A thermodynamic model of sea ice with desalination for climate modeling

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1 Basic Relations

Nearly all of the salt in sea ice exists in brine inclusions, yet measuremens of sea ice salinity are practical only for the bulk material (the ice and brine combined). Further assuming the ice and brine are in thermodynamic equilibrium, the brine salinity S_{br} and temperature T follow the liquidus relation

$$S_{br} = -21.4T - 0.886T^2 - 0.0170T^3 \tag{1}$$

for a third-order fit to labratory experimental data (Notz and Worster, 2009) with temperature in Celsius.

Because the salt trapped in the ice crystal is negligible, the bulk salinity S can be related to the brine salinity by the brine mass fraction ϕ_m:

$$S = \phi_m S_{br}. \tag{2}$$

The brine mass fraction ϕ_m is related to the brine volume fraction ϕ by:

$$\phi_m = \frac{\rho_{br}}{\rho_m}\phi \tag{3}$$

where ρ_{br} is the brine density and ρ_m is the density of the combined ice and brine. The densities are related by volume weighting:

$$\rho_m = \rho_i(1-\phi) + \rho_{br}\phi \tag{4}$$

where ρ_i is the ice density. We assume there are no bubbles in the ice, so the porosity is equal to the brine volume fraction.

It is instructive to consider mass conservation to gain an appreciation for the conservation laws of sea ice (although we don't actually need the equation). The mass tendency is equal to the mass flux through the boundaries:

$$\frac{d}{dt}\int_V \rho_m dV = -\int_S \phi\rho_{br}\mathbf{u_{br}} \cdot \mathbf{n}\, dS \tag{5}$$

where $\mathbf{u_{br}}$ is the brine velocity in brine channels. With Gauss' theorem:

$$\frac{d}{dt}\int_V \rho_m dV = -\int_V \nabla \cdot (\phi\rho_{br}\mathbf{u_{br}}) dV. \tag{6}$$

Eliminating the integrals and substituting from Eq. 4 gives

$$\left(\frac{\rho_i}{\rho_{br}} - 1\right)\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{u_{br}}) = 0. \tag{7}$$

2 Governing Equations

A thermo and haline model of sea ice is governed by three equations: The first law of thermodynamics, the Stefan condition for phase change at the top and bottom surfaces, and conservation of salt. The enthalpy of sea ice is

$$q_m = q_i(1-\phi) + q_{br}\phi \tag{8}$$

where q_i and q_{br} are the volumetric specific enthalpies of ice and brine, resp. The first law is thus written:

$$\frac{\partial q_m}{\partial t} + \nabla(\mathbf{u_{br}}\phi q_{br}) = \nabla(k_m \nabla T) + R \tag{9}$$

where the second term captures heat transported in the brine channels due to brine flow, the third term is the diffusion of heat, and the fourth term R represents the absorption of shortwave radiation. The thermal conductivity for the combined ice and brine $k_m = k_i(1-\phi) + k_{br}\phi$, where k_i and k_{br} are the conductivities of ice and brine, resp.

For a moving boundary model, such as the one developed here, an equation describing phase change at the top and bottom surfaces of the ice is needed:

$$q_m \frac{dh}{dt} = \sum F \tag{10}$$

where h is the ice thickness and F is the net flux into the top and bottom surfaces, including the conductive heat flux into the ice.

Finally conservation of salt is

$$\frac{\partial S}{\partial t} + \nabla\left(\mathbf{u_{br}}\phi S_{br}\right) = \nabla\left(\phi D_{mol} \nabla S_{br}\right) \tag{11}$$

where the second term captures salt transport in the brine channels due to brine flow and the third term is the diffusion of salt. The molecular diffusion of salt $D_{mol} = 6.8 \times 10^{-10}$ m^2 s^{-1} is within the brine.

3 More on Sea Ice Enthalpy

The volumetric specific enthalpies of ice and brine are written

$$q_i = -L_o + \int_{T_o}^{T} c_i(T')dT' \tag{12}$$

and

$$q_{br} = \int_{T_o}^{T} c_{br}(T')dT' \tag{13}$$

in terms of volumetric specific heat capacities of ice c_i and brine c_{br}, the (constant) volumetric latent heat of fusion of fresh ice $L_o = 306$ MJ m^{-3}, and a reference temperature T_o.

