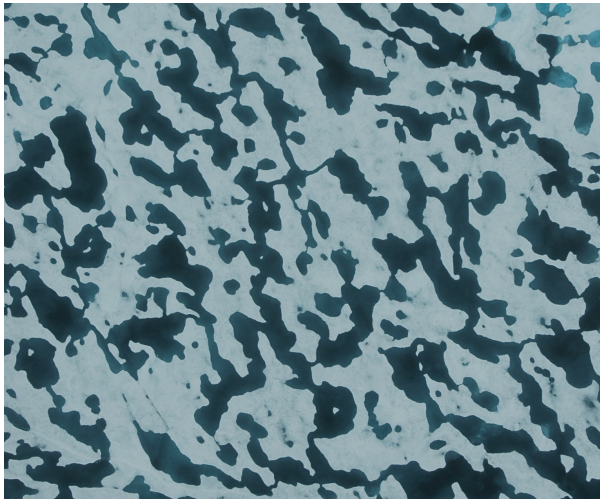


# challenge:

Represent sea ice more realistically in climate models to improve projections.



*How do patterns of dark and light evolve?*



Account for key processes

*e.g. melt pond evolution*

Including PONDS in simulations **LOWERS** predicted sea ice volume over time by 40%.

Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

... and other sub-grid scale structures and processes.

*linkage of scales*

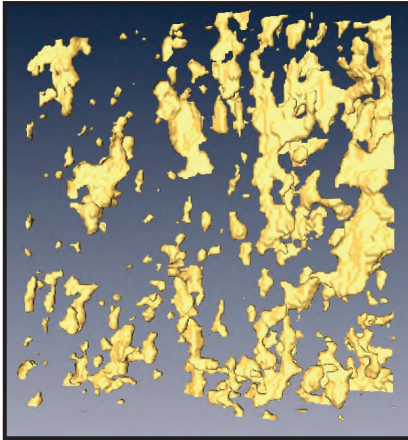
# Sea Ice is a Multiscale Composite Material

## *microscale*

brine inclusions

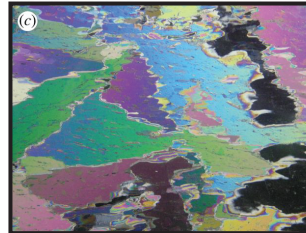
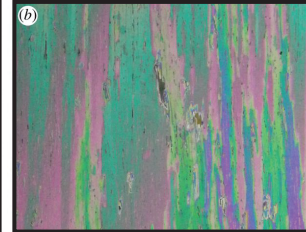
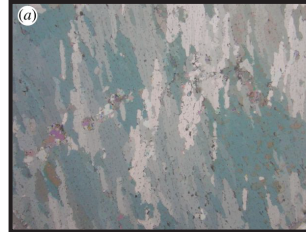


Weeks & Assur 1969



H. Eicken  
Golden et al. GRL 2007

polycrystals

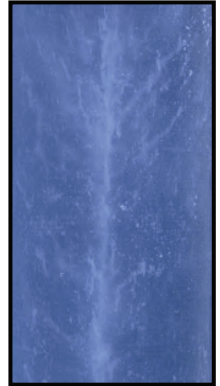


Gully et al. Proc. Roy. Soc. A 2015

brine channels



D. Cole



K. Golden

**millimeters**

**centimeters**

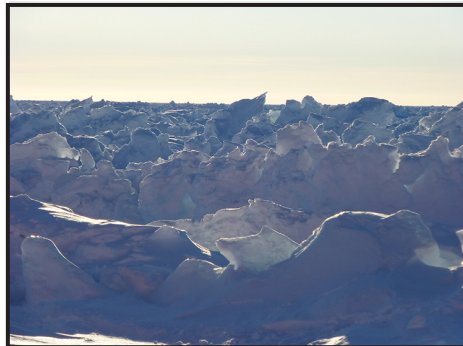
## *mesoscale*

Arctic melt ponds



K. Frey

Antarctic pressure ridges



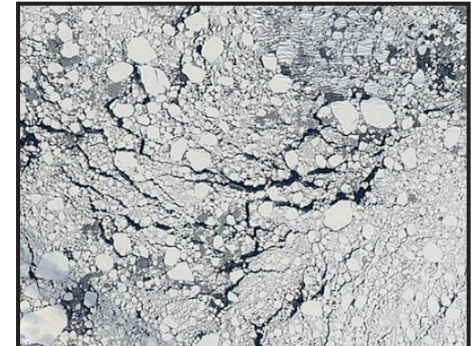
K. Golden

sea ice floes



J. Weller

sea ice pack



NASA

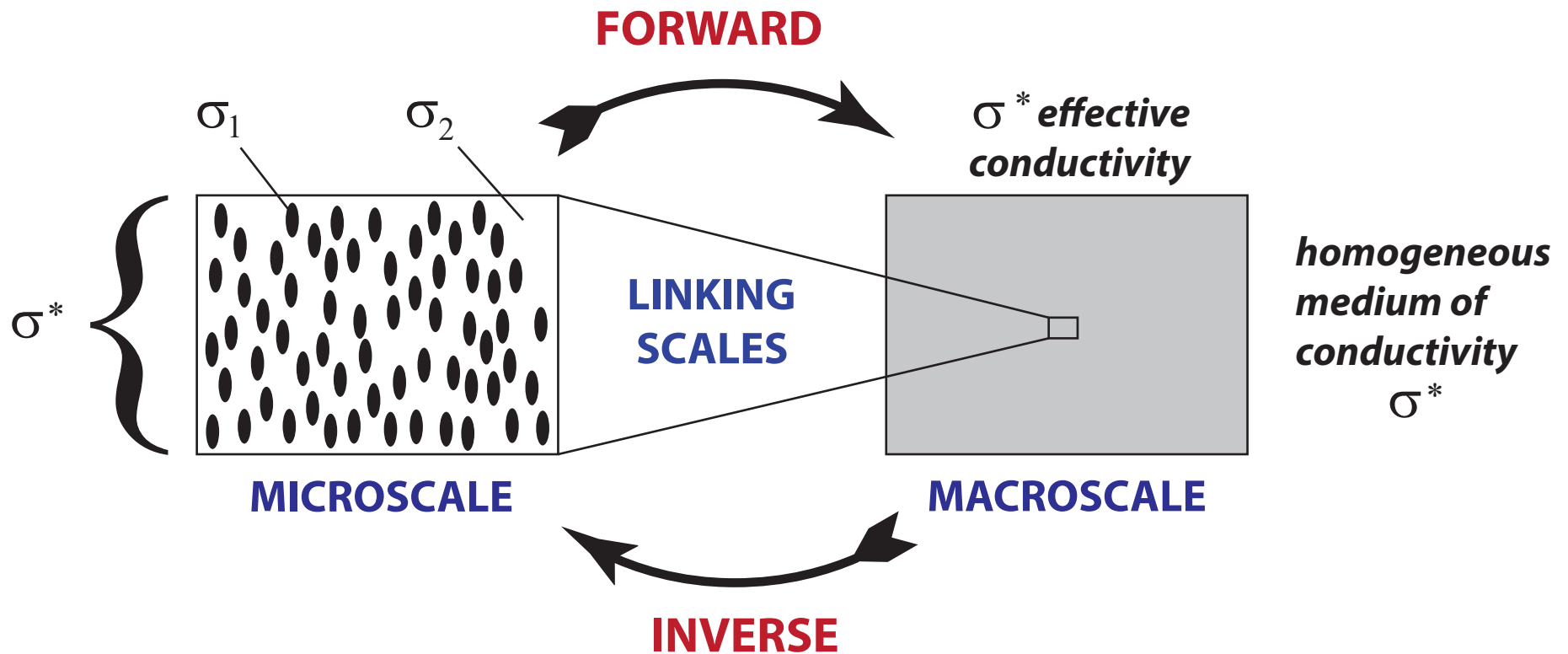
**meters**

**kilometers**

## *macroscale*



# ***HOMOGENIZATION for Composite Materials***



*Maxwell 1873 : effective conductivity of a dilute suspension of spheres*

*Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid*

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*

*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

# Department of Mathematics, University of Utah

## Faculty involved in **Mathematical Aspects of Materials Science**



Andrej Cherkaev  
Professor



Elena Cherkaev  
Professor



Katya Epshteyn  
Associate Professor



Will Feldman  
Assistant Professor



Ken Golden  
Distinguished Professor



Fernando Guevara Vasquez  
Associate Professor



Graeme Milton  
Distinguished Professor



Braxton Osting  
Associate Professor



Andrejs Treibergs  
Professor

## Faculty involved in **Mathematical Fluid Dynamics**



Sasha Balk  
Professor



Aaron Fogelson  
Professor



Christel Hohenegger  
Associate Professor

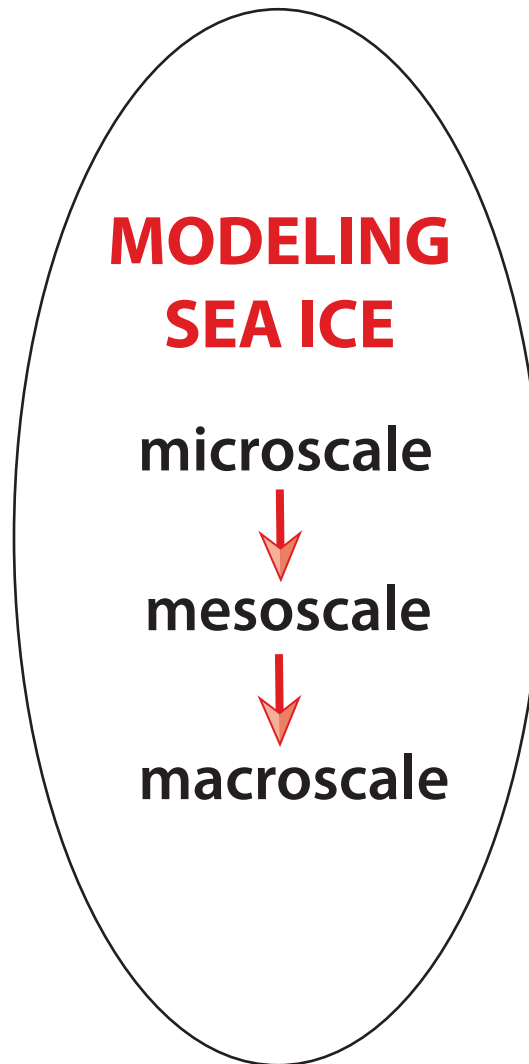


Jingyi Zhu  
Associate Professor



# What is our research about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...



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Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...

## *Inputs, Ingredients*

### COMPOSITE MATERIALS

electrical engineering,  
stealth technology

porous media,  
oil extraction

statistical mechanics  
of ferromagnets

Anderson localization,  
semiconductor physics

random matrix theory

differential equations



**MODELING  
SEA ICE**

microscale

mesoscale

macroscale



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### MODELING SEA ICE

microscale

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macroscale

## *Outputs, Impacts*

### CLIMATE MODELING

sea ice physics  
& biology

composites,  
polycrystals

remote sensing

advection diffusion

biomedical imaging,  
biomaterials, EPS

polar microbial ecology



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*Physics of sea ice drives advances in many areas of science and engineering.*



