

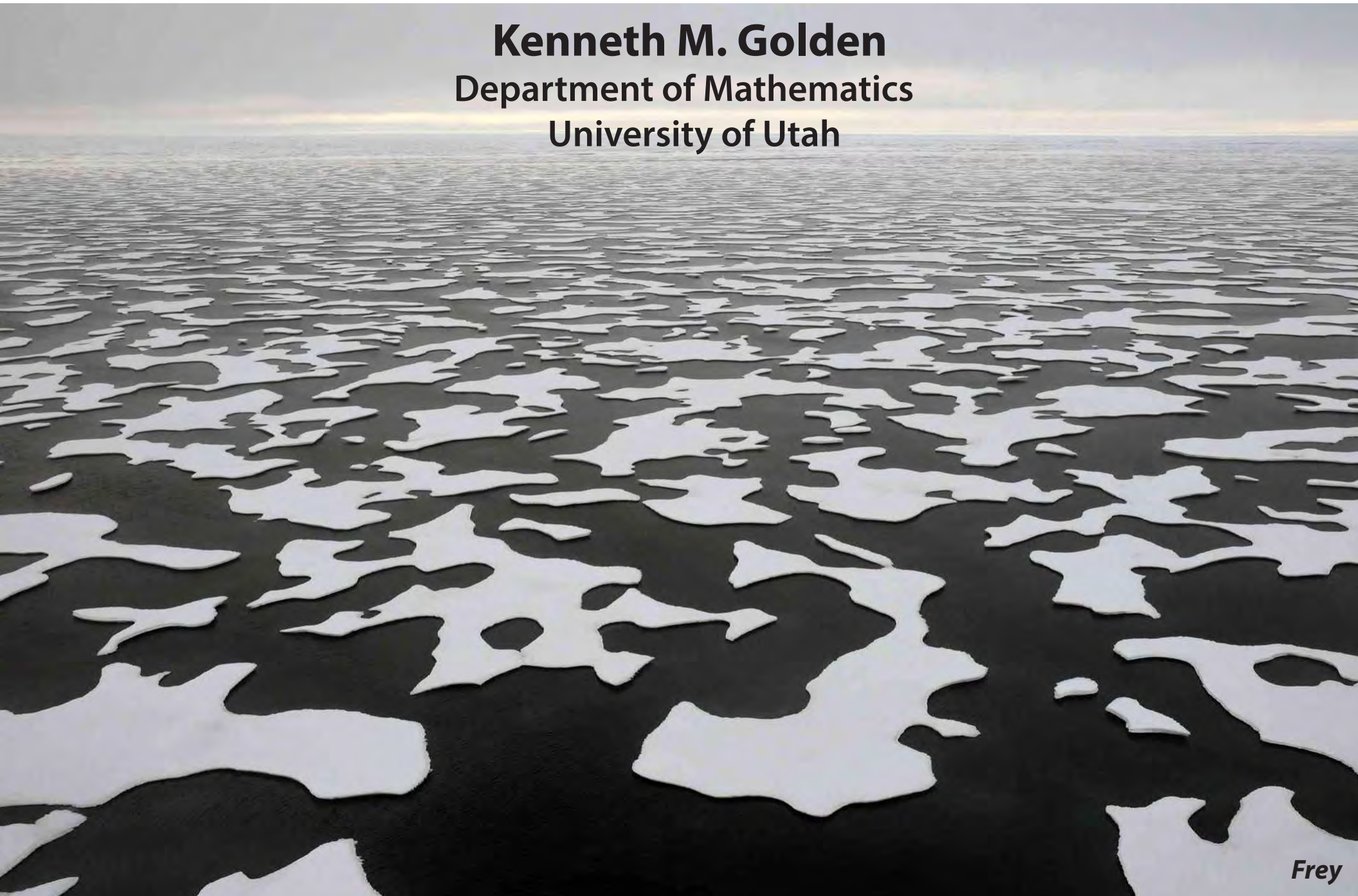
# **MODELING *the* MELT:**

## ***What math tells us about the disappearing polar ice caps***

**Kenneth M. Golden**

Department of Mathematics

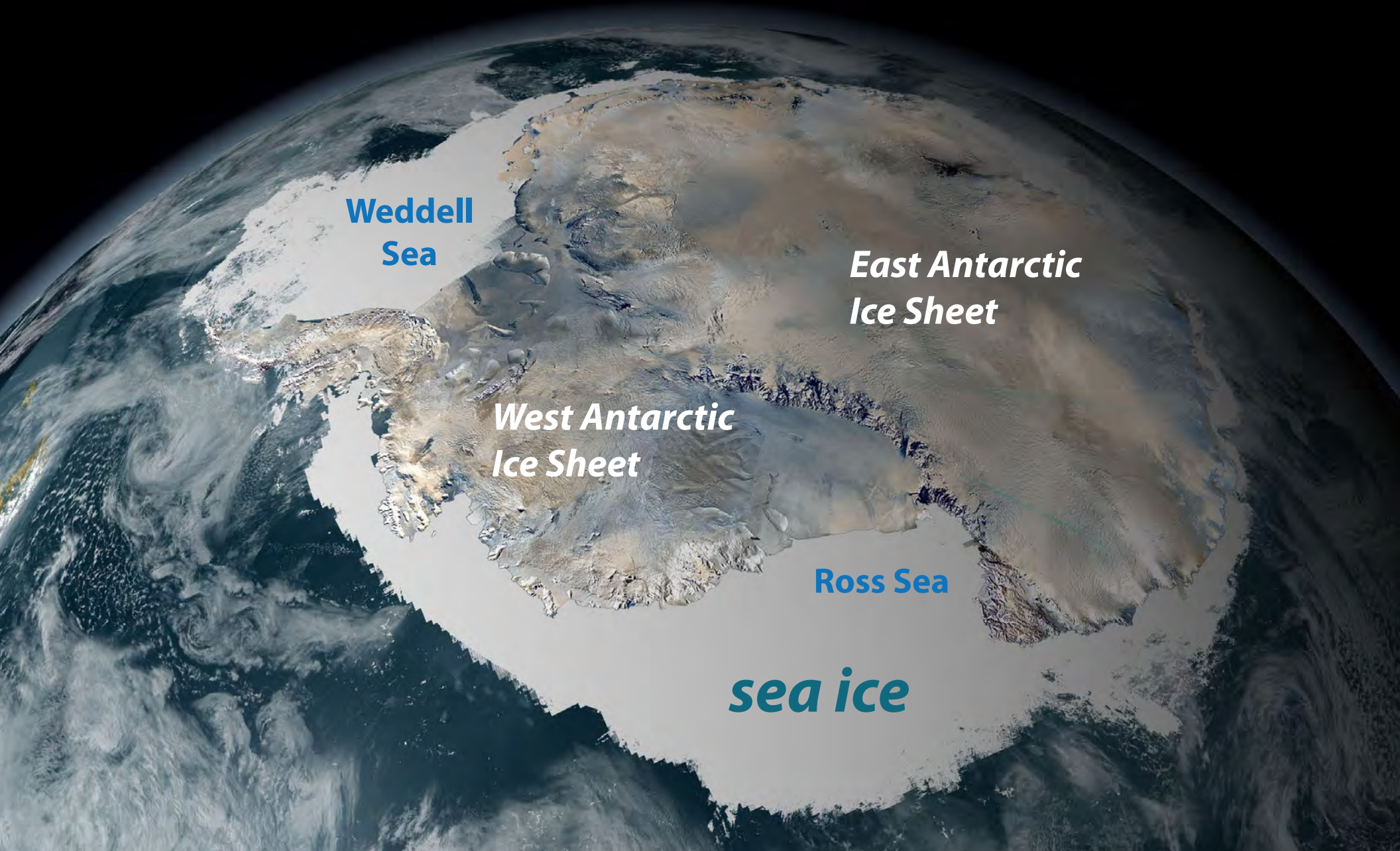
University of Utah





# ***ANTARCTICA***

southern cryosphere



**Weddell  
Sea**