With Eqs 12 and 13, the sea ice enthalpy is

$$q_m = -(1-\phi)L_o + \int_{T_o}^{T} c_i dT' + \phi \int_{T_o}^{T} (c_{br} - c_i) dT'. \tag{14}$$

The last term is small so we neglect it and c_i is nearly constant so we let $c_i = 2.054$ MJ m^{-3} K^{-1} to arrive at the sea ice enthalpy used in our model

$$q_m \approx c_i(T - T_o) - (1-\phi)L_o. \tag{15}$$

4 Comparison with Energy-Conserving Thermodynamics of Bitz and Lipscomb (1999)

Bitz and Lipscomb (1999, henceforth BL99) used nearly the same equation for sea ice enthalpy as Eq. 15 in their model, but they derived it by first formulating an effective heat capacity for sea ice. BL99 assumed a steady bulk salinity profile in the ice and no brine flow, so they could write Eq. 9 as

$$\frac{\partial q^*}{\partial t} = c_{eff}\frac{\partial T}{\partial t} = \nabla(k_{eff}\nabla T) + R. \tag{16}$$

They used the derivation of the heat capacity for sea ice by Ono (1967) to write the effective heat capacity

$$c_{eff} = c_i + \frac{\mu L_o S}{T^2}, \tag{17}$$

by assuming a linearized liquidus relation $S_{br} = -T/\mu$ with $\mu = 0.054$ from Assur (1958). They used an empirical fit for k_{eff} in Eq. 16 from Untersteiner (1964), which is similar but not exactly equal to k_m.

BL99 then derived q^* by integrating the effective heat capacity with respect to temperature from a given temperature to the melting point of sea ice $T_m = -\mu S$ (also the liquidus temperature, but for the bulk salinity):

$$q^* = c_i(T - T_m) - L\left(1 + \frac{\mu S}{T}\right). \tag{18}$$

BL99 actually had a minus sign in their expression (fine since enthalpy is a relative quantity), which they termed the “energy of melting”. Equation 18 is equivalent to Eq. 15 provided $T_o = T_m$, assuming a linear liquidus relation, and neglecting the difference between ϕ and ϕ_m (i.e., $\phi \approx -\mu S/T$, Untersteiner, 1961).

Thus BL99 developed a thermodynamic model approriate for their assumption of a steady salinity profile. Modifications are necessary when salinity is prognostic because

$$\frac{\partial q_m}{\partial t} = \frac{\partial q_m}{\partial T}\frac{\partial T}{\partial t} + \frac{\partial q_m}{\partial S}\frac{\partial S}{\partial t}. \tag{19}$$

Plus it important to include the heat transport by brine flow in the first law, as shown in Eq. 9 (will need to test though) and accuracy is improved from using the cubic liquidus relation. (A linear liquidus relationship gives melting temperatures only accurate to about 50% between -20 and 0°C, personal comm. Dirk Notz, 6 Mar, 2009, as shown in his thesis).

5 Mechanisms that influence salinity

Salt is captured in sea ice upon growth. Recent theoretical, experimental, and field evidence of this process by Notz et al. (2005) showed that the bulk salinity is continuous across an advancing ice-ocean front during growth, and hence salt is not immediately expelled during growth. Instead the primary means of desalination are gravity drainage during the cold season and flushing from surface meltwater during the warm season (Notz and Worster, 2009). Gravity drainage is buoyancy-driven convection in sea ice brine channels (see, e.g., Wettlaufer et al., 1997) that results from persistent heat loss through the top surface. The temperature gradient in the ice produces higher brine salinities near the top surface and thus an unstable density profile (the density at subfreezing temperatures is dominated by salinity). Flushing is driven by the pressure head caused by meltwater pooling at the top surface (Untersteiner, 1968).

There are two additional minor means of desalination. The first is the expulsion of brine from internal pressure that builds-up upon freezing (owing to the lower density of ice than brine). The second is brine pocket migration, which is caused by the molecular diffusion of salt. The magnitude of each desalination process is described, estimated, and discussed by Notz and Worster (2009) and references therein.