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**Cross pollination**

magnets  
radar absorbers  
human bone  
rat brains

**MODELING  
SEA ICE**

*Physics of sea ice drives advances in many areas of science and engineering.*

# How do scales interact in the sea ice system?

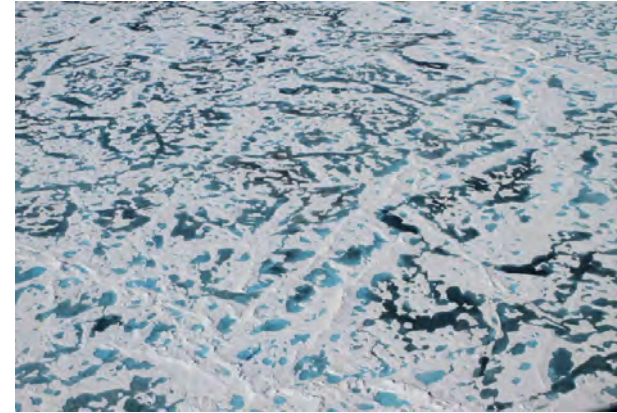


basin scale -  
grid scale  
albedo

NASA

## Linking Scales

km  
scale  
melt  
ponds

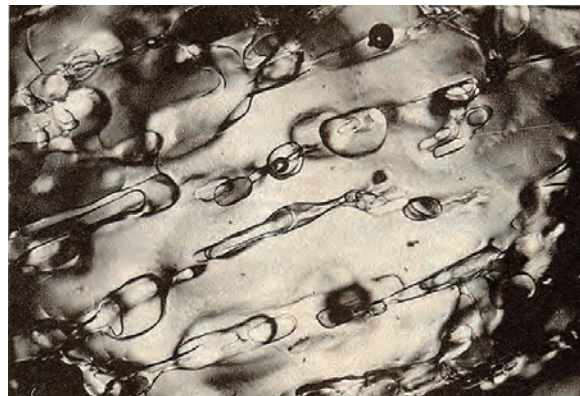


Perovich

## Linking

## Scales

mm  
scale  
brine  
inclusions



meter  
scale  
snow  
topography

***sea ice microphysics***

***fluid transport***



# fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

*evolution of Arctic melt ponds and sea ice albedo*



*nutrient flux for algal communities*



T. Maksym and T. Markus, 2008

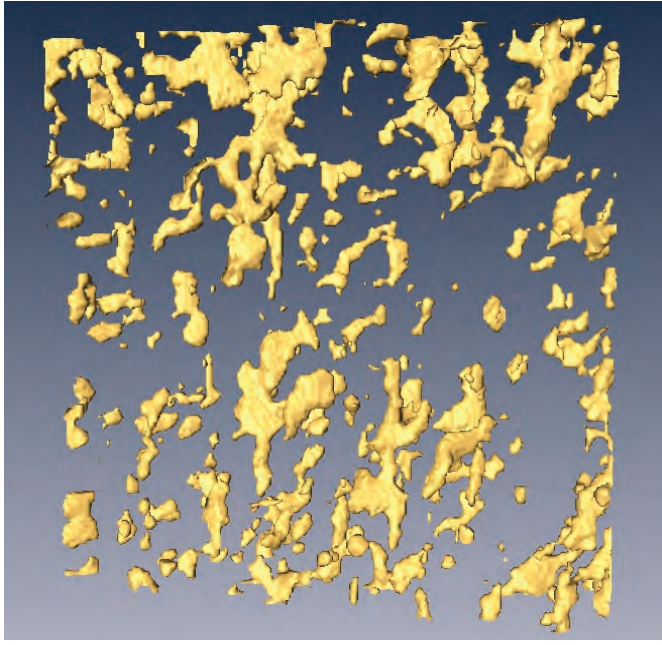
*Antarctic surface flooding  
and snow-ice formation*

September  
snow-ice  
estimates

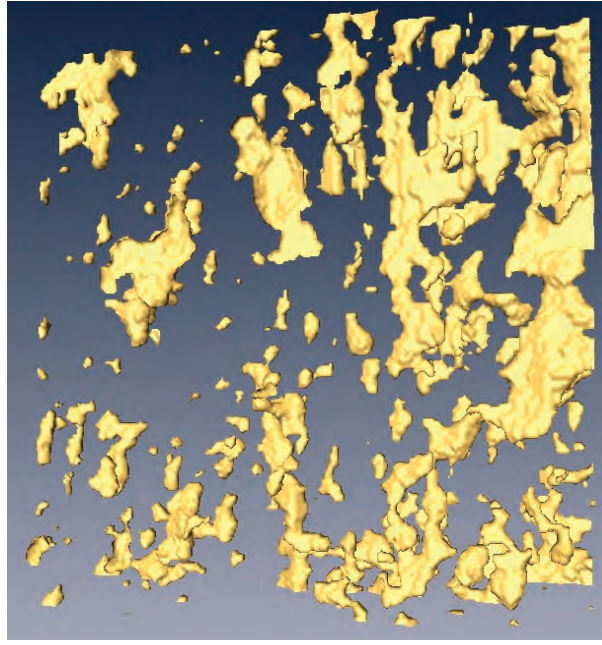
- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO<sub>2</sub>*



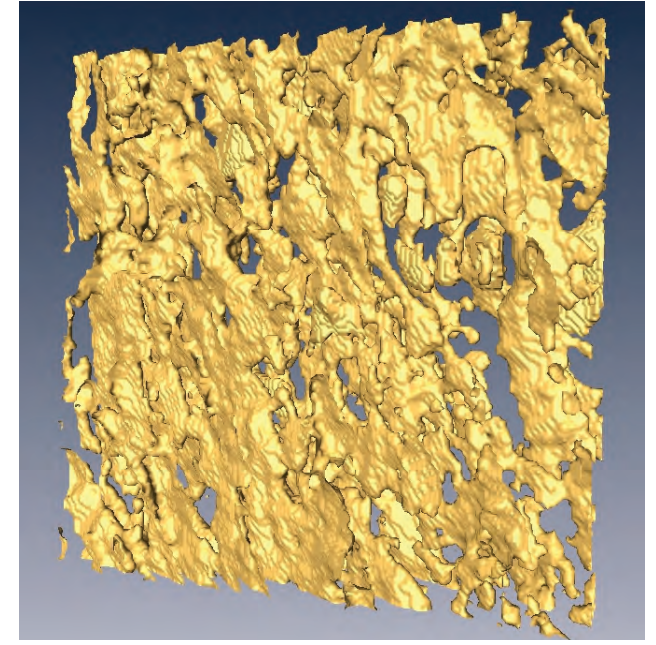
brine volume fraction and **connectivity** increase with temperature



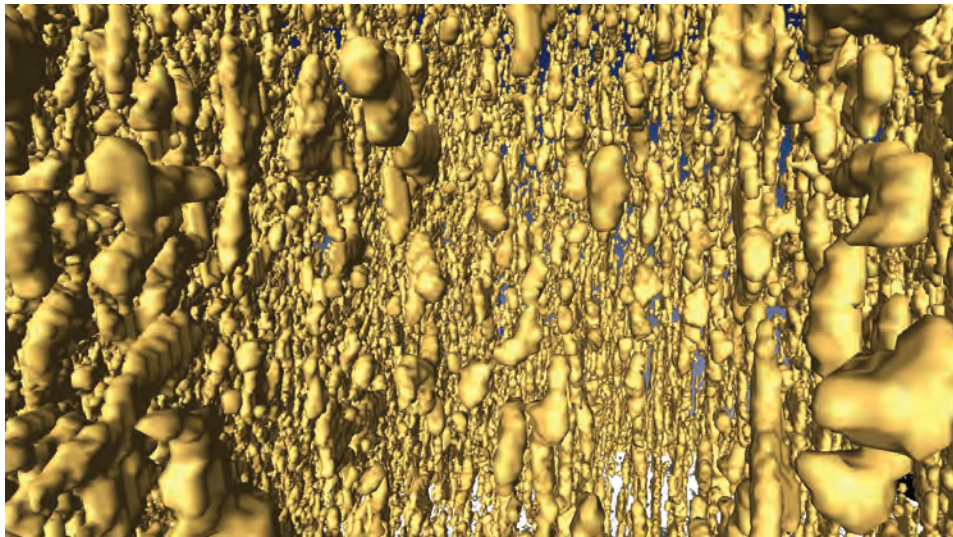
$T = -15\text{ }^{\circ}\text{C}$ ,  $\phi = 0.033$



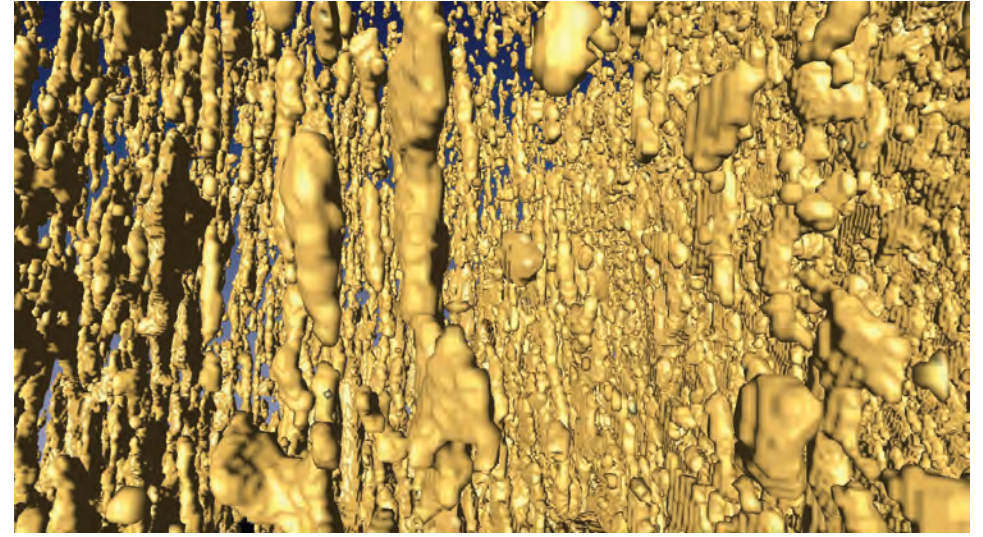
$T = -6\text{ }^{\circ}\text{C}$ ,  $\phi = 0.075$



$T = -3\text{ }^{\circ}\text{C}$ ,  $\phi = 0.143$



$T = -8\text{ }^{\circ}\text{C}$ ,  $\phi = 0.057$

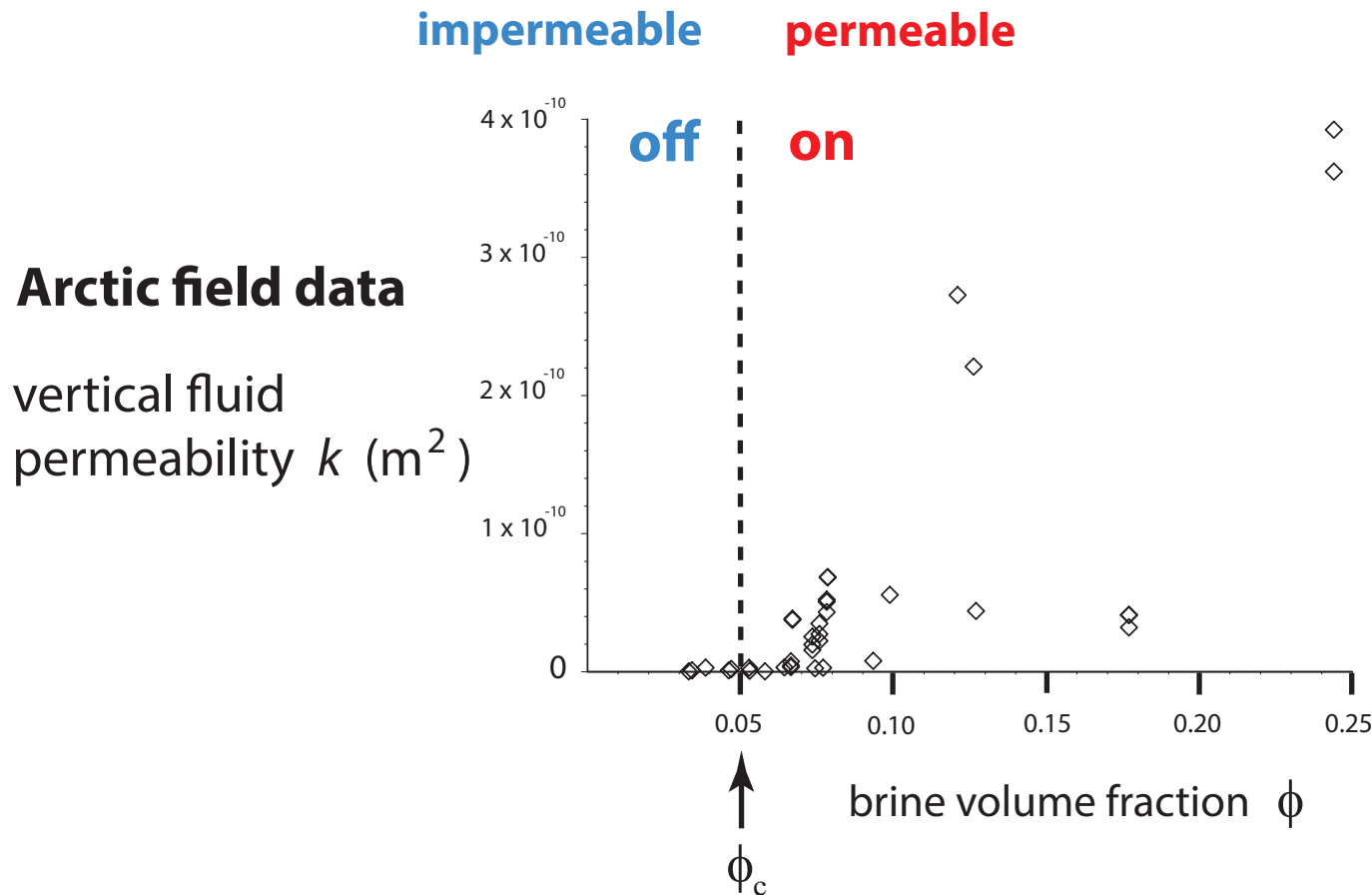


$T = -4\text{ }^{\circ}\text{C}$ ,  $\phi = 0.113$

*X-ray tomography for brine in sea ice*

Golden et al., *Geophysical Research Letters*, 2007

# Critical behavior of fluid transport in sea ice



***“on - off” switch  
for fluid flow***

critical brine volume fraction  $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ \text{C}, S \approx 5 \text{ ppt}$

