On Thinning Ice: What math tells us about disappearing polar sea ice and its ecosystems

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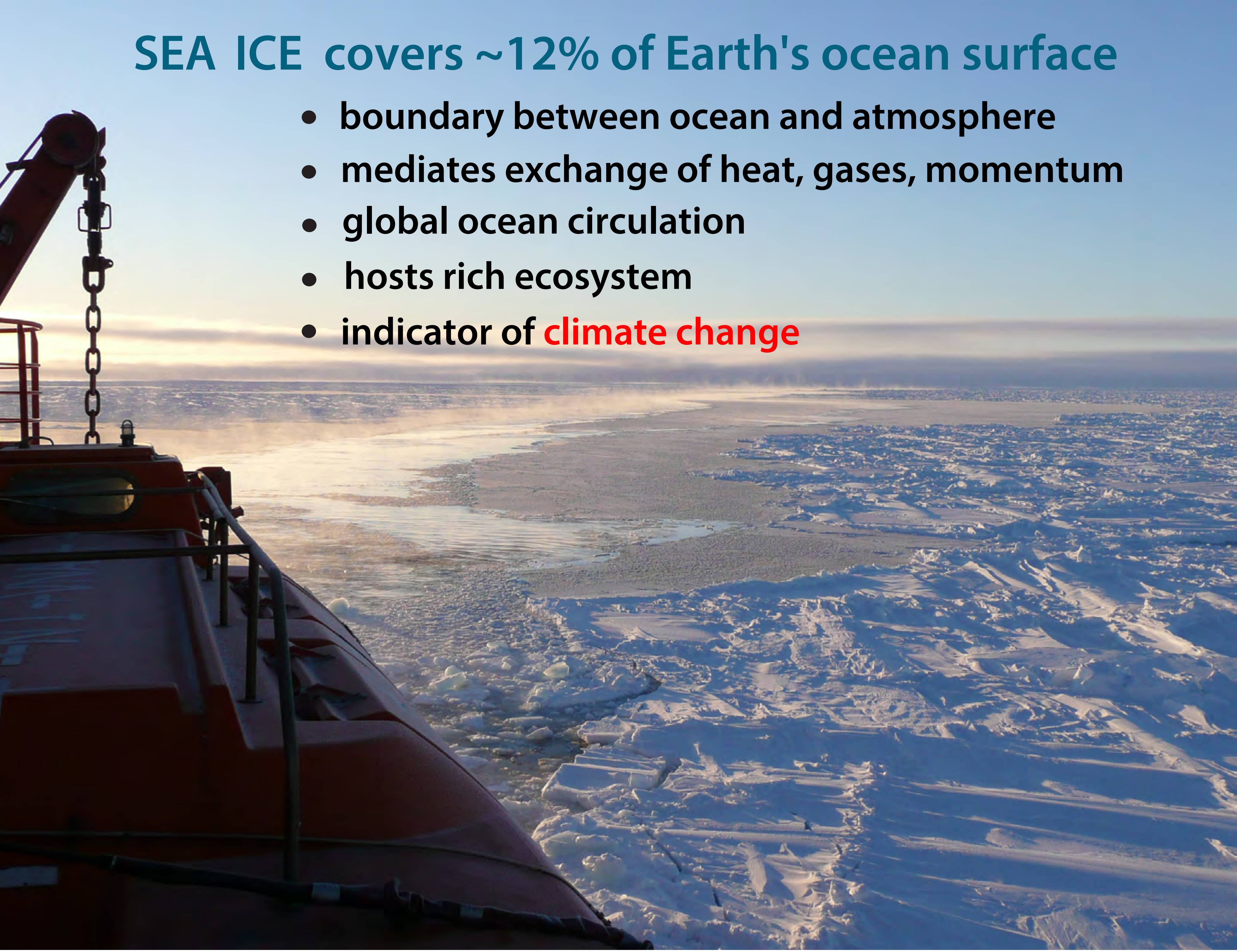
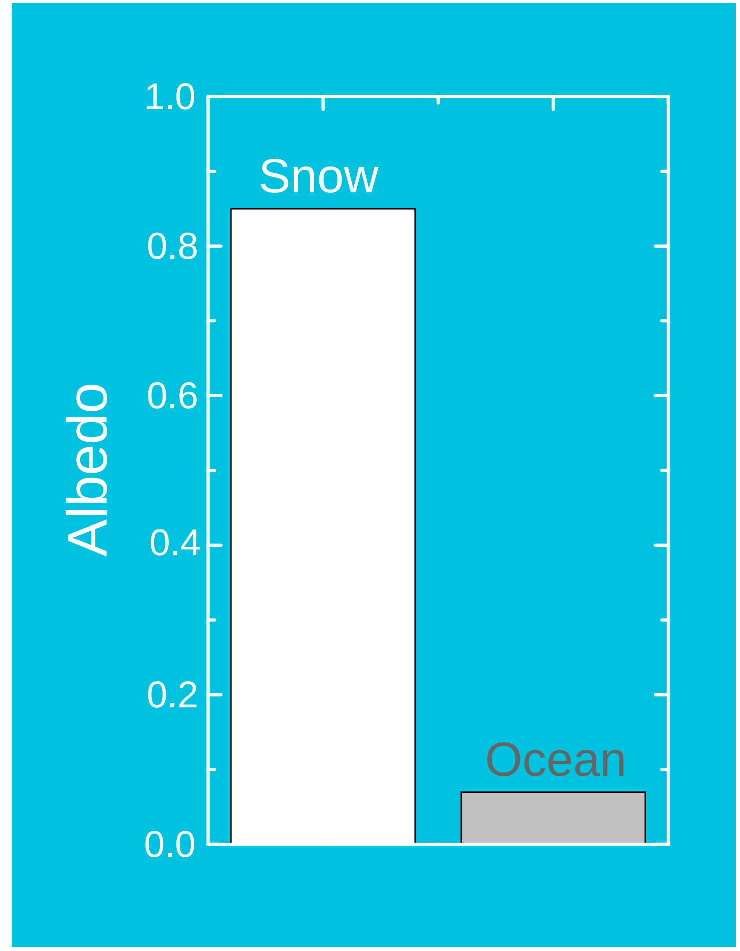
Department of Mathematics
University of Utah

Mathematics Graduate Recruitment Weekend, March 2021


SEA ICE covers ~12% of Earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, gases, momentum
• global ocean circulation
• hosts rich ecosystem
• indicator of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

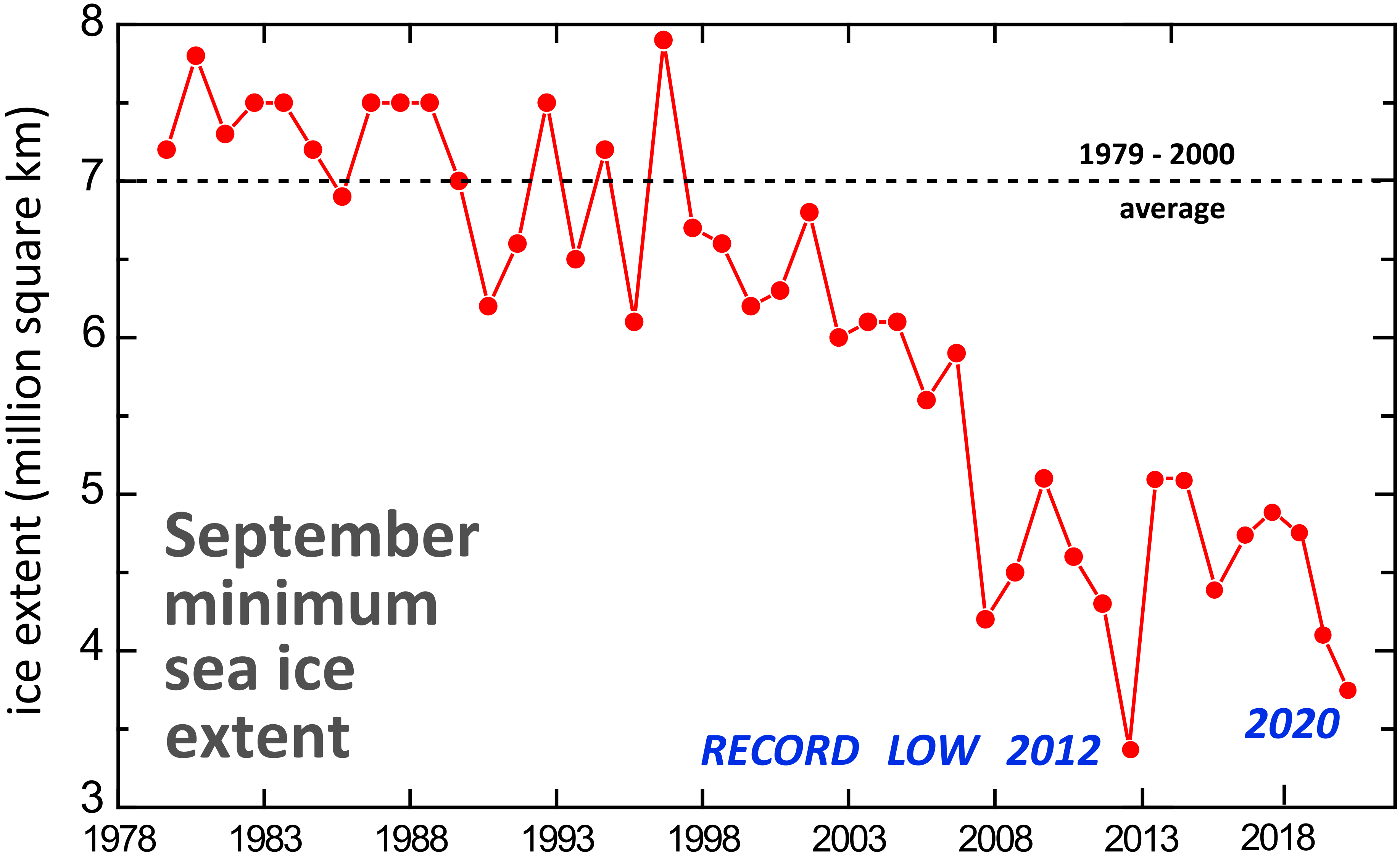
white snow and ice reflect



dark water and land absorb

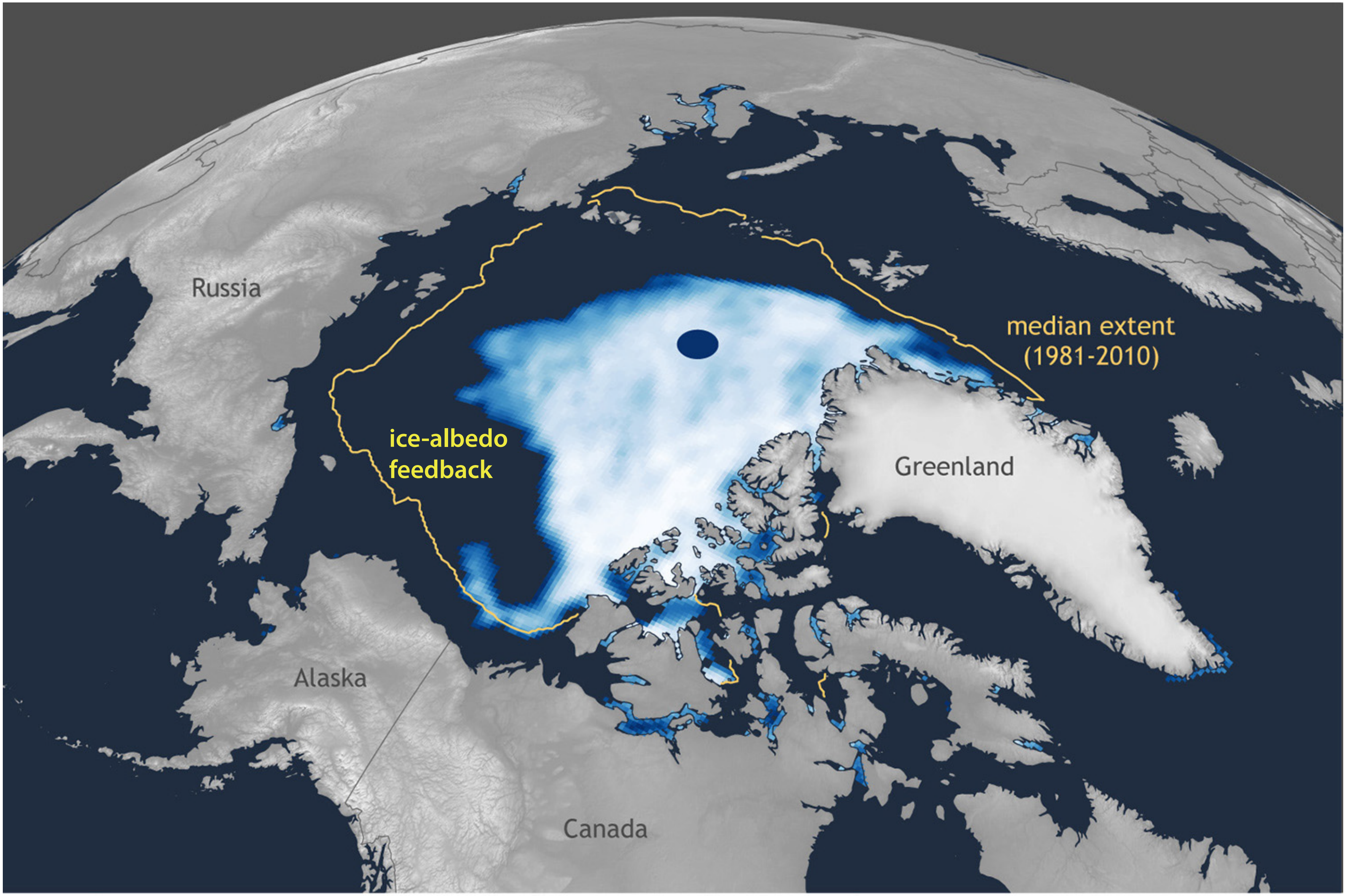
$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