6 Parameterizations for a one-dimensional model

Several recent papers formulate two-dimensional sea ice models that treat fluid motion explicitly (Oertling and Watts, 2004; Petrich et al., 2006), which requires momentum equations in two dimensions and a resolution finer than 1 mm. This kind of resolution is unrealistic for a climate model, especially in the horizontal. Therefore, we must model salinity as a one-dimensional process (within each category of the ice-thickness distribution). Eicken et al. (2002) found that during early melt pond formation, meltwater is mostly lost through lateral transport, rather than percolation through the ice. Pond levels usually rapidly adjust to the sea surface. Notz and Worster (2006) and Notz and Worster (2009) argue that the lateral flushing results in little desalination and therefore it is possible to treat desalination from flusing in one-dimensional. A similar argument can be made for brine drainage. In addition, the permeability, a measure of the effective pore channel area, has been shown to be much greater in the vertical than in the horizontal (Golden et al., 2007).

6.1 Brine Drainage

The product of brine velocity and brine salinity that appears in the transport term in the salt conservation equation can be partitioned into slow and fast processes using Reynold's averaging. The vertical velocity component can then be written:

$$w_{br}(z,t) = \bar{w}_{br}(z,t) + w'(z,t) \tag{20}$$

where z is positive downward and the horizontal and ice-thickness category dependences are supressed. The overbar denotes a moving average over time:

$$\bar{w}_{br}(z,t) = \frac{1}{\tau}\int_0^{\tau} w(z,t+t')dt' \tag{21}$$

where τ is long compared to the timescale of eddy motions but comparable or shorter than the model timestep, which is an hour. (?) We can think of the basic state flow $\bar{w}_{br}(z,t)$ as representing flow from flushing and expulsion and the average influence of eddies. The eddy component is the anomaly from the basic state. The same expansion can be done for S_{br} and the average of their product gives a basic state and eddy flux:

$$\overline{w_{br}S_{br}} = \bar{w}_{br}\bar{S}_{br} + \overline{w'_{br}S'_{br}} \tag{22}$$

assuming no correlation between eddies and basic state components.

The eddy salinity component can be approximated from mixing length theory:

$$S'_{br} = -\xi\frac{\partial \bar{S}_{br}}{\partial z} \tag{23}$$

where a parcel moving in an eddy will maintain the salinity of the base state at a given point over a distance known as the mixing length ξ before mixing with its environment. The downward brine volume flux by eddies is taken as a Darcy law for flow through a porous material

$$w' = \frac{\Pi}{\eta}\left(-\frac{\partial p'}{\partial z} - \rho'_{br}g\right) \tag{24}$$

where Π is the permeability to vertical flows (discussed later), η is the dynamic viscosity, p' and ρ'_{br} are the eddy's pressure and density, and g is the downward acceleration due to gravity (used for sea ice before by Feltham et al. 2002). The downward flow speed of the brine within channel is $w'_{br} = w'/\phi$. If we assume the pressure gradient of eddies in Eq. 24 is hydrostatic and eddies adjusts freely to their surroundings, then

$$\frac{\partial p'}{\partial z} = \frac{\partial \bar{p}}{\partial z} = -g\bar{\rho}_{br} \tag{25}$$

and

$$w'_{br} = \frac{\Pi g}{\bar{\phi}\eta}(\bar{\rho}_{br} - \rho'_{br}) = \frac{\Pi g \beta}{\bar{\phi}\eta}(\bar{S}_{br} - S'_{br}) \tag{26}$$

where $\beta = \partial\rho/\partial S$ is the salinity expansion coefficient.

Thus Reynold's averaging, mixing length theory, and Darcy's law allow us to write the eddy flux convergence as another diffusion term:

$$\frac{\partial}{\partial z}\left(\bar{\phi}\;\overline{w'_{br}S'_{br}}\right) = -\frac{\partial}{\partial z}\left(D_{ed}\frac{\partial \bar{S}_{br}}{\partial z}\right) \tag{27}$$

where the eddy diffusivity is

$$D_{ed} = -\bar{\phi} w'_{br}\xi = \frac{\Pi(\phi(z))g\beta(h-z)}{\eta}\mathcal{R}(S_{br}(z)-\bar{S}_{ocn}), \tag{28}$$

with the ramp function $\mathcal{R}(x)$ equal to x for positive x and zero otherwise. Here we have assumed that eddies mix ocean water up to level z and thus the mixing length ξ is the distance between level z and the bottom of the ice. This diffusivity is equal to the Rayleigh number Ra for sea ice given by Notz and Worster (2009) multiplied by the thermal diffusivity at a level z, except Notz and Worster let the permeability equal the minimum permeability between z and the bottom of the ice. (I think we should do this too.)