***East Antarctic  
Ice Sheet***

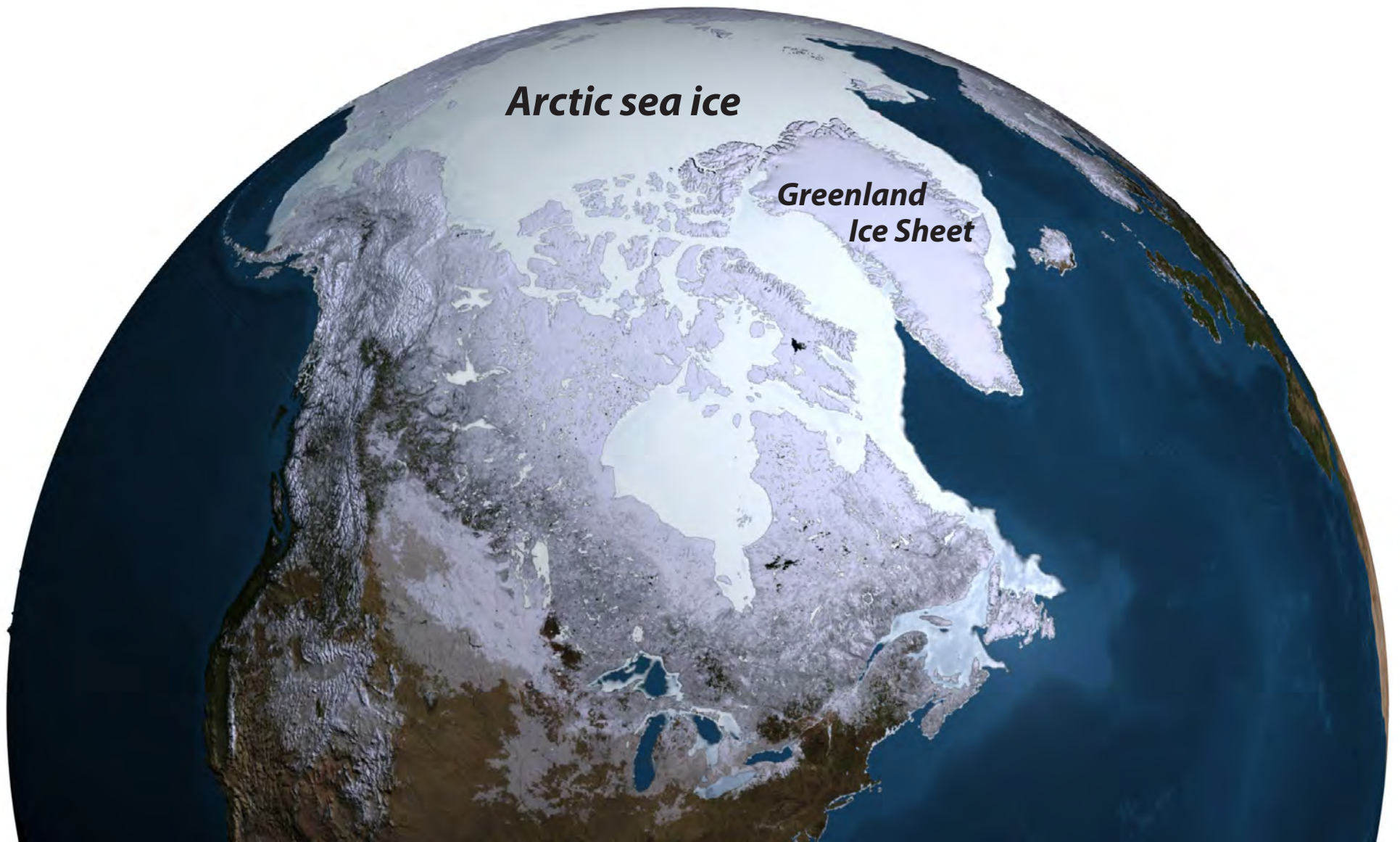
***West Antarctic  
Ice Sheet***

**Ross Sea**

***sea ice***



# **northern cryosphere**





# SEA ICE covers 7 - 10% of earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of **climate change**





# polar ice caps critical to global climate in