**RULE OF FIVES**

**Golden, Ackley, Lytle Science 1998**

**Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007**

**Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009**





# sea ice algal communities

D. Thomas 2004

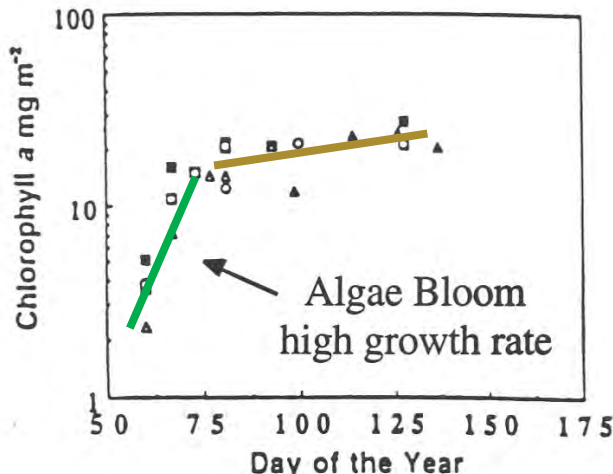
nutrient replenishment  
controlled by ice permeability

biological activity turns on  
or off according to  
*rule of fives*

Golden, Ackley, Lytle      Science 1998

Fritsen, Lytle, Ackley, Sullivan      Science 1994

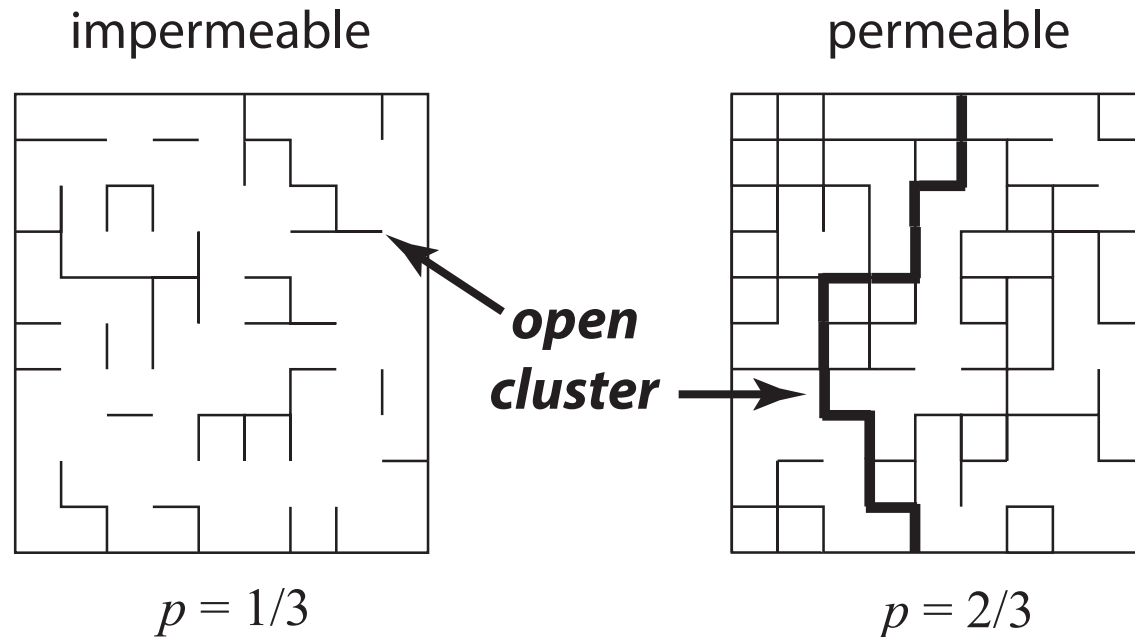
## critical behavior of microbial activity



Convection-fueled algae bloom  
Ice Station Weddell

# percolation theory

***probabilistic theory of connectedness***



bond  $\longrightarrow$  **open** with probability  $p$   
**closed** with probability  $1-p$

**percolation threshold**

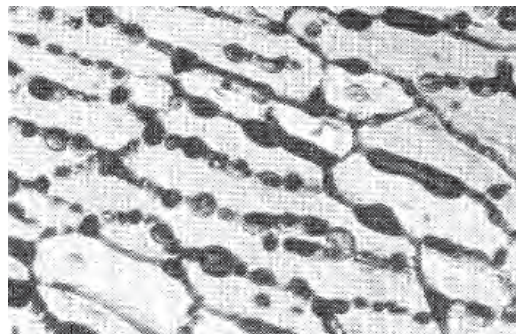
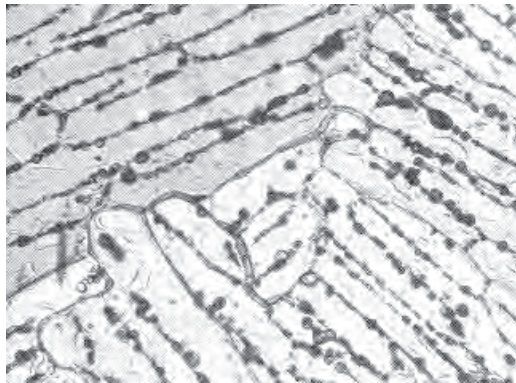
$$p_c = 1/2 \quad \text{for } d = 2$$

smallest  $p$  for which there is an infinite open cluster

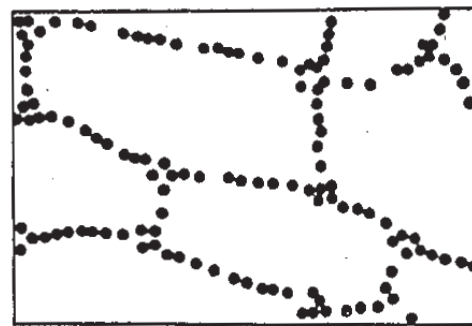
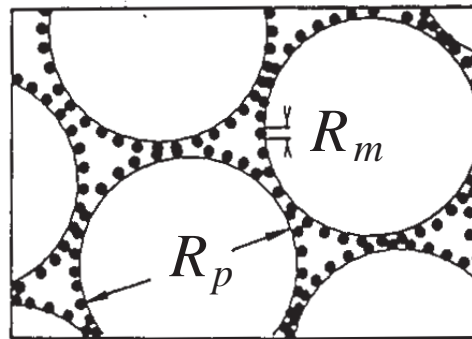
*Continuum* percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5 \%$$

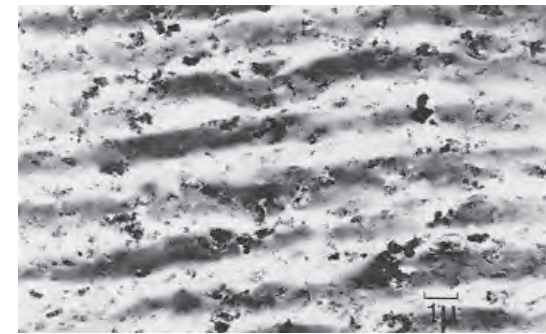
Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed  
powder



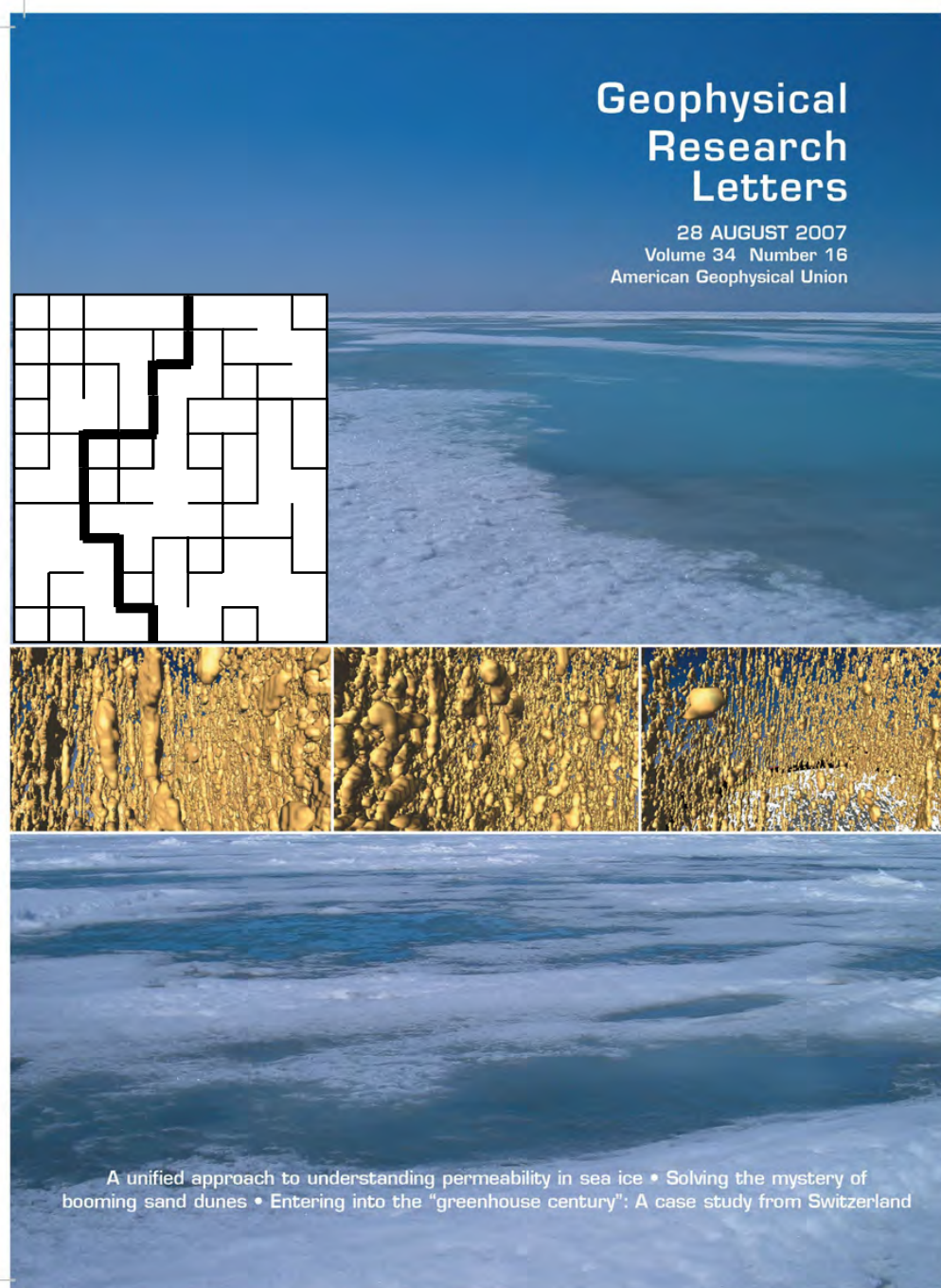
radar absorbing  
composite

**sea ice is radar absorbing**



# Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton\*, Miner, Pringle, Zhu, *Geophysical Research Letters* 2007



