Matlab Melt Pond Documentation

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1 Percolation Threshold of 2D Lattice

1.1 Purpose

The purpose of this program was to demonstrate ability and competency to produce the .5927 percolation threshold value in the 2D lattice case on Matlab. As a base case, it is valuable to understand that the program is working as desired so that we can be sure of results obtained for relevant melt pond generating functions as well as for the examination of actual melt pond images.

1.2 Method

When the program runs, it generates a large number of square matrices for a given probability of occupation, P_c, with each of the sites randomly filled with 0 or 1 according to the probability of occupation. It tests each matrix for percolation (a connected pathway of 1s from top to bottom or from one side to the other) and then calculates the proportion of matrices for a given P_c which exhibited percolation. P_c is ultimately plotted against the proportion of percolating matrices and an arctangent curve is fitted to this output. The inflection point of this graph is solved for and the according P_c value is the percolation threshold of the system.

1.3 Results

The P_c value representing the change between a non-percolating system and one exhibiting percolation as calculated by this program for a 2D lattice system is .5927. This is the widely documented value for the 2D lattice site percolation threshold (Newman and Ziff, 2000), and demonstrates the program's suitability.

2 Percolation Threshold of Real Ponds

2.1 Purpose

This program fulfills one of the main purposes of our project–to define the percolation threshold for melt ponds in the Arctic. By analyzing actual photographs of melt pond formations, we hoped to calculate the percolation threshold by calling water 1 and ice 0.

2.2 Method

Images are first downloaded and then ice is converted to 0s while water is converted to 1s. These black and white images are sliced into smaller subimages which are then tested for percolation. The images are binned by area fraction (the proportion of the image which is water) and plotted with area fraction on the x-axis and the proportion of images in a given bin exhibiting percolation. This was repeated for various subimage widths to find the relation between percolation threshold and subimage size.

2.3 Results

From the plot of percolation threshold and subimage size, it can be seen that as subimage width is increased, the calculated percolation threshold stays level at around 0.3 until subimage width = 1000, and then jumps to around 0.32. Although we were working with large images, the data seems very stable at first and then suddenly jumps to a higher average and becomes more erratic–not the logarithmic curve we were looking for. Thus we expect the percolation threshold to be around 3.2-3.3 as long as no more jumps occurred at higher subimage widths which could not be tested because despite the size of the images used, still more data is req

More image data was not found because the MIZ images used have many have blurry clouds in the way which interferes with the conversion to black and white.

This method is appropriate because of finite size scaling effects. For the 2D lattice problem, as lattice size increases and approaches correlation length, the percolation threshold approaches the theoretical value it will hold at infinite scale. Once lattice length is greater than correlation length, all percolation threshold calculations should be accurate regardless of exact lattice length (Christensen and Moloney, *Complexity and Criticality*, p. 69-78). This may be the effect that is occurring around subimage width of 1000 for the real melt ponds, very well cementing 0.32-0.33 as the correct value for the percolation threshold of the real melt ponds.

3 Percolation Threshold of Brady's Ponds

3.1 Purpose

By calculating the percolation threshold for Brady Bowen's generated pond-like structures, we hope to evaluate the accuracy to which his algorithm models real melt ponds.

3.2 Method

To calculate the percolation threshold for Brady's Ponds, the program first generates a random surface using a program created by Brady Bowen with optimum inputs (0.2, 0.3; a two dimensional vector of red noise constants). After generation of the 3D surface, minimum and maximum points of the surface through the space are calculated, and a plane is raised through the surface at small increments. At each step, the portion of the surface intersected by the plane is checked for top-bottom or side-side percolation and the area fraction is recorded. This is repeated for 150 surfaces and all area and as before, ponds are binned according to their corresponding area fractions. Ultimately, a produced graph outlines the change in proportion of percolating lattices as area fraction increases.

3.3 Results

The program fitted an arctan curve to the produced data and calculated the percolation threshold as .4856, which is close to .5. This hints at a symmetrical property of Brady's ponds (if water was switched for ice, would you get similar results?). Also interesting to note because the bond lattice percolation threshold is .5, which Brady's ponds may be more similar to than the site lattice percolation problem. Although near the suspected percolation threshold for real melt ponds, the generated percolation threshold was not as close as the PDE model. Based solely on this element of percolating systems, Brady's ponds represent a weaker model for real melt pond behavior than PDE.

4 Percolation Threshold of PDE Ponds

4.1 Purpose

By calculating the percolation threshold for the PDE model's generated pond-like structures, we hope to evaluate the accuracy to which this algorithm models real melt ponds.

4.2 Method

To calculate the percolation threshold for the PDE ponds, 3D surfaces are first loaded, then a plane at a certain depth from the top of the surface is singled out for calculation–this plane is treated like the surface of the melt ponds in the Arctic. The program loops through multiple surfaces (40), generating a plane from each, and then bins the data by area fraction, calculating the proportion of percolating planes for each bin. This data is graphed and the inflection point of the fitted arctangent curve is solved for to estimate the percolation threshold at optimal 0.2, 0.3 input values.

This calculation was done for multiple depth criterion to gauge if the depth of water at which you identify surface water as a melt pond has an effect on the percolation threshold. The resulting curve of percolation thresholds versus depth was logarithmic–as the depth criterion increased, so did the percolation threshold. This increase, however, slowed as depth increased, so that the percolation threshold seemed to approach an upper bound.

To identify this upper bound, the relationship between area fraction and depth was evaluated for a single PDE surface. As the depth criterion was increased, the area fraction decreased–at first rapidly, but then more slowly in a linear fashion. This linear relationship began around a depth criterion of 2 centimeters; the logarithmic curve of the first graph identifying the relationship between depth criterion chosen and resulting percolation threshold also seemed to reach an upper limit around 2 centimeters. As the depth criterion varies in the PDE model, the observed changes have a physical basis in melt rates and Darcy's law.

The depth at which the percolation threshold could be reliably calculated for the PDE surfaces only went up to 3 centimeters, because beyond that, there was not enough water on the resulting plane to have any instances of percolation. This is because the generated ponds are parabolic in nature, so as the depth at which water is considered a melt pond increases, the resulting melt ponds diminish in size. The outputted graphs for higher depth criterion consisted of a line of data where the proportion of percolating graphs per bin equals zero, and no arctangent curve could be fitted to discern the percolation threshold.

4.3 Results

At a depth criterion of .02 meters, the PDE model's percolation threshold was calculated as .4152. This is highly similar to the expected value of the real melt ponds and confirms this model's appropriateness in simulating Arctic Melt Pond behavior.

The recorded sensitivity of the PDE model to the depth criterion is also highly interesting and valuable because it reinforces the applicability of satellite imagery for investigating the percolation threshold of melt ponds. Satellite photos usually capture only ponds which are fairly deep (more than 2 centimeters). As the relationship between depth and percolation threshold seen in the PDE model shows, however, any depth beyond 2 centimeters should produce the correct percolation threshold.