reflecting incoming solar radiation



white snow and ice  
reflect

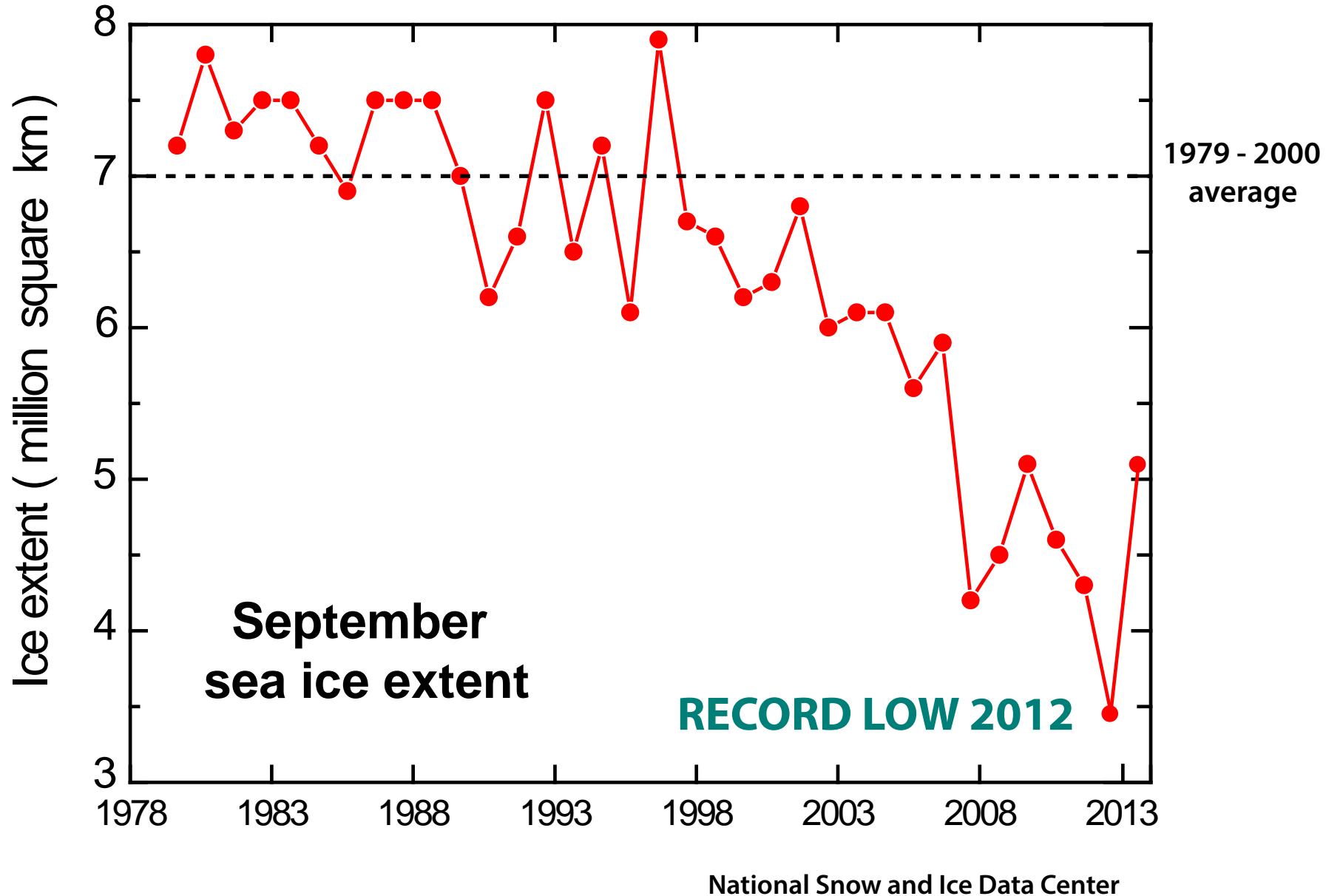


dark water and land  
absorb

$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$



# *the summer Arctic sea ice pack is melting*

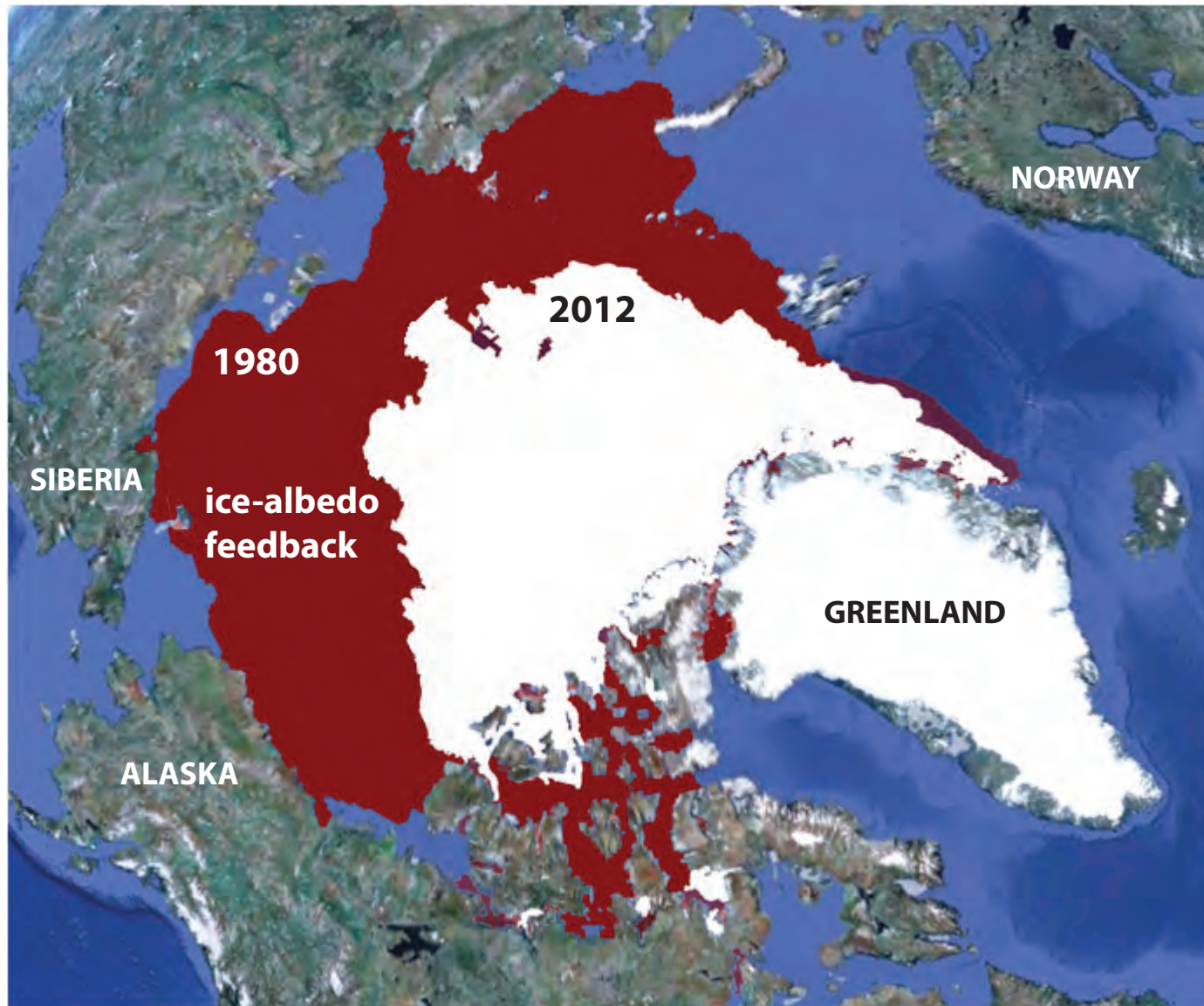