**percolation theory  
for fluid permeability**

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical  
exponent  
 $t$

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

from critical path analysis  
in **hopping conduction**

**hierarchical model**

**rock physics**

**network model**

**rigorous bounds**

**X-ray tomography for  
brine inclusions**

**confirms rule of fives**

*Pringle, Miner, Eicken, Golden*  
*J. Geophys. Res.* 2009

**theories agree closely  
with field data**

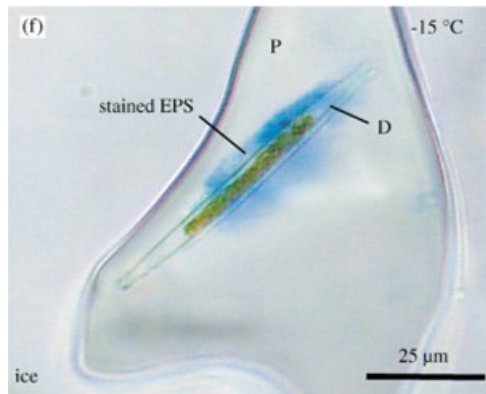
microscale  
governs  
mesoscale  
processes

**melt pond  
evolution**

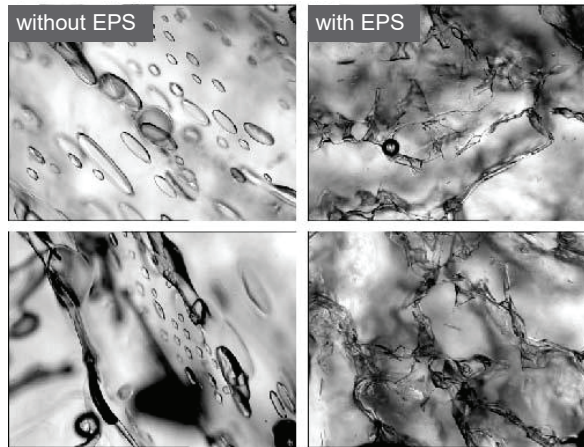
A unified approach to understanding permeability in sea ice • Solving the mystery of  
booming sand dunes • Entering into the "greenhouse century": A case study from Switzerland

# Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

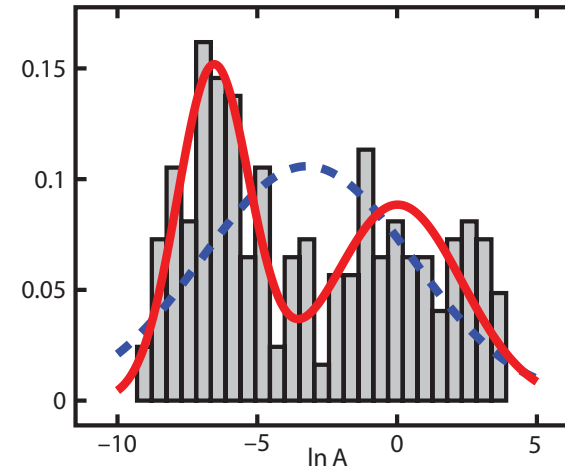
How does EPS affect fluid transport? How does the biology affect the physics?



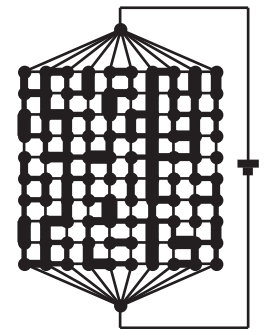
Krembs



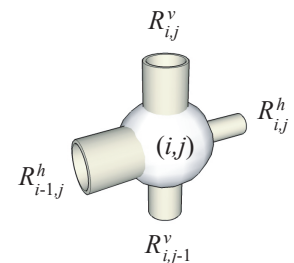
Krembs, Eicken, Deming, PNAS 2011



**RANDOM  
PIPE  
MODEL**



- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability  $k$ ; results predict observed drop in  $k$



Steffen, Epshteyn, Zhu, Bowler, Deming, Golden  
*Multiscale Modeling and Simulation*, 2018

Zhu, Jabini, Golden,  
Eicken, Morris  
*Ann. Glac.* 2006



# Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and  
the Mathematics of  
Transport in Sea Ice

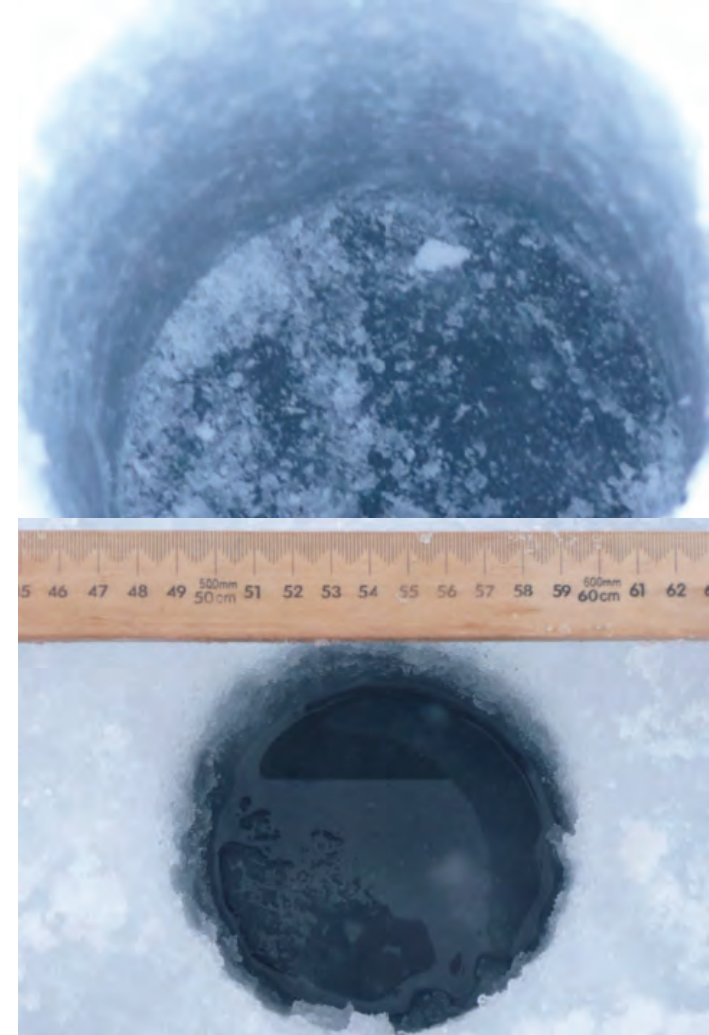
page 562

Mathematics and the  
Internet: A Source of  
Enormous Confusion  
and Great Potential

page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



***measuring  
fluid permeability  
of Antarctic sea ice***

***SIPEX 2007***



**mesoscale**

# *melt pond formation and albedo evolution:*

- *major drivers in polar climate*
- *key challenge for global climate models*

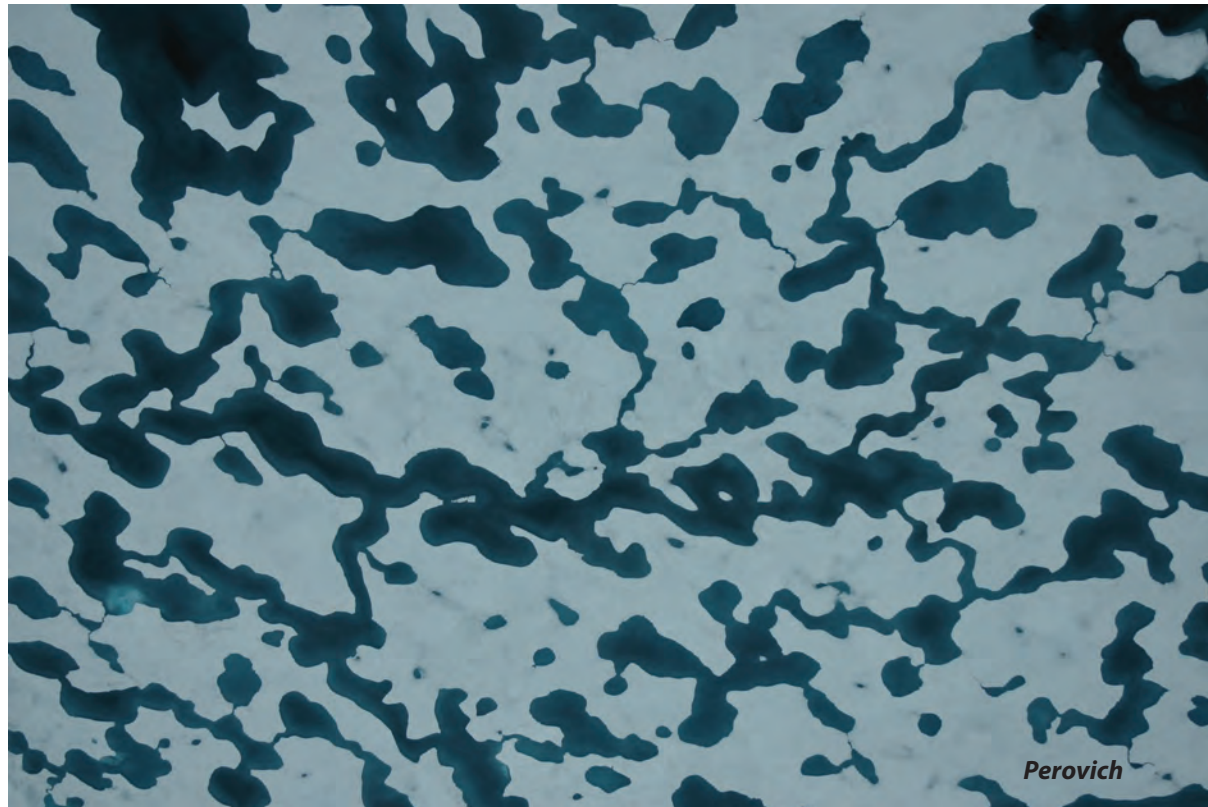
**numerical models of melt pond evolution, including topography, drainage (permeability), etc.**

Lüthje, Feltham,  
Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson,  
Perovich 2009

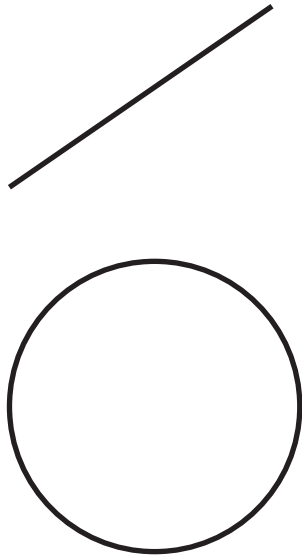
Flocco, Feltham,  
Hunke 2012



**Are there universal features of the evolution similar to phase transitions in statistical physics?**

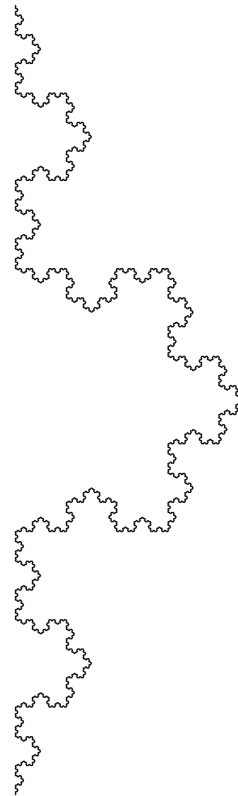
# *fractal curves in the plane*

*they wiggle so much that their dimension is  $>1$*



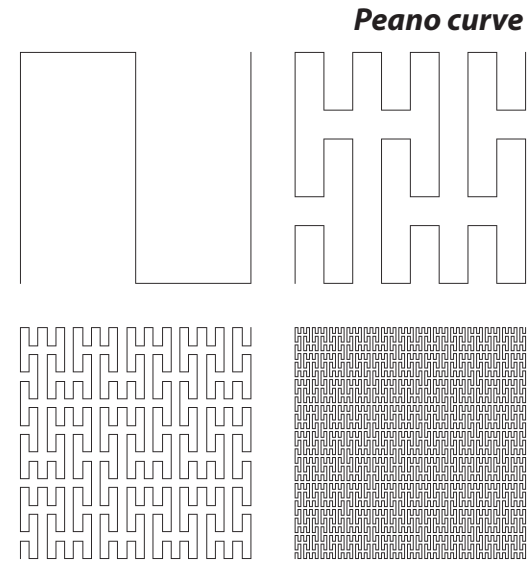
*simple curves*

$D = 1$



*Koch snowflake*

$D = 1.26$



*Peano curve*

*Brownian motion*

*space filling curves*

$D = 2$



# clouds exhibit fractal behavior from 1 to 1000 km

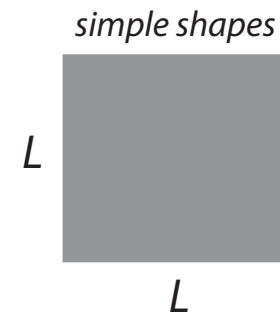
use **perimeter-area** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

