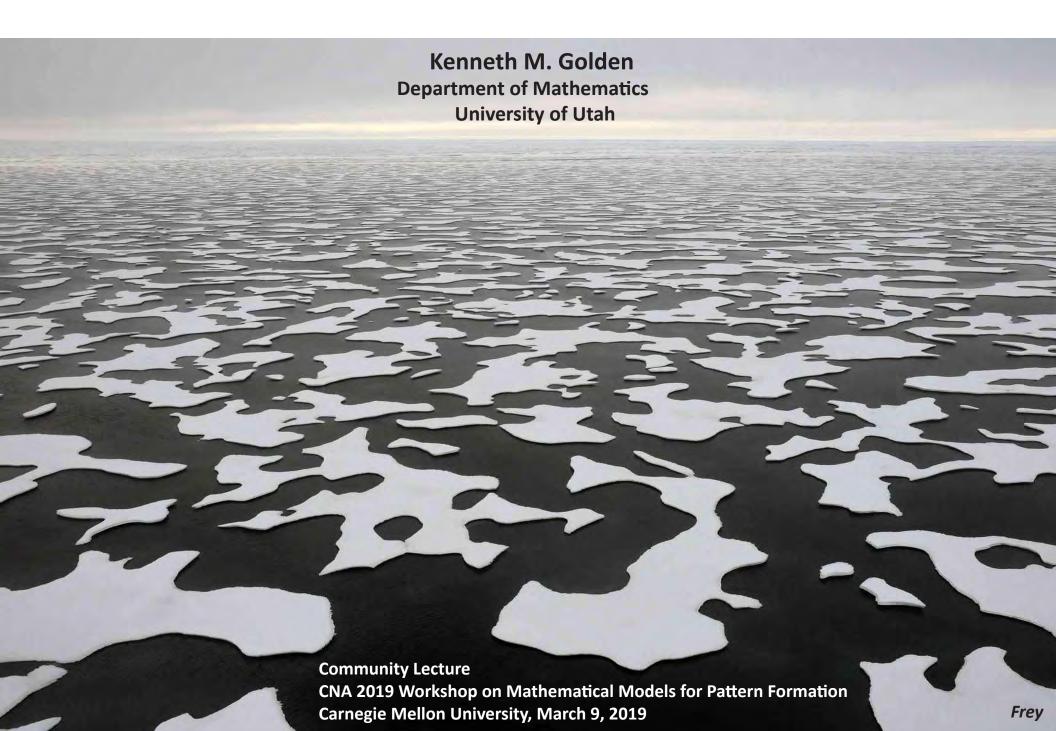
Pattern Formation in Melting Arctic Sea Ice

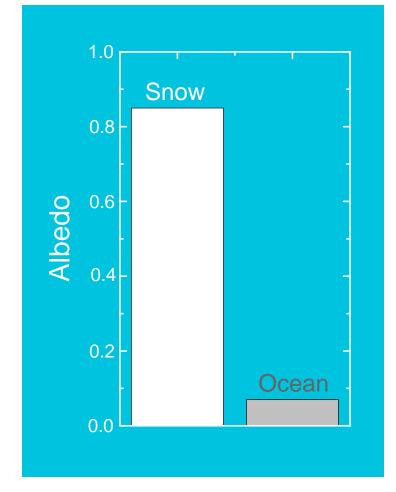


SEA ICE covers ~12% of Earth's ocean surface boundary between ocean and atmosphere mediates exchange of heat, gases, momentum global ocean circulation indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

white snow and ice reflect



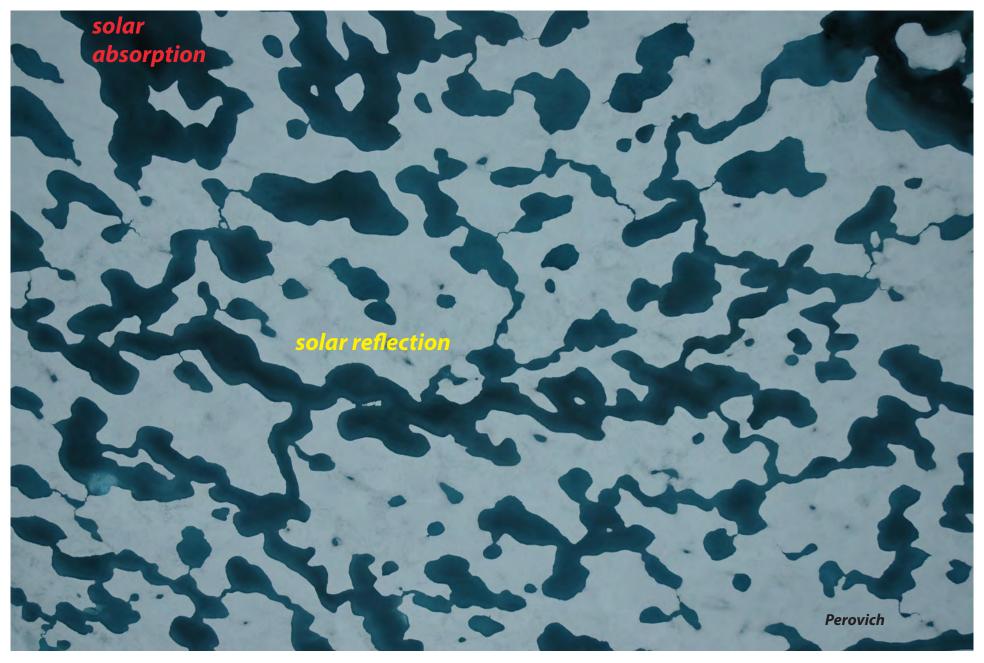




dark water and land absorb

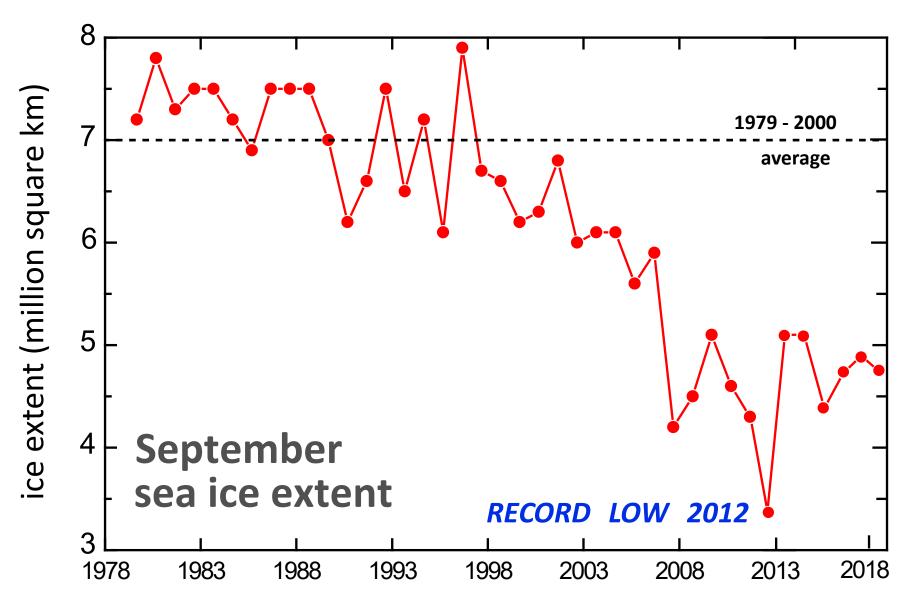
albedo
$$\alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

Arctic melt ponds



melt pond pattern formation and albedo evolution -- major drivers in polar climate key challenge for global climate models

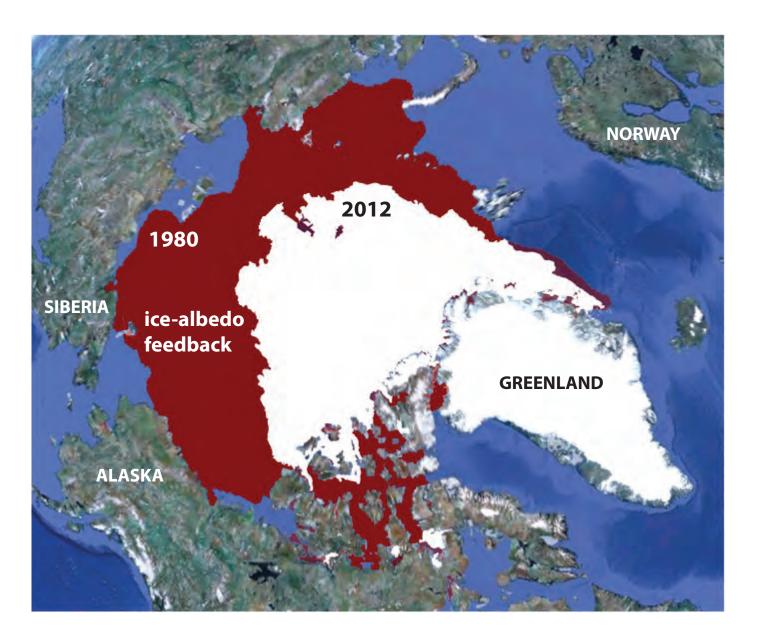
the summer Arctic sea ice pack is melting



Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers

September 2012 -- 3.4 million square kilometers



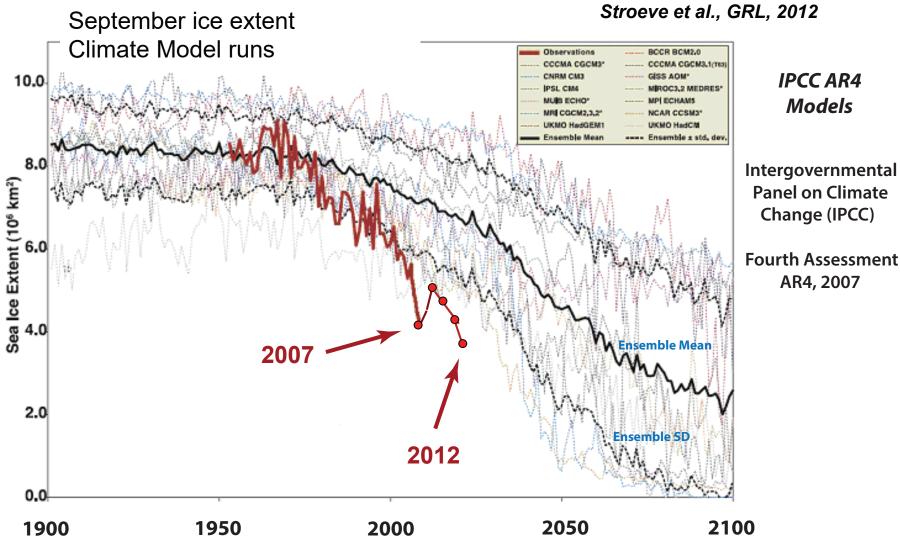


recent losses in comparison to the United States



Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007 Stroeve et al., GRL, 2012

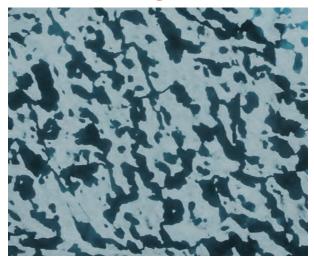


challenge

represent sea ice more realistically in climate models account for key processes

such as melt pond evolution

How do patterns of dark and light evolve?



Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, JGR Oceans 2012

For simulations with ponds September ice volume is nearly 40% lower.

... and other sub-grid scale structures and processes

linkage of scales

What is this talk about?

Tour the fascinating patterns in sea ice structure and how they form on scales from millimeters to tens of kilometers.

How do these patterns influence processes on larger scales?

[Use theories of multiscale composite materials and statistical physics to homogenize - compute effective behavior and improve climate models.]

1. Patterns in sea ice structure

mm brine microstructure fluid flow and cm polycrystalline microstructure EM properties m/km floe structure -- breakage patterns; wave propagation m/km advective patterns -- enhanced diffusion

2. Fractal geometry of melt pond patterns observations, models, under-ice light field and algal blooms

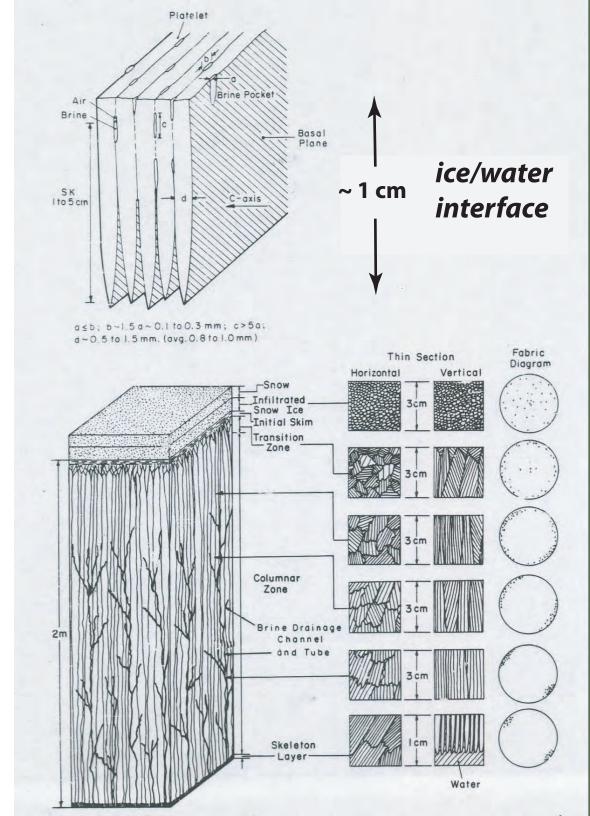
cross-sections of sea ice structure

$$T_{freeze} = -1.8$$
° C

crystallographic texture



vertical thin section



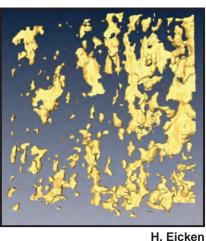
Multiscale Patterns of Sea Ice Structure

sea ice microstructure

brine inclusions



Weeks & Assur 1969



Golden et al. GRL 2007

polycrystals

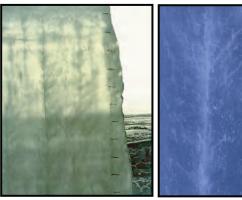






Gully et al. Proc. Roy. Soc. A 2015

brine channels



D. Cole

K. Golden

millimeters

centimeters

sea ice mesostructure

sea ice macrostructure

Arctic melt ponds



Antarctic pressure ridges

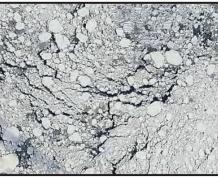


K. Golden



sea ice floes

J. Weller

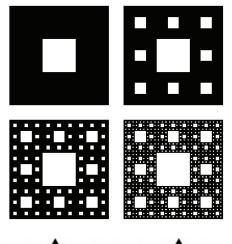


sea ice pack

NASA

meters

kilometers

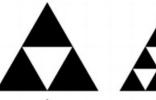




self-similar structure

non-integer dimension

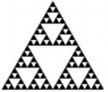


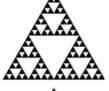














$$D = \frac{\log 3}{\log 2} = 1.585...$$





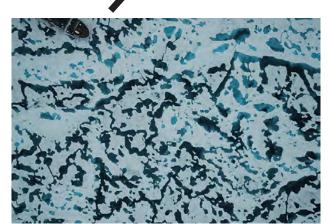


How do scales interact in the sea ice system?



basin scale grid scale albedo

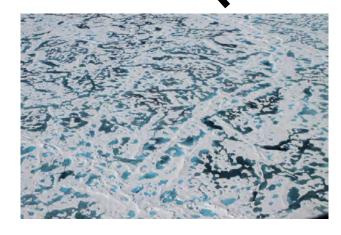
km scale melt ponds



Linking



Linking Scales



km scale melt ponds

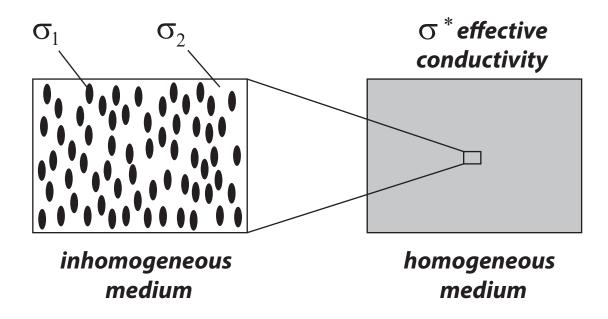
Scales



meter scale snow topography

mm scale brine inclusions

HOMOGENIZATION - Linking Scales in Composites



find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873: effective conductivity of a dilute suspension of spheres Einstein 1906: effective viscosity of a dilute suspension of rigid spheres in a fluid

Wiener 1912: arithmetic and harmonic mean bounds on effective conductivity Hashin and Shtrikman 1962: variational bounds on effective conductivity

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

sea ice microphysics

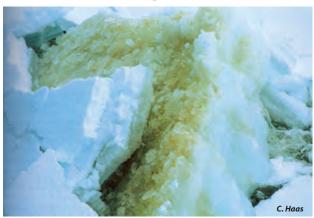
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

