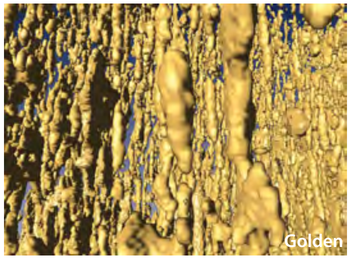
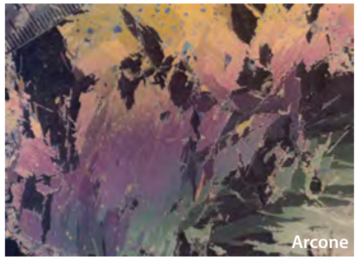
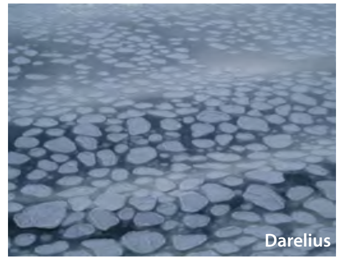
millimeters



centimeters



meters



kilometers

10^3 kilometers

From Micro to Macro in Modeling Sea Ice

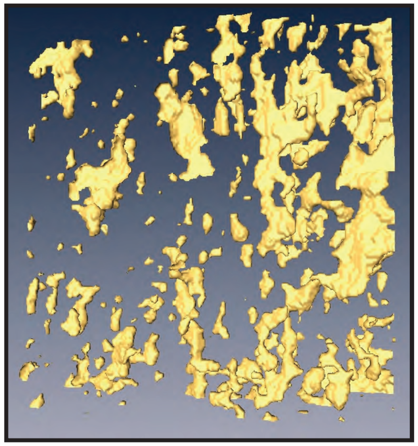
Ken Golden, University of Utah

Sea Ice is a Multiscale Composite Material

microscale

brine inclusions

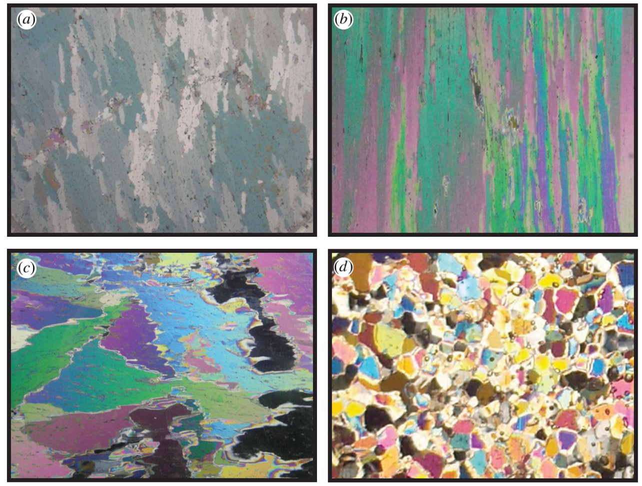
Weeks & Assur 1969



H. Eicken

Golden *et al.* *GRL* 2007

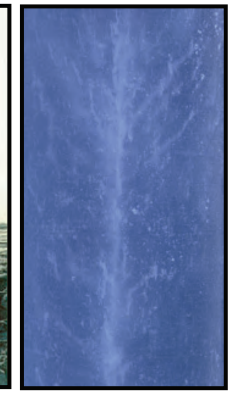
polycrystals



Gully *et al.* *Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

millimeters

centimeters

---

mesoscale

Arctic melt ponds

K. Frey

Antarctic pressure ridges

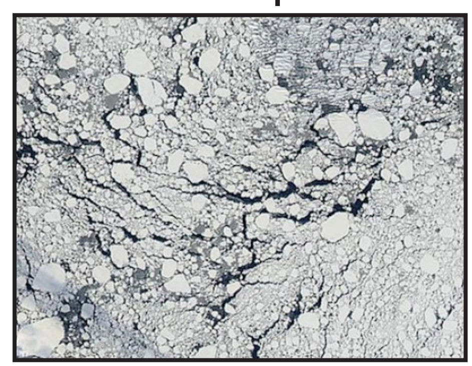
K. Golden

macroscale

sea ice floes

J. Weller

sea ice pack

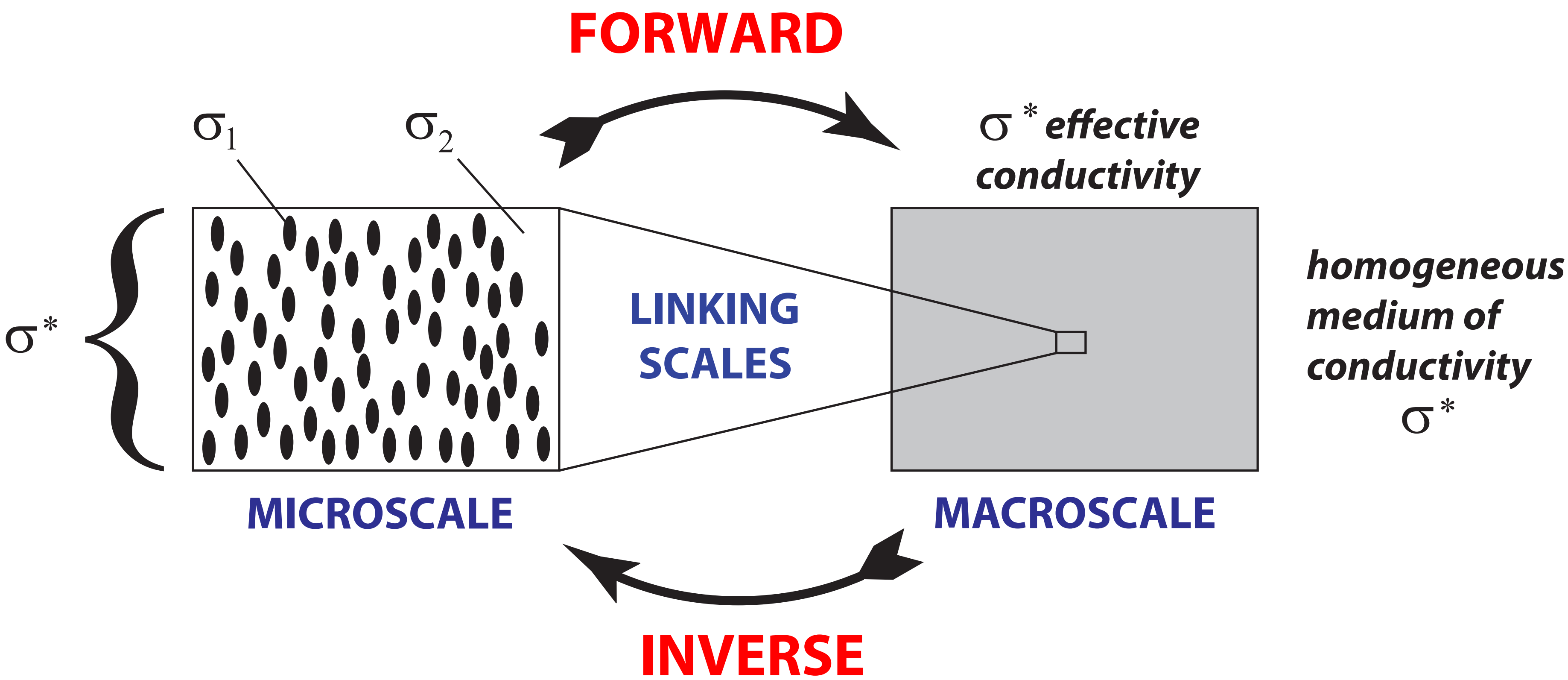


NASA

meters

kilometers

HOMOGENIZATION for Composite Materials



Maxwell 1873, Einstein 1906
Wiener 1912, Hashin and Shtrikman 1962

Polar Ecology and the Physics of Sea Ice

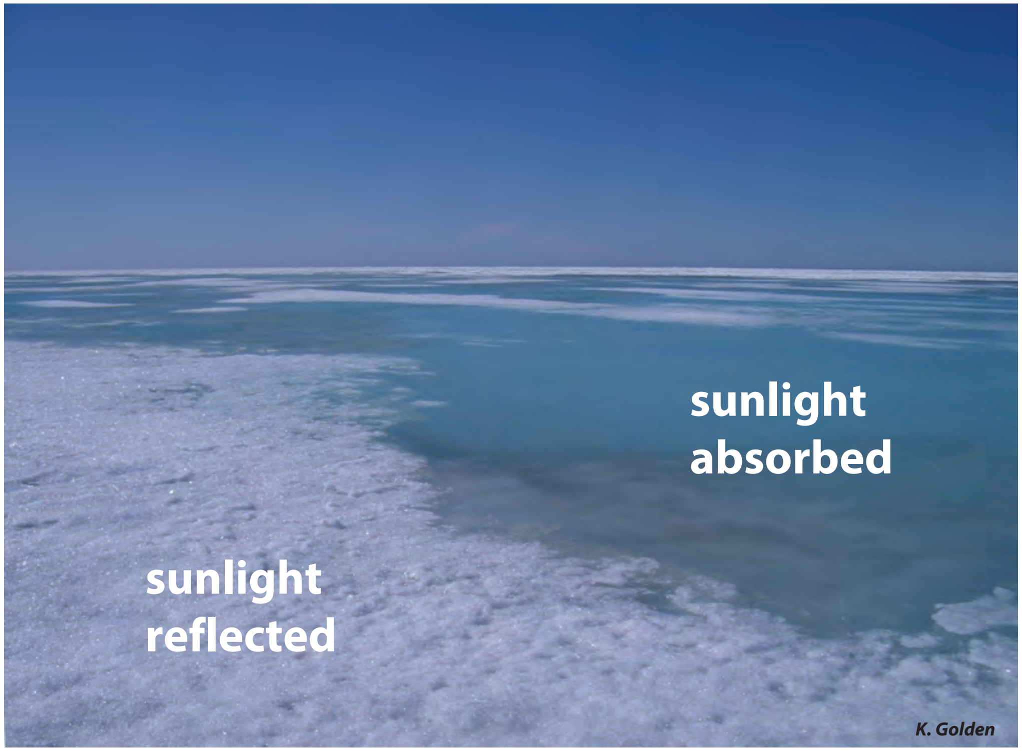
How do sea ice properties affect the life it hosts?

How does life in and on sea ice affect its physical properties?

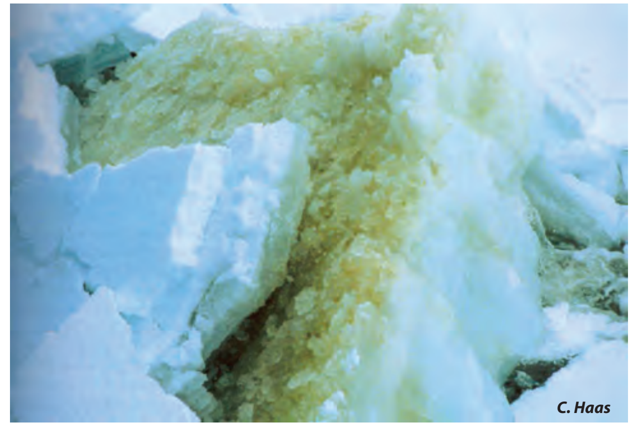
microscale

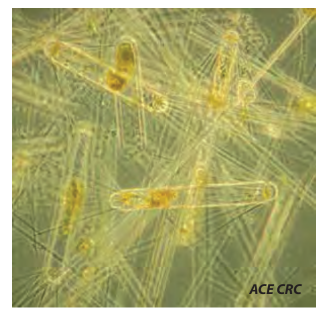
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

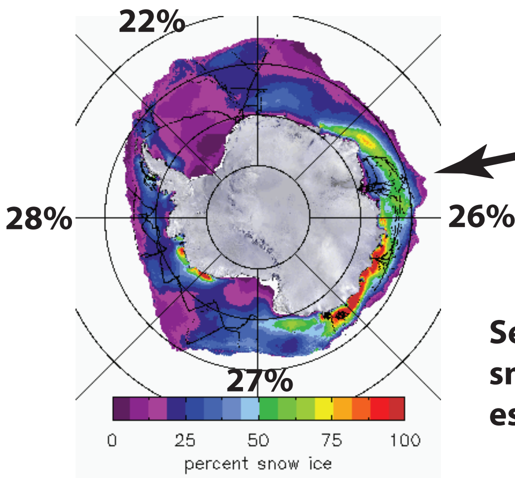
evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







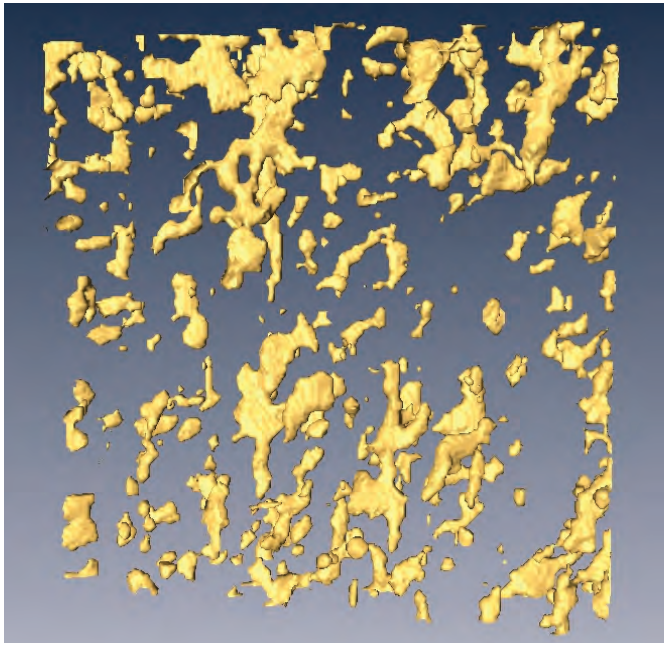
T. Maksym and T. Markus, 2008

September snow-ice estimates

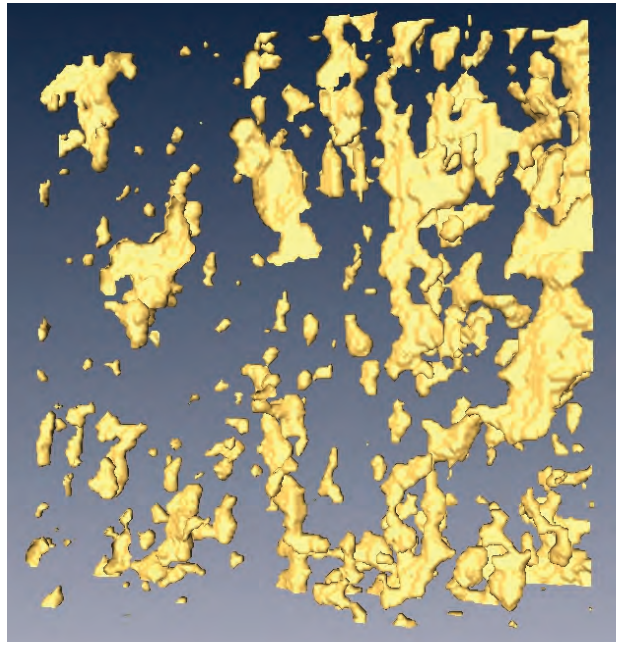
Antarctic surface flooding and snow-ice formation

- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO_2*

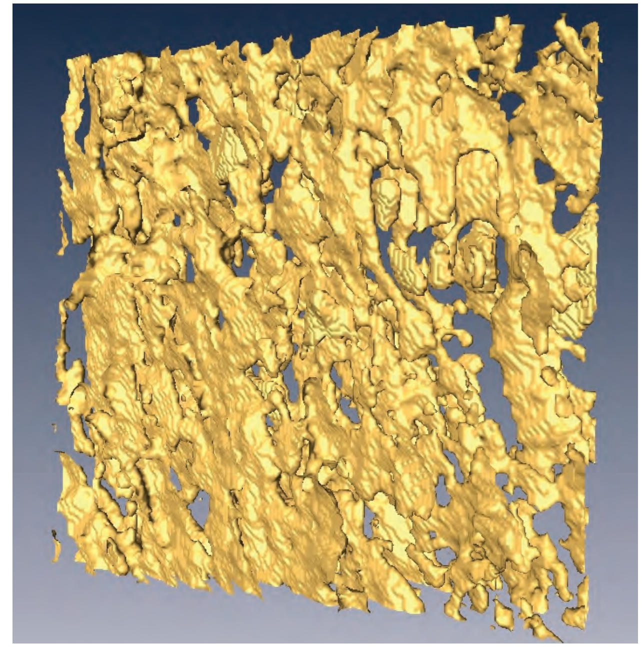
brine volume fraction and ***connectivity*** increase with temperature



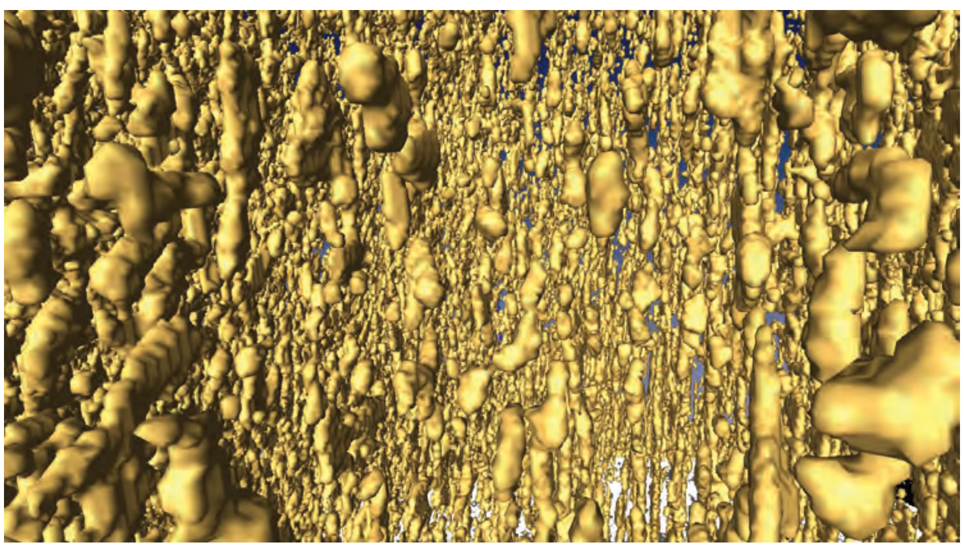
T = -15 °C, $\phi = 0.033$



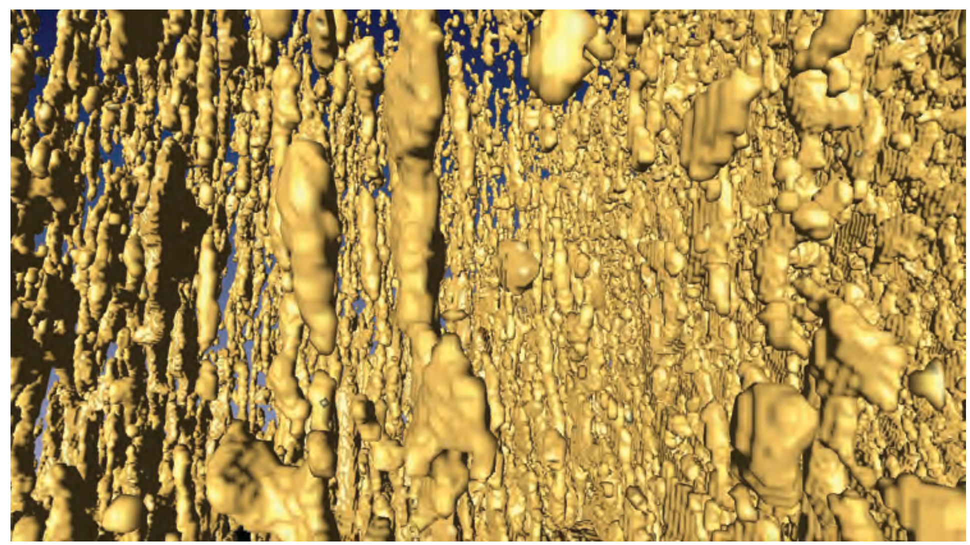
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$



T = -8 °C, $\phi = 0.057$



T = -4 °C, $\phi = 0.113$

X-ray tomography for brine in sea ice

Golden *et al.*, *Geophysical Research Letters*, 2007

Critical behavior of fluid transport in sea ice

impermeable permeable

Arctic field data

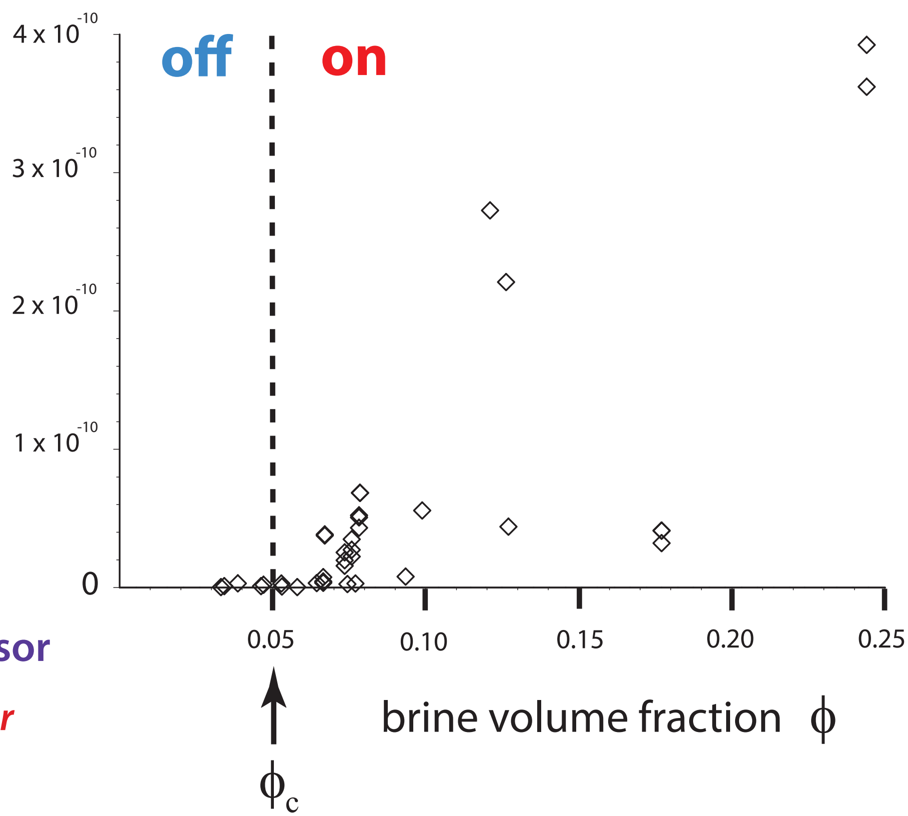
vertical fluid permeability k (m^2)

Darcy's Law

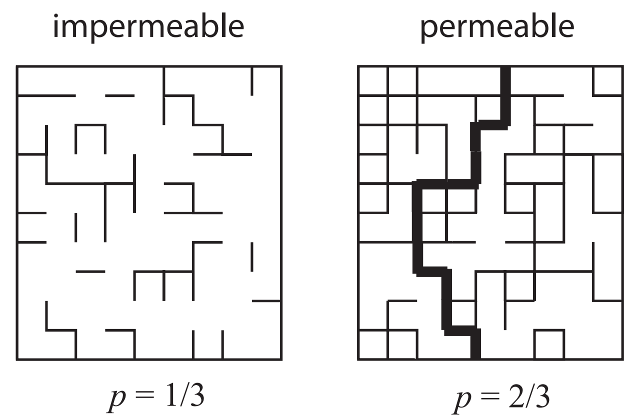
$$\mathbf{v} = -\frac{\mathbf{k}}{\eta}\nabla p$$

$\mathbf{k}$ = fluid permeability tensor

homogenized parameter



"on - off" switch for bulk fluid flow



lattice percolation

FRACTAL percolation clusters

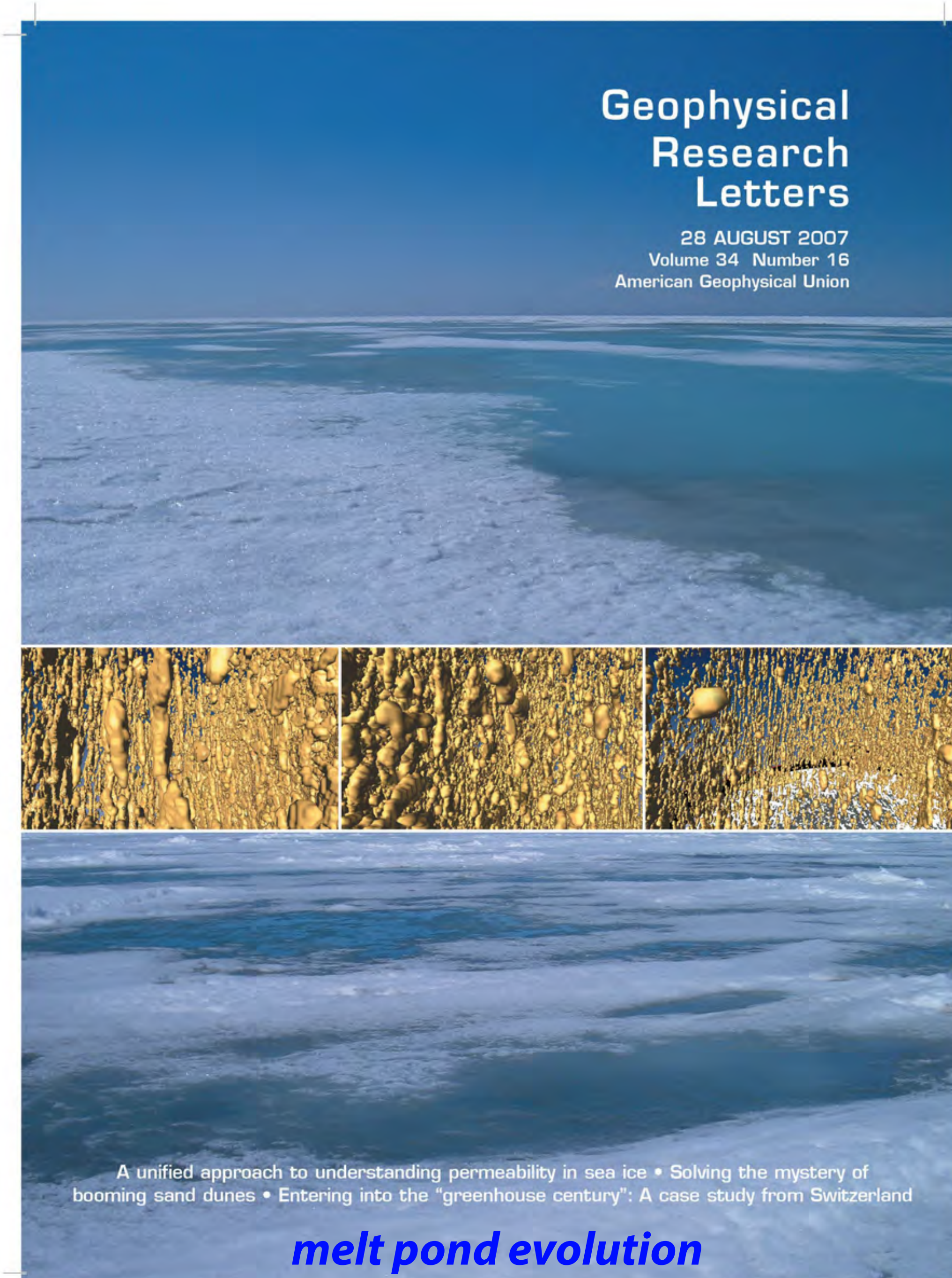
PERCOLATION THRESHOLD $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ C,\ S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



microscale

governs

mesoscale processes

melt pond evolution

percolation theory for fluid permeability

X-ray tomography for brine inclusions

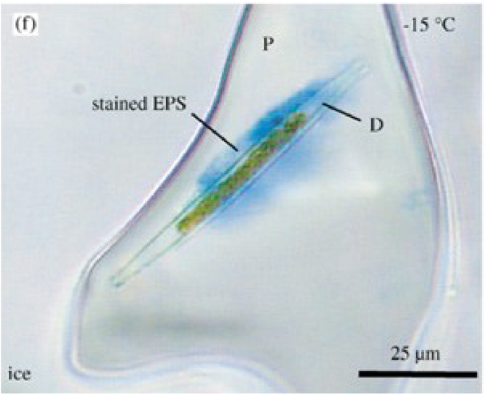
confirms rule of fives

Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009

theory agrees closely with field data

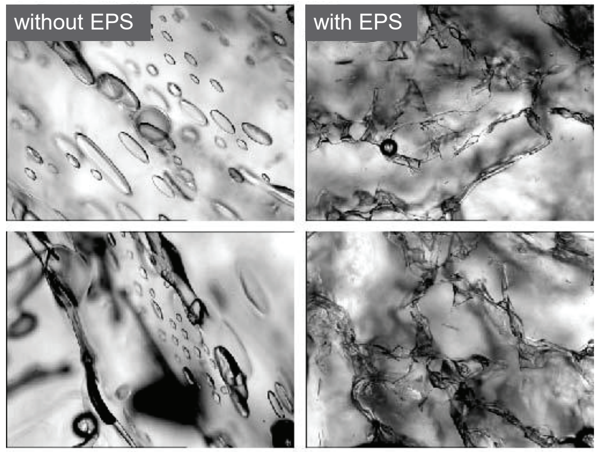
Sea ice algae secrete exopolymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport? How does the biology affect the physics?