# Change in Arctic Sea Ice Extent

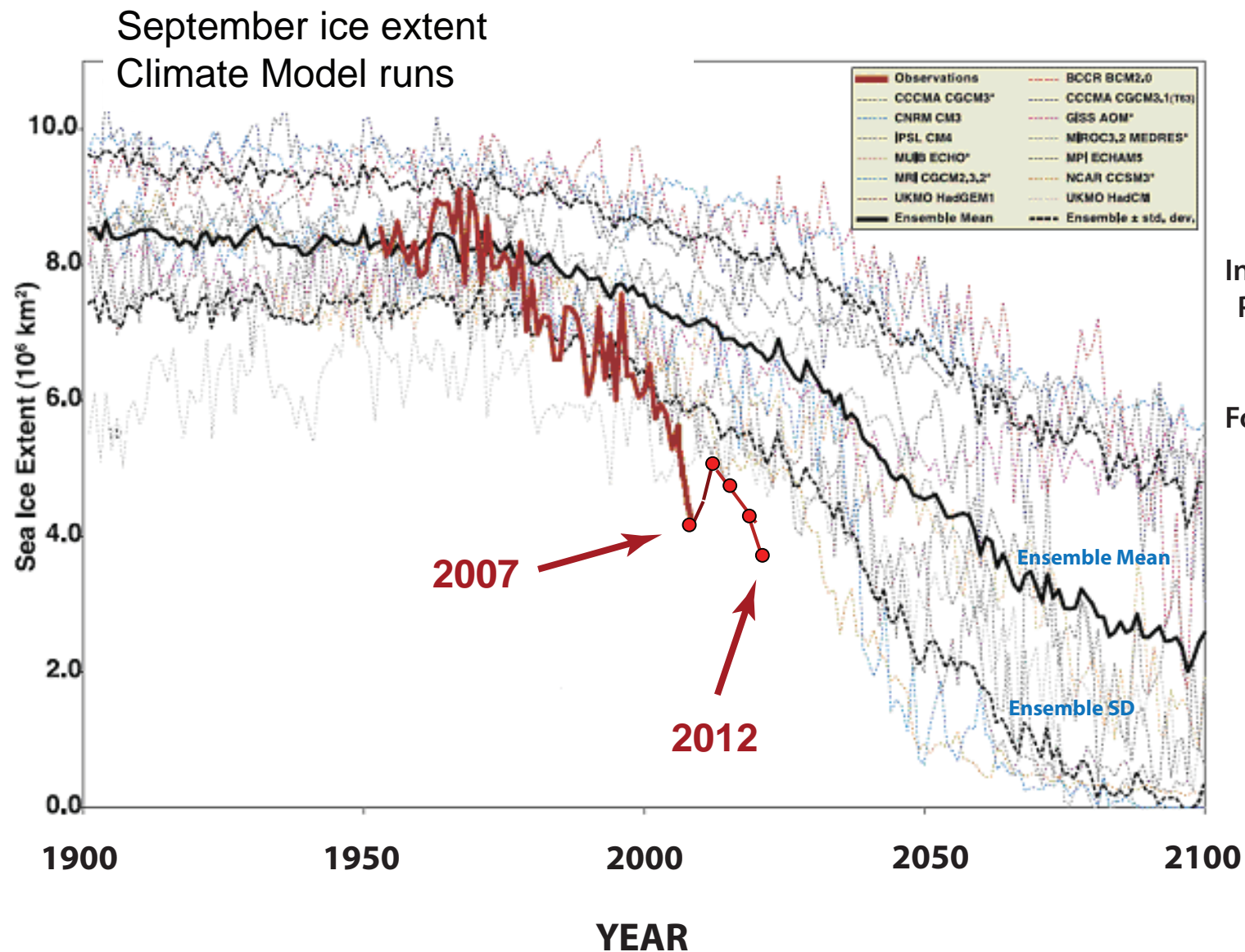
September 1980 -- 7.8 million square kilometers

September 2012 -- 3.4 million square kilometers



# Arctic sea ice decline - faster than predicted by climate models

Stroeve et al., GRL, 2007



**IPCC AR4  
Models**

Intergovernmental  
Panel on Climate  
Change (IPCC)