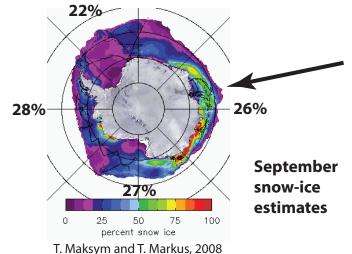


nutrient flux for algal communities





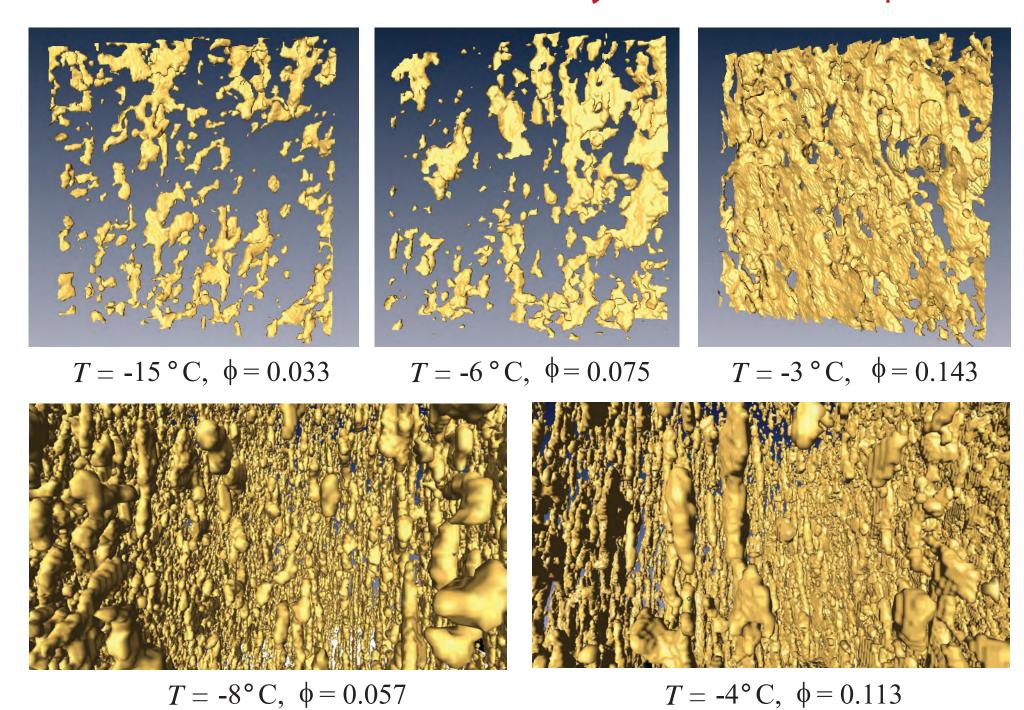




Antarctic surface flooding and snow-ice formation

- evolution of salinity profiles
- ocean-ice-air exchanges of heat, CO₂

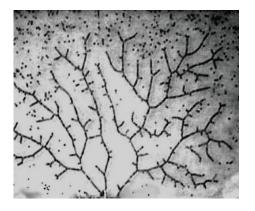
brine volume fraction and *connectivity* increase with temperature



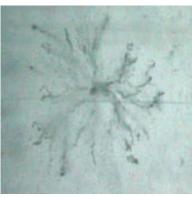
X-ray tomography for brine in sea iceGolden et al., Geophysical Research Letters, 2007



fractal microstructures



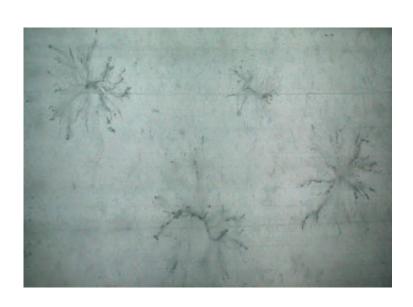
electrorheological fluid with metal spheres



brine channel in sea ice



diffusion limited aggregation



brine channels





fluid permeability k of a porous medium

porous concrete

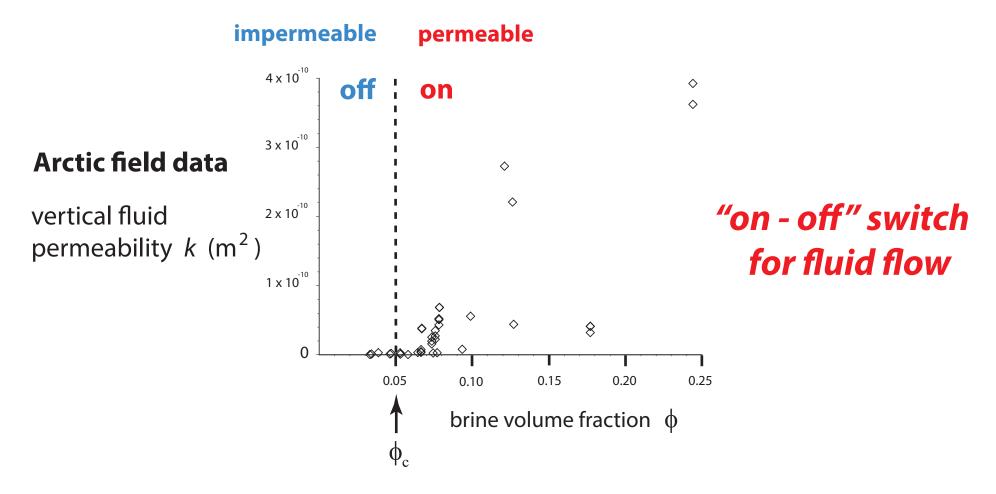


how much water gets through the sample per unit time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Critical behavior of fluid transport in sea ice



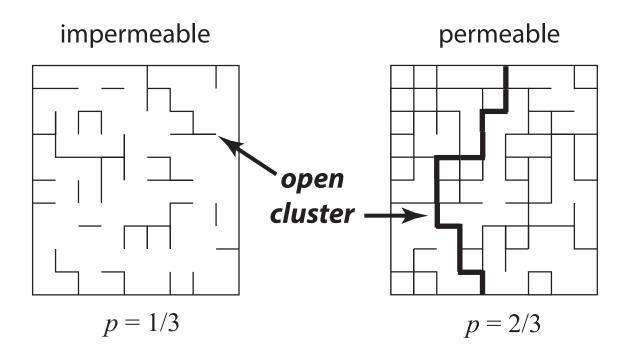
critical brine volume fraction
$$\phi_c \approx 5\%$$
 \longrightarrow $T_c \approx -5^{\circ} \text{C}$, $S \approx 5 \text{ ppt}$

RULE OF FIVES

Golden, Ackley, Lytle Science 1998 Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007 Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

percolation theory

probabilistic theory of connectedness



bond
$$\longrightarrow$$
 open with probability p closed with probability 1-p

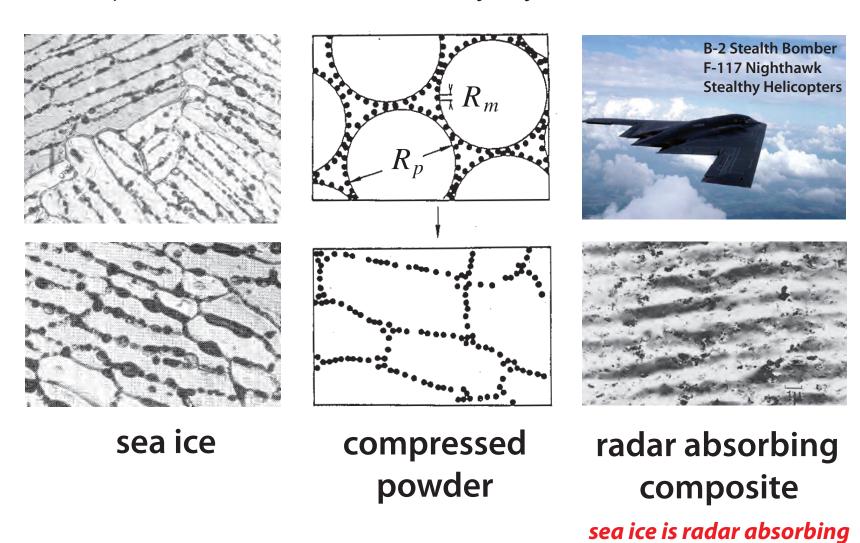
percolation threshold

$$p_c = 1/2$$
 for $d = 2$

smallest p for which there is an infinite open cluster

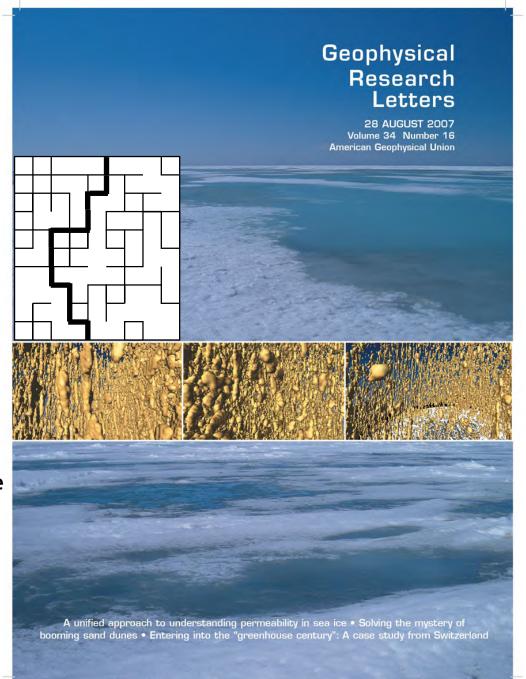
Continuum percolation model for stealthy materials applied to sea ice microstructure explains Rule of Fives and Antarctic data on ice production and algal growth