the summer Arctic sea ice pack is melting



National Snow and Ice Data Center (NSIDC)

Arctic sea ice extent September 15, 2020

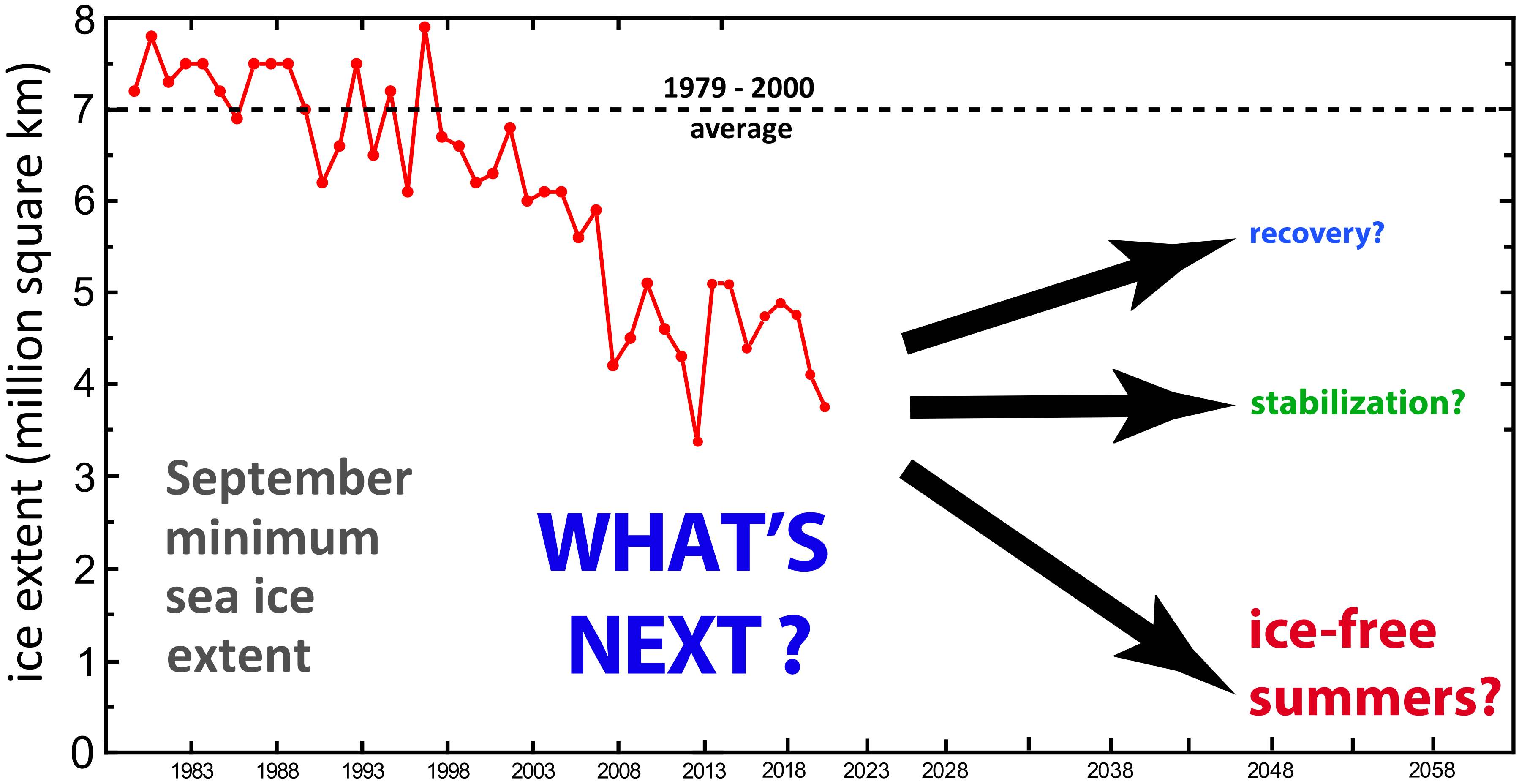


Sea ice concentration (percent)

15 100

NSIDC

Predicting what may come next requires lots of math modeling.

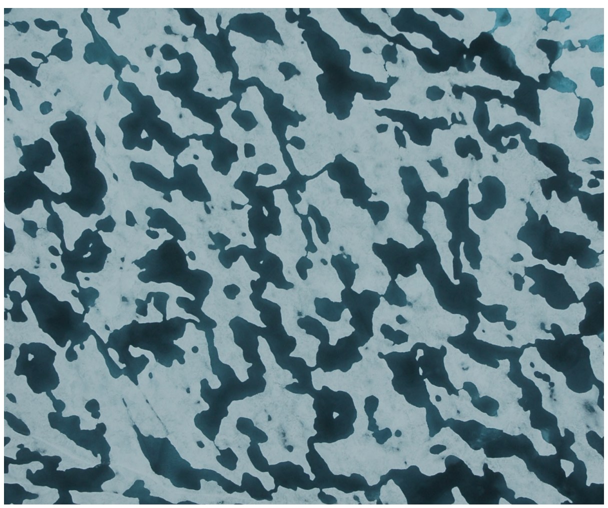


National Snow and Ice Data Center (NSIDC)

challenge:

Represent sea ice more realistically in climate models to improve projections.

How do patterns of dark and light evolve?



Account for key processes

e.g. melt pond evolution

Including PONDS in simulations LOWERS predicted sea ice volume over time by 40%.

Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

... and other sub-grid scale structures and processes.

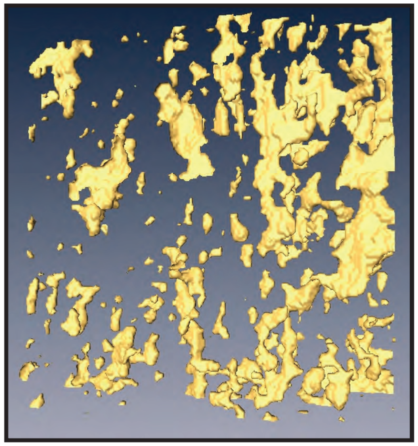
linkage of scales

Sea Ice is a Multiscale Composite Material

microscale

brine inclusions

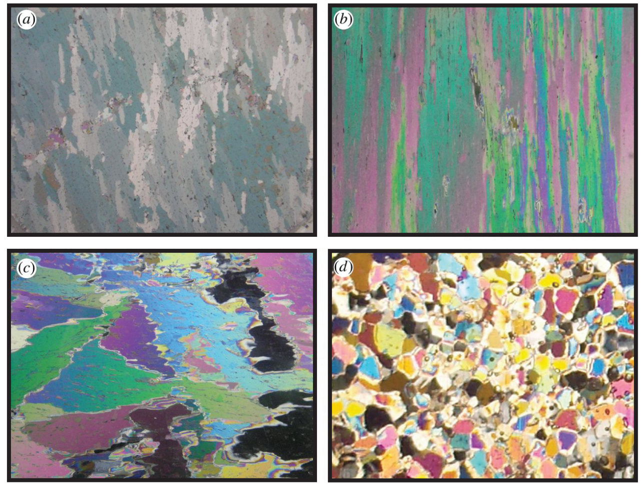
Weeks & Assur 1969



H. Eicken

Golden *et al. GRL* 2007

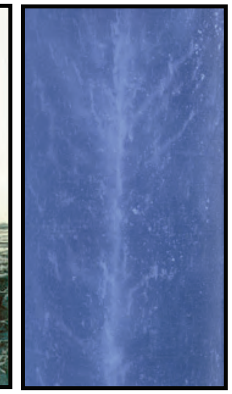
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