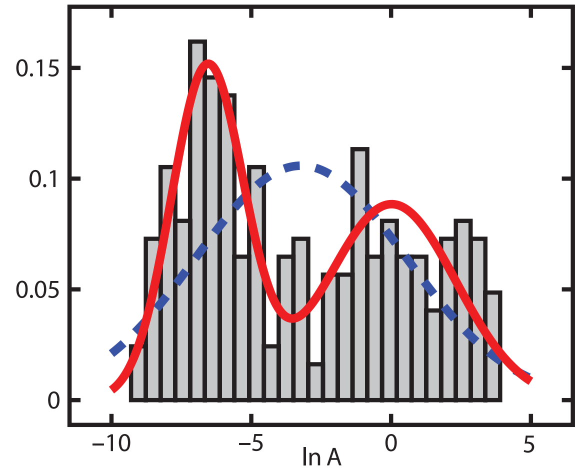


Krembs

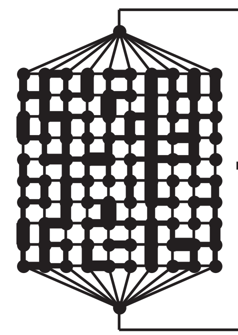
FRACTAL



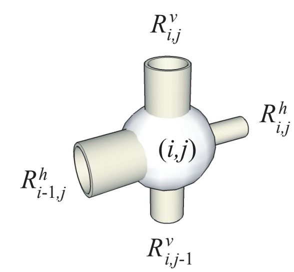
Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability *k;* results predict observed drop in *k*



Zhu, Jabini, Golden, Eicken, Morris
Ann. Glac. 2006

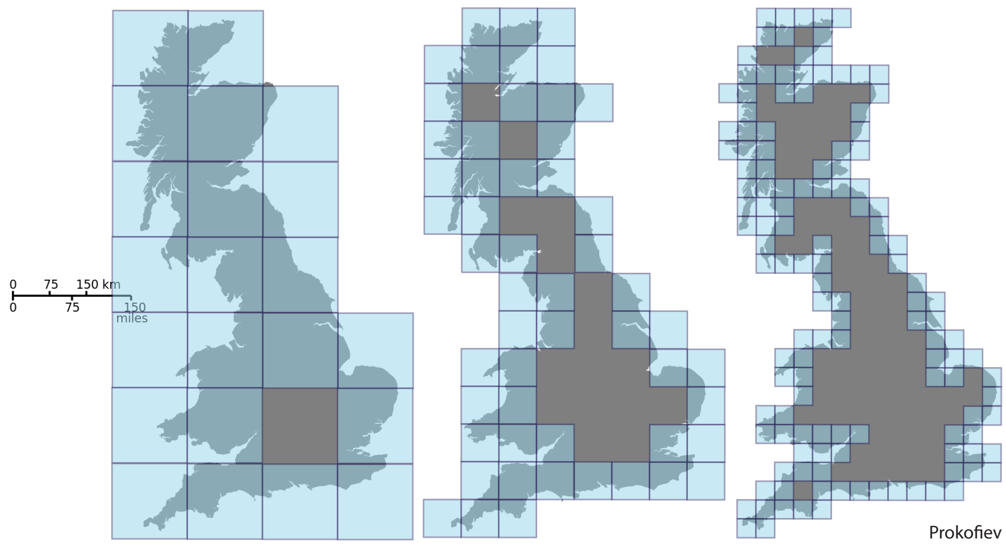
Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

EPS - Algae Model Jajeh, Reimer, Golden, 2024

SIAM News
June 2024

Thermal Evolution of Brine Fractal Geometry in Sea Ice

Nash Ward, Daniel Hallman, Benjamin Murphy, Jody Reimer,
Marc Oggier, Megan O'Sadnick, Elena Cherkaev and Kenneth Golden, 2024

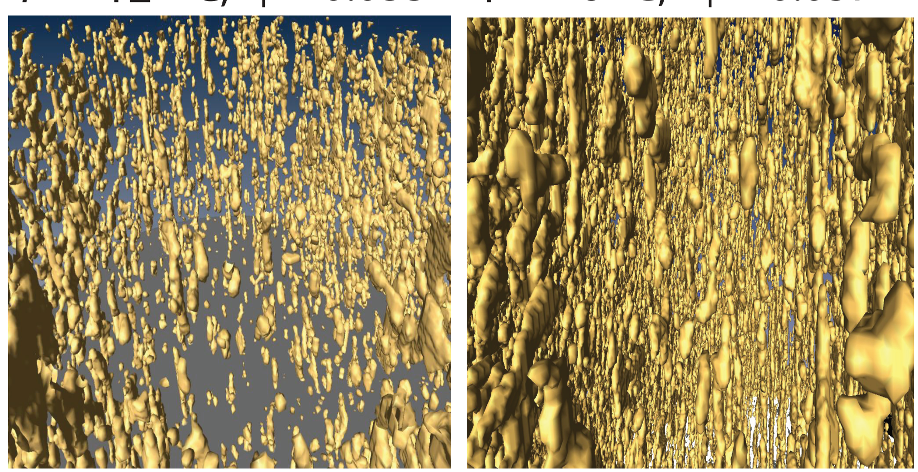


Prokofiev

fractal dimension of the coastline of Great Britain by box counting

$$N(\epsilon) \sim \epsilon^{-D}$$

$T =$ -12° C, $\phi = 0.033$ $T =$ -8° C, $\phi = 0.057$



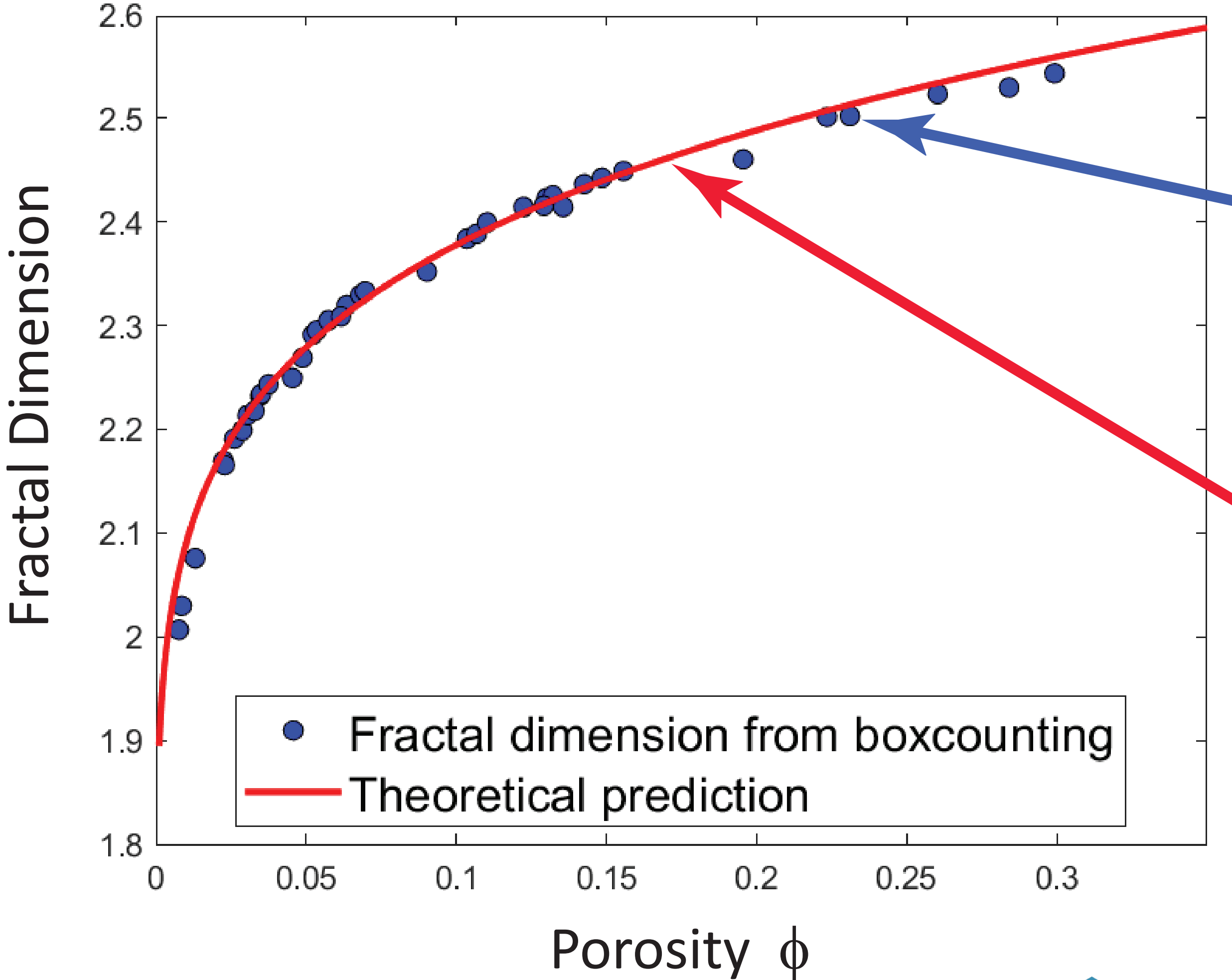
Golden, Eicken, et al. *GRL*, 2007

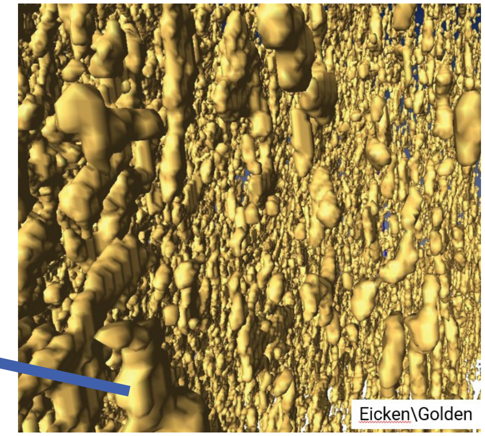
brine channels and inclusions "look" like fractals (from 30 yrs ago)

X-ray computed tomography of brine in sea ice

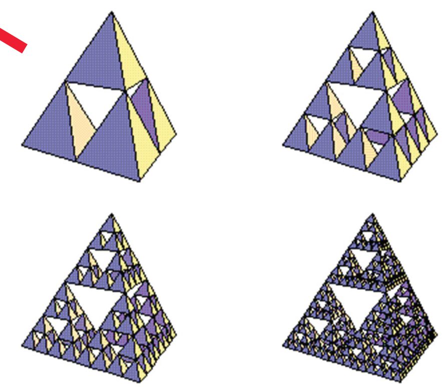
columnar and granular

The first quantitative study of the fractal dimension of brine in sea ice and its strong dependence on temperature and porosity.





Follows same curve as exactly self-similar Sierpinski tetrahedron

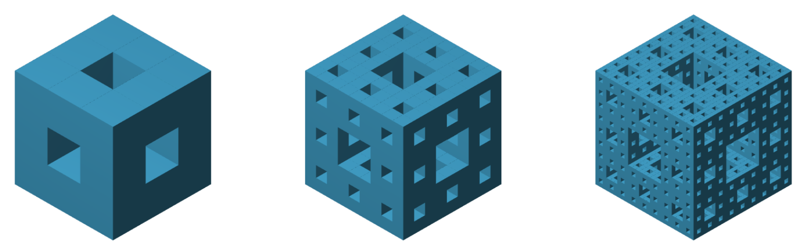


red curve

$$F_d = d_E - \frac{\ln \phi}{\ln(\lambda_{min}/\lambda_{max})}$$

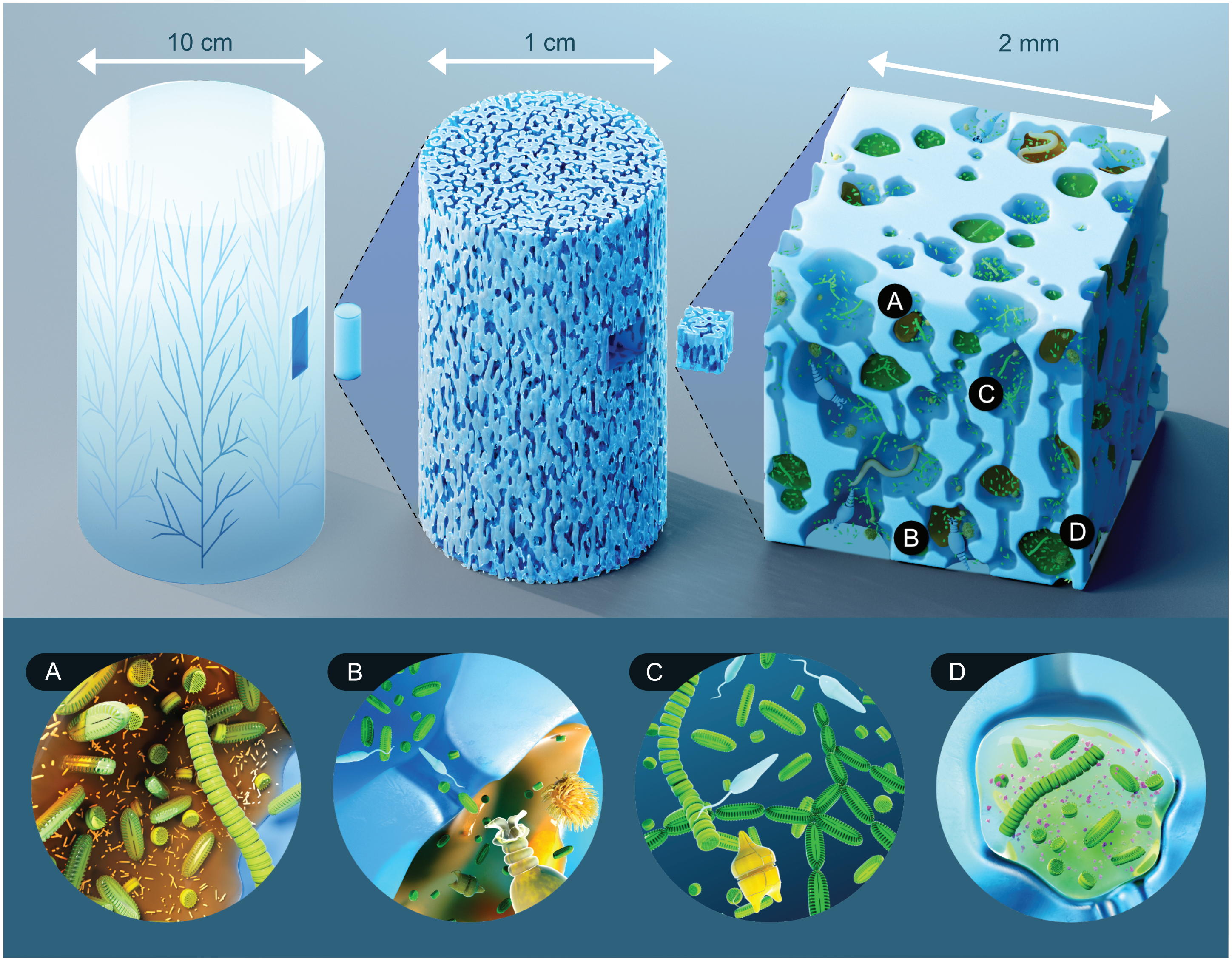
Katz and Thompson,1985; Yu and Li, 2001

discovered for sandstones
statistically self-similar porous media



Fractal geometry of brine in sea ice, Ward, *et al.* 2024

Implications of brine fractal geometry on sea ice ecology and biogeochemistry



Brine inclusions are home to ice endemic organisms, e.g., bacteria, diatoms, flagellates, rotifers, nematodes.