Fourth Assessment  
AR4, 2007

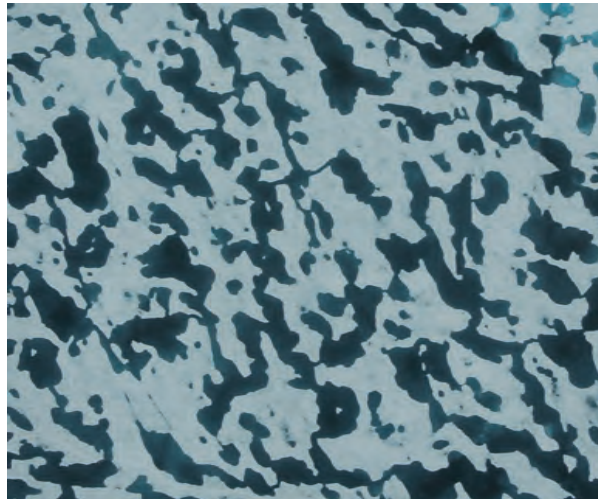


# challenge

represent sea ice more rigorously in climate models

*account for key processes*

*such as melt pond evolution*

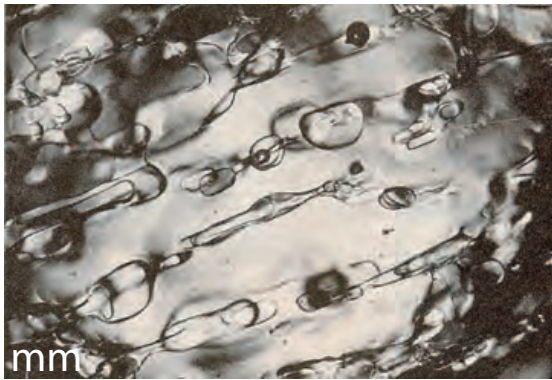


... and other sub-grid scale structures and processes

*linkage of scales*

sea ice displays *multiscale* structure over 10 orders of magnitude

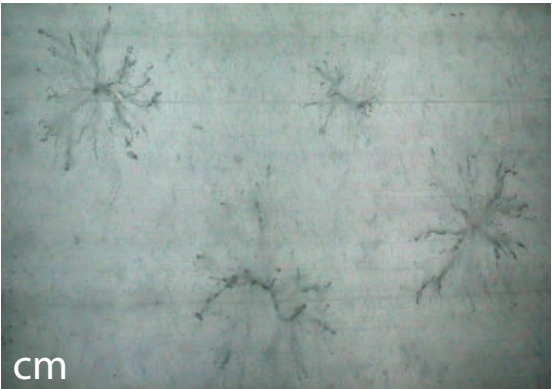
0.1 millimeter



brine inclusions



polycrystals



horizontal

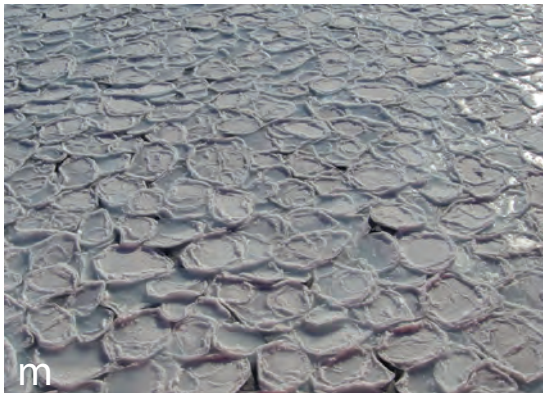


brine channels



vertical

1 meter



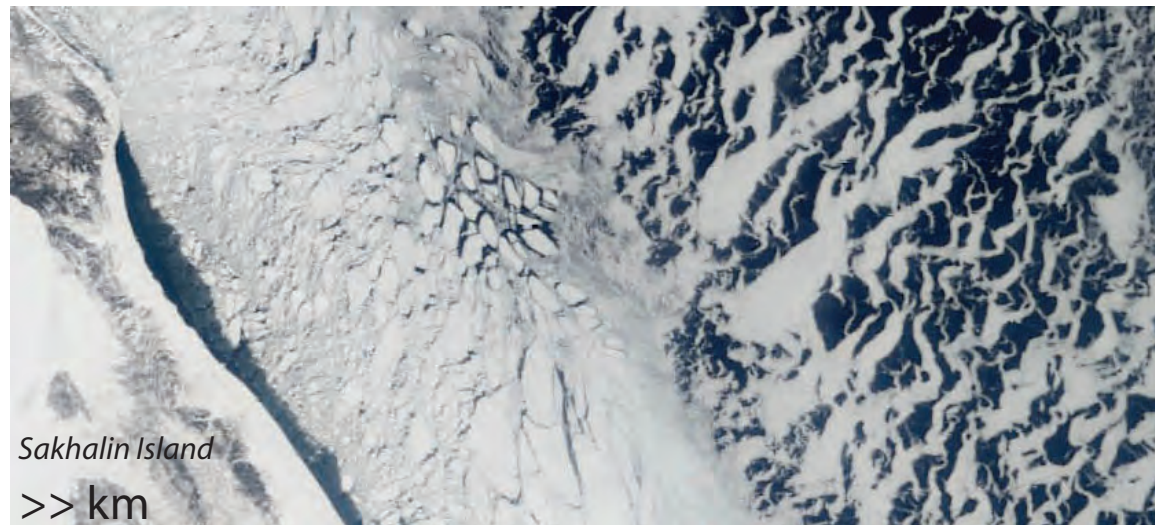
pancake ice



1 meter



100 kilometers

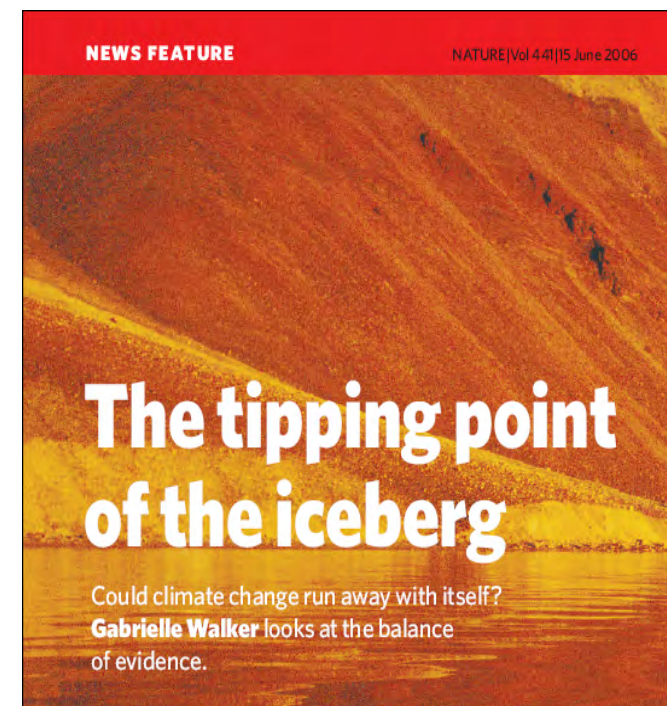
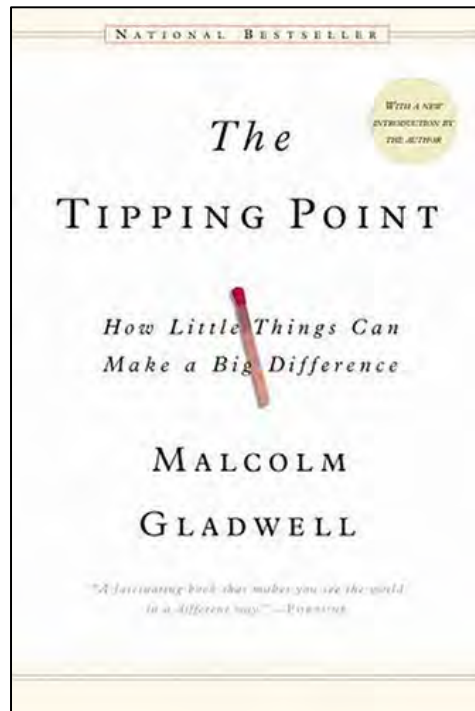


*Sakhalin Island*

>> km

