MODELING SEA ICE in a changing climate

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Research Administration NAKAMA University of Utah 2 February 2018

Frey

ANTARCTICA

southern cryosphere

Weddell Sea

East Antarctic Ice Sheet

West Antarctic Ice Sheet

Ross Sea

sea ice

THE ARCTIC northern cryosphere



SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

white snow and ice reflect







dark water and land absorb

albedo
$$\alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers September 2012 -- 3.4 million square kilometers





recent losses in comparison to the United States



Arctic sea ice decline: faster than predicted by climate models



challenge

represent sea ice more rigorously in climate models

account for key processes such as melt pond evolution



Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, JGR Oceans 2012

For simulations with ponds September ice volume is nearly 40% lower.

... and other sub-grid scale structures and processes *linkage of scales*

sea ice is a multiscale composite displaying structure over 10 orders of magnitude

0.1 millimeter

1 meter



pancake ice

1 meter

100 kilometers



What is this talk about?

HOMOGENIZATION

Using mathematics of composite materials and statistical physics to LINK SCALES in the sea ice system and rigorously compute effective behavior ... to improve climate projections.

- 1. Climate models, sea ice components, homogenization
- 2. Sea ice microphysics and porous media
- 3. EM monitoring of sea ice, remote sensing
- 4. Advection diffusion; polycrystals; waves in the MIZ
- 5. Low order models and basin scale homogenization
- 6. Evolution of Arctic melt ponds, fractal geometry

critical behavior

cross - pollination



Global Climate Models

Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 50 km), using very powerful computers.

key challenge :

incorporating sub - grid scale processes

linkage of scales



sea ice components of GCM's

What are the key ingredients -- or *governing equations* that need to be solved on grids using powerful computers?

1. Ice thickness distribution g(x, y, h, t) evolution equation **dynamics**



nonlinear PDE incorporating ice velocity field ice growth and melting mechanical redistribution - ridging and opening

2. Conservation of momentum, stress vs. strain relation (Hibler 1979)

F = ma for sea ice

Coriolis, air and water drag, floe - floe interactions, ...

dynamics

3. Heat equation for sea ice and snow

evolution of temperature field in ice brine convection

thermodynamics

+ balance of radiative and thermal fluxes on interfaces

(Maykut and Untersteiner 1971)

How do scales interact in the sea ice system?



basin scale grid scale albedo

Linking Scales

km scale melt ponds





km scale melt ponds

Linking

mm scale brine inclusions



Scales



meter scale snow topography

HOMOGENIZATION - Linking Scales in Composites



inhomogeneous medium homogeneous medium

find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

Composite materials in the Boeing 787 Dreamliner



sea ice microphysics

fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







Antarctic surface flooding and snow-ice formation

evolution of salinity profiles
ocean-ice-air exchanges of heat, CO₂

sea ice ecosystem



sea ice algae support life in the polar oceans

fluid permeability k of a porous medium



porous

concrete

how much water gets through the sample per unit time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Darcy's Law for slow viscous flow in a porous medium



 $\mathbf{k} =$ fluid permeability tensor

Critical behavior of fluid transport in sea ice



RULE OF FIVES

Golden, Ackley, Lytle Science 1998Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009



sea ice algal communities

D. Thomas 2004

nutrient replenishment controlled by ice permeability

biological activity turns on or off according to *rule of fives*

Golden, Ackley, Lytle

Science 1998

Fritsen, Lytle, Ackley, Sullivan Science 1994

critical behavior of microbial activity



Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond \longrightarrow *open with probability p closed with probability 1-p*

percolation threshold $p_c = 1/2$ for d = 2

smallest *p* for which there is an infinite open cluster

Continuum percolation model for *stealthy* materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

 $\phi_c \approx 5\%$ Golden, Ackley, Lytle, *Science*, 1998



sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice Golden, Eicken, Heaton, Miner, Pringle, Zhu



rigorous bounds percolation theory hierarchical model network model

field data

X-ray tomography for brine inclusions

unprecedented look at thermal evolution of brine phase and its connectivity

controls

micro-scale

macro-scale processes

brine connectivity (over cm scale)

8 x 8 x 2 mm



-15 °C, $\phi = 0.033$ -6 °C, $\phi = 0.075$ -3 °C, $\phi = 0.143$

X-ray tomography confirms percolation threshold

3-D images 3-D graph ores and throats nodes and edges

analyze graph connectivity as function of temperature and sample size

- use finite size scaling techniques to confirm rule of fives
- order parameter data from a natural material

Pringle, Miner, Eicken, Golden, J. Geophys. Res. 2009

lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2 \qquad \text{critical} \\ \text{exponent} \\ k_0 = 3 \times 10^{-8} \text{ m}^2 \qquad t$$

- exponent is UNIVERSAL lattice value $t \approx 2.0$
- sedimentary rocks like sandstones also exhibit universality
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0

$$y = \log k \xrightarrow{-7}_{-8}_{-10}$$

theory: $y = 2 \times -7.5$
 $y = \log k \xrightarrow{-9}_{-10}_{-11}_{-12}_{-13}_{-14}_{-12}_{-13}_{-14}_{-15}_{-2.2 -2}_{-2.2 -2}_{-1.8 -1.6 -1.4 -1.2 -1}_{-1.6 -1.4 -1.2 -1}_{x = \log(\phi - 0.05)}$

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport?



Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



 $R_{i,j}^{h} \xrightarrow{R_{i,j}^{v}} R_{i,j}^{h}$

Zhu, Jabini, Golden, Eicken, Morris *Ann. Glac*. 2006

- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution;
 Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability k.
- Rigorous bound on *k* for bimodal distribution of pore sizes

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden Multiscale Modeling and Simulation, 2018

How does the biology affect the physics?

Microbial Habitability in the Icy Moons of Jupiter and Saturn

Ruby Bowers David Morison Ken Golden

Enceladus

NASA



Where might we look for microbes?

In the layer where there are "brine" inclusions!

~ bottom 200 m

Better yet....

10 km

In the layer where the fluid phase percolates, allowing convection, nutrient transport,...

- Icy shell of frozen 1% MgSO₄ ocean melting point -0.31°C eutectic point -3.7°C
- 1-D thermal model (linear temperature profile) surface temperature -173°C (100 K)



What is the rule of fives for magnesium sulfate sea ice?

Remote sensing of sea ice



sea ice thickness ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

8*

effective complex permittivity (dielectric constant, conductivity)



brine volume fraction brine inclusion connectivity Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?



 p_1 , p_2 = volume fractions of brine and ice

Theory of Effective Electromagnetic Behavior of Composites analytic continuation method

Forward Homogenization Bergman (1978), Milton (1979), Golden and Papanicolaou (1983) *Theory of Composites*, Milton (2002)

> **composite geometry** (spectral measure µ)



Stieltjes integral representation

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s-z} \qquad s = \frac{1}{1 - \epsilon_1/\epsilon_2} \qquad \xrightarrow{\text{complex s-plane}}$$

spectral measure of self adjoint operator X ΓX
mass = p₁
higher moments depend on *n*-point correlations
Golden and Papanicolaou 1983



1

Inverse Homogenization Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001) McPhedran, McKenzie, Milton (1982), *Theory of Composites*, Milton (2002)



recover brine volume fraction, brine connectivity, etc.



inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ϵ^*

> rigorous inverse bound on spectral gap

Orum, Cherkaev, Golden Proc. Roy. Soc. A, 2012 inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden Physica B, 2007

SEA ICE



young healthy trabecular bone



spectral characterization of porous microstructures in human bone

reconstruct spectral measures from complex permittivity data



m





use regularized inversion scheme

apply spectral measure analysis of brine connectivity and spectral inversion to electromagnetic monitoring of osteoporosis

Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

Spectral computations for Arctic melt ponds



Ben Murphy Elena Cherkaev Ken Golden 2017

eigenvalue statistics for transport tend toward the UNIVERSAL Wigner-Dyson distribution as the "conducting" phase percolates



metal / insulator transition localization

Anderson 1958 Mott 1949 Shklovshii et al 1993 Evangelou 1992

Anderson transition in wave physics: quantum, optics, acoustics, water waves, ...

we find a surprising analog

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017





transition to universal eigenvalue statistics (GOE) extended states, mobility edges

-- but without wave interference or scattering effects ! --

Bounds on the complex permittivity of polycrystalline materials by analytic continuation

> Adam Gully, Joyce Lin, Elena Cherkaev, Ken Golden

 Stieltjes integral representation for effective complex permittivity

Milton (1981, 2002), Barabash and Stroud (1999), ...

- Forward and inverse bounds
- Applied to sea ice using two-scale homogenization
- Inverse bounds give method for distinguishing ice types using remote sensing techniques





Proc. Roy. Soc. A 8 Feb 2015

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PROCEEDINGS A



An invited review commemorating 350 years of scientific publishing at the Royal Society

A method to distinguish between different types of sea ice using remote sensing techniques A computer model to determine how a human should walk so as to expend the least energy



advection enhanced diffusion

effective diffusivity

sea ice floes diffusing in ocean currents diffusion of pollutants in atmosphere salt and heat transport in ocean heat transport in sea ice with convection

advection diffusion equation with a velocity field $\,ec u\,$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$
$$\vec{\nabla} \cdot \vec{u} = 0$$
$$homogenize$$
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* effective diffusivity_

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017 Murphy, Cherkaev, Zhu, Xin, Golden, 2018