*S. Lovejoy, Science, 1982*

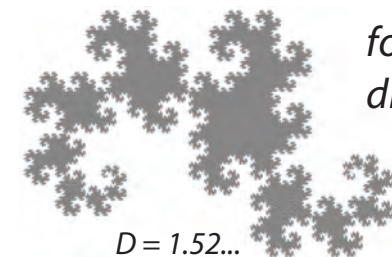


$$P \sim \sqrt{A}$$



$$A = L^2$$
$$P = 4L = 4\sqrt{A}$$

$$P \sim \sqrt{A}^D$$



for fractals with dimension  $D$

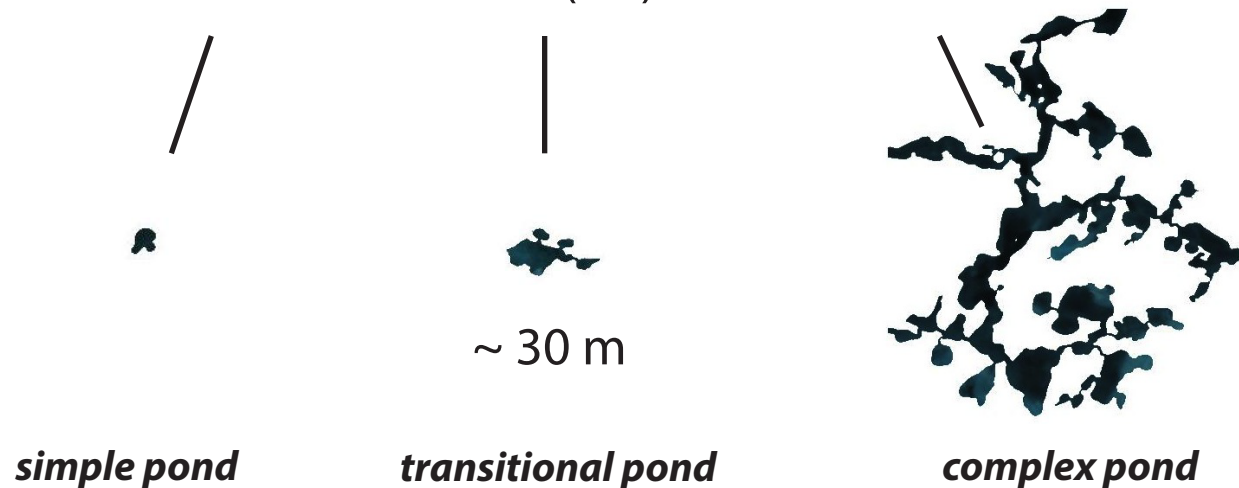
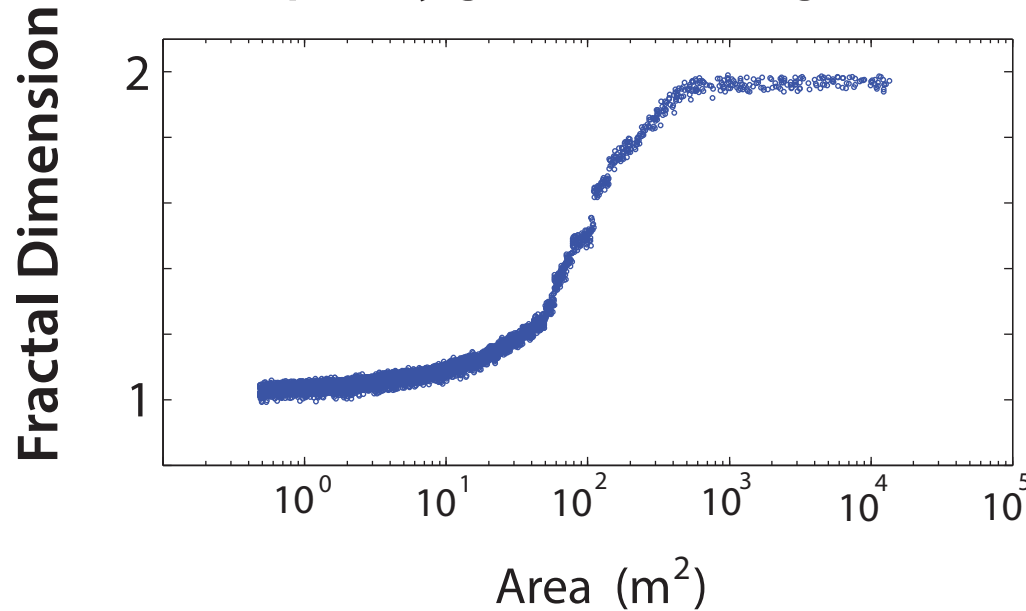
$D = 1.52...$

# *Transition in the fractal geometry of Arctic melt ponds*

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

*The Cryosphere, 2012*

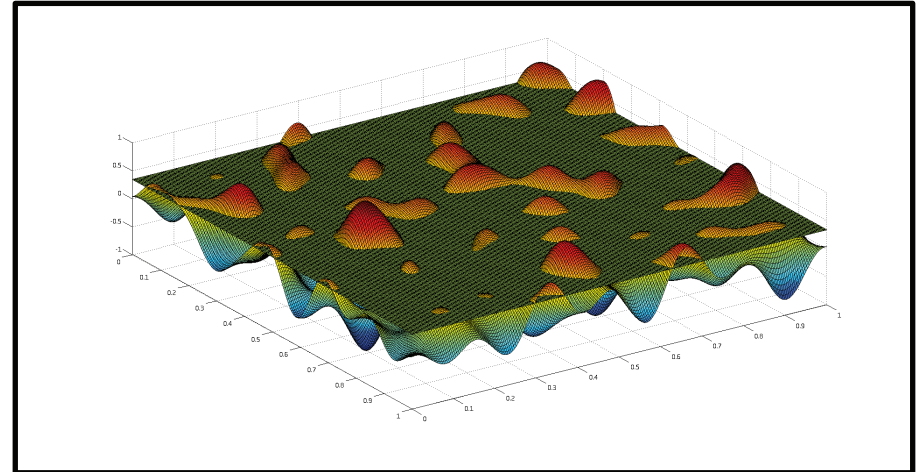
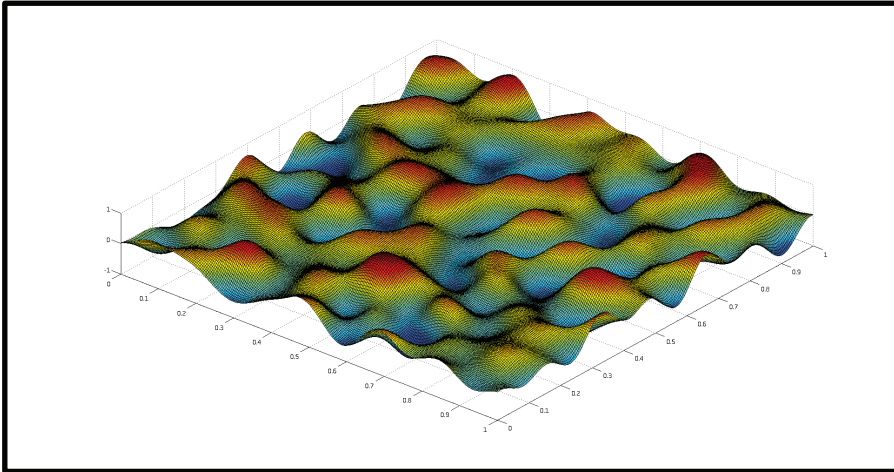
complexity grows with length scale



# Continuum percolation model for melt pond evolution

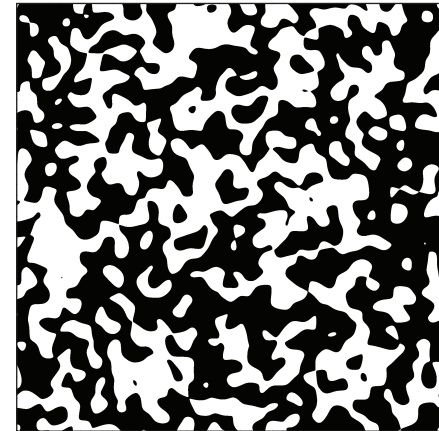
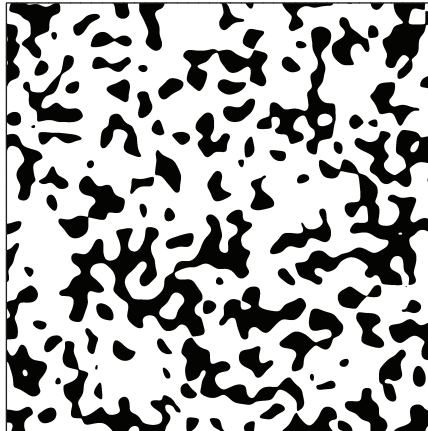
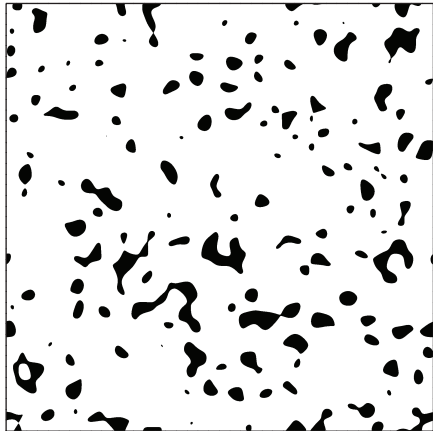
## *level sets of random surfaces*

*Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018*



random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds



*electronic transport in disordered media*

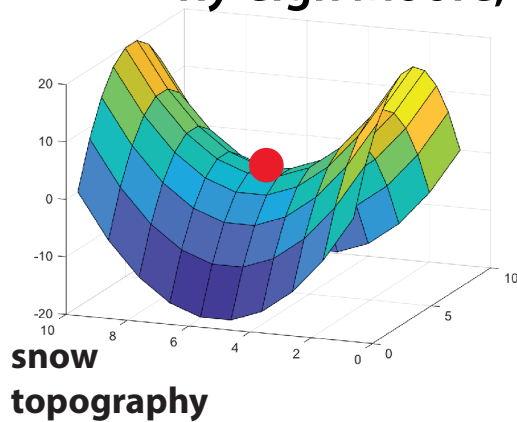
*diffusion in turbulent plasmas*

*Isichenko, Rev. Mod. Phys., 1992*



# Saddle Points, Morse Theory and the Fractal Geometry of Melt Ponds

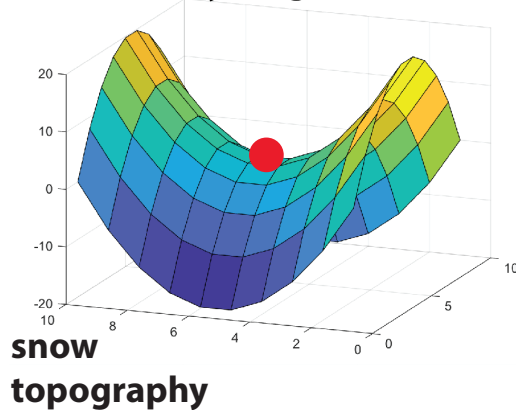
Ryleigh Moore, Jacob Jones, Dane Gollero, Court Strong, Ken Golden 2021



As ponds coalesce at saddle points, fractal dimension proxy isoperimetric quotient  $P^2/4\pi A$  jumps, driving the transition.

# Saddle Points, Morse Theory and the Fractal Geometry of Melt Ponds

Ryleigh Moore, Jacob Jones, Dane Gollero, Court Strong, Ken Golden 2021



As ponds coalesce at saddle points, fractal dimension proxy isoperimetric quotient  $P^2/4\pi A$  jumps, driving the transition.