Finally we can write salt conservation in one dimension as

$$\frac{\partial \bar{S}}{\partial t} + \frac{\partial}{\partial z}\left(\bar{S}\bar{w}_{br}\right) = \frac{\partial}{\partial z}\left((\bar{\phi} D_{mol} + D_{ed})\frac{\partial \bar{S}_{br}}{\partial z}\right). \tag{29}$$

It is convenient to transform to a normalized coordinate $\tilde{z} = (z - z_t)/h$ with $h = z_b - z_t$ (see, Taylor and Feltham, 2004; Huwald et al., 2005) to incorporate the effect of growth and melt on the salinity in terms of a normalized speed of the moving boundaries:

$$\tilde{\omega} = \frac{1-\tilde{z}}{h}\,\frac{dz_t}{dt} - \frac{\tilde{z}}{h}\,\frac{dz_b}{dt}\,. \tag{30}$$

Salt conservation can thus be written:

$$\frac{\partial(h\bar{S})}{\partial t} + \frac{\partial}{\partial \tilde{z}}\left(h\bar{S}(\tilde{\omega} + \tilde{w}_{br})\right) = \frac{\partial}{\partial \tilde{z}}\left(\frac{\bar{\phi} D_{mol} + D_{ed}}{h}\frac{\partial \bar{S}_{br}}{\partial \tilde{z}}\right), \tag{31}$$

where $\tilde{w}_{br} = \bar{w}_{br}/h$. It is also convenient to define an effective salt flow that results molecular diffusion

$$\tilde{W} = -\frac{\rho_m D_{mol}}{\rho_{br}\bar{S}_{br}h^2}\frac{\partial \bar{S}_{br}}{\partial \tilde{z}} \tag{32}$$

and substituting for $\bar{\phi}$ from Eqs. 2 and 3:

$$\frac{\partial(h\bar{S})}{\partial t} + \frac{\partial}{\partial \tilde{z}}\left(h\bar{S}(\tilde{W} + \tilde{\omega} + \tilde{w}_{br})\right) = \frac{\partial}{\partial \tilde{z}}\left(\frac{D_{ed}}{h}\frac{\partial \bar{S}_{br}}{\partial \tilde{z}}\right). \tag{33}$$

Eddies influence heat transport as well as salt transport and thus we also Reynold's average the first law of thermodynamics. It is first helpful to rewrite the heat transport in brine channels using the linear liquidus relations as (is this decent?)

$$c_{br}\nabla(\mathbf{u_{br}}\phi T) \approx \mu c_{br}\frac{\rho_m}{\rho_{br}}\nabla(\mathbf{u_{br}}\phi S_{br}). \tag{34}$$

Thus

$$\frac{\partial(h\bar{q})}{\partial t} + \frac{\partial(h\bar{q}\tilde{\omega})}{\partial \tilde{z}} + \mu c_{br}\frac{\rho_m}{\rho_{br}}\frac{\partial}{\partial \tilde{z}}\left(h\bar{S}\tilde{w}_{br} - \frac{D_{ed}}{h}\frac{\partial \bar{S}_{br}}{\partial \tilde{z}}\right) = \frac{\partial}{\partial \tilde{z}}\left(\frac{k_m}{h}\frac{\partial(h\bar{T})}{\partial \tilde{z}}\right) + R \tag{35}$$

6.2 Flushing

When meltwater pools on sea ice, the meltwater may percolate through the ice and flush out brine. As already explained, the level of meltwater is rarely above the sea surface and therefore the rate of drainage must equal the change in freeboard:

$$\bar{w}_{br} = -\frac{1}{\bar{\phi}}\frac{\rho_x}{\rho_{br}}\mathcal{R}\left(-\frac{dh_x}{dt}\right). \tag{36}$$

where the x indicates either snow or ice, whichever is melting. The flushing rate is assumed to be zero, however, if the porosity is below 5% at any level in the ice.

6.3 Permeability

A number of relations have been used to relate the permeability to porosity. Currently I am using the formula given by Freitag

$$\Pi = 10^{-17}(10^3\phi)^{3.1} \tag{37}$$

in m^2. But I plan to investigate the model sensitivity to this assumtion.

7 Numerical solutions

$$\tag{38}$$

$$\tag{39}$$

$$\tag{40}$$

$$\tag{41}$$

$$\tag{42}$$

$$\tag{43}$$

$$\tag{44}$$

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