 $\phi_c \approx 5 \%$ Golden, Ackley, Lytle, *Science*, 1998



Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



percolation theory

$$k(\phi) = k_0 (\phi - 0.05)^2$$
 critical exponent
$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

hierarchical model network model rigorous bounds

agree closely with field data

X-ray tomography for brine inclusions

unprecedented look at thermal evolution of brine phase and its connectivity

confirms rule of fives

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

controls

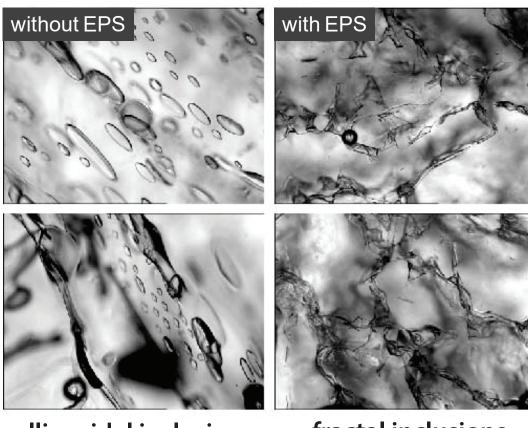
micro-scale

macro-scale

processes

Sea ice algae secrete extracellular polymeric substances (EPS)

EPS changes brine microstructure



ellipsoidal inclusions

fractal inclusions

numerical model bounds on fluid permeability

Krembs, Eicken, Deming PNAS 2011

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden *Multiscale Modeling and Simulation*, 2018

How does the biology affect the physics?

Remote sensing of sea ice











sea ice thickness ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

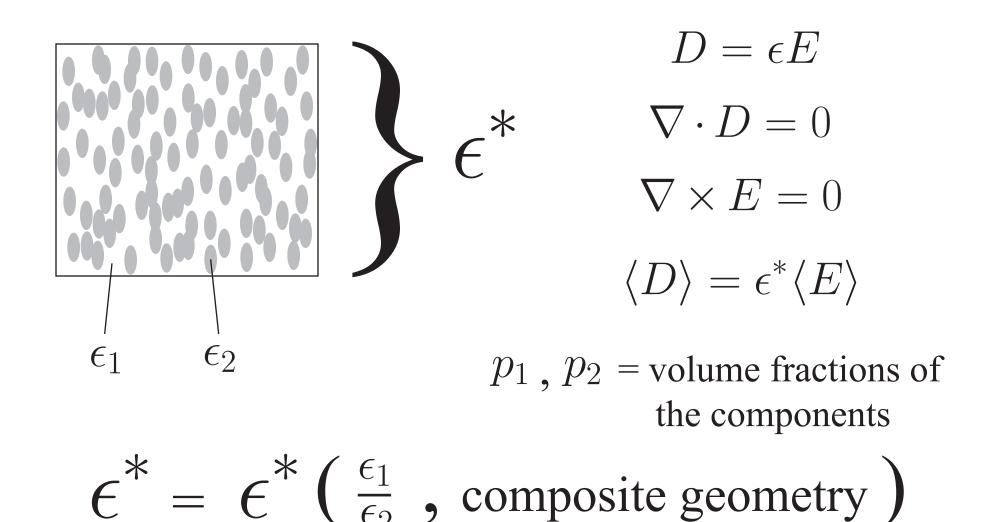
٤*

effective complex permittivity (dielectric constant, conductivity)



brine volume fraction brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

Analytic Continuation Method

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)

Stieltjes integral representation for homogenized parameter

separates geometry from parameters

$$F(s)=1-\frac{\epsilon^*}{\epsilon_2}=\int_0^1\frac{d\mu(z)}{s-z} \qquad \qquad s=\frac{1}{1-\epsilon_1/\epsilon_2}$$
 material parameters

• spectral measure of self adjoint operator
$$\Gamma \chi$$

$$\mu$$
 - • mass = p_1

• higher moments depend on *n*-point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla \cdot$$

 $\chi = {\rm characteristic} \, {\rm function}$ of the brine phase

$$E = s (s + \Gamma \chi)^{-1} e_k$$

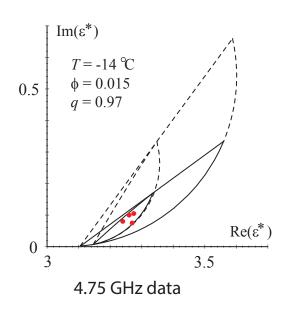
$\Gamma \chi$: microscale \rightarrow macroscale

$\Gamma \chi$ links scales

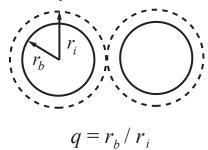
Golden and Papanicolaou, Comm. Math. Phys. 1983

forward and inverse bounds on the complex permittivity of sea ice

forward bounds



matrix particle (Bruno 91)

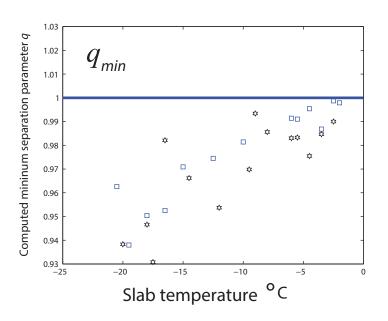


Golden JGR 95, IMA 97

inverse bounds and recovery of brine porosity

Cherkaev and Golden, Waves in Random Media 1998 Gully, Backstrom, Eicken, Golden Physica B, 2007

inverse bounds



inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ϵ^*

rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden Proc. Roy. Soc. A, 2012

SEA ICE

HUMAN BONE

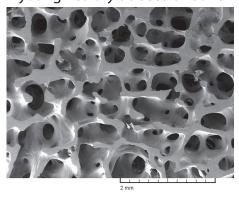


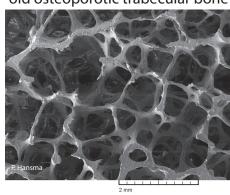


spectral characterization of porous microstructures in human bone

young healthy trabecular bone

old osteoporotic trabecular bone





reconstruct spectral measures from complex permittivity data

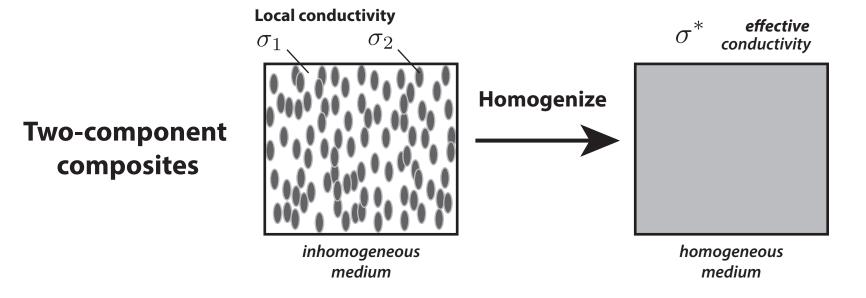
use regularized inversion scheme

apply spectral measure analysis of brine connectivity and spectral inversion to electromagnetic monitoring of osteoporosis

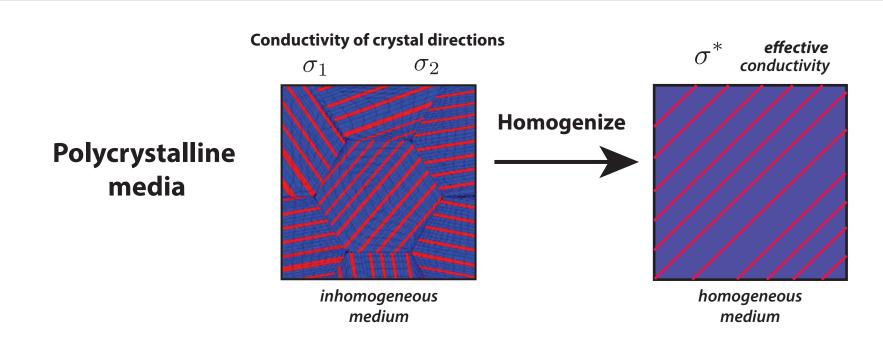
Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