# ***tipping points in the mainstream***

**climate tipping points** — September Arctic sea ice cover ??



**Melting of the Greenland ice sheet ?**

**Melting of the West Antarctic ice sheet ?**

**Permafrost and tundra loss, leading to the release of methane ?**

**Shutoff of N. Atlantic thermohaline conveyor (Gulf Stream) ? ●●●**

*Lenton, et al., PNAS 2008*

*Eisenman*

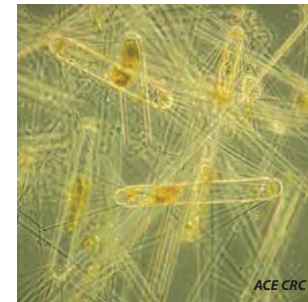


# fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

*evolution of Arctic melt ponds and sea ice albedo*



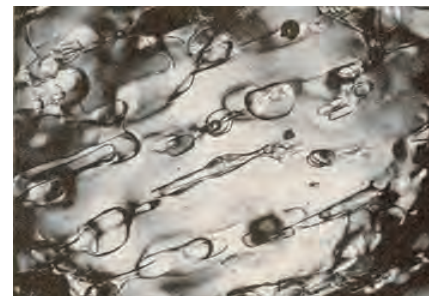
*nutrient flux for algal communities*



- *drainage of brine and melt water*
- *ocean-ice-air exchanges of heat, CO<sub>2</sub>*
- *Antarctic surface flooding and snow-ice formation*
- *evolution of salinity profiles*

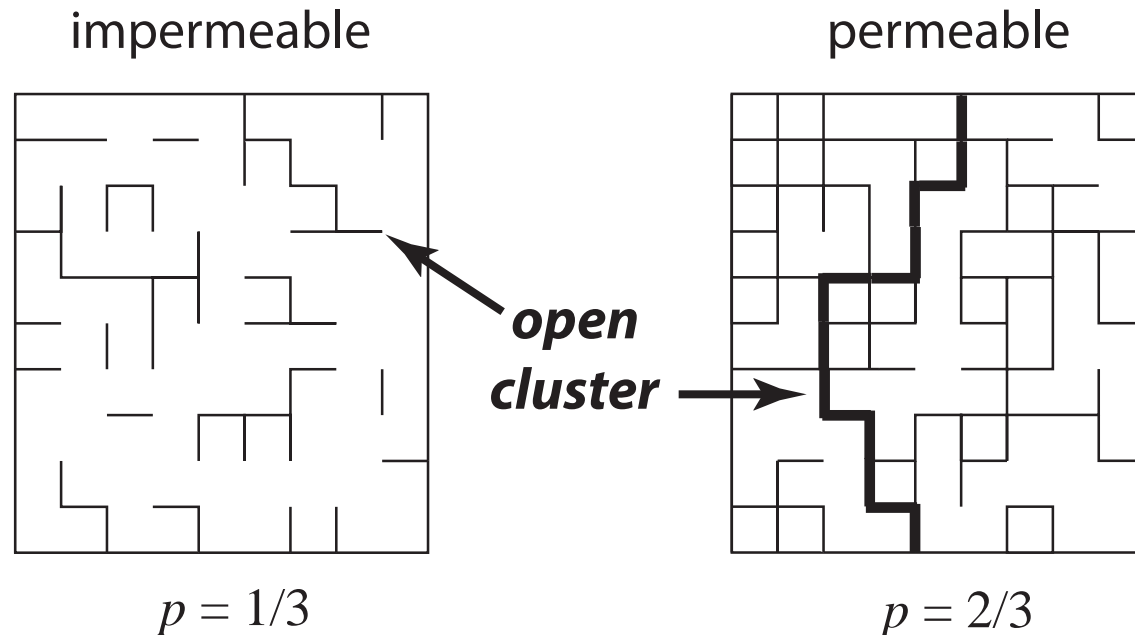


linkage of scales



# percolation theory

***probabilistic theory of connectedness***



bond  $\longrightarrow$  **open** with probability  $p$   
**closed** with probability  $1-p$