The habitability of sea ice for these organisms is inextricably linked to its complex brine geometry.

(A) Many sea ice organisms attach themselves to inclusion walls; inclusions with a higher fractal dimension have greater surface area for colonization.

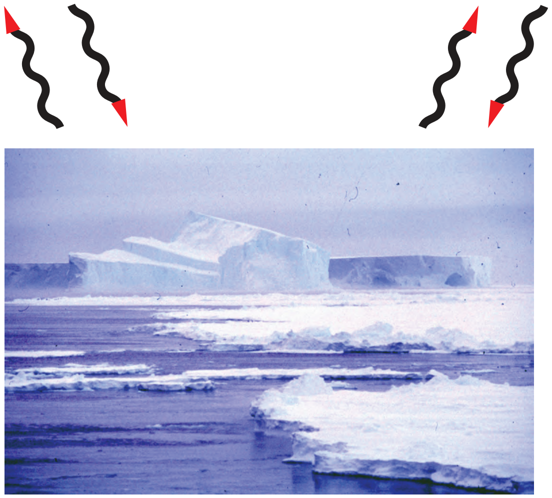
(B) Narrow channels prevent the passage of larger organisms, leading to refuges where smaller organisms can multiply without being grazed, as in (C).

(D) Ice algae secrete extracellular polymeric substances (EPS) which alter incusion geometry and may further increase the fractal dimension.

Remote Sensing of Sea Ice

with radar, microwaves, ...

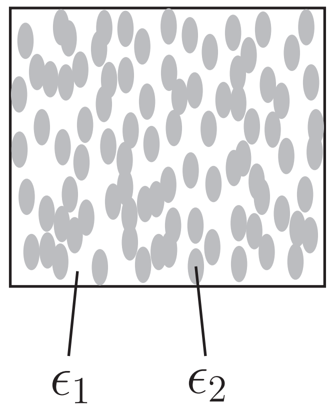
interaction of EM waves with brine and polycrystalline microstructures, rough surfaces



INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data ε^*

Effective complex permittivity of a composite in the quasistatic (long wavelength) limit



$\Big\} \; \epsilon^*$

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

electrical conductivity
thermal conductivty
magnetic permeability
diffusivity

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \, , \text{ composite geometry} \right)$$

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

Analytic Continuation Method for Homogenization

Bergman 1978, Milton 1979, Golden & Papanicolaou 1983, Milton 2002

Stieltjes integrals for homogenized parameters separate component parameters from geometry

geometry

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z} \qquad s = \frac{1}{1 - \epsilon_1/\epsilon_2}$$

material parameters

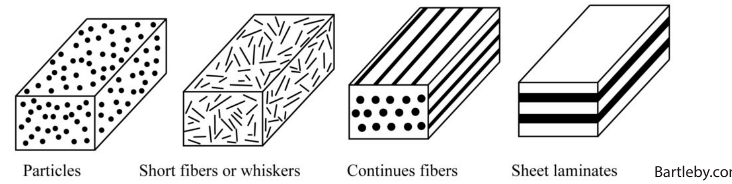
- spectral measure of self adjoint operator $\Gamma\chi$ (matrix)

  $\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$

  $\chi =$ characteristic function of the brine phase

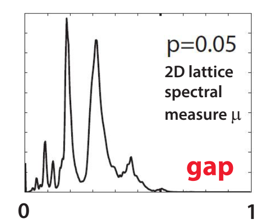
- $E = s\,(s + \Gamma\chi)^{-1} e_k$ **resolvent**
- **bounds in the complex plane; approximations inverse bounds to recover porosity, connectivity**

complexities of mixture geometry



distilled

distilled



spectral measure mass $= p_1$

spectral properties of operator (matrix) ~ quantum states, energy levels for atoms

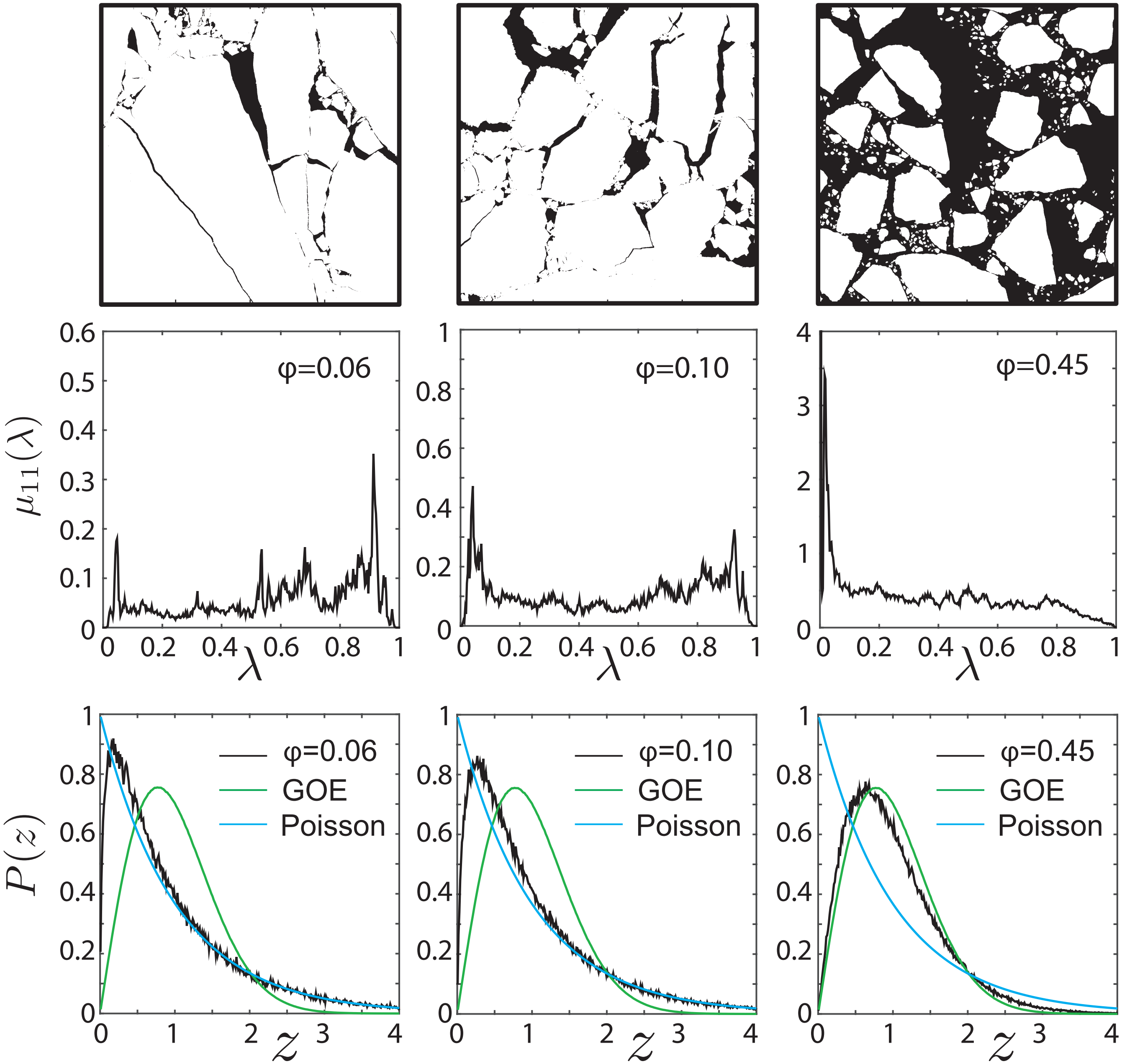
eigenvectors **eigenvalues**

$\Gamma\chi$: microscale ⟶ macroscale

Spectral computations for sea ice floe configurations

spectral measures

eigenvalue spacing distributions



RANDOM MATRIX THEORY

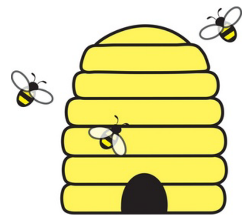
UNIVERSAL Wigner-Dyson distribution

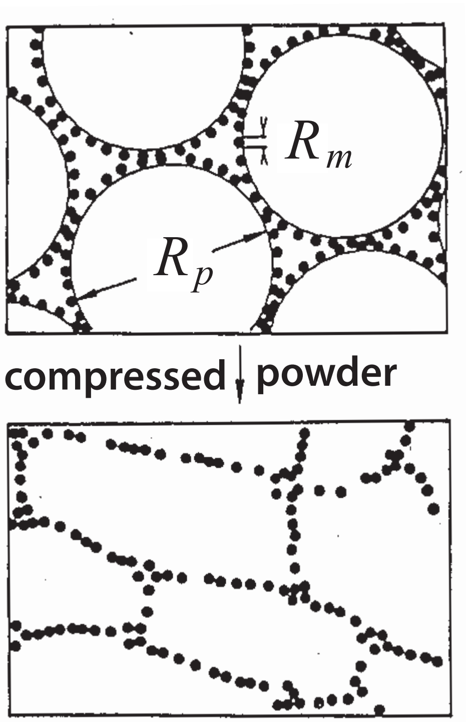
Anderson localization transition

uncorrelated → **level repulsion**

Murphy, Cherkaev, Golden, *Phys. Rev. Lett.* 2017; Murphy, Cherkaev, Hohenegger, Golden, *Comm. Math. Sci.* 2015

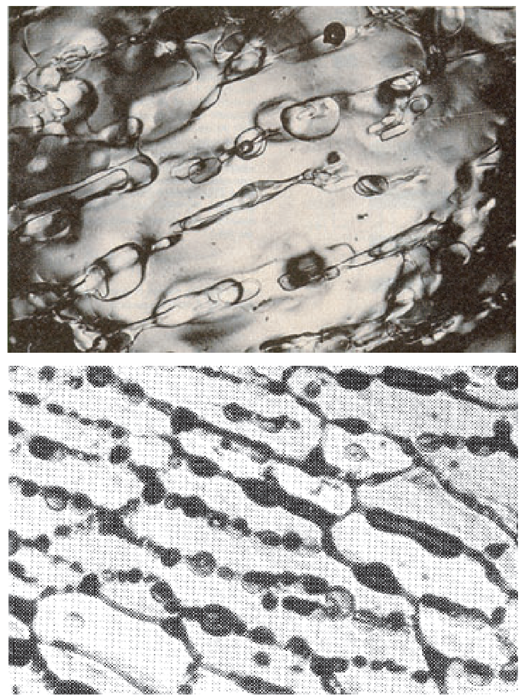
cross pollination





radar absorbing coating

Kusy & Turner
Nature 1971

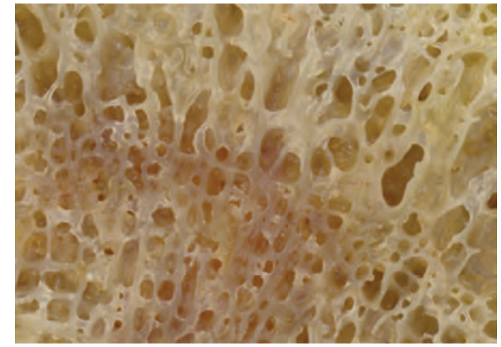


sea ice

Golden, Ackley, Lytle
Science 1998

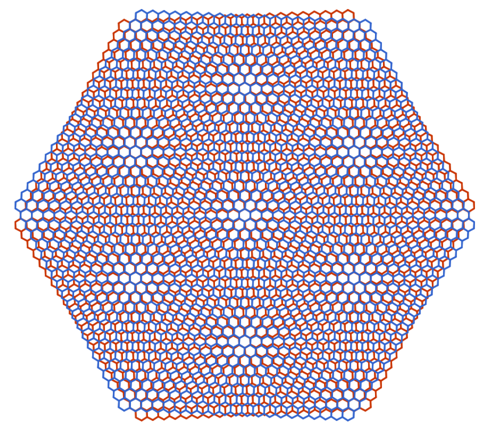
Rule of Fives
fluid flow

spectral analysis & RMT



human bone

Golden, Murphy, Cherkaev
J. Biomechanics 2011



twisted bilayer composites

Morison, Murphy, Cherkaev, Golden
Communications Physics 2022

stealth technology, climate science, medical imaging, twistronics

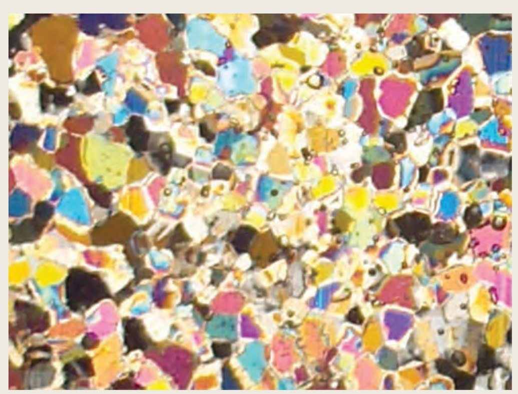
Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**

  Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**

  orientation statistics
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



Proc. Roy. Soc. A 8 Feb 2015

ISSN 1364-5021 | Volume 471 | Issue 2174 | 8 February 2015

PROCEEDINGS A

350 YEARS OF SCIENTIFIC PUBLISHING

Proc. R. Soc. A | Volume 471 | Issue 2174 | 8 February 2015

An invited review commemorating 350 years of scientific publishing at the Royal Society

A method to distinguish between different types of sea ice using remote sensing techniques

A computer model to determine how a human should walk so as to expend the least energy

THE ROYAL SOCIETY PUBLISHING

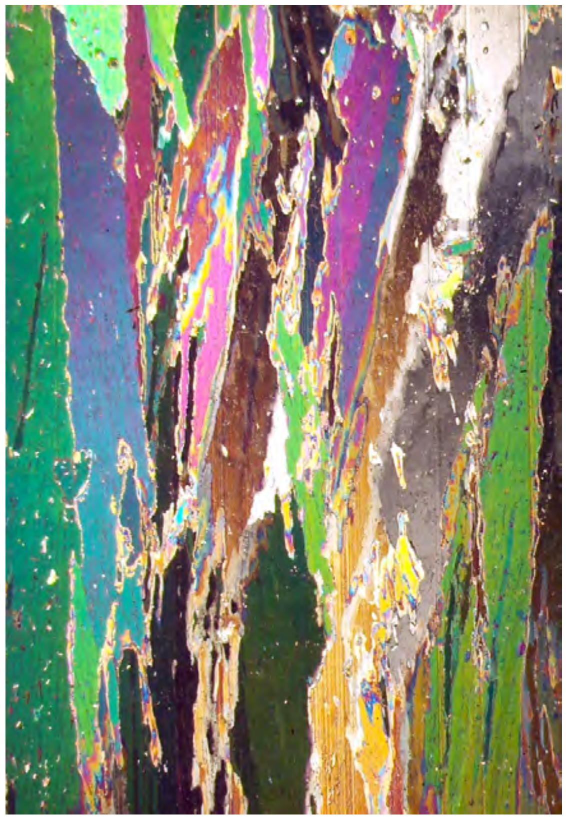
21/01/15 1:47 PM

higher threshold for fluid flow in granular sea ice

microscale details impact "mesoscale" processes

nutrient fluxes for microbes
melt pond drainage
snow-ice formation

columnar



5%

granular

10%

Golden, Sampson, Gully, Lubbers, Mosier, Tison 2024

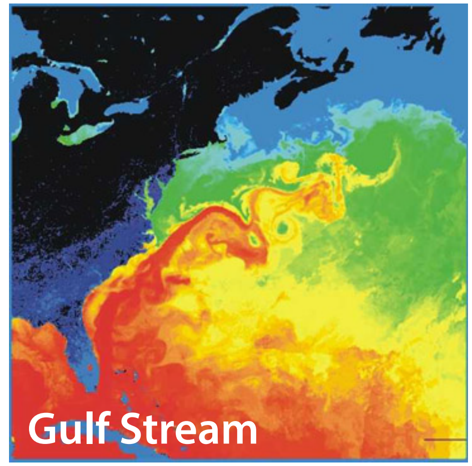
electromagnetically distinguish ice types
inverse homogenization for polycrystals

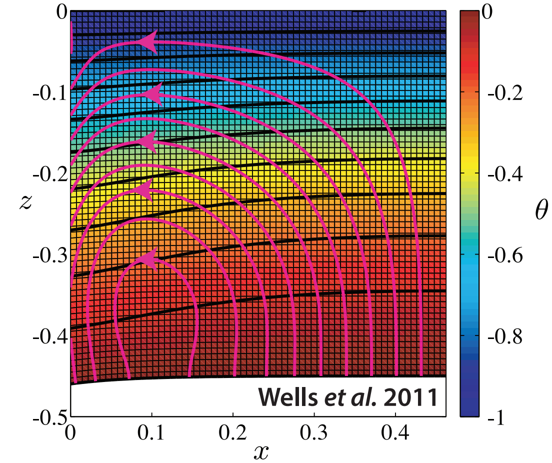
mesoscale

advection enhanced diffusion

effective diffusivity

nutrient and salt transport in sea ice
heat transport in sea ice with convection
sea ice floes in winds and ocean currents
tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere





advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

↓ ***homogenize***

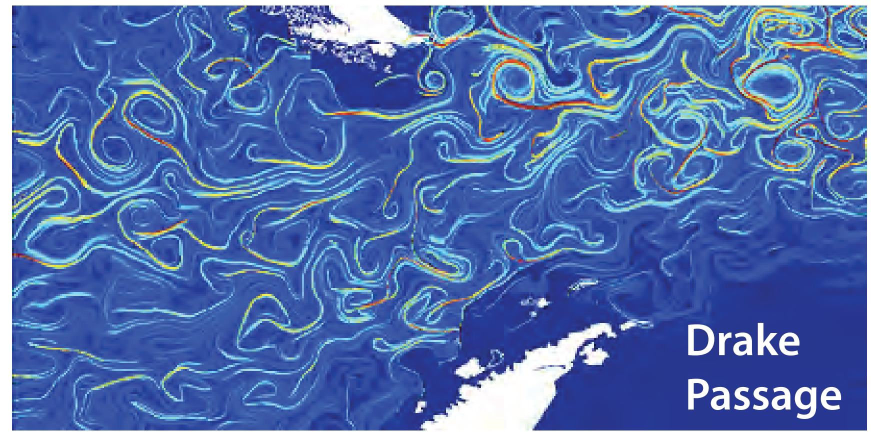
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* ***effective diffusivity***

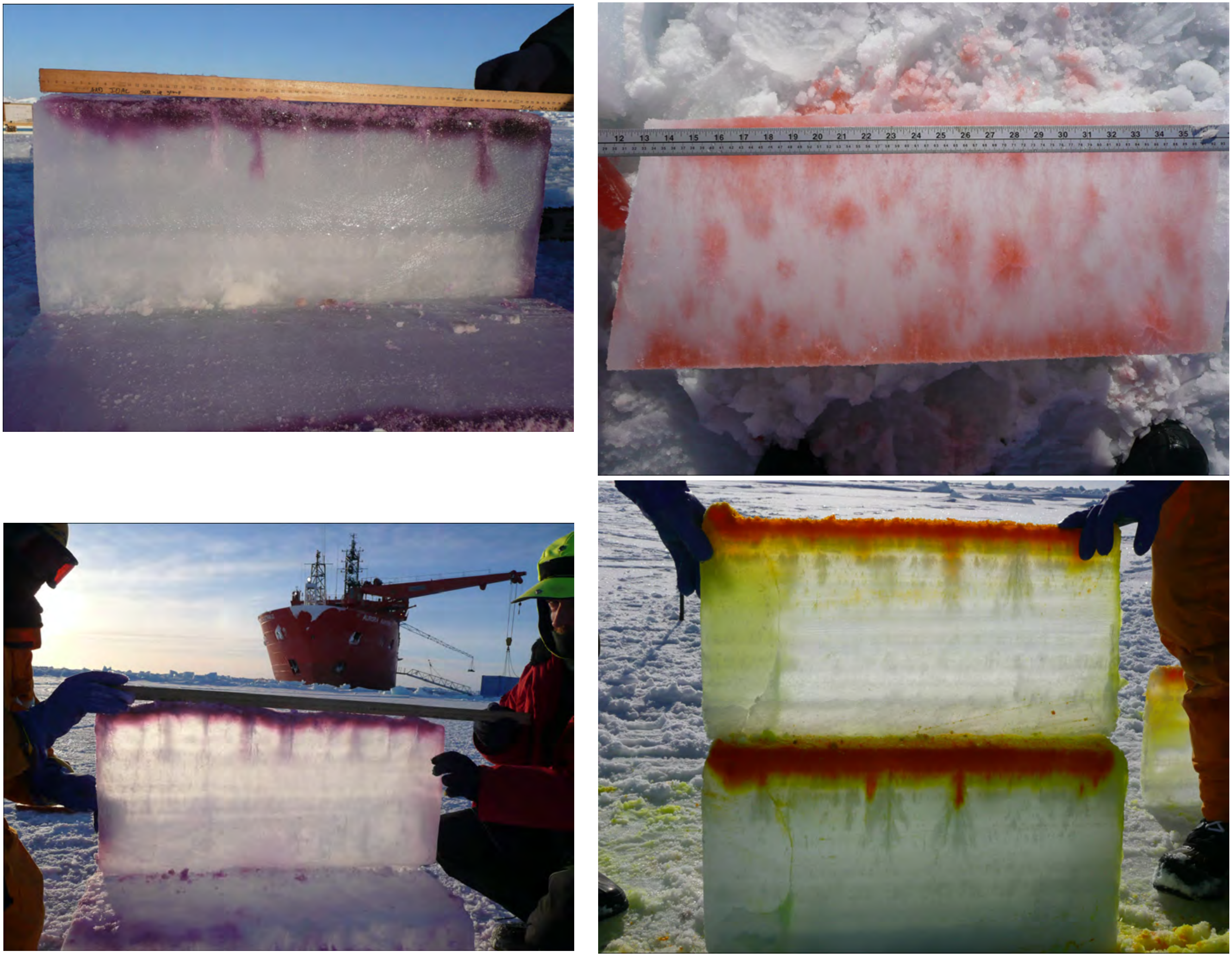
Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017
Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020



tracers flowing through inverted sea ice blocks



Stieltjes Integral Representation for Advection Diffusion

Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020

$$\kappa^* = \kappa\left(1 + \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2}\right), \quad F(\kappa) = \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2}$$

- μ is a positive definite measure corresponding to the spectral resolution of the self-adjoint operator $i\Gamma H \Gamma$
- $H =$ stream matrix , $\kappa =$ local diffusivity
- $\Gamma := -\nabla(-\Delta)^{-1}\nabla\cdot$, Δ is the Laplace operator
- $i\Gamma H \Gamma$ is bounded for time independent flows
- $F(\kappa)$ is analytic off the spectral interval in the κ-plane

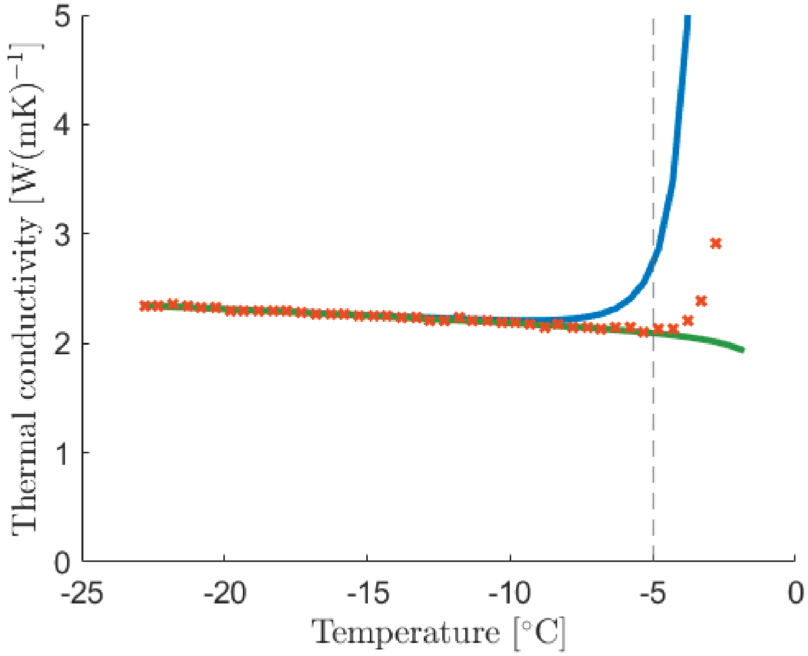
rigorous framework for numerical computations of spectral measures and effective diffusivity for model flows

new integral representations, theory of moment calculations

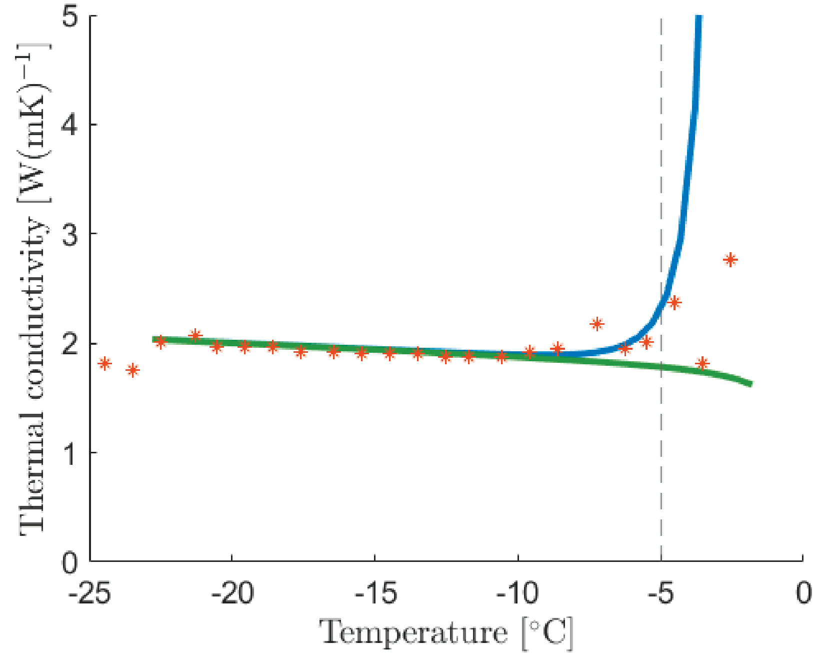
separation of material properties and flow field

Bounds on Convection Enhanced Thermal Transport

simulations



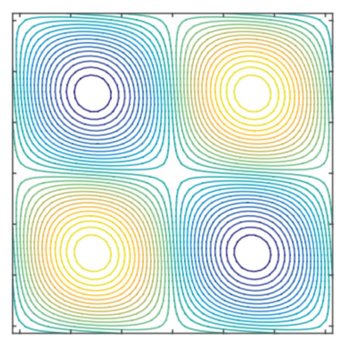
data [Trodahl et al., 2001]