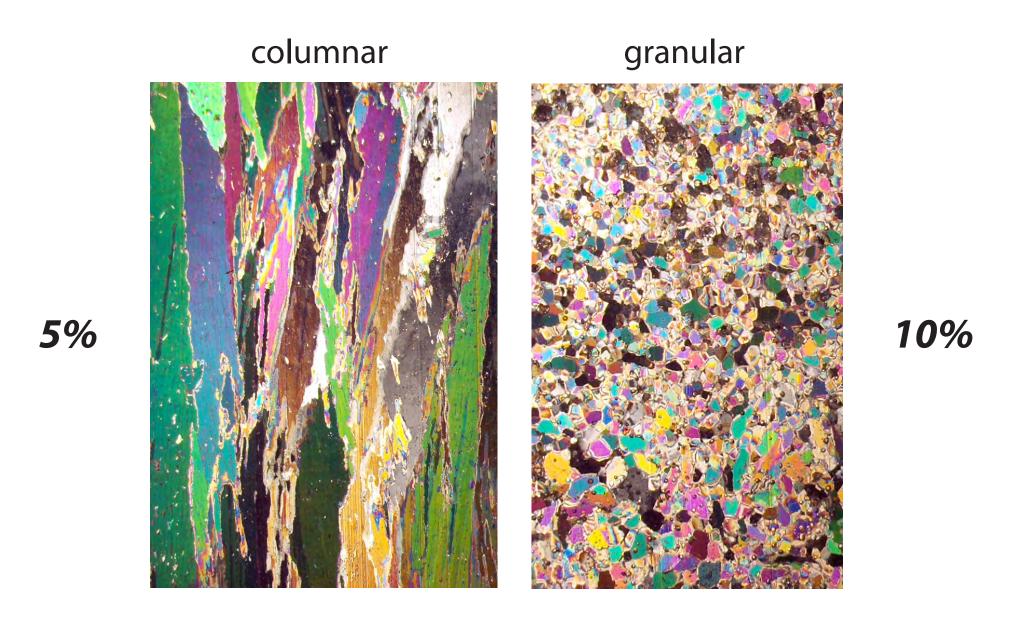
Homogenization for composite materials



Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium



higher threshold for fluid flow in Antarctic granular sea ice



Golden, Sampson, Gully, Lubbers, Tison 2019

Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin, Elena Cherkaev, Ken Golden

Stieltjes integral representation for effective complex permittivity

Milton (1981, 2002), Barabash and Stroud (1999), ...

- Forward and inverse bounds
- Applied to sea ice using two-scale homogenization
- Inverse bounds give method for distinguishing ice types using remote sensing techniques





Proc. Roy. Soc. A 8 Feb 2015

ISSN 1364-5021 | Volume 471 | Issue 2174 | 8 February 2015

PROCEEDINGS A



An invited review commemorating 350 years of scientific publishing at the Royal Society A method to distinguish between different types of sea ice using remote sensing techniques A computer model to determine how a human should walk so as to expend the least energy



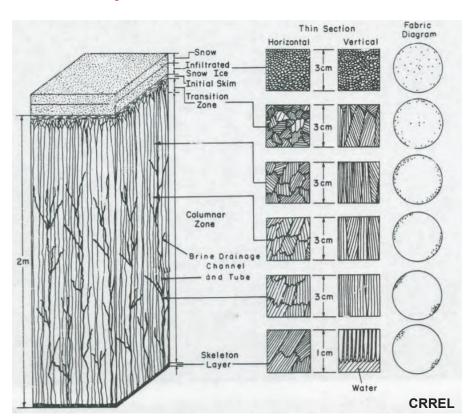
Rigorous bounds on the complex permittivity tensor of sea ice with polycrystalline anisotropy within the horizontal plane

McKenzie McLean, Elena Cherkaev, Ken Golden 2019

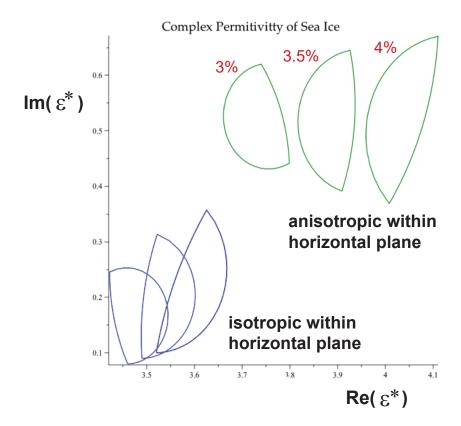
motivated by

Weeks and Gow, 1979: c-axis alignment in Arctic fast ice off Barrow Golden and Ackley, 1981: radar propagation model in aligned sea ice

input: orientation statistics



output: bounds



sea ice floe structure



sea ice formation











pancake ice





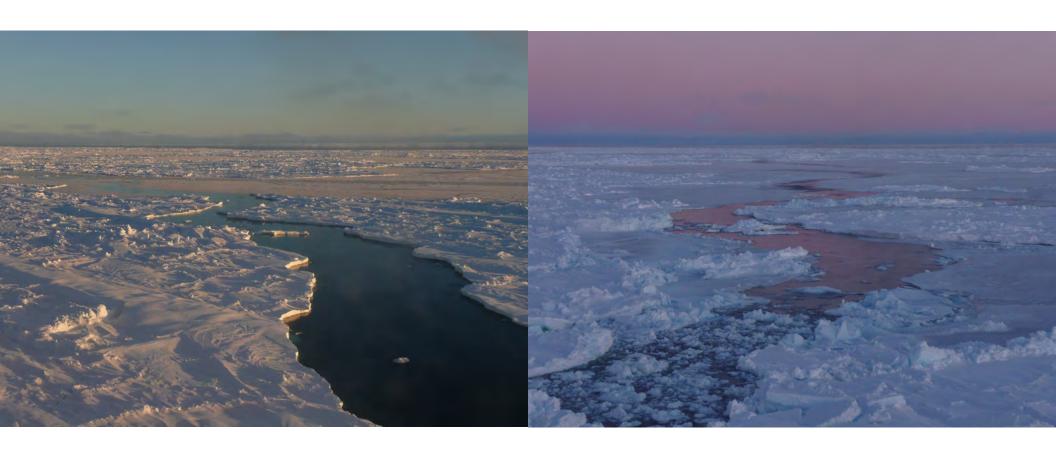


sea ice dynamics plate tectonics on a fast time scale





leads

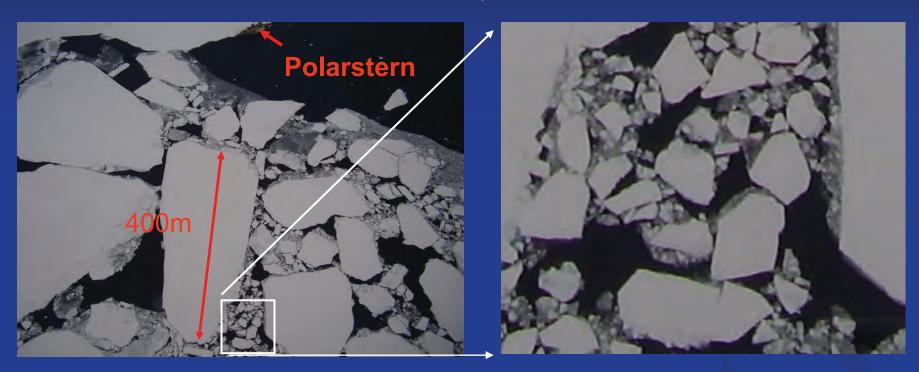


heat flows directly from ocean to atmosphere

The sea ice pack has fractal structure.