**percolation threshold**

$$p_c = 1/2 \quad \text{for } d = 2$$

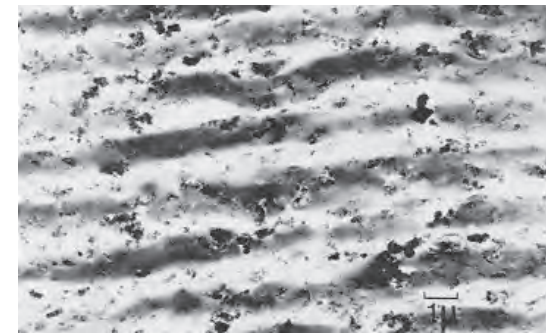
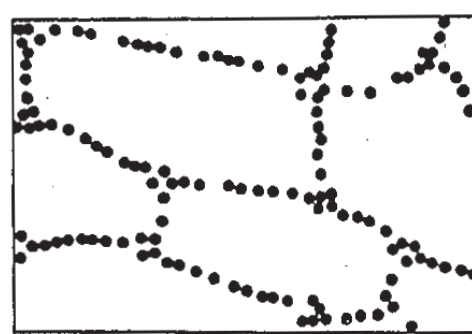
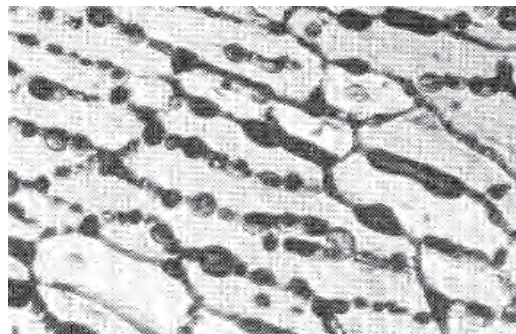
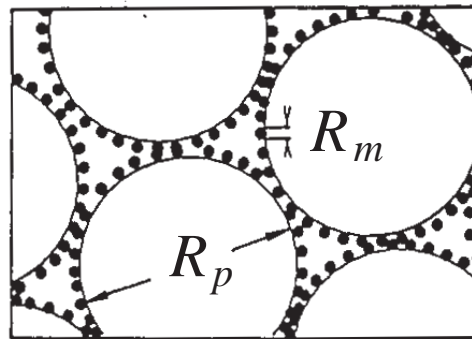
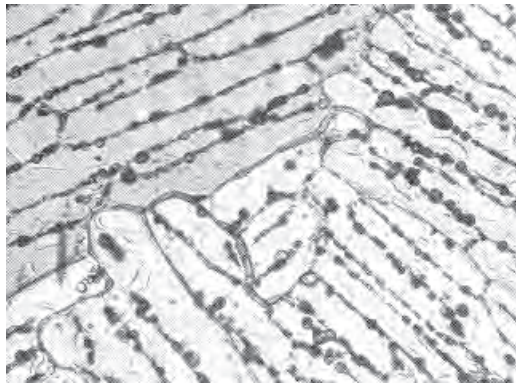
smallest  $p$  for which there is an infinite open cluster



*Continuum* percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5 \%$$

Golden, Ackley, Lytle, *Science*, 1998



sea ice

compressed  
powder

radar absorbing  
composite

**sea ice is radar absorbing**



***rigorous bounds  
percolation theory  
hierarchical model  
network model***

***field data***

**X-ray tomography for  
brine inclusions**

***unprecedented look  
at thermal evolution  
of brine phase and  
its connectivity***

micro-scale  
controls  
macro-scale  
processes

A unified approach to understanding permeability in sea ice • Solving the mystery of booming sand dunes • Entering into the "greenhouse century": A case study from Switzerland



# Remote sensing of sea ice



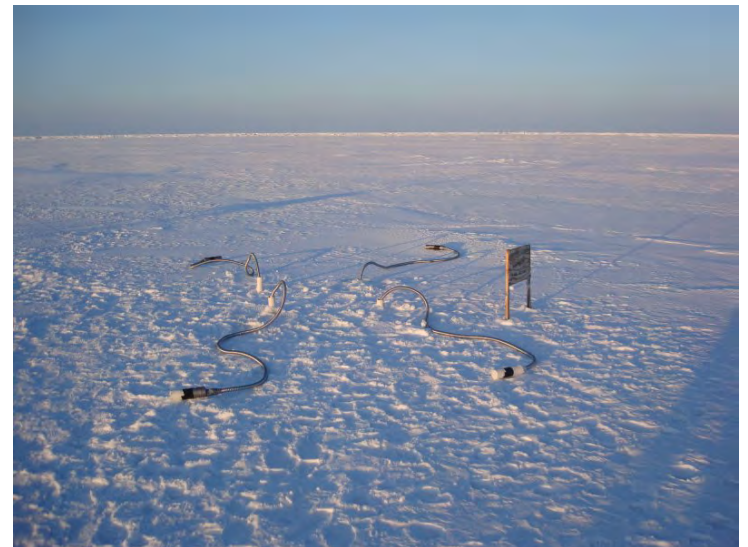
*sea ice thickness*  
*ice concentration*

## **INVERSE PROBLEM**

Recover sea ice  
properties from  
electromagnetic  
(EM) data

$$\epsilon^*$$

effective complex permittivity  
(dielectric constant, conductivity)



*brine volume fraction*  
*brine inclusion connectivity*

# Arctic and Antarctic field experiments

*develop electromagnetic methods  
of monitoring fluid transport and  
microstructural transitions*

extensive measurements of fluid and  
electrical transport properties of sea ice:

**2007    Antarctic    SIPEX**

**2010    Antarctic    McMurdo Sound**

**2011    Arctic        Barrow AK**

**2012    Arctic        Barrow AK**

**2012    Antarctic    SIPEX II**

**2013    Arctic        Barrow AK**

**2014    Arctic        Chukchi Sea**





# Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and  
the Mathematics of  
Transport in Sea Ice

page 562

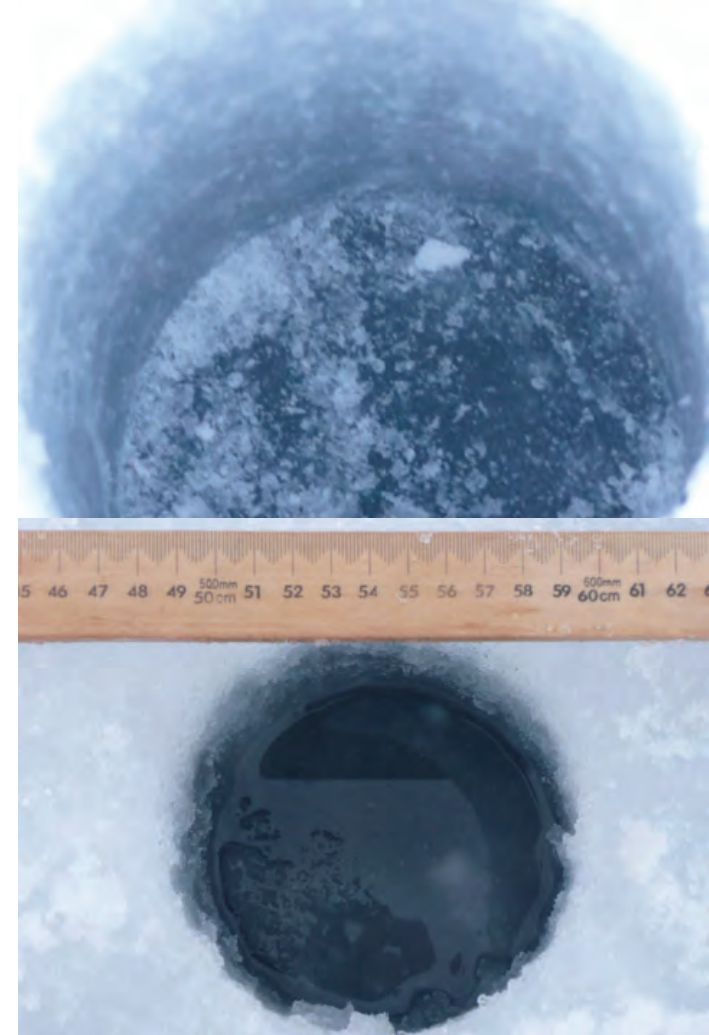
Mathematics and the  
Internet: A Source of  
Enormous Confusion  
and Great Potential

page 586



*photo by Jan Lieser*

*Real analysis in polar coordinates (see page 613)*



***measuring  
fluid permeability  
of Antarctic sea ice***

***SIPEX 2007***

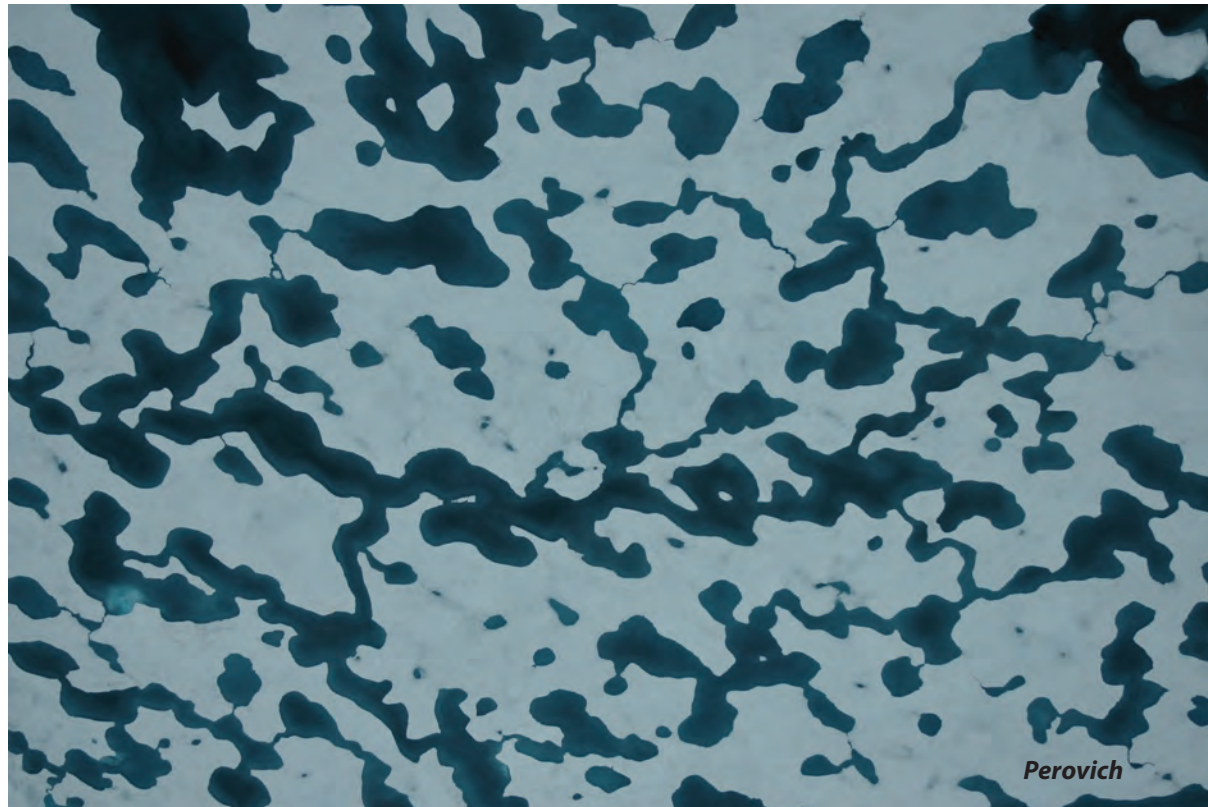
# *melt pond formation and albedo evolution:*

- *major drivers in polar climate*
- *key challenge for global climate models*

**numerical models of melt pond evolution, including topography, drainage (permeability), etc.**

Lüthje, Feltham,  
Taylor, Worster 2006  
Flocco, Feltham 2007

Skyllingstad, Paulson,  
Perovich 2009  
Flocco, Feltham,  
Hunke 2012



**Are there universal features of the evolution similar to phase transitions in statistical physics?**