Monte-Carlo simulations of SDE with temperature dependent Péclet number P

strength of advection B = κP/2π

Euler-Maruyama and subsampling methods for SDE



cat's eye flow model for brine convective flow

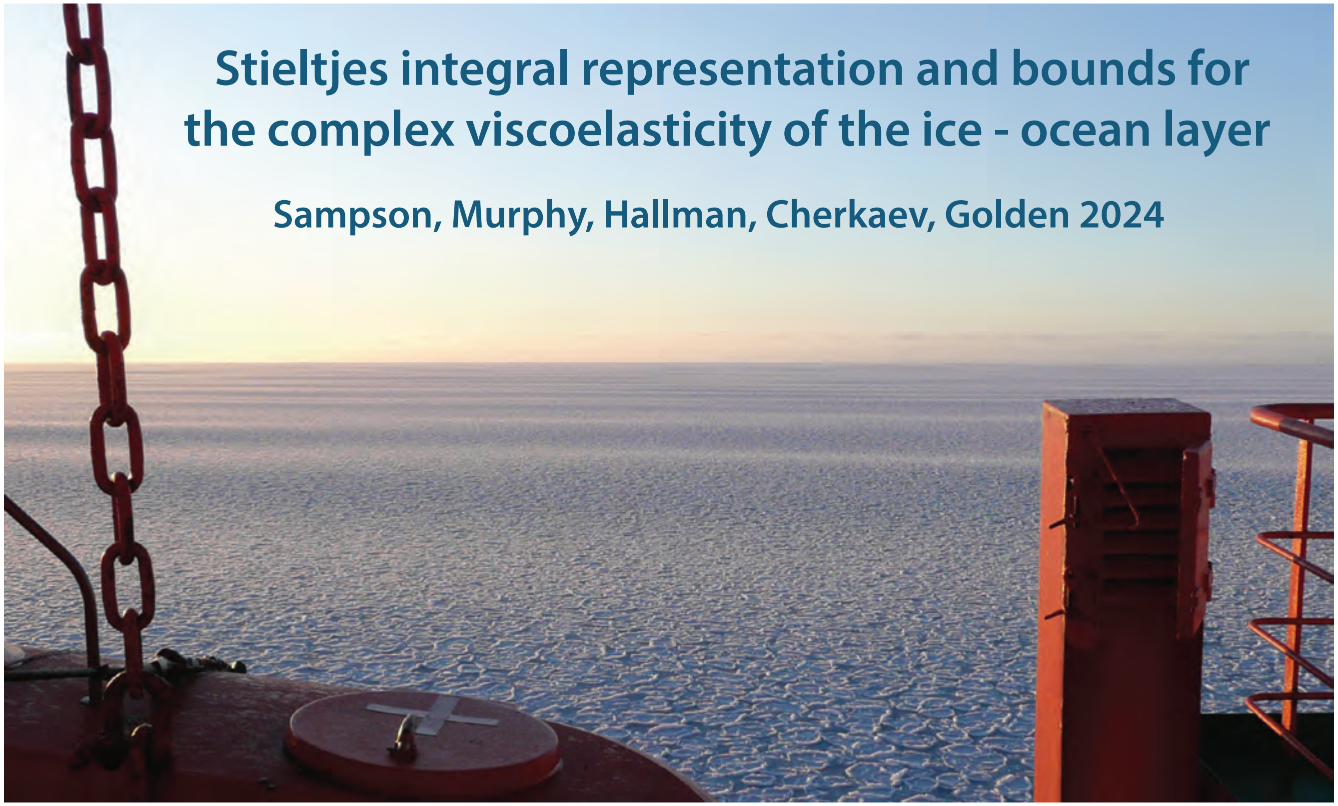
Rigorous Padé approximant bounds in terms of P using Stieltjes integral + analytic continuation method for the measure

Darcy velocity v = 0.5 [m/s]

Kraitzman, Hardenbrook, Dinh, Murphy, Cherkaev, Zhu, & Golden, 2024

Proc. Royal Soc. A, in press

ocean wave propagation through the sea ice pack



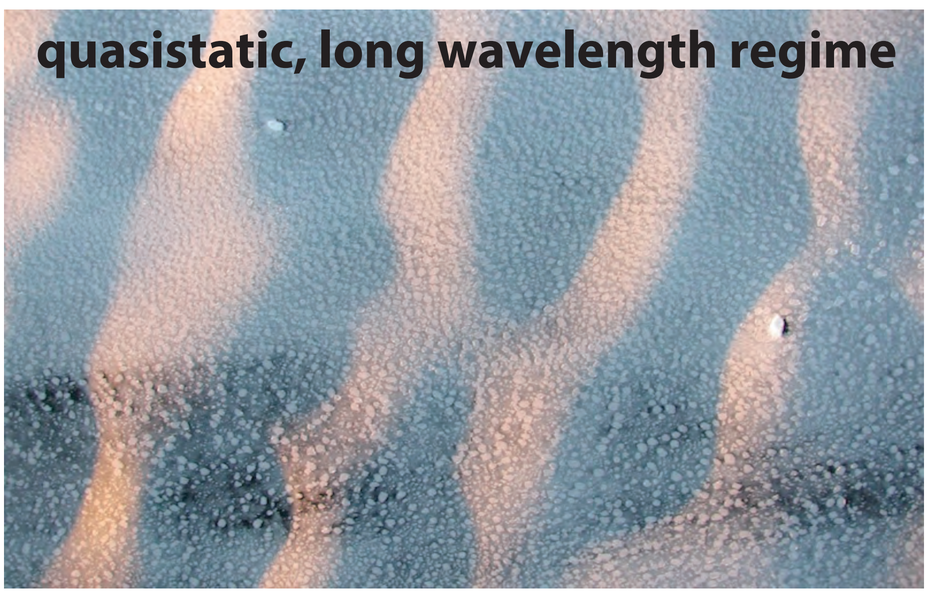
- wave-ice interactions critical to growth and melting processes
- break-up; pancake promotion floe size distribution

effective layer parameter previously fit to wave data

Keller 1998
Mosig, Montiel, Squire 2015
Wang, Shen 2012

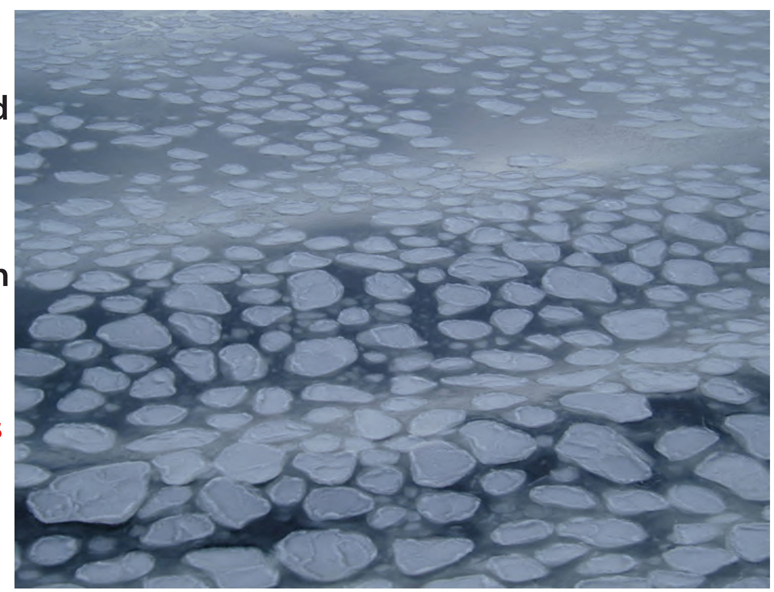
Analytic Continuation Method

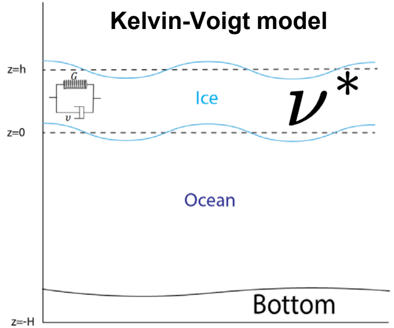
Bergman 1978, Milton 1979
Golden and Papanicolaou 1983
Milton, *Theory of Composites* 2002



homogenized parameter depends on sea ice concentration and ice floe geometry

like EM waves





Single effective rheological parameter (Mosig et al. 2015)

$$\nu^* = G - i\omega\rho v$$

Effective complex shear modulus

Integral representation

$$\frac{\nu^*}{\nu_2} = ||\epsilon_s^0||^2(1 - F(s))$$

$$F(s) = \int_0^1 \frac{d\mu(\lambda)}{s-\lambda}$$

divergence-free deviatoric stress

$$\nabla \cdot \sigma_s = 0$$

microscale

$$\sigma_s = 2\nu\epsilon_s$$

$$\nu(\vec{x}) = \chi_1\nu_1 + \chi_2\nu_2$$

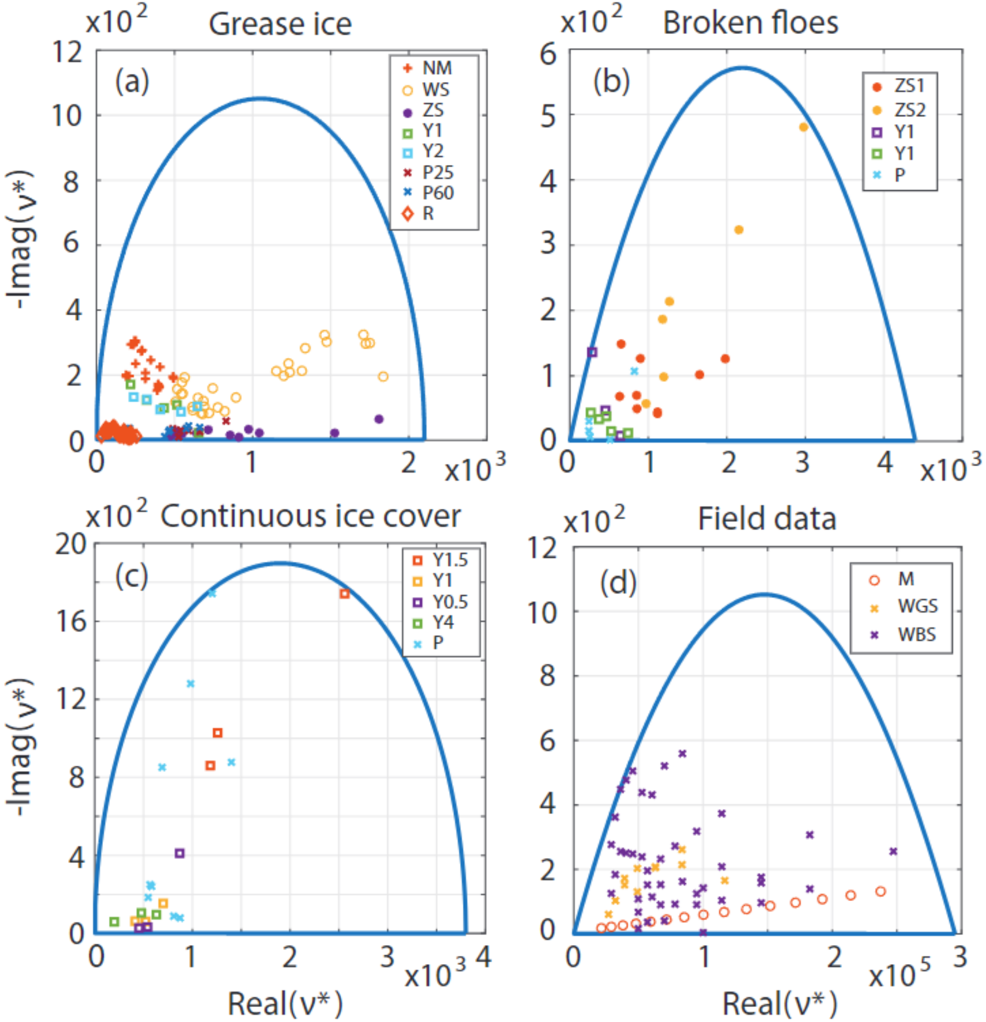
macroscale

$$\langle\sigma_s\rangle = 2\nu^*\epsilon_s^0$$

$$\langle\epsilon_s\rangle = \epsilon_s^0$$

Forward bounds for the effective viscoelasticity are fitted to well known wave-ice datasets, including *Wadhams et al. 1988, Newyear & Martin 1997, Wang & Shen 2010, Meylan et al. 2014*, and several others!

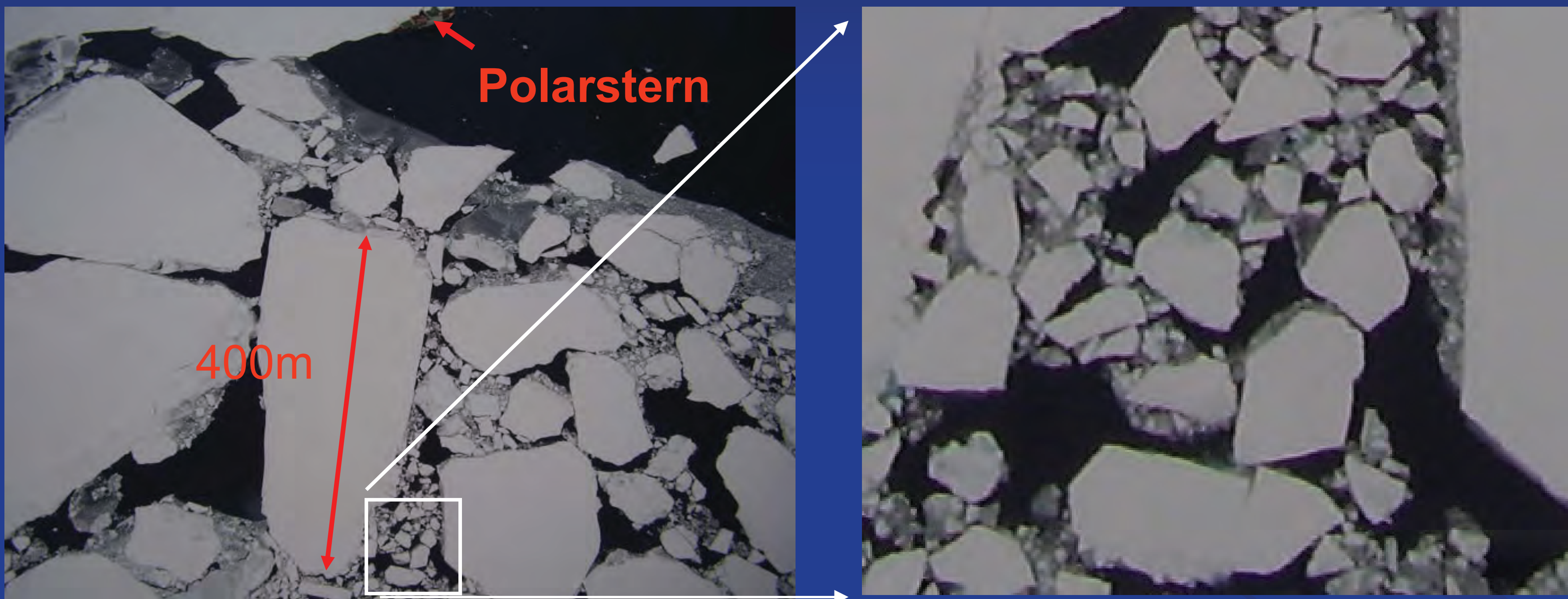
G	P	C	ρ	v	g	u
shear modulus	pressure	elasticity	density	kinematic viscosity	gravity	displacement



The sea ice pack has fractal structure.