Self-similarity of sea ice floes

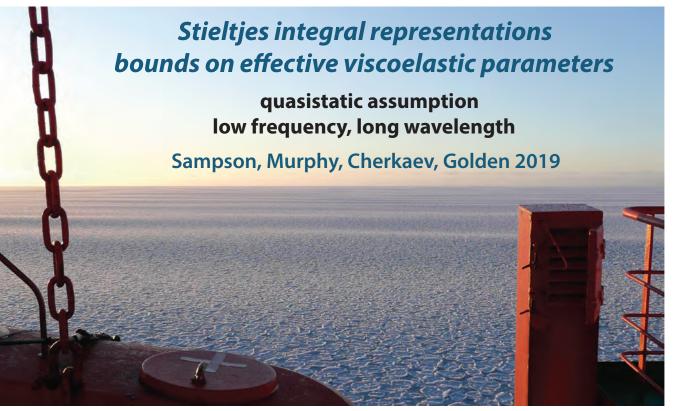
Weddell Sea, Antarctica

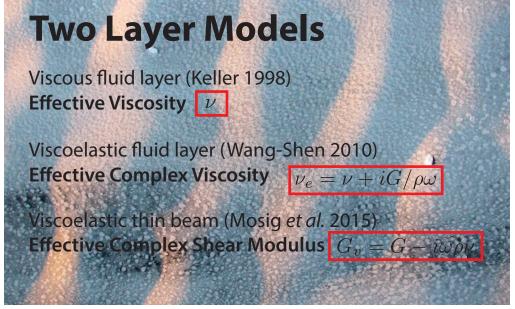


fractal dimensions of Okhotsk Sea ice pack smaller scales D~1.2, larger scales D~1.9

Toyota, et al. Geophys. Res. Lett. 2006 Rothrock and Thorndike, J. Geophys. Res. 1984

wave propagation in the marginal ice zone



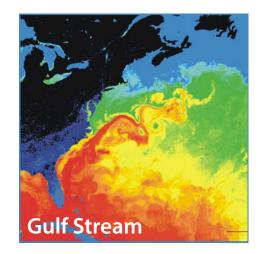


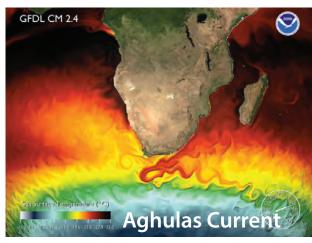




advection enhanced diffusion effective diffusivity

sea ice floes diffusing in ocean currents diffusion of pollutants in atmosphere salt and heat transport in ocean heat transport in sea ice with convection





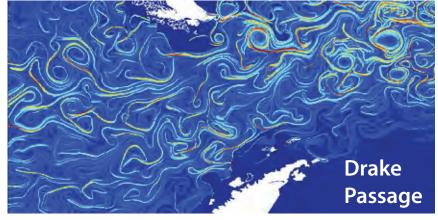
advection diffusion equation with a velocity field $ec{u}$

 κ^* effective diffusivity

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

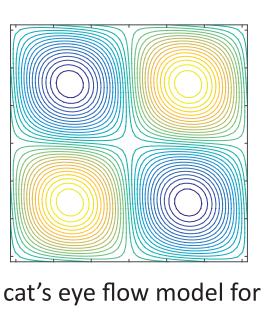
Murphy, Cherkaev, Xin, Zhu, Golden, Ann. Math. Sci. Appl. 2017 Murphy, Cherkaev, Zhu, Xin, Golden, J. Math Phys. 2019



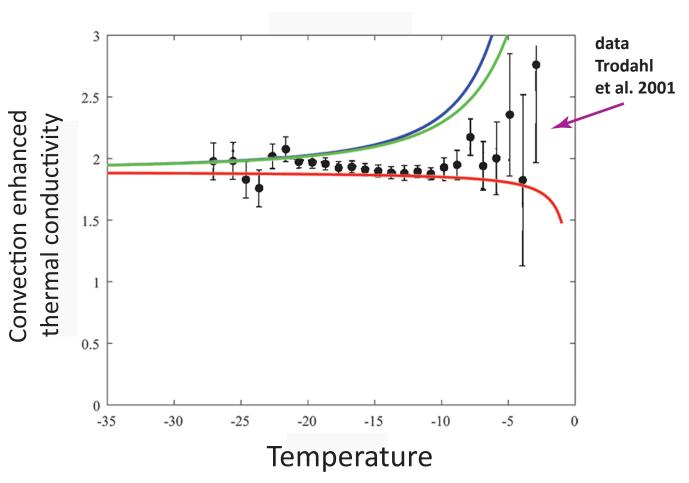


Rigorous bounds on convection enhanced thermal conductivity of sea ice

Kraitzman, Hardenbrook, Dinh, Murphy, Zhu, Cherkaev, Golden 2019



brine convection cells



rigorous Padé bounds from Stieltjes integral + analytical calculations of moments of measure

Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007 Antarctic SIPEX

2010 Antarctic McMurdo Sound

2011 Arctic Barrow AK

2012 Arctic Barrow AK

2012 Antarctic SIPEX II

2013 Arctic Barrow AK

2014 Arctic Chukchi Sea



Notices

of the American Mathematical Society

Climate Change and the Mathematics of

page 562

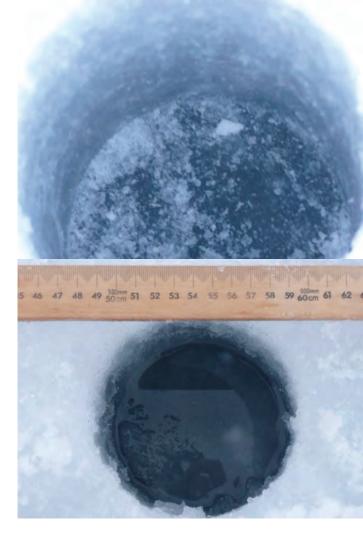
May 2009

Mathematics and the **Enormous Confusion** and Great Potential

page 586



Volume 56, Number 5



measuring fluid permeability of Antarctic sea ice

SIPEX 2007

tracers flowing through inverted sea ice blocks



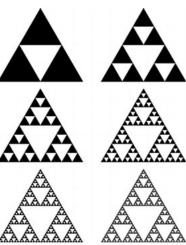






fractals and multiscale structure





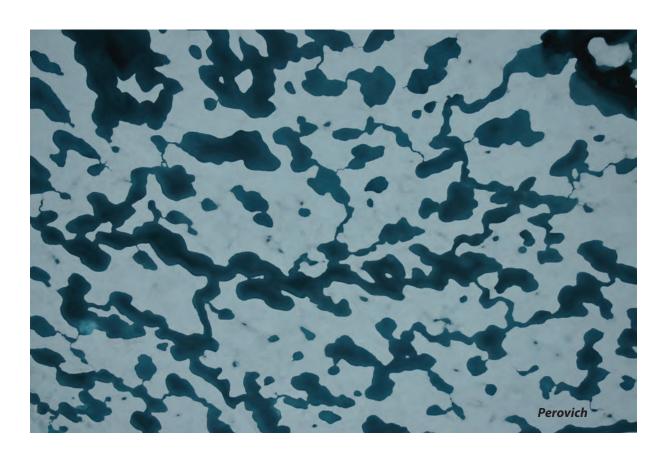
melt pond formation and albedo evolution:

- major drivers in polar climate
- key challenge for global climate models

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006 Flocco, Feltham 2007

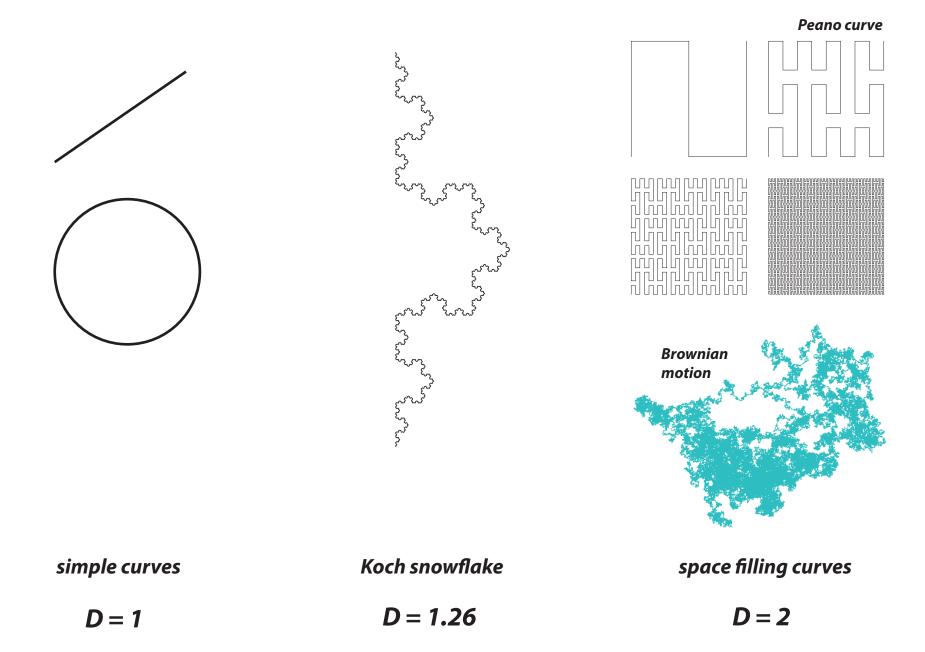
Skyllingstad, Paulson, Perovich 2009 Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