millimeters

centimeters

---

mesoscale

Arctic melt ponds

K. Frey

Antarctic pressure ridges

K. Golden

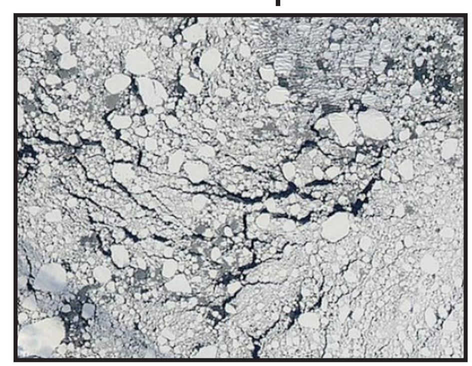
meters

macroscale

sea ice floes

J. Weller

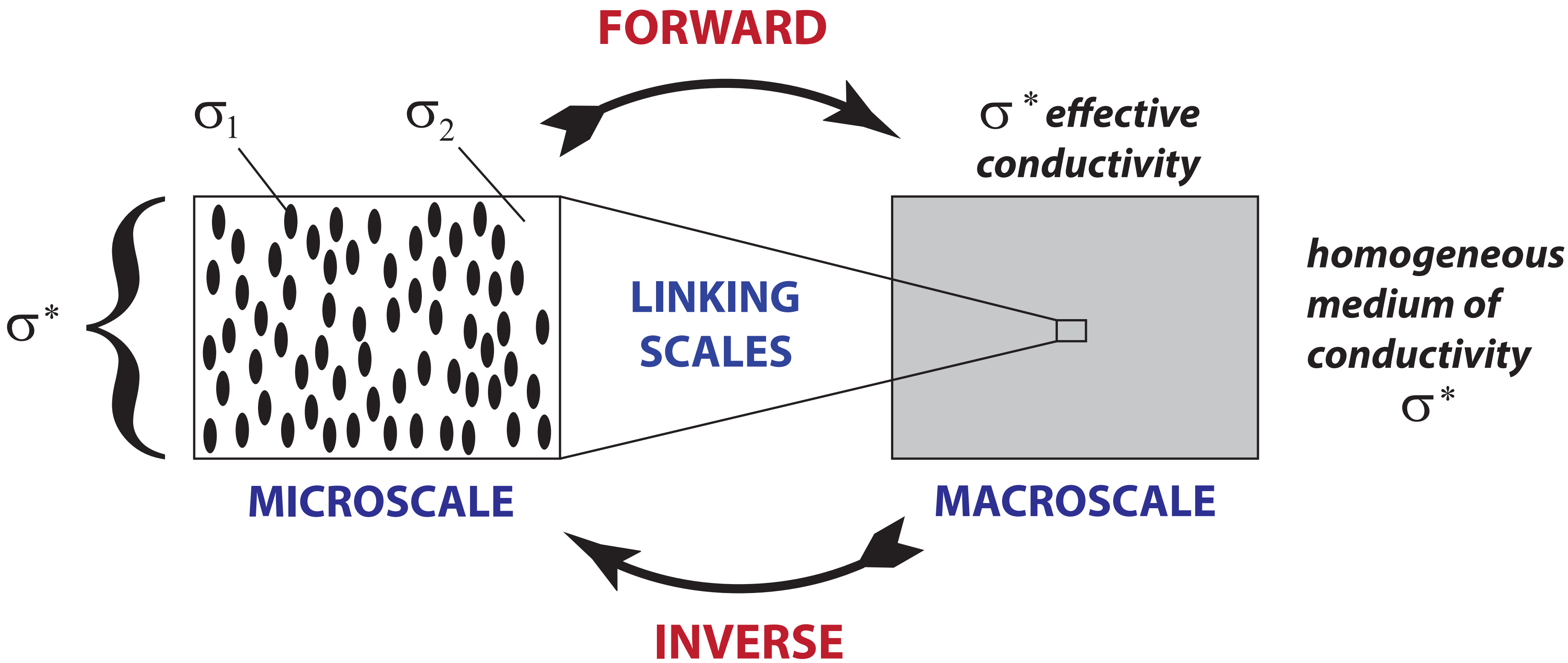
sea ice pack



NASA

kilometers

HOMOGENIZATION for Composite Materials



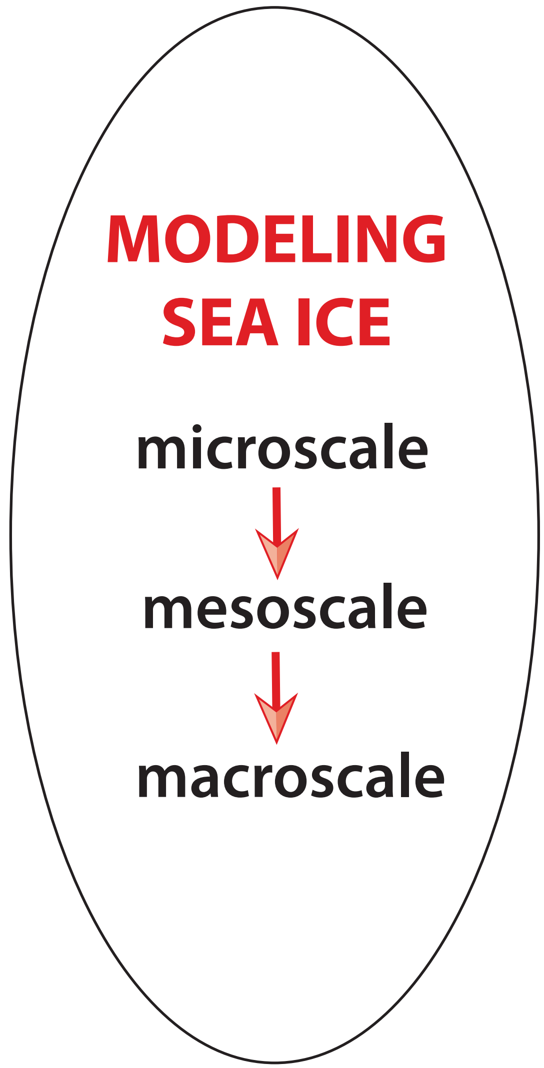
Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is this talk about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...



A tour of key sea ice processes on micro, meso, and macro scales.

What is our research about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...

Inputs, Ingredients

COMPOSITE MATERIALS

- electrical engineering, stealth technology
- porous media, oil extraction
- statistical mechanics of ferromagnets
- Anderson localization, semiconductor physics
- random matrix theory
- differential equations

MODELING SEA ICE

- microscale
- mesoscale
- macroscale

Outputs, Impacts

CLIMATE MODELING

- sea ice physics & biology
- composites, polycrystals
- remote sensing
- advection diffusion
- biomedical imaging, biomaterials, EPS
- polar microbial ecology

What is our research about?

Inputs, Ingredients

COMPOSITE MATERIALS

electrical engineering,
stealth technology

porous media,
oil extraction

statistical mechanics
of ferromagnets

Anderson localization,
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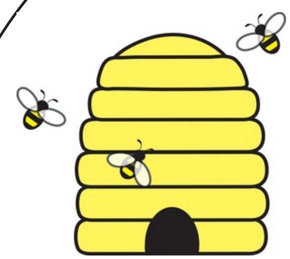
random matrix theory

differential equations



cross pollination

magnets
radar absorbers
human bone
rat brains

MODELING SEA ICE

Outputs, Impacts

CLIMATE MODELING

sea ice physics
& biology

composites,
polycrystals

remote sensing

advection diffusion

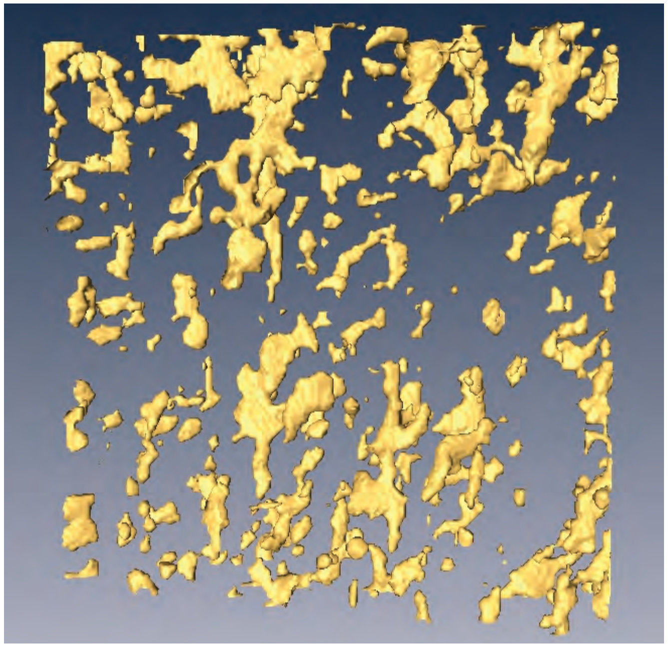
biomedical imaging,
biomaterials, EPS

polar microbial ecology

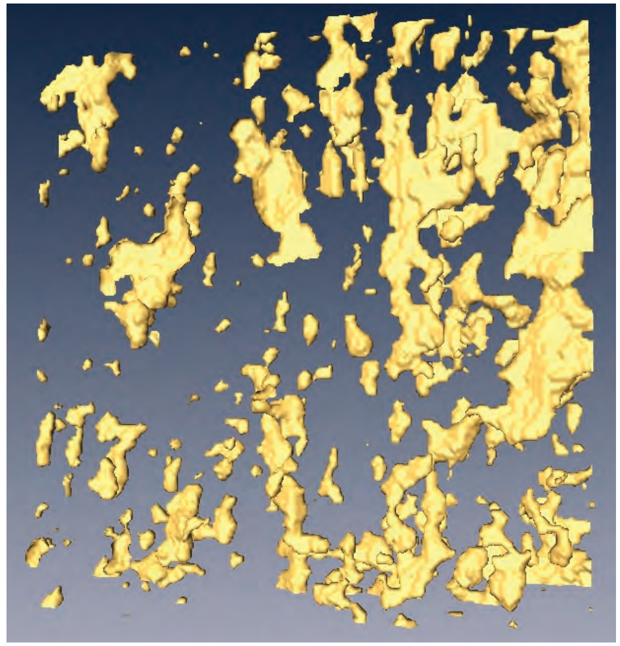
Modeling sea ice drives advances in many areas of science and engineering.

microscale

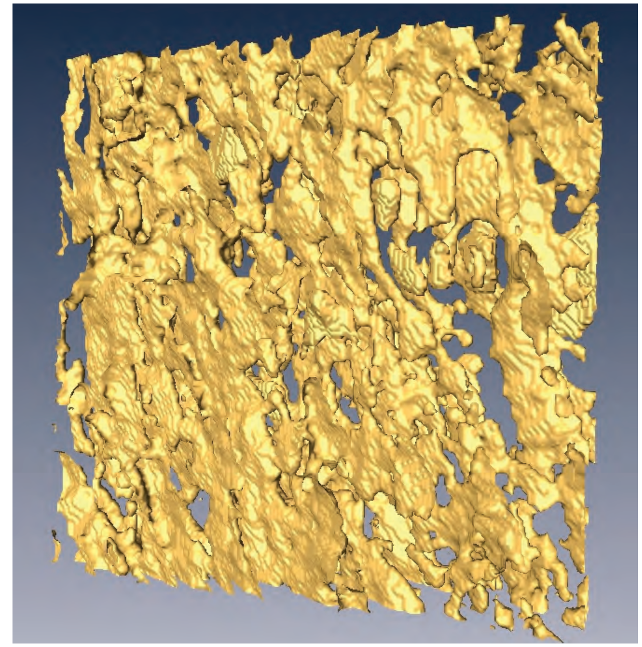
brine volume fraction and ***connectivity*** increase with temperature



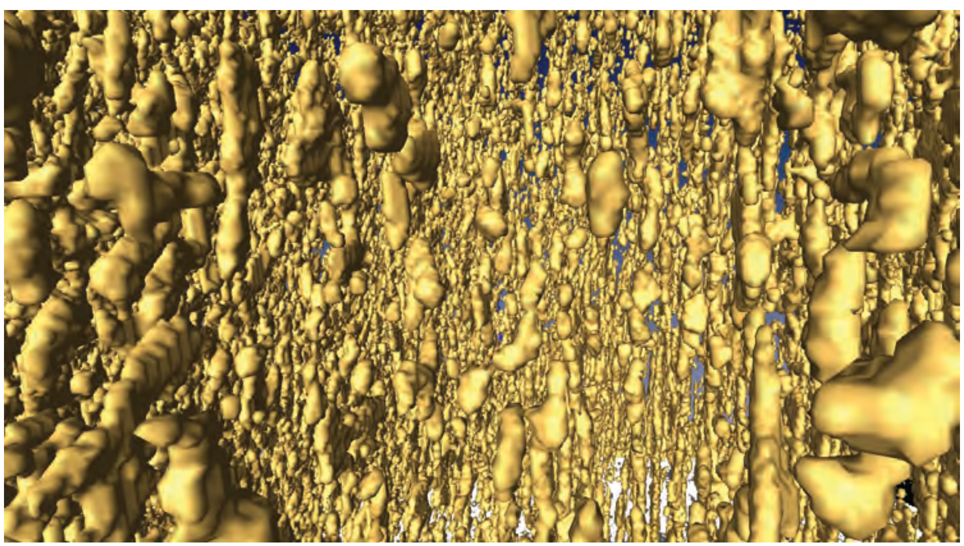
T = -15 °C, $\phi = 0.033$



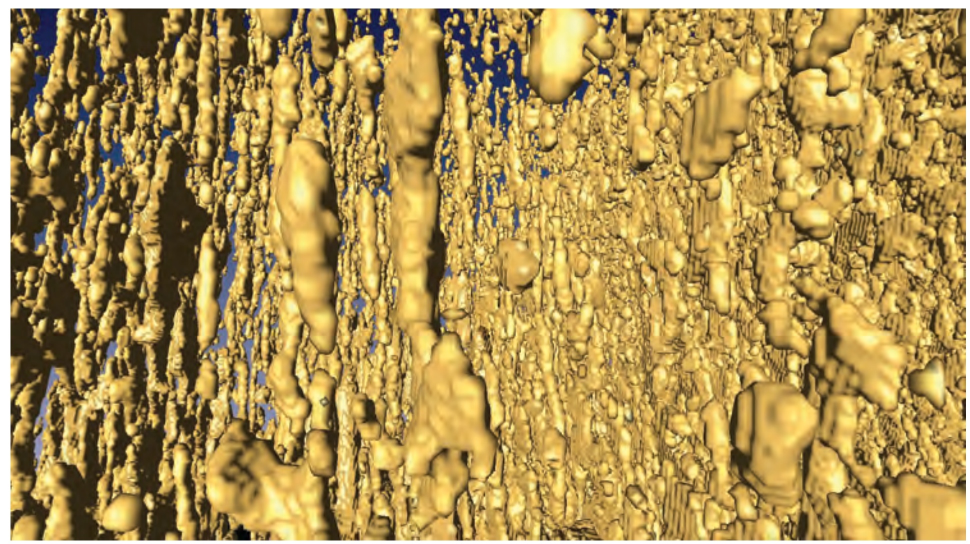
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$