Self-similarity of sea ice floes

Weddell Sea, Antarctica



fractal dimensions of Okhotsk Sea ice pack
smaller scales D~1.2, larger scales D~1.9

fractal dim. *vs.* floe size exponent
Adam Dorsky, Nash Ward, Ken Golden 2024

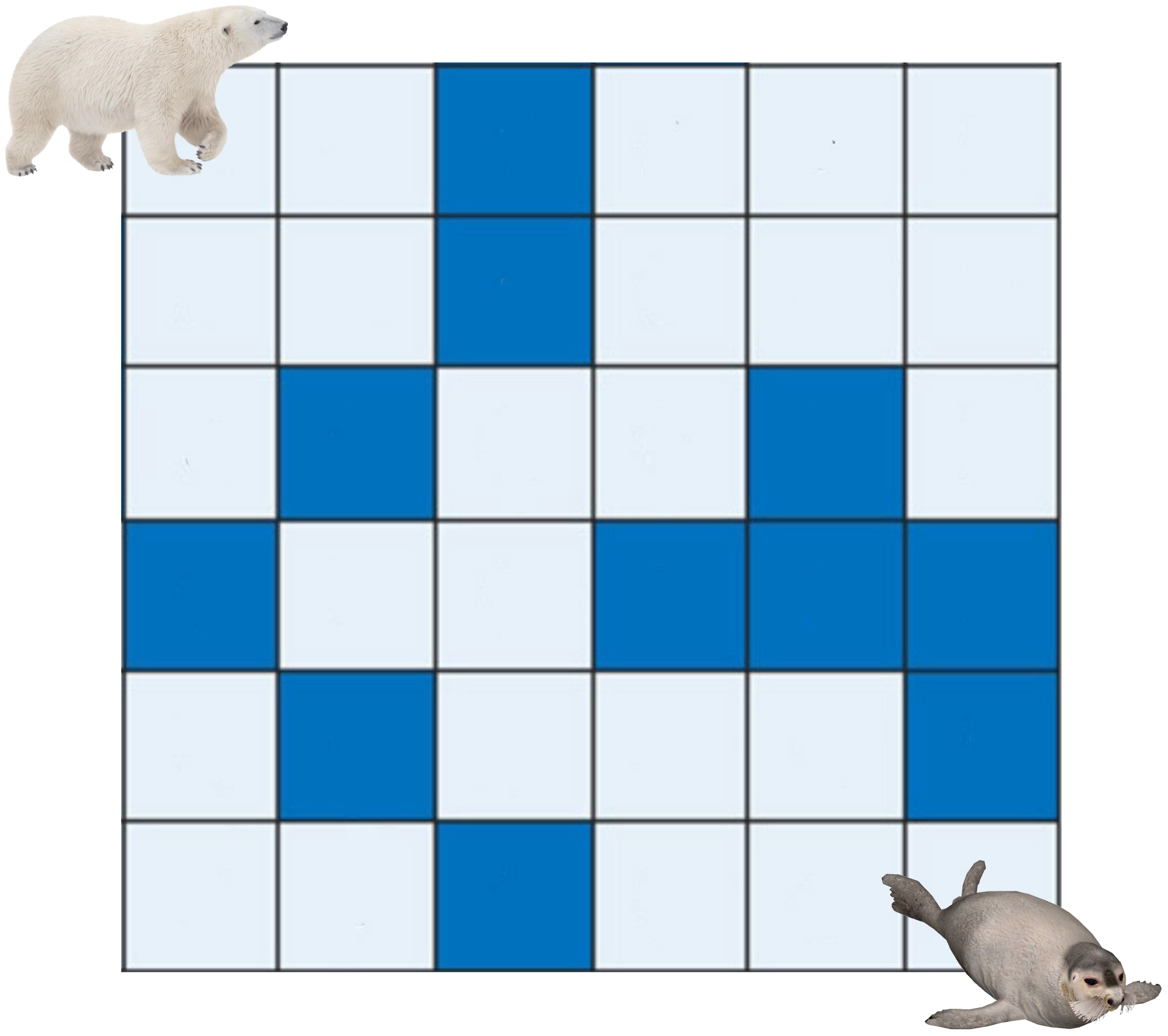
Toyota, *et al. Geophys. Res. Lett.* 2006
Rothrock and Thorndike, *J. Geophys. Res. 1984*

Optimal Movement of a Polar Bear in a Heterogenous Icescape

Nicole Forrester, Jody Reimer, Ken Golden 2024

Polar bears expend 5X more energy swimming than walking on sea ice.

As sea ice is lost, how do polar bears optimize their movement to save energy and survive?

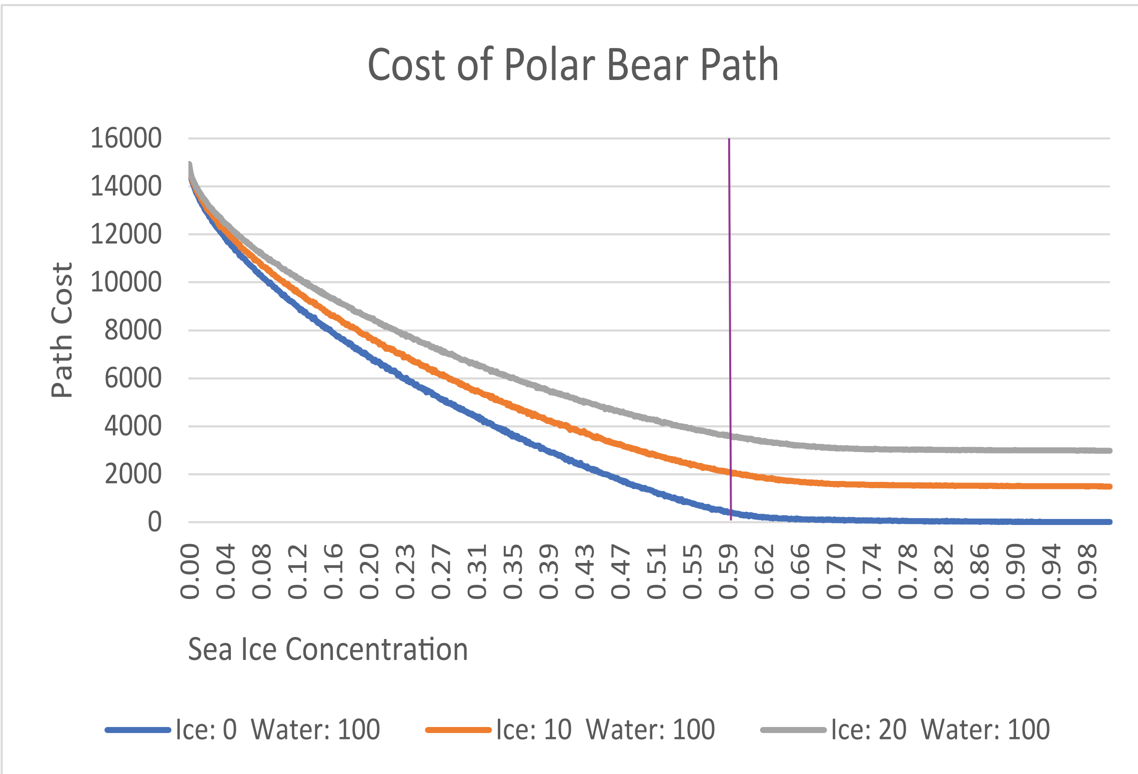


https://pluspng.com/img-png/polar-bear-png-polar-white-bear-png-1000.png
https://zt2downloadlibrary.fandom.com/wiki/Bearded_Seal_(Tamara_Henson)

Polar Bear Percolation

To study the importance of ice connectedness, we exaggerate the data by setting the cost of walking on ice to 0 with the cost of swimming still at 5.

$C(p)$



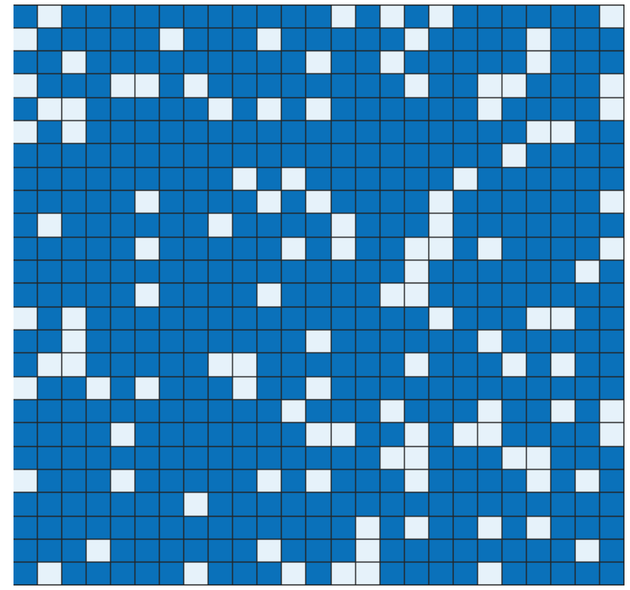
← $h = 0.2$

← $h = 0.1$

← $h = 0$

$$h = \frac{c_i}{c_w}$$

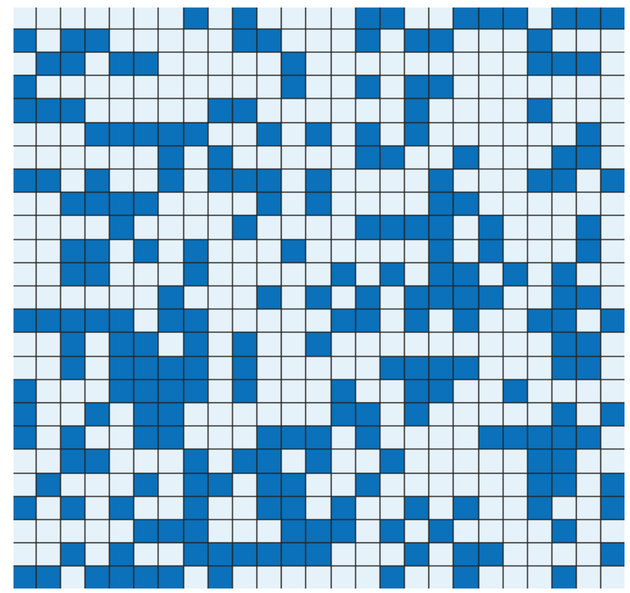
ratio of local “conductivities”



20% Ice

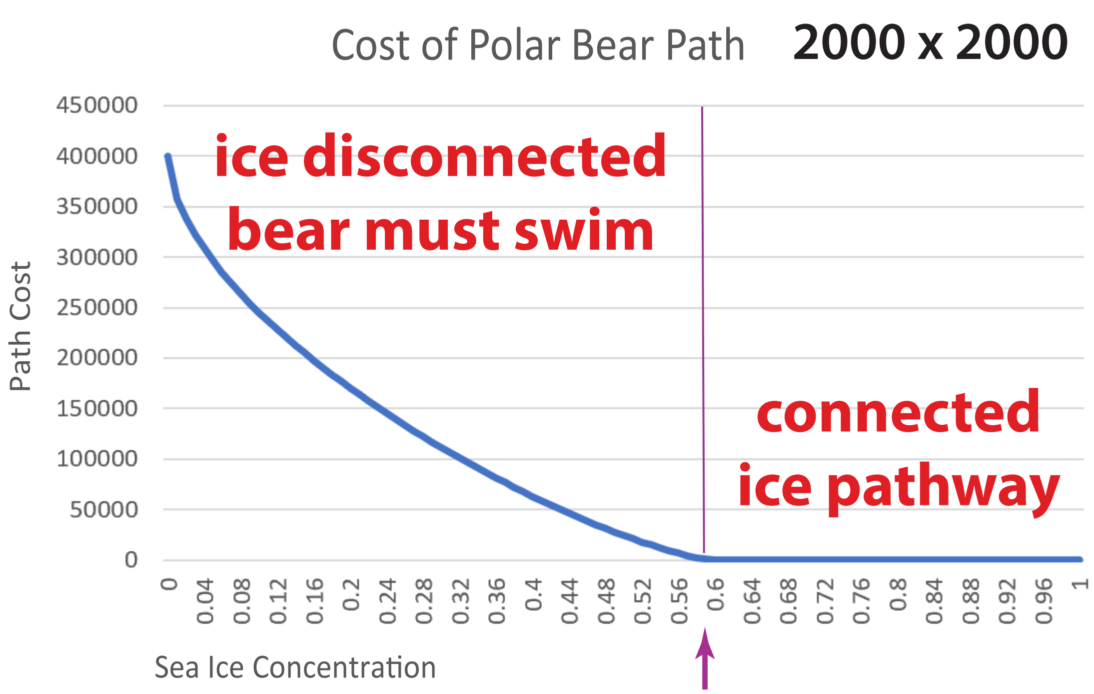
site percolation threshold

$p_c = 0.59$ for $d = 2$



60% Ice

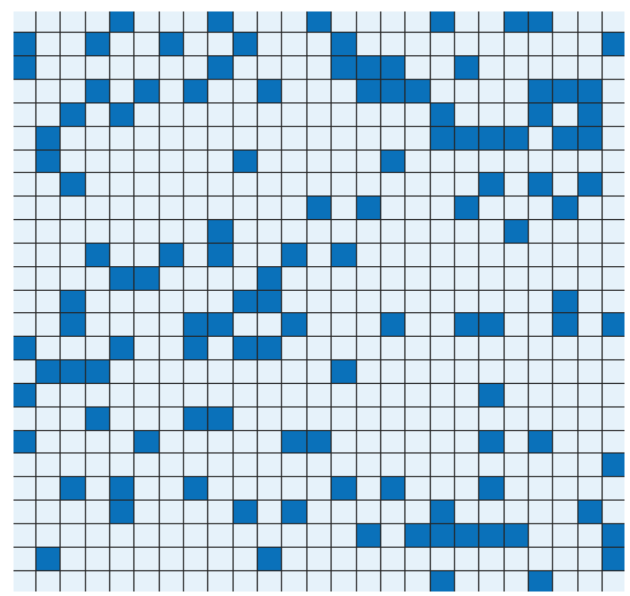
$C(p)$



p_c

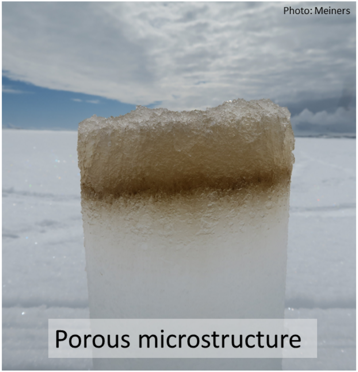
← $h = 0$

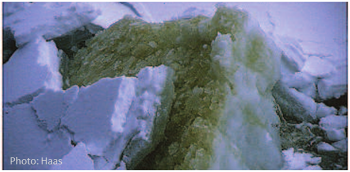
Polar Bear Critical Exponent

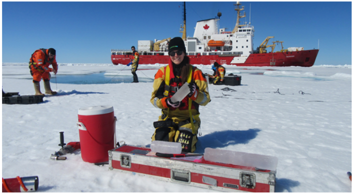


80% Ice

SEA ICE ALGAE high level of local heterogeneity







Can we improve agreement between algae models and data?