Storm-induced sea-ice breakup and the implications for ice extent

Kohout et al., Nature 2014

- during three large-wave events, significant wave heights did not decay exponentially, enabling large waves to persist deep into the pack ice.
- Iarge waves break sea ice much farther from the ice edge than would be predicted by the commonly assumed exponential decay





ice extent compared with significant wave height

Waves have strong influence on both the floe size distribution and ice extent.

ocean swells propagating through a vast field of pancake ice

HOMOGENIZATION: long wave sees an effective medium, not individual floes



wave propagation in the marginal ice zone





Marginal Ice Zone

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



transitional region between dense interior pack (*c* > 80%) sparse outer fringes (*c* < 15%)

MIZ WIDTH fundamental length scale of ecological and climate dynamics

Strong, *Climate Dynamics* 2012 Strong and Rigor, *GRL* 2013 How to objectively measure the "width" of this complex, non-convex region?

Objective method for measuring MIZ width motivated by medical imaging and diagnostics



Arctic Marginal Ice Zone

crossection of the cerebral cortex of a rodent brain

analysis of different MIZ WIDTH definitions

Strong, Foster, Cherkaev, Eisenman, Golden J. Atmos. Oceanic Tech. 2017

> Strong and Golden Society for Industrial and Applied Mathematics News, April 2017

Filling the polar data gap

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km 06 January 1985

Gap radius: 311 km 30 August 2007



fill with harmonic function satisfying satellite BC's plus stochastic term

Strong and Golden, *Remote Sensing* 2016 Strong and Golden, *SIAM News* 2017

Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	Antarctic	SIPEX
2010	Antarctic	McMurdo Sound
2011	Arctic	Barrow AK
2012	Arctic	Barrow AK
2012	Antarctic	SIPEX II
2013	Arctic	Barrow AK
2014	Arctic	Chukchi Sea



Notices Anterior Mathematical Society

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and the Mathematics of Transport in Sea Ice

page 562

Mathematics and the Internet: A Source of Enormous Confusion and Great Potential

page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



measuring fluid permeability of Antarctic sea ice

SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

columnar

5%

granular



10%

Golden, Sampson, Gully, Lubbers, Tison 2018

tracers flowing through inverted sea ice blocks







fractals and multiscale structure



melt pond formation and albedo evolution:

- major drivers in polar climate
- key challenge for global climate models

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006 Flocco, Feltham 2007 Skyllingstad, Paulson, Perovich 2009 Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

fractal curves in the plane

they wiggle so much that their dimension is >1



Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012



small simple ponds coalesce to form large connected structures with complex boundaries



melt pond percolation

results on percolation threshold, correlation length, cluster behavior

Anthony Cheng (Hillcrest HS), Dylan Webb (Skyline HS), Court Strong, Ken Golden

Continuum percolation model for melt pond evolution level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds







electronic transport in disordered media

diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

melt pond evolution depends also on large-scale "pores" in ice cover

drainage vortex

photo courtesy of C. Polashenski and D. Perovich

Melt pond connectivity enables vast expanses of melt water to drain down seal holes, thaw holes, and leads in the ice.

Network modeling of Arctic melt ponds

Barjatia, Tasdizen, Song, Sampson, Golden (sci-матн) Cold Regions Science and Tecnology, 2016



develop algorithms to map images of melt ponds onto

random resistor networks

graphs of nodes and edges with edge conductances

edge conductance ~ neck width

compute effective horizontal fluid conductivity



"melt ponds" are clusters of magnetic spins that align with the applied field

predictions of fractal transition, pond size exponent Ma, Sudakov, Strong, Golden 2018

Ising model results

Minimize Ising Hamiltonian energy

Random magnetic field represents snow topography; interaction term represents horizontal heat transfer.

Melt ponds – metastable islands of like spins in our random field Ising model.



pond size distribution exponent

observed -1.5 (*Perovich, et al 2002*)

model -1.58



The lattice constant *a* must be small relative to the 10-20 m length scales prominent in sea ice and snow topography. We set a=1 m as the length scale above which important spatially correlated fluctuations occur in the power spectrum of snow topography.





Arrigo et al., Science 2012

melt ponds act as *WINDOWS*

allowing light through sea ice



bloom

no bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances*, 2017

(2015 AMS MRC, Snowbird)

The Melt Pond Conundrum:

How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE) aboard USCGC Healy





Conclusions

- 1. Summer Arctic sea ice is melting rapidly, and melt ponds and other processes must be accounted for in order to predict melting rates.
- 2. Fluid flow through sea ice mediates melt pond evolution and many processes important to climate change and polar ecosystems.
- 3. Statistical physics and homogenization help *link scales*, provide rigorous methods for finding effective behavior, and advance how sea ice is represented in climate models.
- 4. Critical behavior (in many forms) is inherent in the climate system.
- 5. Field experiments are essential to developing relevant mathematics.
- 6. Our research will help to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

THANK YOU

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Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999