T = -8 °C, $\phi = 0.057$



T = -4 °C, $\phi = 0.113$

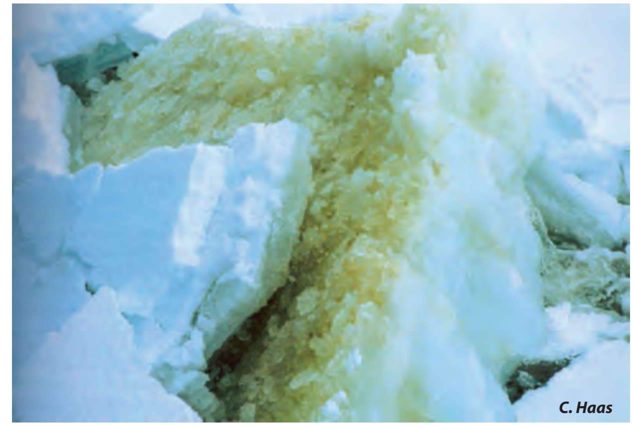
X-ray tomography for brine in sea ice

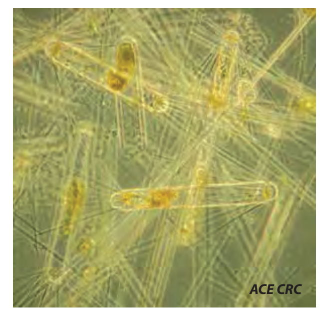
Golden *et al.*, *Geophysical Research Letters*, 2007

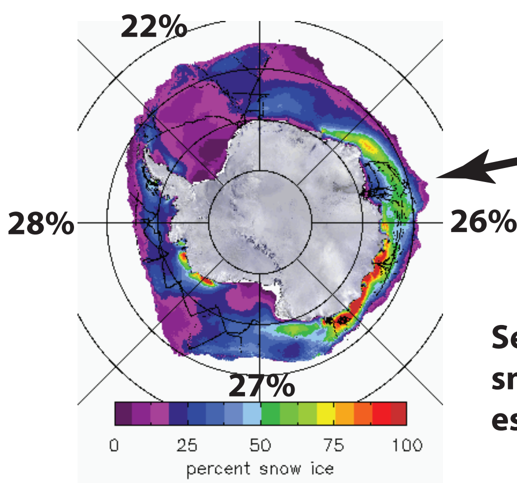
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

nutrient flux for algal communities







T. Maksym and T. Markus, 2008

September snow-ice estimates

Antarctic surface flooding and snow-ice formation

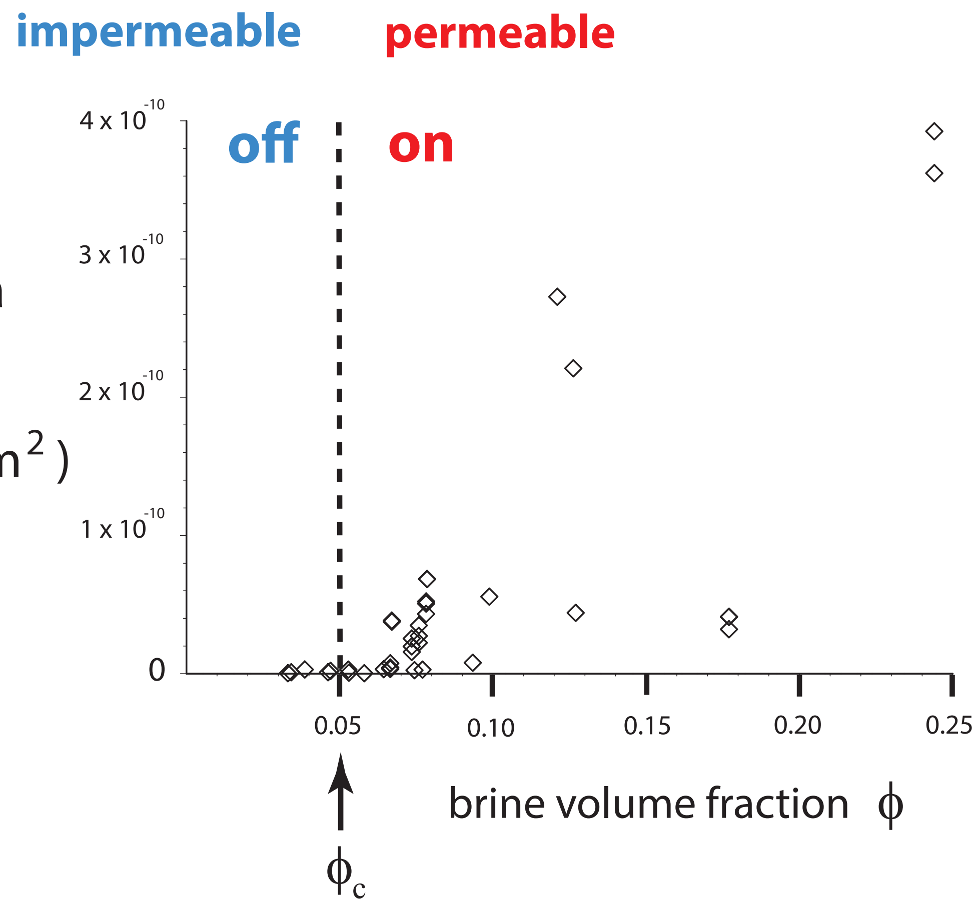
- evolution of salinity profiles

- ocean-ice-air exchanges of heat, CO_2

Critical behavior of fluid transport in sea ice

Arctic field data

vertical fluid permeability k (m^2)



"on - off" switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\%$ $\longleftrightarrow$ $T_c \approx -5^\circ C$, $S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

sea ice algal communities

D. Thomas 2004

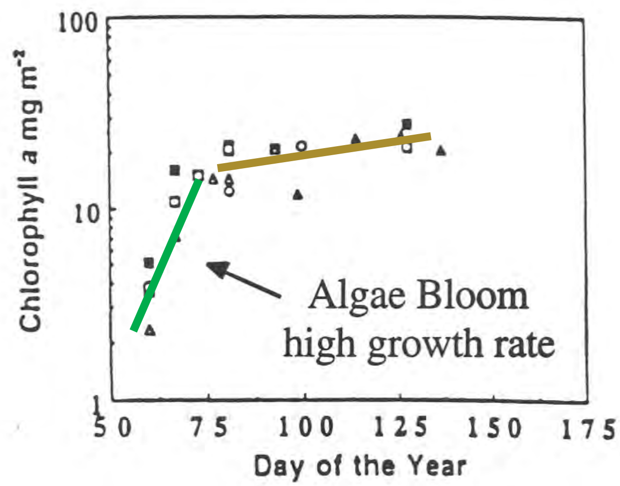
nutrient replenishment controlled by ice permeability

biological activity turns on or off according to ***rule of fives***

Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

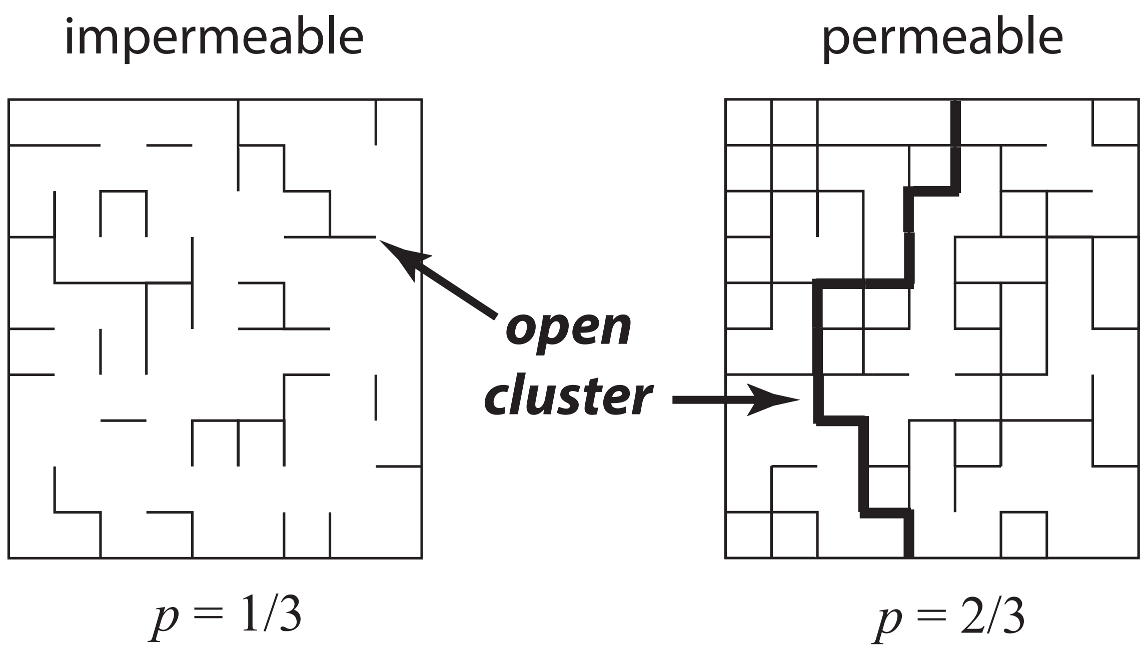
critical behavior of microbial activity



Convection-fueled algae bloom
Ice Station Weddell

percolation theory

probabilistic theory of connectedness



bond → **open** *with probability p*
closed *with probability 1-p*

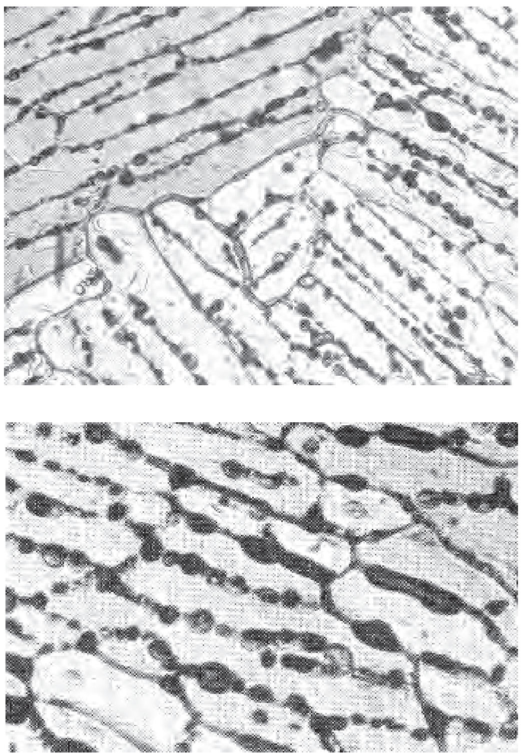
percolation threshold

$$p_c = 1/2 \quad \text{for} \quad d = 2$$

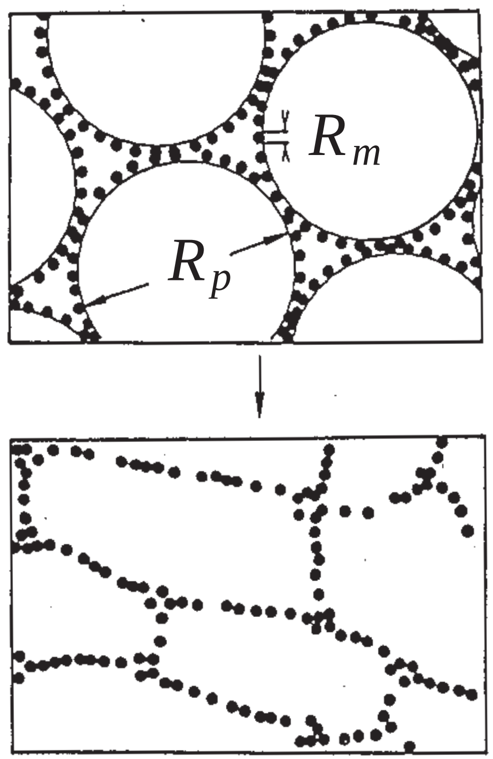
smallest *p* for which there is an infinite open cluster