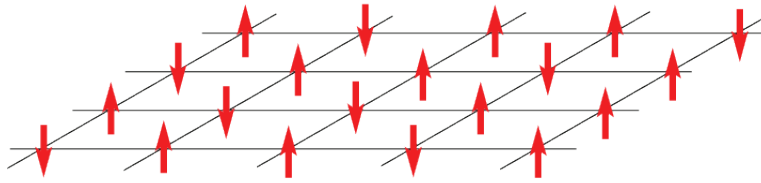
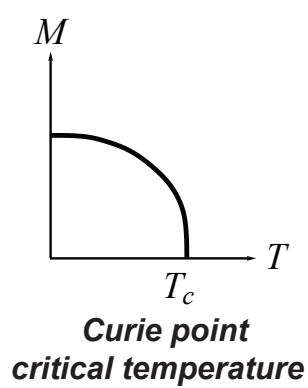
**Ryleigh Moore**  
Department of Mathematics  
University of Utah

Multidisciplinary drifting Observatory  
for the Study of Arctic Climate (**MOSAiC**)

MOSAiC School aboard the icebreaker *RV Akademik Federov*

20 grad students from around the world  
(**3 from U.S., 1 mathematician**)

# Ising Model for a Ferromagnet



applied  
magnetic  
field



$$s_i = \begin{cases} +1 & \text{spin up} \\ -1 & \text{spin down} \end{cases} \quad \begin{matrix} \text{blue} \\ \text{white} \end{matrix}$$

$$\mathcal{H} = -H \sum_i s_i - J \sum_{\langle i,j \rangle} s_i s_j$$

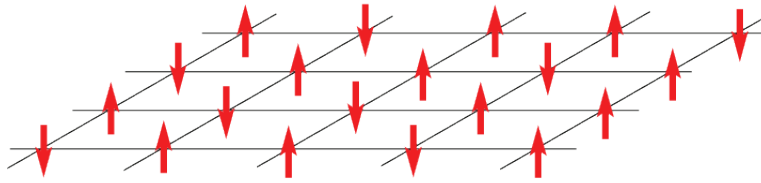
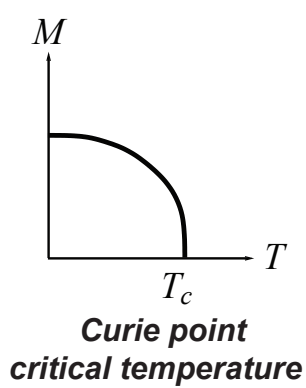
nearest neighbor Ising Hamiltonian

$$M(T, H) = \lim_{N \rightarrow \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

effective magnetization



# Ising Model for a Ferromagnet



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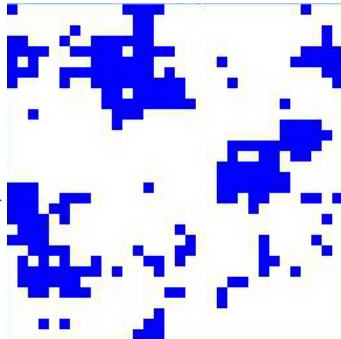
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nearest neighbor Ising Hamiltonian

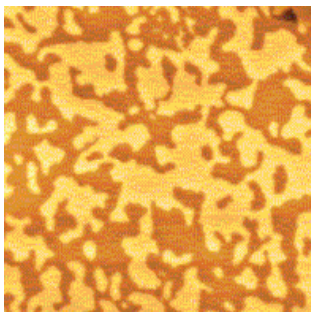
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effective magnetization

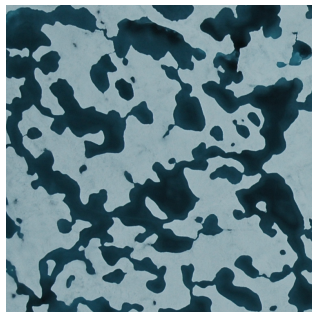
islands of like spins



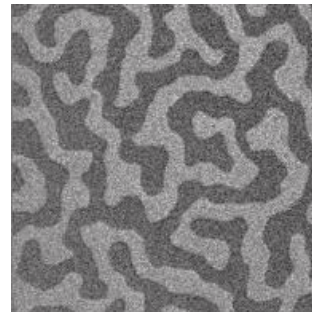
energy is lowered when nearby spins align with each other, forming **magnetic domains**



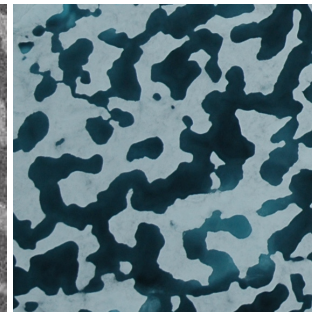
magnetic domains in cobalt



melt ponds (Perovich)



magnetic domains in cobalt-iron-boron



melt ponds (Perovich)

# Ising model for ferromagnets $\longrightarrow$ Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

$$\mathcal{H} = - \sum_i^N H_i s_i - J \sum_{\langle i,j \rangle}^N s_i s_j \quad s_i = \begin{cases} \uparrow & +1 \text{ water (spin up)} \\ \downarrow & -1 \text{ ice (spin down)} \end{cases}$$

random magnetic field  
represents snow topography

magnetization  $M$       pond area fraction  $F = \frac{(M+1)}{2}$       only nearest neighbor patches interact

$\sim$  *albedo*

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system “flows” toward metastable equilibria.

*Order from Disorder*

# Ising model for ferromagnets $\longrightarrow$ Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

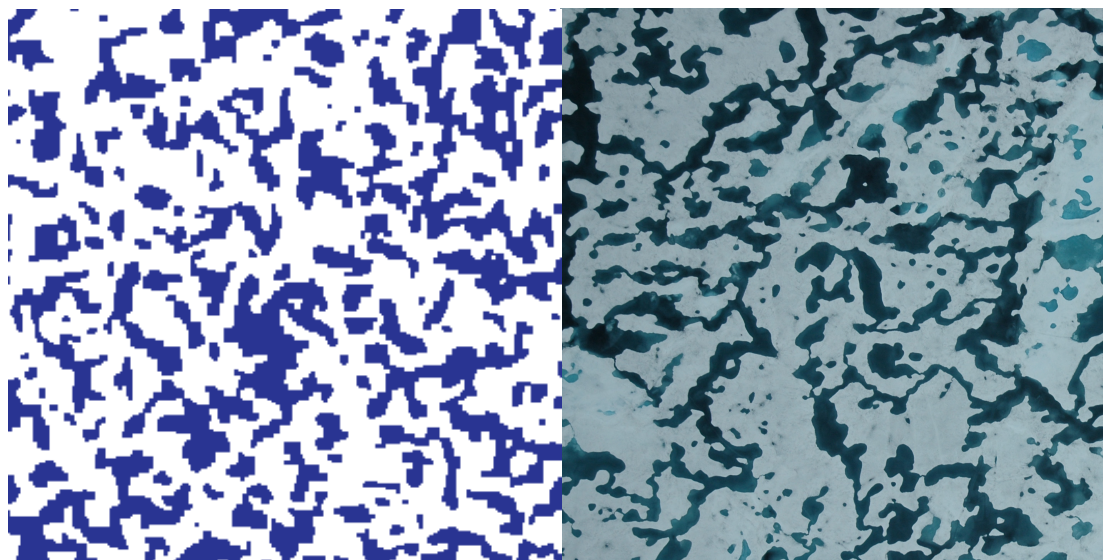
$$\mathcal{H} = - \sum_i^N H_i s_i - J \sum_{\langle i,j \rangle}^N s_i s_j \quad s_i = \begin{cases} \uparrow & +1 \text{ water (spin up)} \\ \downarrow & -1 \text{ ice (spin down)} \end{cases}$$

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 *$\sim$  albedo*

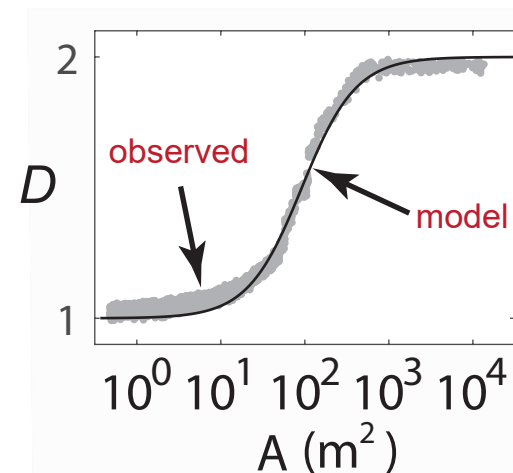
Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system “flows” toward metastable equilibria.

## *Order from Disorder*



Ising  
model

melt pond  
photo (Perovich)



pond size  
distribution exponent

observed -1.5

(Perovich, et al. 2002)

model -1.58

*Scientific American  
EOS, PhysicsWorld, ...*

**ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data**

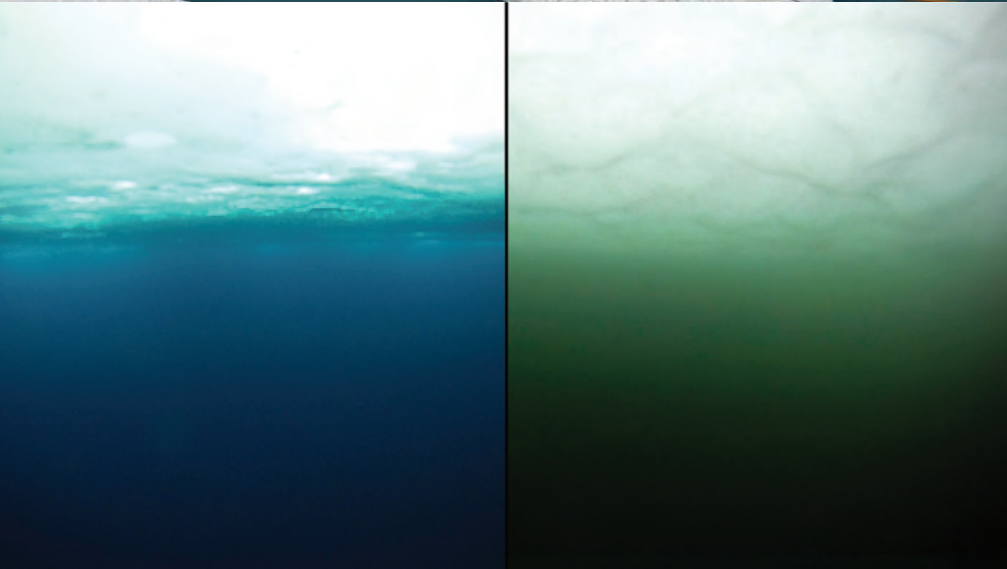




Perovich

Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

## WINDOWS



no bloom

bloom

massive under-ice **algal bloom**

Arrigo et al., *Science* 2012

***Have we crossed into a new ecological regime?***

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances* 2017

The effect of melt pond geometry on the distribution of solar energy under first year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden  
*Geophys. Res. Lett.* 2019

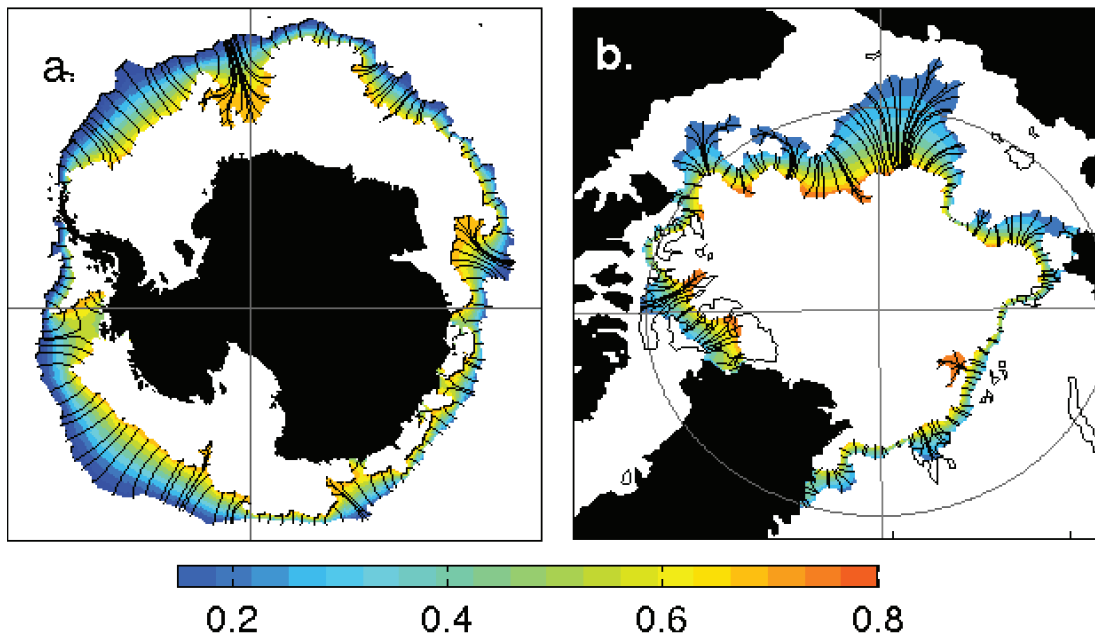
(2015 AMS MRC)

