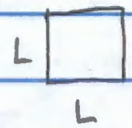
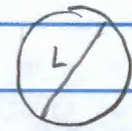
Fractal Dimension
Sierpinski Gasket

two dim



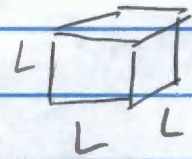


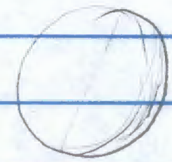
Mass = Area = L^2

Mass $= \pi \left(\frac{L}{2}\right)^2 = \left(\frac{\pi}{4}\right) L^2$

$M = 2^2 = 4$

three dim





Mass = Vol = L^3

Mass $= \frac{4}{3}\pi \left(\frac{L}{2}\right)^3 = \left(\frac{\pi}{6}\right) L^3$

$m = 2^3 = 8$

$$M = cL^d$$

d = dimension

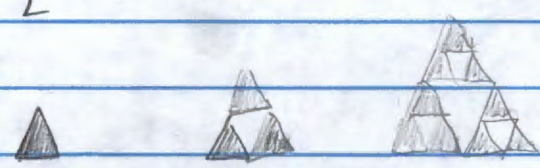
density $\rho = \frac{M}{L^2} = cL^{d-2}$

$d = 2$

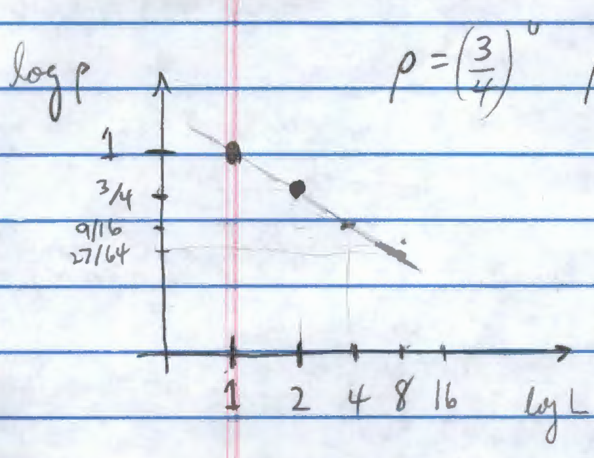
For a fractal replace d by d_f

fractal dimension

$\log \rho = \underbrace{(d_f - 2)}_{\text{slope}} \log L + \text{const}$



$L = 2^0$	$L = 2^1$	$L = 2^2$	
$M = 3^0$	$M = 3^1$	$M = 3^2$	- - - -
$\rho = \left(\frac{3}{4}\right)^0$	$\rho = \left(\frac{3}{4}\right)^1$	$\rho = \left(\frac{3}{4}\right)^2$	$\boxed{\rho \to 0 \text{ as } L \to \infty}$



$$\text{slope} = \frac{\log(3/4) - \log 1}{\log 2 - \log 1} = \frac{\log 3 - 2\log 2}{\log 2}$$

$$= \frac{\log 3}{\log 2} - 2$$

$d_f = 1.58\ldots$