Continuum percolation model for *stealthy* materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

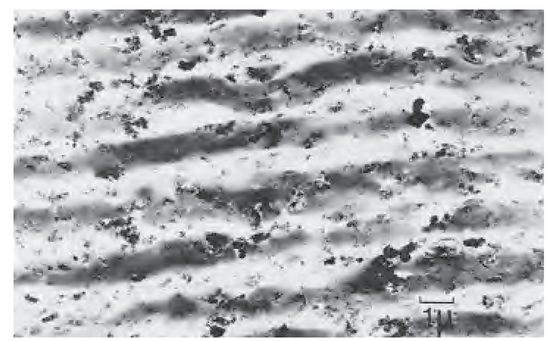
$\phi_c \approx 5\ \%$ Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed powder

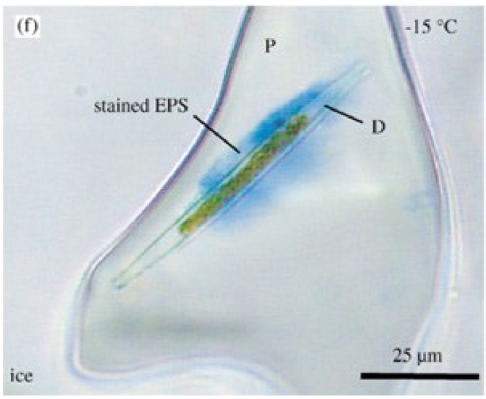


radar absorbing composite

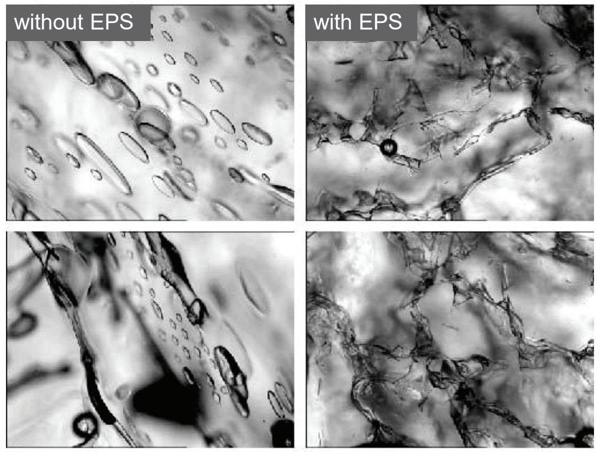
sea ice is radar absorbing

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

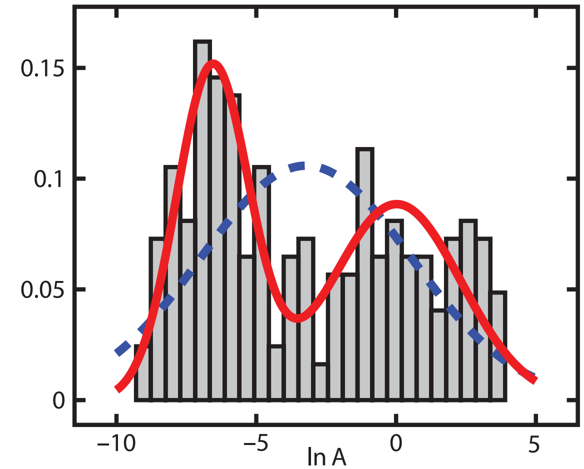
How does EPS affect fluid transport? **How does the biology affect the physics?**



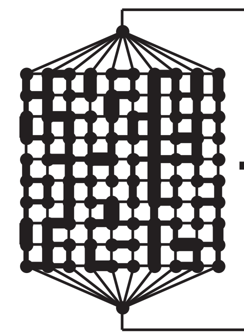
Krembs

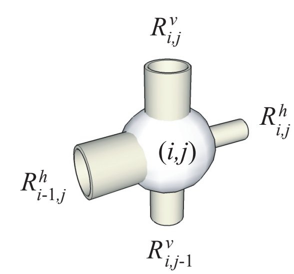


Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL





- 2D random pipe model with bimodal distribution of pipe radii

- Rigorous bound on permeability *k;* results predict observed drop in *k*

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
***Multiscale Modeling and Simulation*, 2018**

Zhu, Jabini, Golden, Eicken, Morris
***Ann. Glac.* 2006**

mesoscale

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

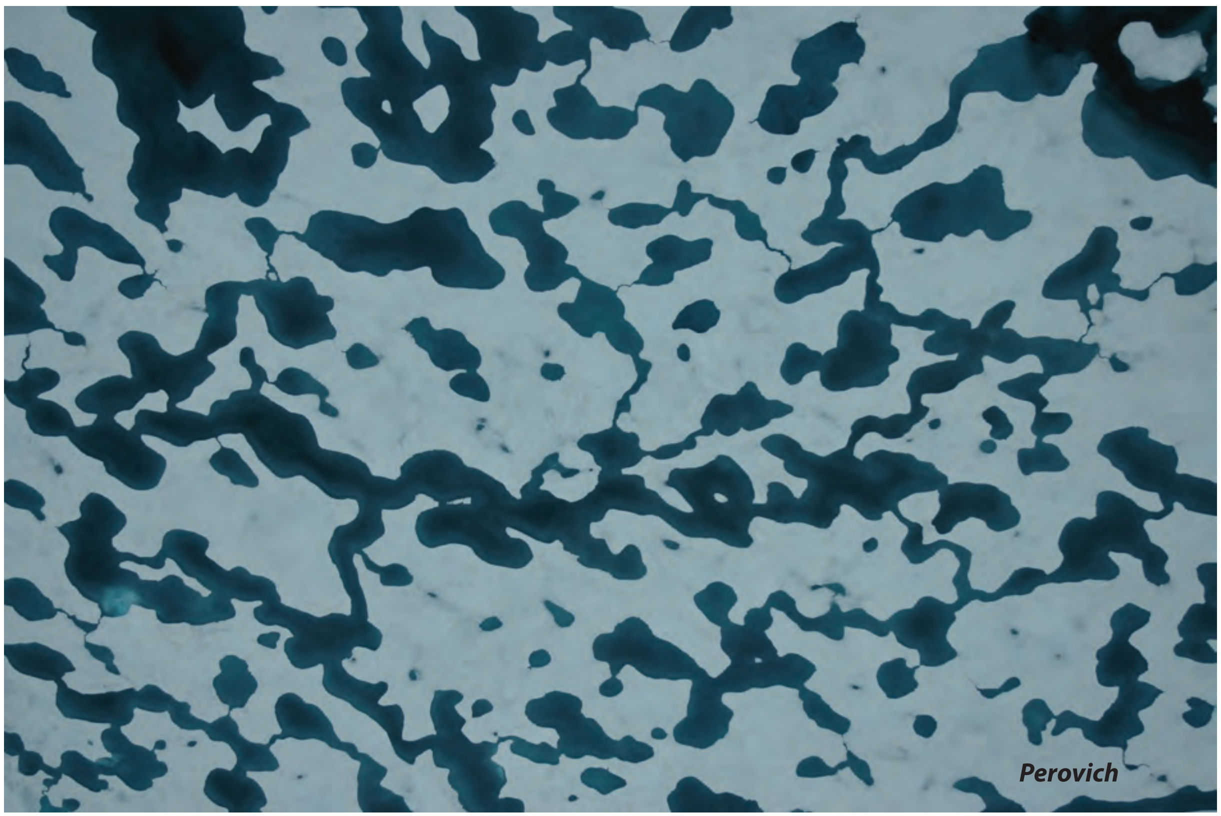
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

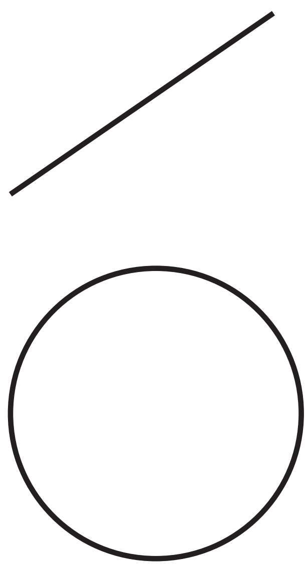
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