**macroscale**

# Marginal Ice Zone

## MIZ

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



### MIZ WIDTH

fundamental length scale of  
ecological and climate dynamics

Strong, *Climate Dynamics* 2012

Strong and Rigor, *GRL* 2013

transitional region between  
dense interior pack ( $c > 80\%$ )  
sparse outer fringes ( $c < 15\%$ )

**How to objectively  
measure the “width”  
of this complex,  
non-convex region?**



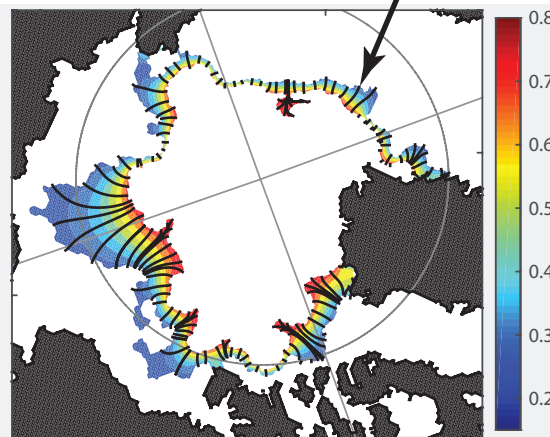
# Objective method for measuring MIZ width motivated by medical imaging and diagnostics

Strong, *Climate Dynamics* 2012  
Strong and Rigor, *GRL* 2013

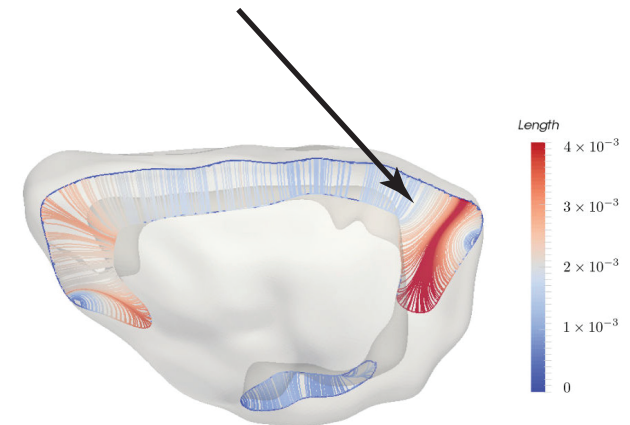
**39% widening**  
**1979 - 2012**

**“average” lengths of streamlines**

streamlines of a solution  
to Laplace’s equation



**Arctic Marginal Ice Zone**



**cross-section of the  
cerebral cortex of a rodent brain**

## ***analysis of different MIZ WIDTH definitions***

Strong, Foster, Cherkaev, Eisenman, Golden  
*J. Atmos. Oceanic Tech.* 2017

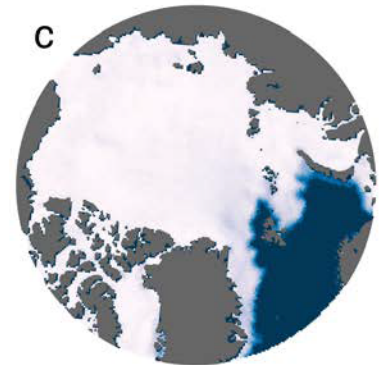
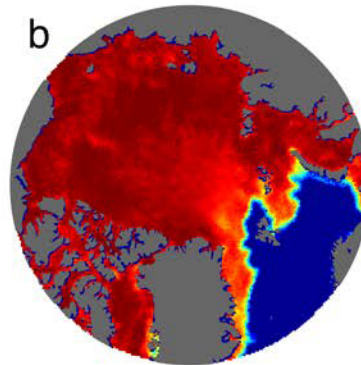
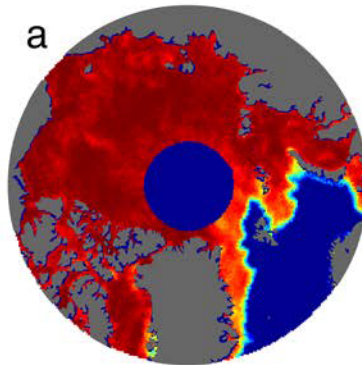
Strong and Golden  
*Society for Industrial and Applied Mathematics News*, April 2017

# Filling the polar data gap with partial differential equations

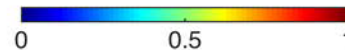
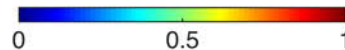
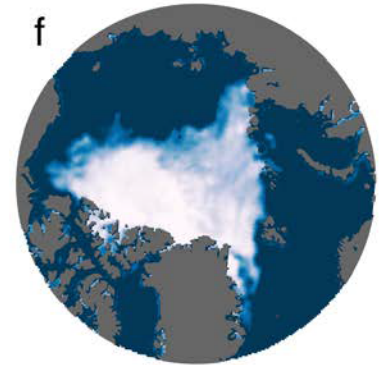
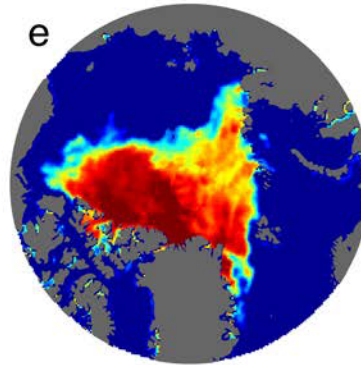
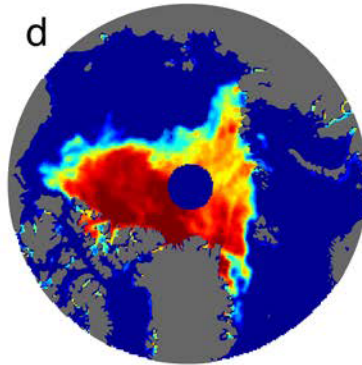
hole in satellite coverage  
of sea ice concentration field

previously assumed  
ice covered

Gap radius: 611 km  
06 January 1985



Gap radius: 311 km  
30 August 2007



$$\Delta\psi=0$$

fill with harmonic function satisfying  
satellite BC's plus stochastic term

Strong and Golden, *Remote Sensing* 2016  
Strong and Golden, *SIAM News* 2017

NOAA/NSIDC Sea Ice Concentration CDR  
product update will use our PDE method.

# Conclusions

1. Sea ice is a fascinating multiscale composite with structure similar to many other natural and man-made materials.
2. Mathematical methods developed for sea ice advance the theory of composites and other areas of science and engineering.
3. **Homogenization and statistical physics help *link scales in sea ice and composites***; provide rigorous methods for finding effective behavior; advance sea ice representations in climate models.
4. **Fluid flow** through sea ice mediates **melt pond evolution** and many processes important to climate change and polar ecosystems.
5. Field experiments are essential to developing relevant mathematics.
6. Our research is helping to **improve projections of climate change**, the fate of Earth's sea ice packs, and the ecosystems they support.



# University of Utah Sea Ice Modeling Group (2017-2021)

**Senior Personnel:** Ken Golden, Distinguished Professor of Mathematics  
Elena Cherkaev, Professor of Mathematics  
Court Strong, Associate Professor of Atmospheric Sciences  
Ben Murphy, Adjunct Assistant Professor of Mathematics

**Postdoctoral Researchers:** Noa Kraitzman (now at ANU), Jody Reimer

**Graduate Students:** Kyle Steffen (now at UT Austin with Clint Dawson)  
Christian Sampson (now at UNC Chapel Hill with Chris Jones)  
Huy Dinh (now a sea ice MURI Postdoc at NYU/Courant)  
Rebecca Hardenbrook  
David Morison (Physics Department)  
Ryleigh Moore  
Delaney Mosier  
Daniel Hallman

**Undergraduate Students:** Kenzie McLean, Jacqueline Cinella Rich,  
Dane Gollero, Samir Suthar, Anna Hyde,  
Kitsel Lusted, Ruby Bowers, Kimball Johnston,  
Jerry Zhang, Nash Ward, David Gluckman

**High School Students:** Jeremiah Chapman, Titus Quah, Dylan Webb

---

**Sea Ice Ecology Group**    Postdoc Jody Reimer, Grad Student Julie Sherman,  
Undergraduates Kayla Stewart, Nicole Forrester

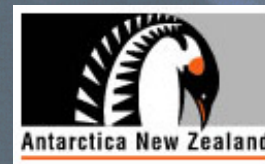
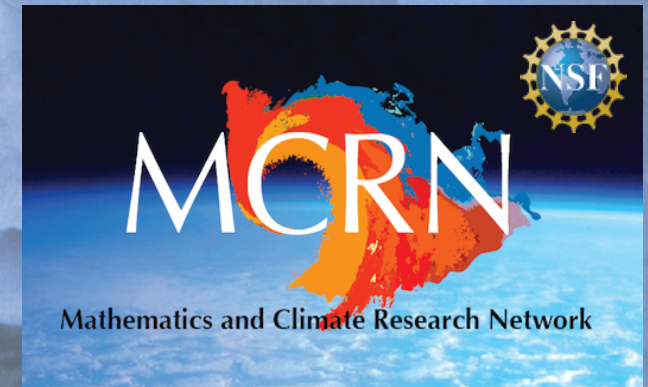
# THANK YOU

## Office of Naval Research

Applied and Computational Analysis Program  
Arctic and Global Prediction Program

## National Science Foundation

Division of Mathematical Sciences  
Division of Polar Programs



***Buchanan Bay, Antarctica    Mertz Glacier Polynya Experiment    July 1999***



# Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship *Aurora Australis* without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 185km off the coast.

The extent of the damage is not known.

Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an inert gas.

The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known.

The 25 crew and 54 expeditioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year but at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Div-



A file photo of the *Aurora Australis* in Antarctica.

ision's Hobart office.

He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available," Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The *Aurora Australis* is owned by P&O Australia and chartered by the Antarctic Div-

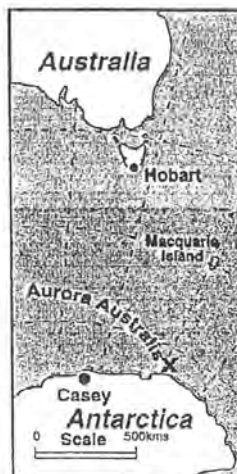
ision for about \$11 million a year.

P&O Australia managing director Richard Hein said yesterday the company was assessing the situation and a number of rescue options were being considered.

It was too early to say whether P&O would be liable for the cost of the aborted mission.

The vessel left Hobart last Wednesday for a seven-week voyage mainly to study a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.



Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

## Fire strands Antarctic ship in sea ice

AN engine room fire has disabled the icebreaker *Aurora Australis* in sea ice, deep in Antarctic waters.

There were no injuries and the ship was not in danger after Tuesday night's fire.

Australian Antarctic Division director Mr Rex Moncur said. But Mr Moncur said he expected it would have to abandon its pioneering mid-winter voyage to the edge of the Ant-

arctic continent and return to Hobart for repairs.

The cause of the fire was not known but the engines have been turned off, with the ship 100 nautical miles from the Antarctic coast.

### THE CANBERRA TIMES

Thursday 23 July 1998

Page 4

## Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Australian icebreaker *Aurora Australis* in sea ice, deep in Antarctic waters.

Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected *Aurora Australis* would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs.

The fire had been extinguished and the engines were turned off, leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good.

Crew had to wear breathing apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully assessed.

The *Aurora*, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

Mr Moncur said, the cause of the fire was not yet known.

2:45 am July 22, 1998

"Please don't be alarmed but we have an uncontrolled fire in the engine room ...."

about 10 minutes later ...

"Please don't be alarmed but we're lowering the lifeboats ...."

*Sydney Morning Herald*  
23 July, 1998

### ICEBREAKER BURNS

A pioneering \$2-million Australian scientific voyage to the mid-winter Antarctic polynya is expected to be scrapped following an engine room fire on the *Aurora Australis* yesterday. The 54 people on board were forced on deck in the







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# Notices

of the American Mathematical Society

November 2020

Volume 67, Number 10



AMERICAN  
MATHEMATICAL  
SOCIETY

Advancing research. Creating connections.