fractal curves in the plane

they wiggle so much that their dimension is >1





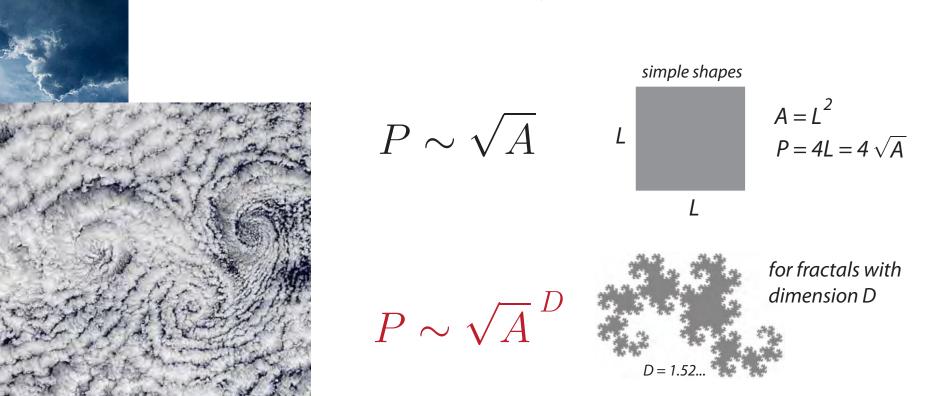
30th Congressional District, Texas, 1991-1996



clouds exhibit fractal behavior from 1 to 1000 km



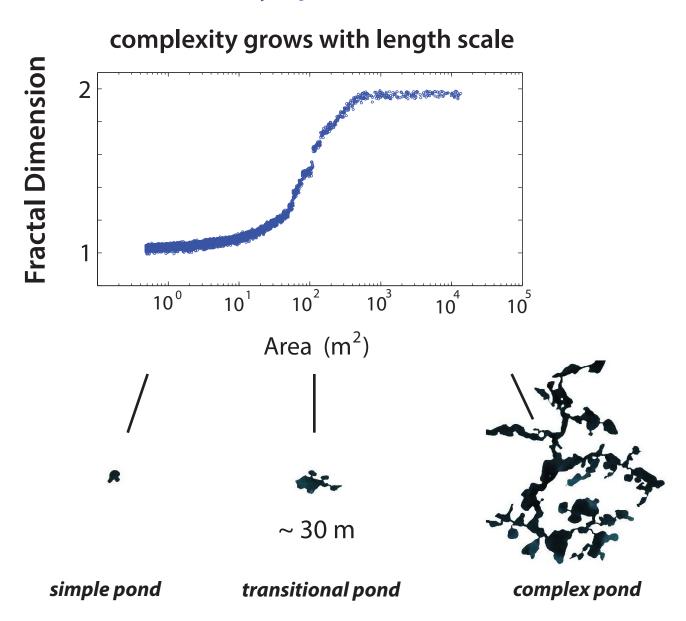
S. Lovejoy, Science, 1982



Transition in the fractal geometry of Arctic melt ponds

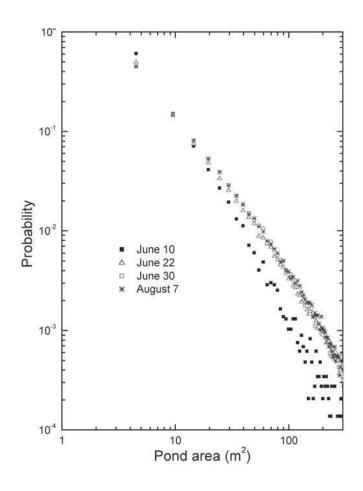
Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012



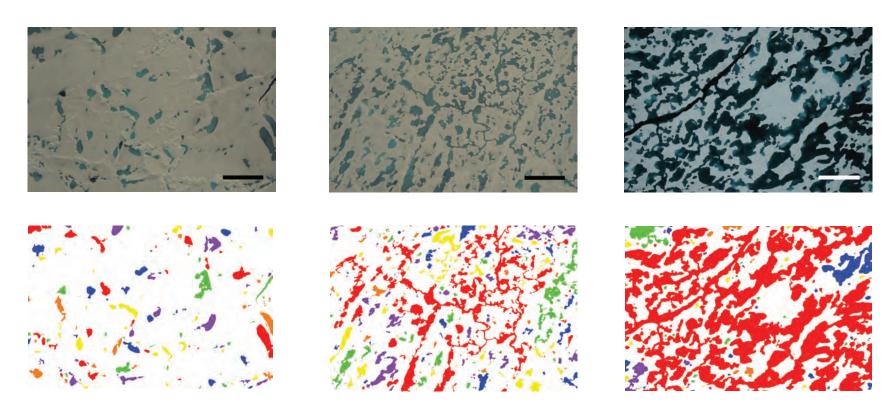
Power law scaling of pond size distribution

Image analysis reveals that the probability distribution of the pond area exhibits power law scaling with exponent about -3/2.



[Perovich, Tucker & Ligett, JGR, 2002]

small simple ponds coalesce to form large connected structures with complex boundaries



melt pond percolation

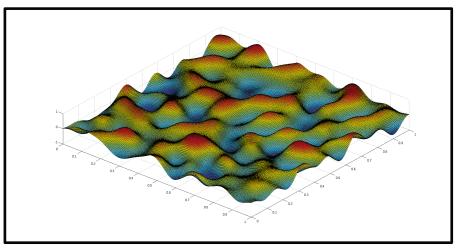
results on percolation threshold, correlation length, cluster behavior

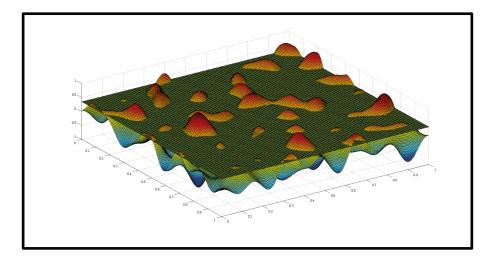
Anthony Cheng (Hillcrest HS), Dylan Webb (Skyline HS), Court Strong, Ken Golden

Continuum percolation model for melt pond evolution

level sets of random surfaces

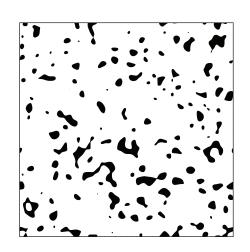
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018

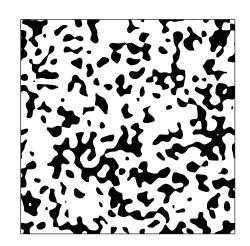


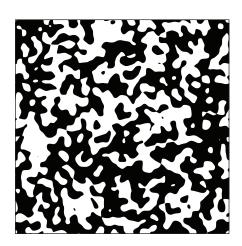


random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds



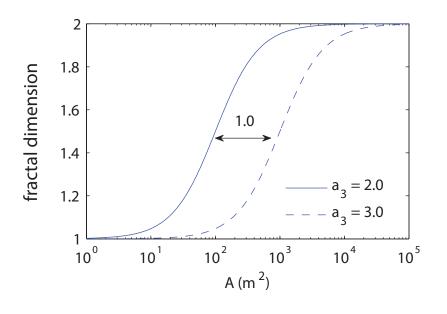


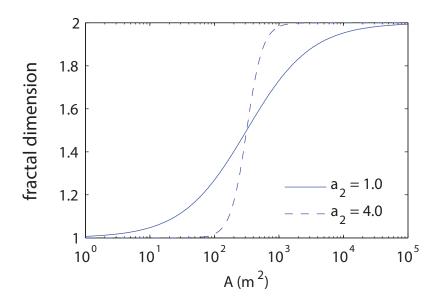


electronic transport in disordered media

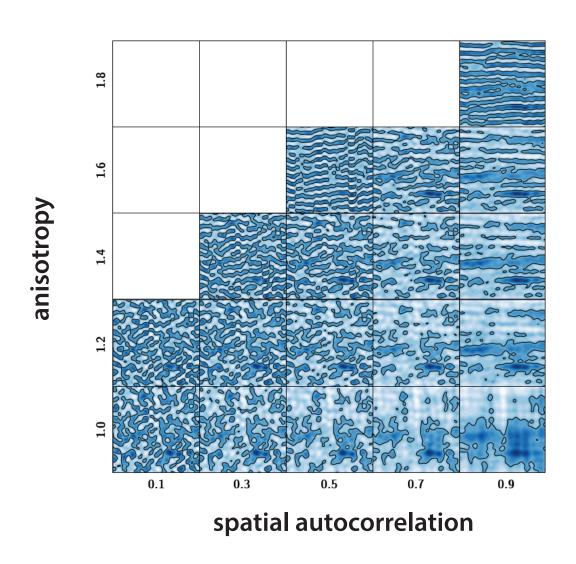
diffusion in turbulent plasmas

fractal dimension curves depend on statistical parameters defining random surface