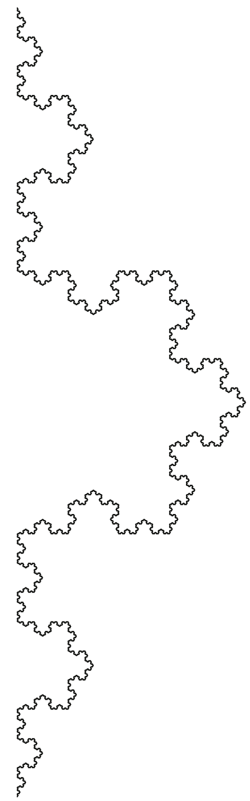
fractal curves in the plane

they wiggle so much that their dimension is >1



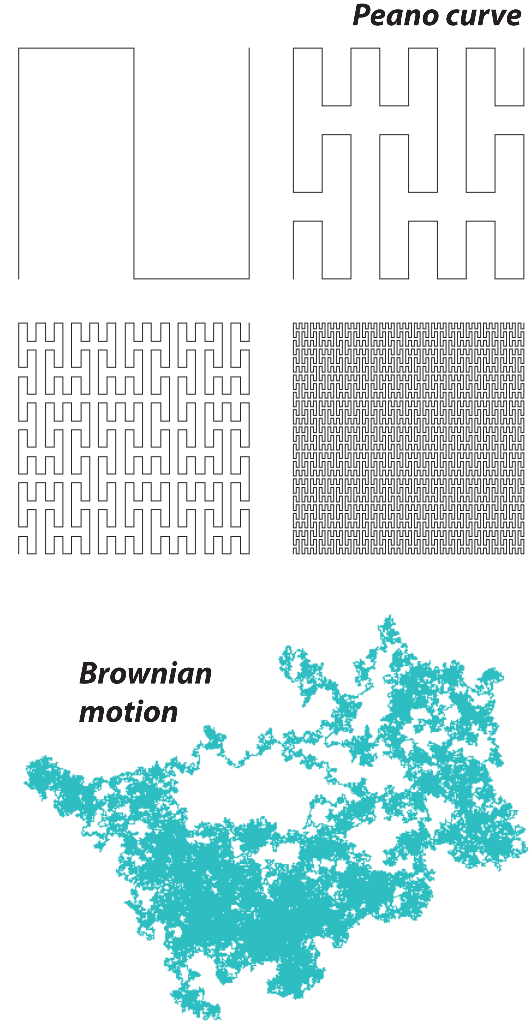
simple curves

***D* = 1**



Koch snowflake

***D* = 1.26**



space filling curves

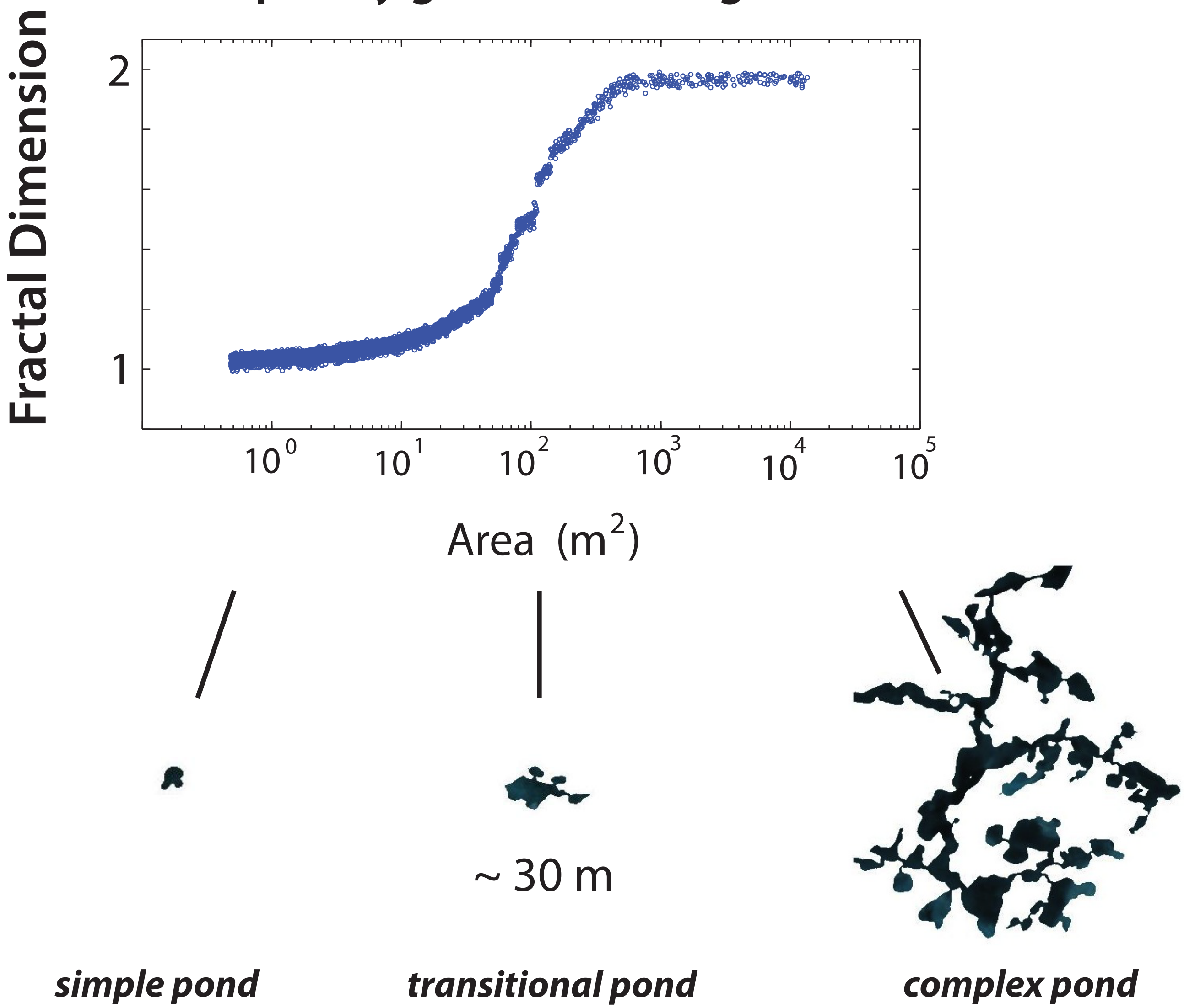
***D* = 2**

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

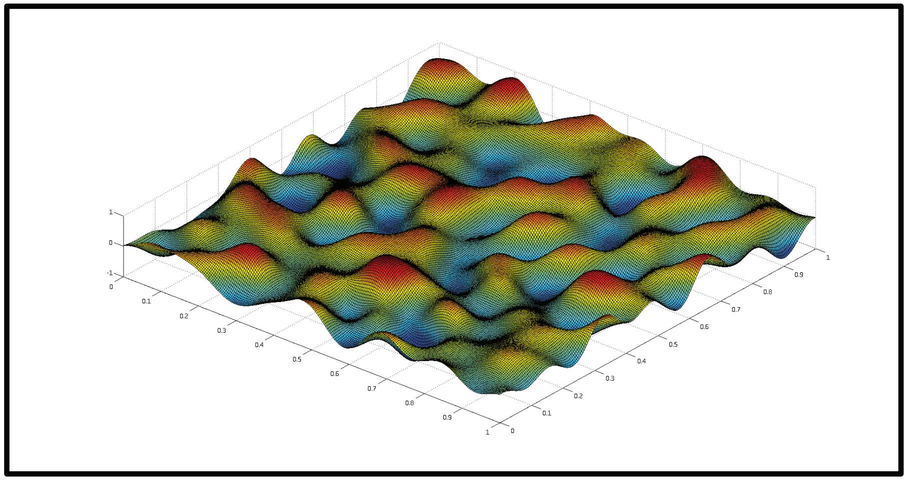
complexity grows with length scale

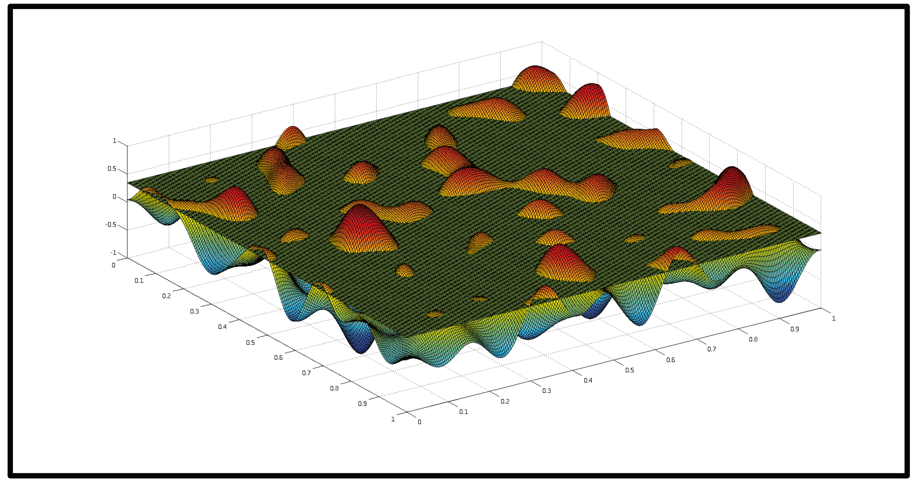


Continuum percolation model for melt pond evolution

level sets of random surfaces

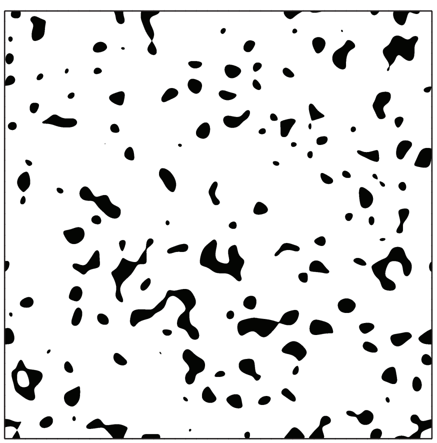
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018

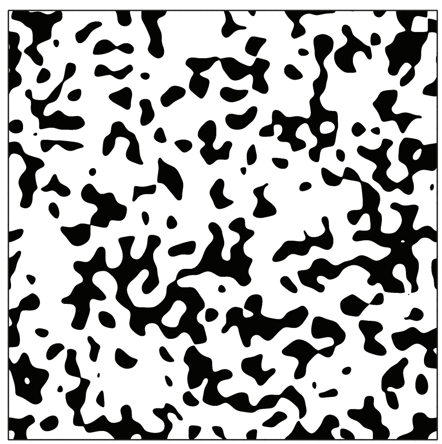


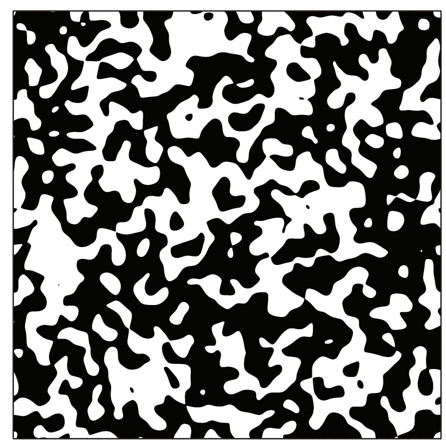


random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds

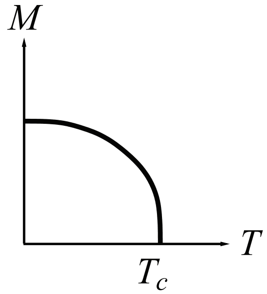




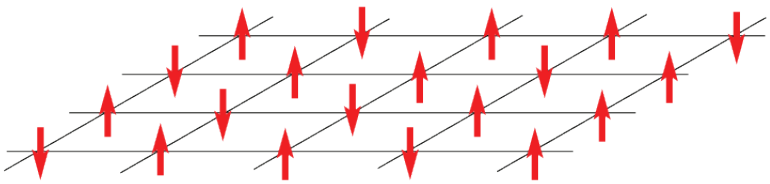


electronic transport in disordered media *diffusion in turbulent plasmas* *Isichenko, Rev. Mod. Phys., 1992*

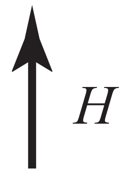
Ising Model for a Ferromagnet



Curie point
critical temperature



applied magnetic field



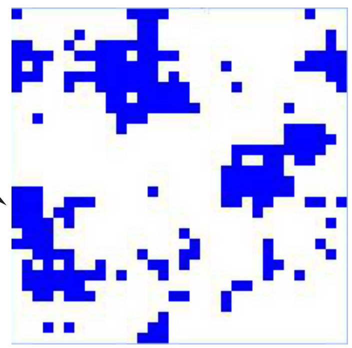
$$s_i = \begin{cases} +1 & \text{spin up} \\ -1 & \text{spin down} \end{cases}$$

blue
white

$$\mathcal{H} = -H \sum_i s_i - J \sum_{<i,j>} s_i s_j$$

nearest neighbor Ising Hamiltonian

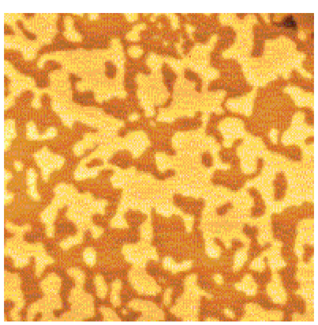
islands of like spins



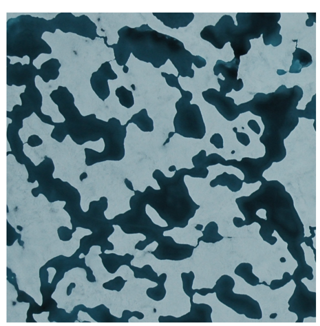
$$M(T,H) = \lim_{N\to\infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

effective magnetization

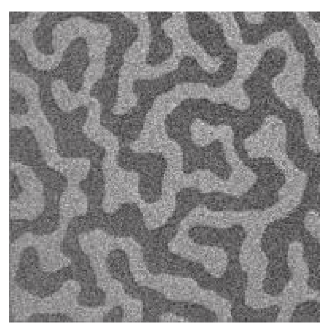
energy is lowered when nearby spins align with each other, forming **magnetic domains**



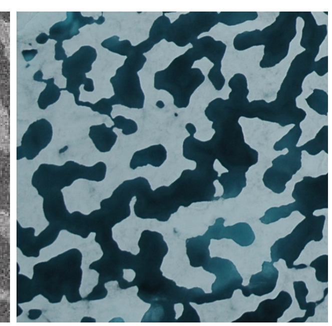
magnetic domains in cobalt



melt ponds (Perovich)



magnetic domains in cobalt-iron-boron



melt ponds (Perovich)

Ising model for ferromagnets ⟶ Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

$$\mathcal{H} = -\sum_{i}^{N} H_i\, s_i - J \sum_{<i,j>}^{N} s_i s_j \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow & -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

random magnetic field represents snow topography

magnetization M

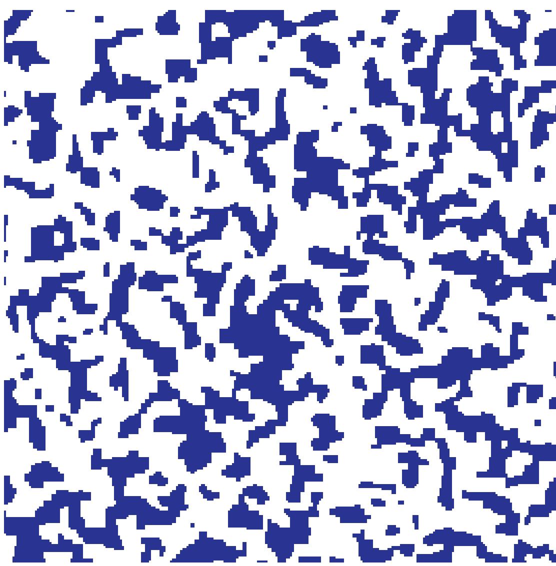
pond area fraction $F = \dfrac{(M+1)}{2}$

~ albedo

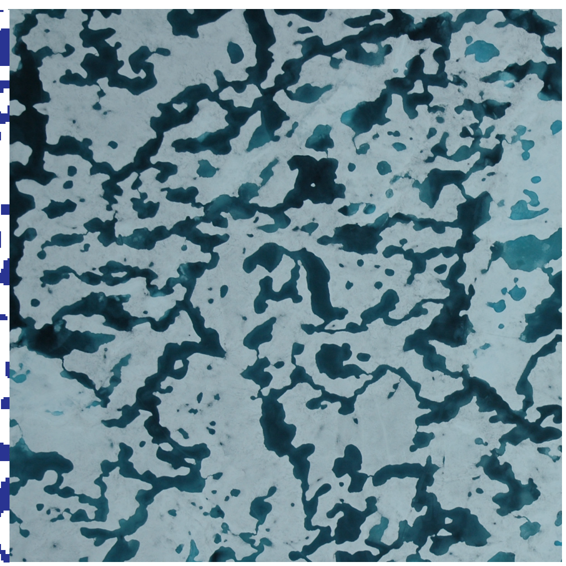
only nearest neighbor patches interact

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system "flows" toward metastable equilibria.