*The cover is based on "Modeling Sea Ice,"  
page 1535.*



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# Modeling Sea Ice



*Kenneth M. Golden, Luke G. Bennetts,  
Elena Cherkaev, Ian Eisenman, Daniel Feltham,  
Christopher Horvat, Elizabeth Hunke,  
Christopher Jones, Donald K. Perovich,  
Pedro Ponte-Castañeda, Courtenay Strong,  
Deborah Sulsky, and Andrew J. Wells*

---

*Kenneth M. Golden is a Distinguished Professor of Mathematics at the University of Utah. His email address is [golden@math.utah.edu](mailto:golden@math.utah.edu).*

*Luke G. Bennetts is an associate professor of applied mathematics at the University of Adelaide. His email address is [luke.bennetts@adelaide.edu.au](mailto:luke.bennetts@adelaide.edu.au).*

*Elena Cherkaev is a professor of mathematics at the University of Utah. Her email address is [elena@math.utah.edu](mailto:elena@math.utah.edu).*

*Ian Eisenman is an associate professor of climate, atmospheric science, and physical oceanography at the Scripps Institution of Oceanography at the University of California San Diego. His email address is [eisenman@ucsd.edu](mailto:eisenman@ucsd.edu).*

*Daniel Feltham is a professor of climate physics at the University of Reading. His email address is [d.l.feltham@reading.ac.uk](mailto:d.l.feltham@reading.ac.uk).*

*Christopher Horvat is a NOAA Climate and Global Change Postdoctoral Fellow at the Institute at Brown for Environment and Society at Brown University. His email address is [christopher\\_horvat@brown.edu](mailto:christopher_horvat@brown.edu).*

*Elizabeth Hunke is a deputy group leader, T-3 fluid dynamics and solid mechanics group at the Los Alamos National Laboratory. Her email address is [elclare@lanl.gov](mailto:elclare@lanl.gov).*

*Christopher Jones is a Bill Guthridge Distinguished Professor of Mathematics*

*at the University of North Carolina, Chapel Hill. His email address is [ckrtj@unc.edu](mailto:ckrtj@unc.edu).*

*Donald K. Perovich is a professor of engineering at the Thayer School of Engineering at Dartmouth College. His email address is [donald.k.perovich@dartmouth.edu](mailto:donald.k.perovich@dartmouth.edu).*

*Pedro Ponte-Castañeda is a Raymond S. Markowitz Faculty Fellow and professor of mechanical engineering and applied mechanics and of mathematics at the University of Pennsylvania. His email address is [ponte@seas.upenn.edu](mailto:ponte@seas.upenn.edu).*

*Courtenay Strong is an associate professor of atmospheric sciences at the University of Utah. His email address is [court.strong@utah.edu](mailto:court.strong@utah.edu).*

*Deborah Sulsky is a professor of mathematics and statistics and of mechanical engineering at the University of New Mexico. Her email address is [sulsky@math.unm.edu](mailto:sulsky@math.unm.edu).*

*Andrew J. Wells is an associate professor of physical climate science at the University of Oxford. His email address is [Andrew.Wells@physics.ox.ac.uk](mailto:Andrew.Wells@physics.ox.ac.uk).*

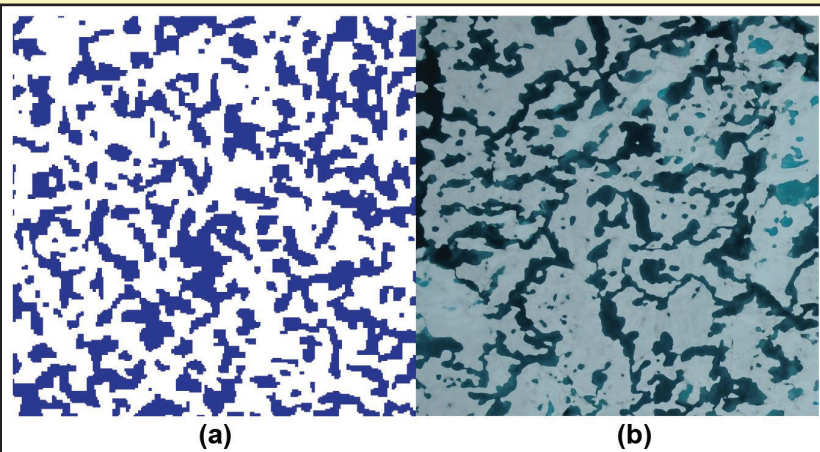
*Communicated by Notices Associate Editor Reza Malek-Madani.*

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## Special Issue on the Mathematics of Planet Earth

Read about the application of mathematics and computational science to issues concerning invasive populations, Arctic sea ice, insect flight, and more in this Planet Earth **special issue!**



**Figure 3.** Comparison of real Arctic melt ponds with metastable equilibria in our melt pond Ising model. **3a.** Ising model simulation. **3b.** Real melt pond photo. Figure 3a courtesy of Yiping Ma, 3b courtesy of Donald Perovich.

Vast labyrinthine ponds on the surface of melting Arctic sea ice are key players in the polar climate system and upper ocean ecology. Researchers have adapted the Ising model, which was originally developed to understand magnetic materials, to study the geometry of meltwater's distribution over the sea ice surface. In an article on page 5, Kenneth Golden, Yiping Ma, Courtenay Strong, and Ivan Sudakov explore model predictions.

## Controlling Invasive Populations in Rivers

By Yu Jin and Suzanne Lenhart

Flow regimes can change significantly over time and space and strongly impact all levels of river biodiversity, from the individual to the ecosystem. Invasive species in rivers—such as bighead and silver carp, as well as quagga and zebra mussels—continue to cause damage. Management of these species may include targeted adjustment of flow rates in rivers, based on recent research that examines the effects of river morphology and water flow on rivers' ecological statuses. While many previous methodologies rely on habitat suitability models or oversimplification of the hydrodynamics, few studies have focused on the integration of ecological dynamics into water flow assessments.

Earlier work yielded a hybrid modeling approach that directly links river hydrology with stream population models [3]. The hybrid model's hydrodynamic component is based on the water depth in a gradually varying river structure. The model derives the steady advective flow from this structure and relates it to flow features like water discharge, depth, velocity, cross-

sectional area, bottom roughness, bottom slope, and gravitational acceleration. This approach facilitates both theoretical understanding and the generation of quantitative predictions, thus providing a way for scientists to analyze the effects of river fluctuations on population processes.

When a population spreads longitudinally in a one-dimensional (1D) river with spatial heterogeneities in habitat and temporal fluctuations in discharge, the resulting hydrodynamic population model is

$$N_t = -A_t(x, t) \frac{N}{A(x, t)} + \frac{1}{A(x, t)} \left( D(x, t) A(x, t) N_x \right)_x - \frac{Q(t)}{A(x, t)} N_x + rN \left( 1 - \frac{N}{K} \right)$$

$$\begin{aligned} N(0, t) &= 0 & \text{on } (0, T), x = 0, \\ N_x(L, t) &= 0 & \text{on } (0, T), x = L, \\ N(x, 0) &= N_0(x) & \text{on } (0, L), t = 0 \end{aligned}$$

(1)

See **Invasive Populations** on page 4

## Modeling Resource Demands and Constraints for COVID-19 Intervention Strategies

By Erin C.S. Acquesta, Walt Beyeler, Pat Finley, Katherine Klise, Monear Makvandi, and Emma Stanislawski

As the world desperately attempts to control the spread of COVID-19, the need for a model that accounts for realistic trade-offs between time, resources, and corresponding epidemiological implications is apparent. Some early mathematical models of the outbreak compared trade-offs for non-pharmaceutical interventions [3], while others derived the necessary level of test coverage for case-based interventions [4] and demonstrated the value of prioritized testing for close contacts [7].

Isolated analyses provide valuable insights, but real-world intervention strategies are interconnected. Contact tracing is the lynchpin of infection control [6] and forms the basis of prioritized testing. Therefore, quantifying the effectiveness of contact tracing is crucial to understanding the real-life implications of disease control strategies.

### Contact Tracing Demands

Contact tracers are skilled, culturally competent interviewers who apply their knowledge of disease and risk factors when notifying people who have come into contact with COVID-19-infected individuals. They also continue to monitor the situation after case investigations [1].

Case investigation consists of four steps:

1. Identify and notify cases
2. Interview cases
3. Locate and notify contacts
4. Monitor contacts.

Most health departments are implementing case investigation, contact identification, and quarantine to disrupt COVID-19 transmission. The timeliness of contact tracing is constrained by the length of the infectious period, the turn-around time for testing and result reporting, and the ability to successfully reach and interview patients and their contacts. The European Centre for Disease Prevention and Control approximates that contact tracers spend one to two hours conducting an interview [2]. Estimates regarding the timelines of other steps are limited to subject matter expert elicitation and can vary based on cases' access to phone service or willingness to participate in interviews.

### Bounded Exponential

The fundamental structure of our model follows traditional susceptible-exposed-infected-recovered (SEIR) compartmental modeling [5]. We add an asymptomatic population  $A$ , a hospitalized population  $H$ , and disease-related deaths  $D$ , as well as corresponding quarantine states. We define the states  $\{S_i, E_i, A_i, I_i, H, R, D\}_{i=0,1}$  for our compartments, such that  $i=0$  and  $i=1$

correspond to unquarantined and quarantined respectively. Rather than focus on the dynamics that are associated with the state transition diagram in Figure 1, we introduce a formulation for the real-time demands on contact tracers' time as a function of infection prevalence, while also respecting constraints on resources.

When the work that is required to investigate new cases and monitor existing contacts exceeds available resources, a backlog develops. To simulate this backlog, we introduce a new compartment  $C$  for tracking the dynamic states of cases:

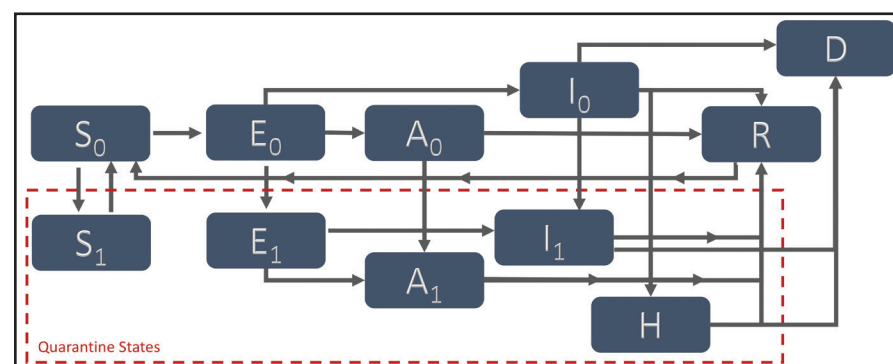
$$\frac{dC}{dt} = [flow_{in}] - [flow_{out}].$$

Flow into the backlog compartment, represented by  $[flow_{in}]$ , reflects case identification that is associated with the following transitions in the model:

- The rate of random testing:  $q_{rA}(t)A_0(t) \rightarrow A_1(t)$  and  $q_{rI}(t)I_0(t) \rightarrow I_1(t)$
- Testing triggered by contact tracing:  $q_{tA}(t)A_0(t) \rightarrow A_1(t)$ ,  $q_{tI}(t)I_0(t) \rightarrow I_1(t)$ , and  $q_{tE}(t)E_i(t) \rightarrow \{A_i(t), I_i(t)\}$
- The population that was missed by the non-pharmaceutical interventions that require hospitalization:  $\tau_{IH}(t)I_0(t) \rightarrow H(t)$ .

Here,  $q_{rs}(t)$  defines the time-dependent rate of random testing,  $q_{ts}(t)$  signifies the time-dependent rate of testing that is triggered by contact tracing, and  $\tau_{IH}$  is the inverse of the expected amount of time for which an infected individual is symptomatic before hospitalization. These terms collectively provide the simulated number of newly-identified positive COVID-19 cases. However, we also need the average number of contacts per case. We thus define function  $\mathcal{K}(\kappa, T_s, \phi_\kappa)$  that depends on the average number of contacts a day ( $\kappa$ ), the average number of days for which an individual is infectious before going into isolation ( $T_s$ ), and the likelihood that the individual

See **COVID-19 Intervention** on page 3



**Figure 1.** Disease state diagram for the compartmental infectious disease model. Figure courtesy of the authors.

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