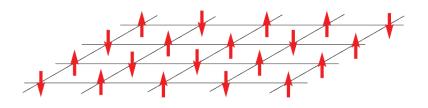




Coefficients of Fourier surface chosen to produce topography with given autocorrelation and anisotropy



Ising Model for a Ferromagnet



$$S_i = \begin{cases} +1 & \text{spin up} \\ -1 & \text{spin down} \end{cases}$$

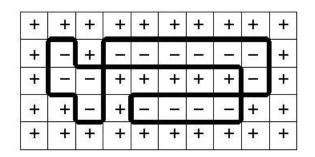


$$\mathcal{H}_{\omega} = -H \sum_{i} s_{i} - J \sum_{\langle i,j \rangle} s_{i} s_{j}$$

nearest neighbor Ising Hamiltonian

for any configuration $\omega \in \Omega = \{-1, 1\}^N$ of the spins

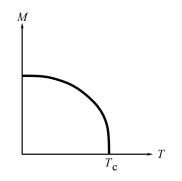
ferromagnetic interaction $J \ge 0$



magnetization

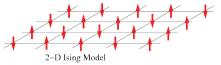
$$M(T, H) = \lim_{N \to \infty} \frac{1}{N} \left\langle \sum_{j} s_{j} \right\rangle = -\frac{\partial f}{\partial H}$$

homogenized parameter like effective conductivity



Curie point critical temperature

Ising model for ferromagnets --- Ising model for melt ponds



Ma, Sudakov, Strong, Golden 2019

$$\mathcal{H}_{\omega} = -J \sum_{\langle i,j \rangle}^{N} s_i s_j - \sum_{i}^{N} H_i s_i$$

$$\mathcal{H}_{\omega} = -J \sum_{\langle i,j \rangle}^{N} s_i s_j - \sum_{i}^{N} H_i s_i \qquad s_i = \begin{cases} \uparrow & +1 & \text{water (spin up)} \\ \downarrow & -1 & \text{ice (spin down)} \end{cases}$$

random magnetic field represents snow topography

magnetization
$$M = \lim_{N \to \infty} \frac{1}{N} \left\langle \sum_{j} s_{j} \right\rangle$$
 pond coverage $\underbrace{(M+1)}_{2}$

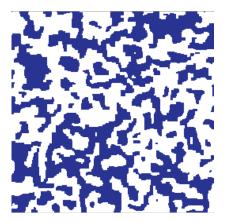
pond coverage
$$(M+1)$$
~ albedo 2

only nearest neighbor patches interact

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system "flows" toward metastable equilibria.

Melt ponds are metastable islands of like spins.

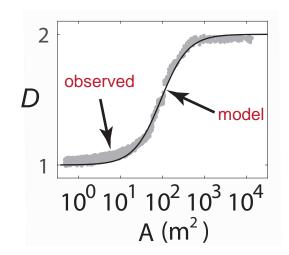
Order from Disorder



Ising model



melt pond photo (Perovich)



pond size distribution exponent

observed -1.5

(Perovich, et al. 2002)

model -1.58

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data



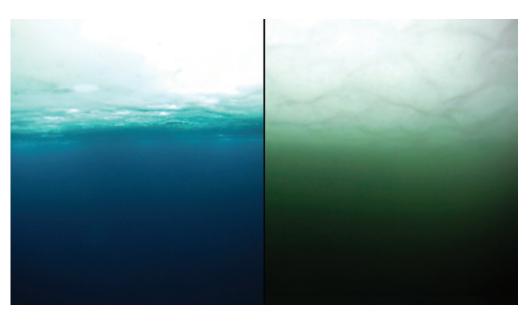
2011 massive under-ice algal bloom

Arrigo et al., Science 2012

melt ponds act as

WINDOWS

allowing light through sea ice



no bloom

bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, lams, Schroeder, Flocco, Feltham, *Science Advances*, 2017

The distribution of solar energy under ponded sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, 2019

(2015 AMS MRC)

the view from underneath —







melt ponds are WINDOWS

light reaches the upper ocean



Perovich

The Melt Pond Conundrum:

How can ponds form on top of sea ice that is highly permeable?

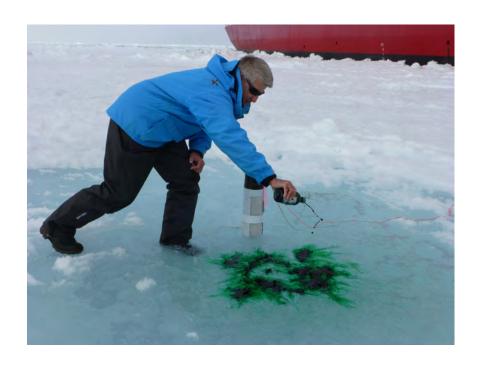
C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE) aboard USCGC Healy





Conclusions

- 1. Sea ice forms complex patterns over a wide range of scales.
- 2. Homogenization and statistical physics help link information on microstructural patterns to larger scale effective behavior; advancing how sea ice is represented in climate models.
- 3. **Stieltjes integral representations** provide powerful methods for homogenization of sea ice structures and processes.
- 4. Melt ponds display fascinating fractal behavior; statistical physics models (percolation, Ising) account for observations.
- 5. This work is helping to improve projections of the fate of Earth's sea ice packs and the ecosystems they support.

THANK YOU

Office of Naval Research

Arctic and Global Prediction Program

Applied and Computational Analysis Program

National Science Foundation

Division of Polar Programs









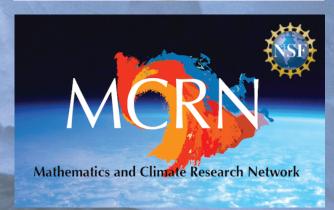












Fire endangers

Thurs 23 July 1998

Fire strands Antarctic ship in sea ice

AN engine more fire has Australian Anteretic Div- arctic continent and return disabled the leabreaker Ausora Australia in sea ico, deep

ision director Mr Rex to Hobart for repairs. Moncur said. But Mr Moncur said he expected it would have to abandon its

The cause of the fire was not known but the engines would have to abandon its have been turned off, with pioneering mid-winter voy- the ship 100 nautical miles age to the edge of the Ant- from the Antaretic coast.

the ship was not in danger after Tuesday night's fire,

THE CANBERRA TIMES Thursday 23 July 1998 Page 4

Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Austra: lian icebreaker Aurora Australis in sea ice, deep in Antarctic

Australian Antarctic Divi-

But Mr Moncur said he expected Aurora Australis would have to abandon its ploneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs.

The fire had been extinguished and the engines were turned off, leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good.

Crew had to wear breathing apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully assessed.

The Aurora, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause beavy, salt-laden water to sink to the bottom.

Mr Moncur said, the cause of the fire was not yet known.



A file photo of the Aurora Australis in Antarctica.

ision's Hobart office.

BY DAVID CARRIGG

Antarctic coast.

Australian Antarctic Division director Rex Moncur said the fire was extinguished by flood-

ing the engine room with an

The gas had to be cleared before crew wearing breathing

apparatus could enter and as-

He said it could be some time

before the extent of damage was

The 25 crew and 54 expedi-

tioners, mostly from Hobart, would wear thermal clothing

and stay below decks to keep

"There is always a risk of becoming ice-bound in these

waters at this time of the year

rut at this stage we don't expect

to launch a rescue mission from

The ship was in regular radio

contact with the Antarctic Div-

Hobart," Mr Moncur said.

not known.

inert gas.

sess the situation.

He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available," Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The Aurora Australis is owned by P&O Australia and charted by the Antarctic Division for about \$11 million

Australia managing director Richard Hein said yesterday the company was assessing the situation and a number of rescue options were being

It was too early to say whether P&O would be liable for the cost of the aborted

The vessel left Hobart last Wednesday for a seven-week voyage mainly to study a polyn-ya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.

Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

Antarctica

Casev

Australia

Hobart

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

2:45 am July 22, 1998

"Please don't be alarmed but we have an uncontrolled fire in the engine room"

about 10 minutes later ...

"Please don't be alarmed but we're lowering the lifeboats"

Sydney Morning Herald 23 July, 1998

ICEBREAKER BURNS

A ploneering 2 million as Australian scientific voyage to the mid-winter Antarous package is expected to be scrapped following an engine-grow fire on the Aurora Australis yesterday. The 54 people on board were locked on decicin me

in Antarotic waters

There were no injuries and

sion director Rex Moneur said there were no injuries and the ship was not in danger after Tuesday night's fire.