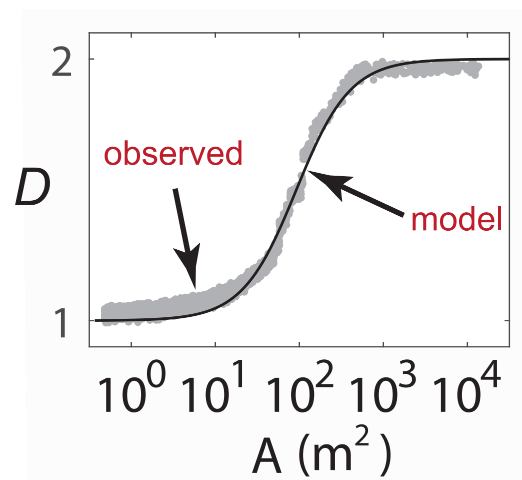
Order from Disorder



Ising model



melt pond photo (Perovich)



pond size distribution exponent

observed -1.5

(Perovich, *et al.* 2002)

model -1.58

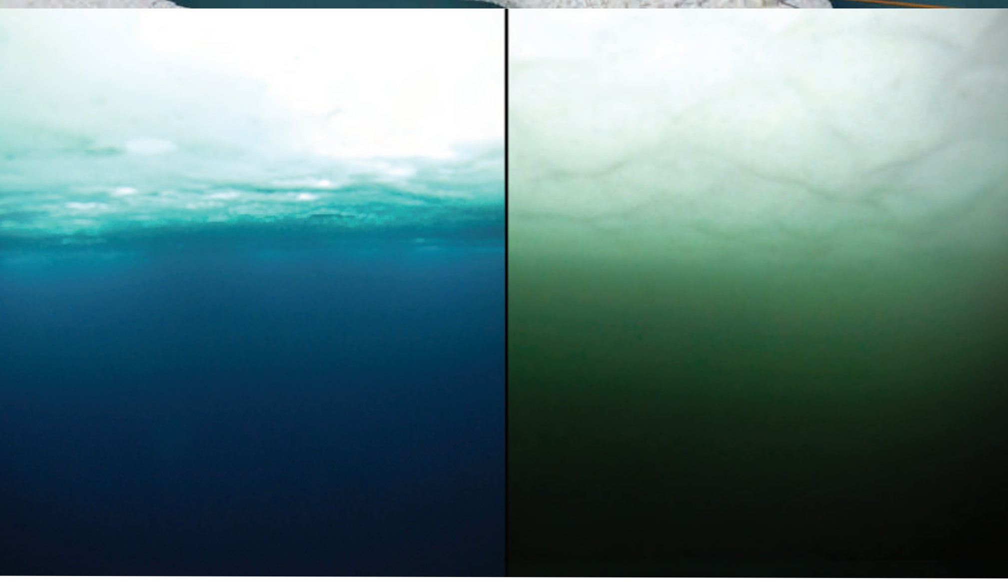
Scientific American
EOS, PhysicsWorld, ...

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data

Perovich

Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

WINDOWS



no bloom bloom

massive under-ice algal bloom

Arrigo et al., *Science* 2012

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, ***Science Advances*** 2017

The effect of melt pond geometry on the distribution of solar energy under first year sea ice

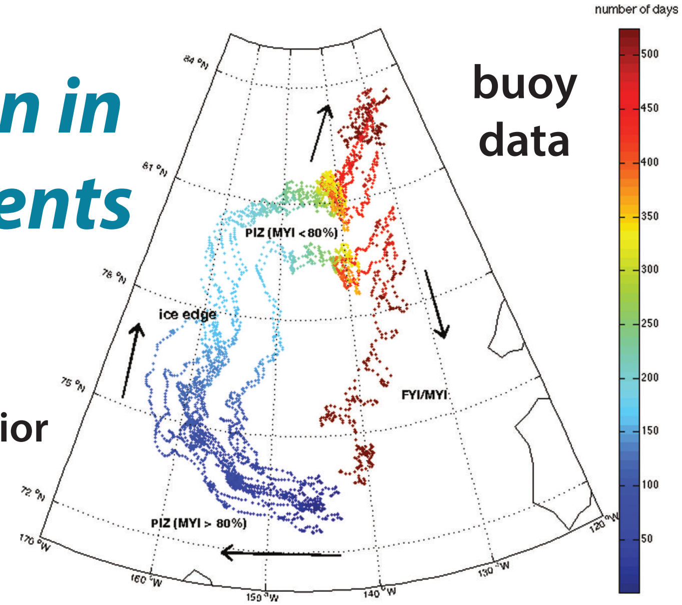
Horvat, Flocco, Rees Jones, Roach, Golden

Geophys. Res. Lett. 2019

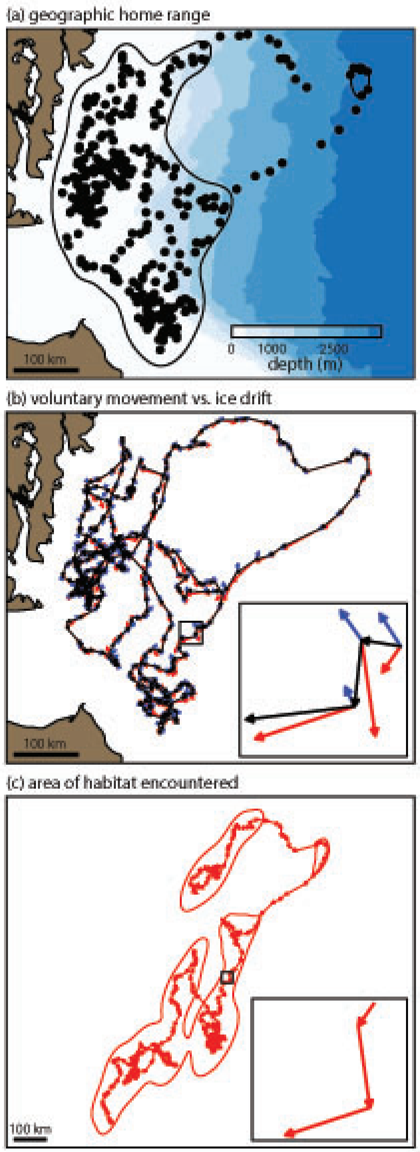
(2015 AMS MRC)

macroscale

Ice floe diffusion in winds and currents



on short time scales floes exhibit Brownian-like behavior



- Effective behavior is purely diffusive, sub-diffusive or super-diffusive depending on ice pack and advective conditions - Hurst exponent.

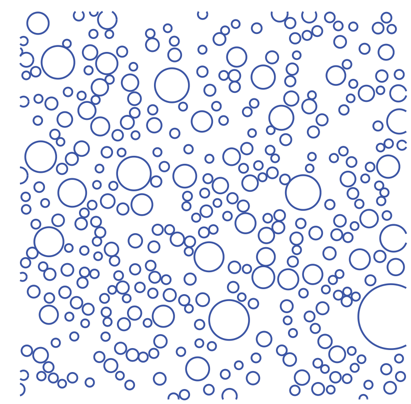
On sea-ice dynamical regimes in the Arctic Ocean

Jennifer Lukovich, Jennifer Hutchings, David Barber, *Ann. Glac.* 2015

Anomalous diffusion and sea ice dynamics

Huy Dinh, Ben Murphy, Elena Cherkaev, Ken Golden 2021

floe-scale model - crowding jamming, advective forcing



sea ice concentration = 0.3

Home ranges in moving habitats: polar bears and sea ice

Marie Auger-Méthé, Mark Lewis, Andrew Derocher, *Ecography*, 2016

What is our research about?

Developing mathematical models of sea ice structures, processes and ecosystems.

Rigorously compute effective or collective behavior
multiscale homogenization
Improve climate models and projections of polar sea ice and the ecosystems they support

sea ice → **physics of climate**
sea ice → **polar marine ecosystems**

Solving problems in physics and biology of sea ice drives advances in mathematics of composite materials, transport phenomena, porous media, inverse problems, biophysics.

What kind of math do we use?

homogenization theory for partial differential equations

stochastic processes, advection diffusion

percolation theory, statistical mechanics

dynamical systems and bifurcation theory

functional analysis, complex analysis, spectral theory

random matrix theory

inverse problems

learning “hidden physics”

www.math.utah.edu/~golden/resources/grad_recruitment_2021/

two PDFs on sea ice physics and biology
3 minute movie on Antarctic expedition
opening video from Frontiers of Science
NAMS overview on sea ice modeling 2020

University of Utah Sea Ice Modeling Group (2017-2021)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics
Elena Cherkaev, Professor of Mathematics
Court Strong, Associate Professor of Atmospheric Sciences
Ben Murphy, Adjunct Assistant Professor of Mathematics

Postdoctoral Researchers: Noa Kraitzman (now at ANU), Jody Reimer

Graduate Students: Kyle Steffen (now at UT Austin with Clint Dawson)
Christian Sampson (now at UNC Chapel Hill with Chris Jones)
Huy Dinh (now a sea ice MURI Postdoc at NYU/Courant)
Rebecca Hardenbrook
David Morison (Physics Department)
Ryleigh Moore
Delaney Mosier
Daniel Hallman

Undergraduate Students: Kenzie McLean, Jacqueline Cinella Rich,
Dane Gollero, Samir Suthar, Anna Hyde,
Kitsel Lusted, Ruby Bowers, Kimball Johnston,
Jerry Zhang, Nash Ward, David Gluckman

High School Students: Jeremiah Chapman, Titus Quah, Dylan Webb

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Sea Ice Ecology Group Postdoc Jody Reimer, Grad Student Julie Sherman,
Undergraduates Kayla Stewart, Nicole Forrester

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The cover is based on "Modeling Sea Ice," page 1535.

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ace CRC · Antarctic Climate & Ecosystems Cooperative Research Centre

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UAF · University of Alaska Fairbanks

The University of Utah

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999

2 — THE MERCURY Thursday, July 23, 1998

Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship Aurora Australis without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 185km off the coast.

The extent of the damage is not known.

Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an inert gas.

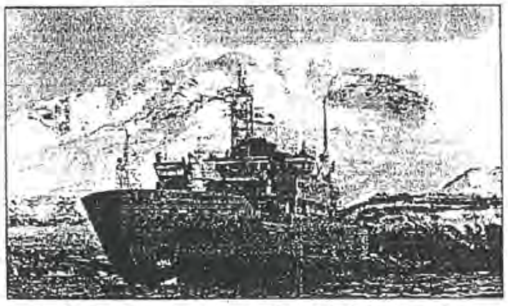
The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known.

The 25 crew and 54 expeditioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year but at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Division's Hobart office.



A file photo of the Aurora Australis in Antarctica.

He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available," Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

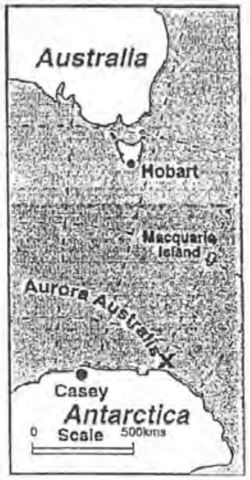
The Aurora Australis is owned by P&O Australia and charted by the Antarctic Division for about $11 million a year.

P&O Australia managing director Richard Hein said yesterday the company was assessing the situation and a number of rescue options were being considered.

It was too early to say whether P&O would be liable for the cost of the aborted mission.

The vessel left Hobart last Wednesday for a seven-week voyage mainly to study a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.



Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

THE ADVERTISER (Adelaide)
Thurs 23 July 1998 Page 14

Fire strands Antarctic ship in sea ice

AN engine room fire has disabled the icebreaker Aurora Australis in sea ice, deep in Antarctic waters.

There were no injuries and the ship was not in danger after Tuesday night's fire, Australian Antarctic Division director Mr Rex Moncur said. But Mr Moncur said he expected it would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent and return to Hobart for repairs.

The cause of the fire was not known but the engines have been turned off, with the ship 100 nautical miles from the Antarctic coast.

THE CANBERRA TIMES
Thursday 23 July 1998 Page 4

Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Australian icebreaker Aurora Australis in sea ice, deep in Antarctic waters.

Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected Aurora Australis would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs.

The fire had been extinguished and the engines were turned off, leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good.

Crew had to wear breathing apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully assessed.

The Aurora, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

Mr Moncur said, the cause of the fire was not yet known.

Sydney Morning Herald
23 July, 1998

ICEBREAKER BURNS

A pioneering $2 million Australian scientific voyage to the mid-winter Antarctic pack ice is expected to be scrapped following an engine-room fire on the Aurora Australis yesterday.

The 54 people on board were forced on deck in the

2:45 am July 22, 1998

``Please don't be alarmed but we have an uncontrolled fire in the engine room ...."

about 10 minutes later ...

``Please don't be alarmed but we're lowering the lifeboats ...."

Conclusions

1. Sea ice is a fascinating multiscale composite with structure similar to many other natural and man-made materials.

2. Mathematical methods developed for sea ice advance the theory of composites and other areas of science and engineering.

3. Homogenization and statistical physics help ***link scales in sea ice and composites***; provide rigorous methods for finding effective behavior; advance sea ice representations in climate models.

4. Fluid flow through sea ice mediates melt pond evolution and many processes important to climate change and polar ecosystems.

5. Field experiments are essential to developing relevant mathematics.

6. Our research is helping to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

Modeling Sea Ice

Kenneth M. Golden, Luke G. Bennetts, Elena Cherkaev, Ian Eisenman, Daniel Feltham, Christopher Horvat, Elizabeth Hunke, Christopher Jones, Donald K. Perovich, Pedro Ponte-Castañeda, Courtenay Strong, Deborah Sulsky, and Andrew J. Wells

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Daniel Feltham is a professor of climate physics at the University of Reading. His email address is d.l.feltham@reading.ac.uk.

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Communicated by Notices *Associate Editor Reza Malek-Madani.*

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November 2020

Special Issue on the Mathematics of Planet Earth

Read about the application of mathematics and computational science to issues concerning invasive populations, Arctic sea ice, insect flight, and more in this Planet Earth ***special issue****!*

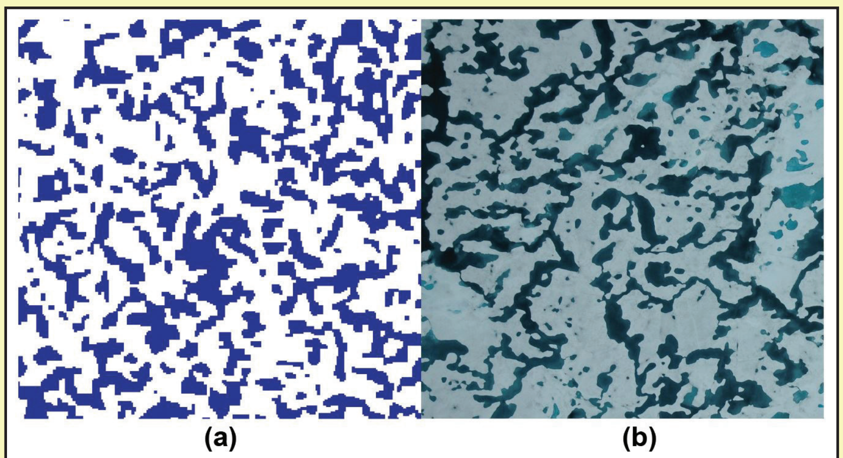

Figure 3. *Comparison of real Arctic melt ponds with metastable equilibria in our melt pond Ising model.* **3a.** *Ising model simulation.* **3b.** *Real melt pond photo. Figure 3a courtesy of Yiping Ma, 3b courtesy of Donald Perovich.*

Vast labyrinthine ponds on the surface of melting Arctic sea ice are key players in the polar climate system and upper ocean ecology. Researchers have adapted the Ising model, which was originally developed to understand magnetic materials, to study the geometry of meltwater's distribution over the sea ice surface. In an article on page 5, Kenneth Golden, Yiping Ma, Courtenay Strong, and Ivan Sudakov explore model predictions.

Controlling Invasive Populations in Rivers

By Yu Jin and Suzanne Lenhart

Flow regimes can change significantly over time and space and strongly impact all levels of river biodiversity, from the individual to the ecosystem. Invasive species in rivers—such as bighead and silver carp, as well as quagga and zebra mussels—continue to cause damage. Management of these species may include targeted adjustment of flow rates in rivers, based on recent research that examines the effects of river morphology and water flow on rivers' ecological statuses. While many previous methodologies rely on habitat suitability models or oversimplification of the hydrodynamics, few studies have focused on the integration of ecological dynamics into water flow assessments.

Earlier work yielded a hybrid modeling approach that directly links river hydrology with stream population models [3]. The hybrid model's hydrodynamic component is based on the water depth in a gradually varying river structure. The model derives the steady advective flow from this structure and relates it to flow features like water discharge, depth, velocity, cross-sectional area, bottom roughness, bottom slope, and gravitational acceleration. This approach facilitates both theoretical understanding and the generation of quantitative predictions, thus providing a way for scientists to analyze the effects of river fluctuations on population processes.

When a population spreads longitudinally in a one-dimensional (1D) river with spatial heterogeneities in habitat and temporal fluctuations in discharge, the resulting hydrodynamic population model is

$$N_t = -A_t(x,t)\frac{N}{A(x,t)} + \frac{1}{A(x,t)}\Big(D(x,t)A(x,t)N_x\Big)_x - \frac{Q(t)}{A(x,t)}N_x + rN\left(1-\frac{N}{K}\right)$$

$$\begin{aligned} &N(0,t)=0 && \text{on } (0,T), x=0,\\ &N_x(L,t)=0 && \text{on } (0,T), x=L,\\ &N(x,0)=N_0(x) && \text{on } (0,L), t=0 \end{aligned} \qquad (1)$$

See **Invasive Populations** *on page 4*

Modeling Resource Demands and Constraints for COVID-19 Intervention Strategies

By Erin C.S. Acquesta, Walt Beyeler, Pat Finley, Katherine Klise, Monear Makvandi, and Emma Stanislawski

As the world desperately attempts to control the spread of COVID-19, the need for a model that accounts for realistic trade-offs between time, resources, and corresponding epidemiological implications is apparent. Some early mathematical models of the outbreak compared trade-offs for non-pharmaceutical interventions [3], while others derived the necessary level of test coverage for case-based interventions [4] and demonstrated the value of prioritized testing for close contacts [7].

Isolated analyses provide valuable insights, but real-world intervention strategies are interconnected. Contact tracing is the lynchpin of infection control [6] and forms the basis of prioritized testing. Therefore, quantifying the effectiveness of contact tracing is crucial to understanding the real-life implications of disease control strategies.

Contact Tracing Demands

Contact tracers are skilled, culturally competent interviewers who apply their knowledge of disease and risk factors when notifying people who have come into contact with COVID-19-infected individuals. They also continue to monitor the situation after case investigations [1].

Case investigation consists of four steps:

1. Identify and notify cases
2. Interview cases
3. Locate and notify contacts
4. Monitor contacts.

Most health departments are implementing case investigation, contact identification, and quarantine to disrupt COVID-19 transmission. The timeliness of contact tracing is constrained by the length of the infectious period, the turn-around time for testing and result reporting, and the ability to successfully reach and interview patients and their contacts. The European Centre for Disease Prevention and Control approximates that contact tracers spend one to two hours conducting an interview [2]. Estimates regarding the timelines of other steps are limited to subject matter expert elicitation and can vary based on cases' access to phone service or willingness to participate in interviews.

Bounded Exponential

The fundamental structure of our model follows traditional susceptible-exposed-infected-recovered (SEIR) compartmental modeling [5]. We add an asymptomatic population A, a hospitalized population H, and disease-related deaths D, as well as corresponding quarantine states. We define the states $\{S_i, E_i, A_i, I_i, H, R, D\}_{i=0,1}$ for our compartments, such that $i=0$ and $i=1$ correspond to unquarantined and quarantined respectively. Rather than focus on the dynamics that are associated with the state transition diagram in Figure 1, we introduce a formulation for the real-time demands on contact tracers' time as a function of infection prevalence, while also respecting constraints on resources.

When the work that is required to investigate new cases and monitor existing contacts exceeds available resources, a backlog develops. To simulate this backlog, we introduce a new compartment C for tracking the dynamic states of cases:

$$\frac{dC}{dt} = [flow_{in}] - [flow_{out}].$$

Flow into the backlog compartment, represented by $[flow_{in}]$, reflects case identification that is associated with the following transitions in the model:

– The rate of random testing: $q_{rA}(t)A_0(t) \to A_1(t)$ and $q_{rI}(t)I_0(t) \to I_1(t)$

– Testing triggered by contact tracing: $q_{tA}(t)A_0(t) \to A_1(t)$, $q_{tI}(t)I_0(t) \to I_1(t)$, and $q_{tE}(t)E_1(t) \to \{A_1(t), I_1(t)\}$

– The population that was missed by the non-pharmaceutical interventions that require hospitalization: $\tau_{IH}(t)I_0(t) \to H(t)$.

Here, $q_{r*}(t)$ defines the time-dependent rate of random testing, $q_{t*}(t)$ signifies the time-dependent rate of testing that is triggered by contact tracing, and τ_{IH} is the inverse of the expected amount of time for which an infected individual is symptomatic before hospitalization. These terms collectively provide the simulated number of newly-identified positive COVID-19 cases. However, we also need the average number of contacts per case. We thus define function $\mathcal{K}(\kappa, T_S, \phi_\kappa)$ that depends on the average number of contacts a day (κ), the average number of days for which an individual is infectious before going into isolation (T_S), and the likelihood that the individual

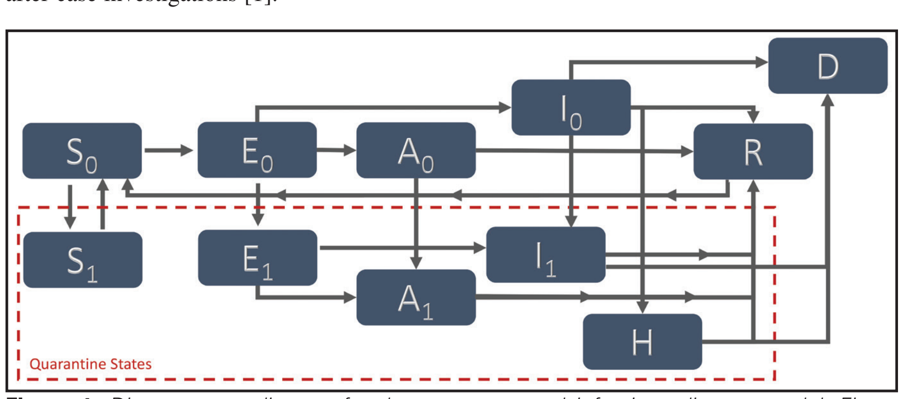

Figure 1. *Disease state diagram for the compartmental infectious disease model. Figure courtesy of the authors.*

See **COVID-19 Intervention** *on page 3*

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