80% of polar bear diet can be traced to ice algae*.

* Brown TA, et al. (2018). *PloS one*, 13(1), e0191631

Received: 7 June 2022 | Revised: 1 August 2022 | Accepted: 1 August 2022
DOI: 10.1111/ele.14095

ECOLOGY LETTERS cnrs WILEY

METHOD

Uncertainty quantification for ecological models with random parameters

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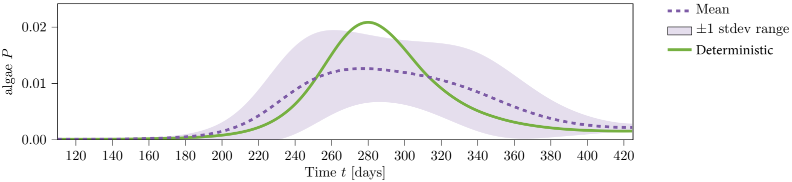
Abstract
There is often considerable uncertainty in parameters in ecological models. This uncertainty can be incorporated into models by treating parameters as random variables with distributions, rather than fixed quantities. Recent advances in uncertainty quantification methods, such as polynomial chaos approaches, allow for the analysis of models with random parameters. We introduce these methods with a motivating case study of sea ice algal blooms in heterogeneous environments. We compare Monte Carlo methods with polynomial chaos techniques to help understand the dynamics of an algal bloom model with random parameters.

N-P Model

Introduce polynomial chaos approach to widely used ecological ODE models, but with random parameters.

Ecological insights



- lower peak bloom intensity
- longer bloom duration
- able to compare variance to data

Inverse Problem: given algal and nutrient data, recover growth rate distribution
Anthony Lee, Jody Reimer, Akil Narayan, Ken Golden 2024

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

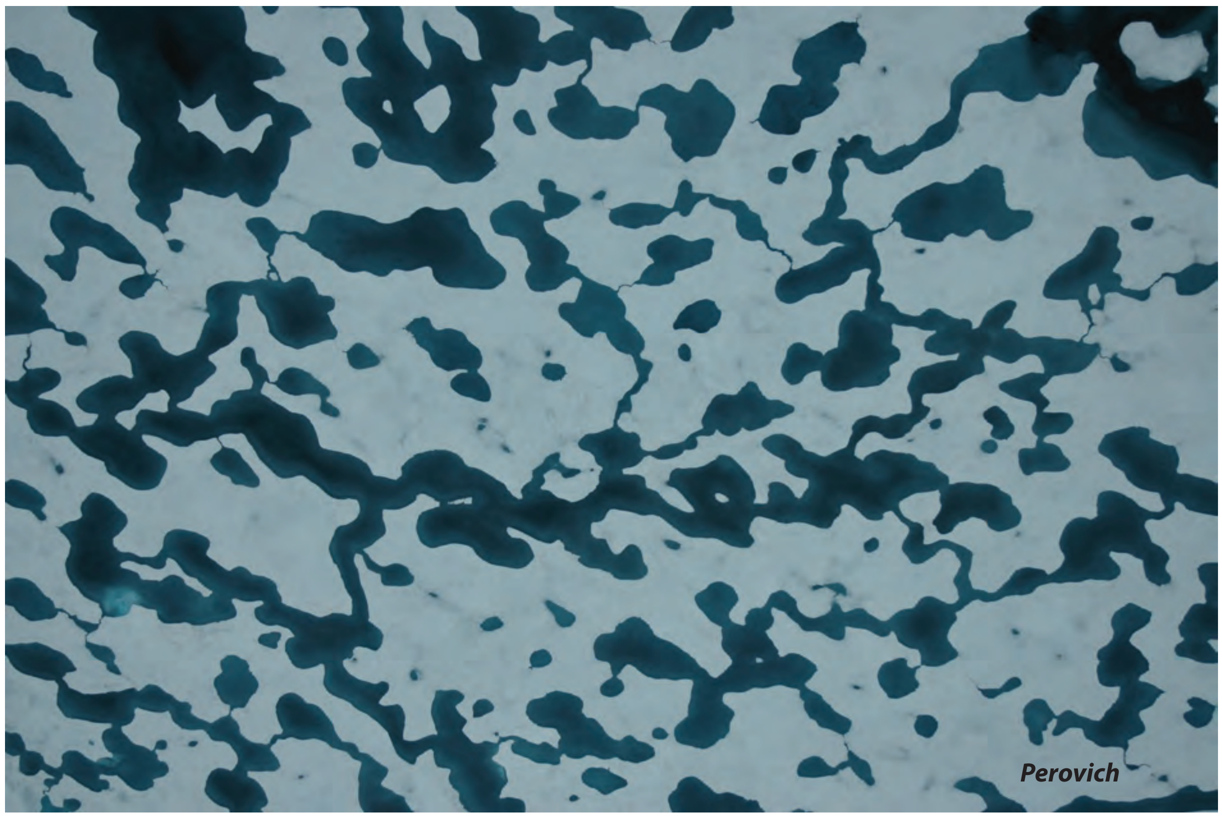
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

Flocco, Feltham, Hunke 2012



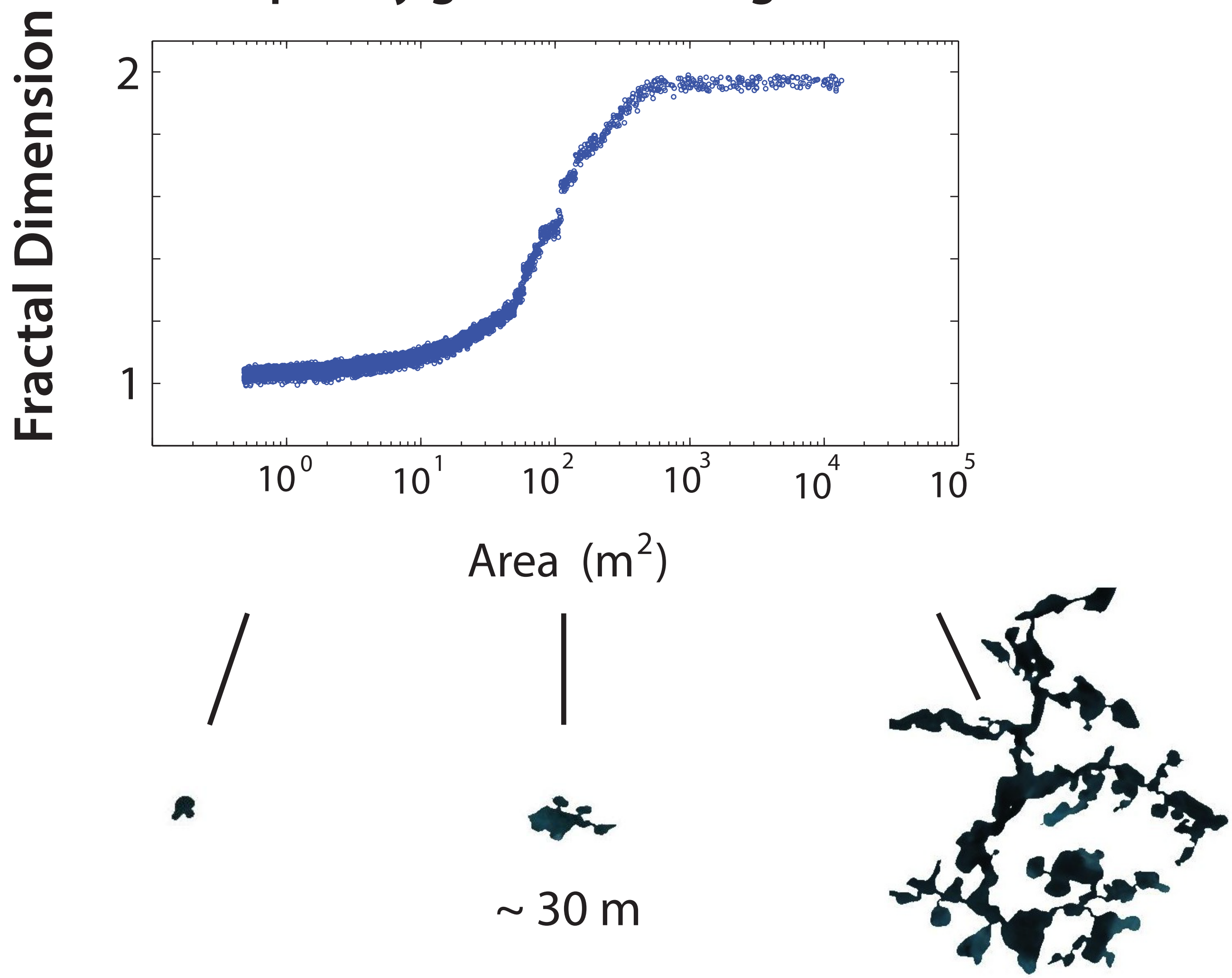
Are there universal features of the evolution similar to phase transitions in statistical physics?

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

complexity grows with length scale

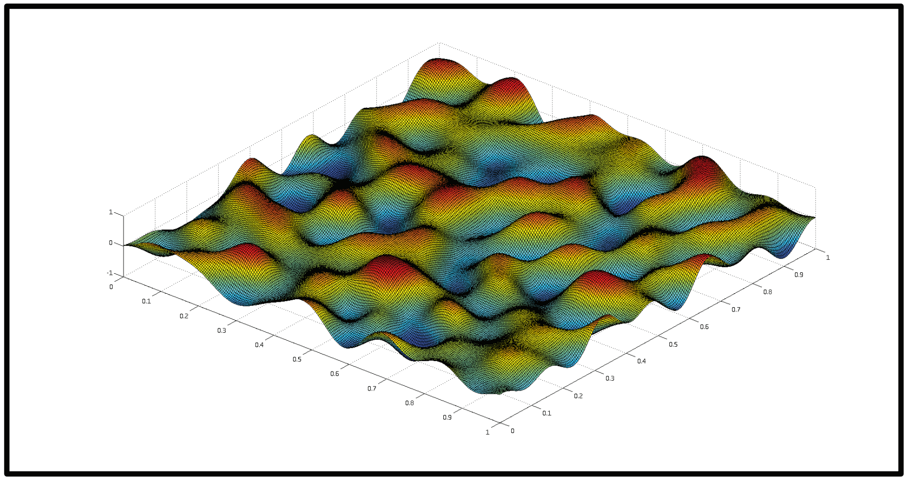


simple pond ***transitional pond*** ***complex pond***

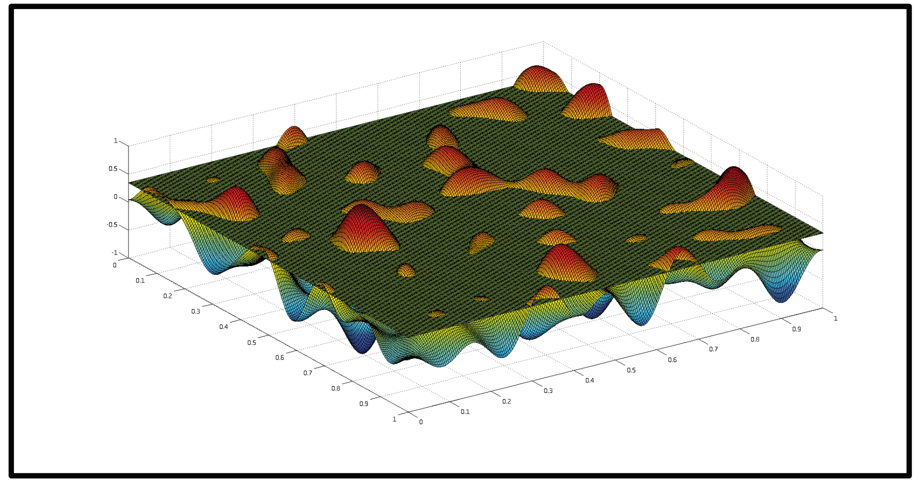
Continuum percolation model for melt pond evolution

level sets of random surfaces

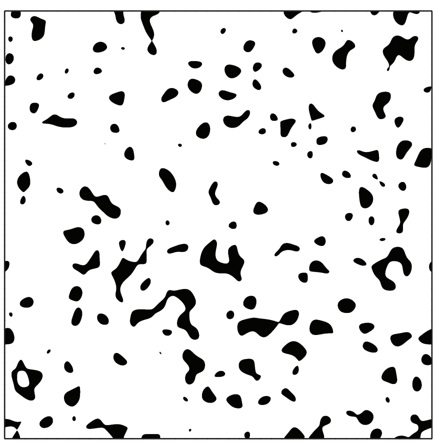
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018

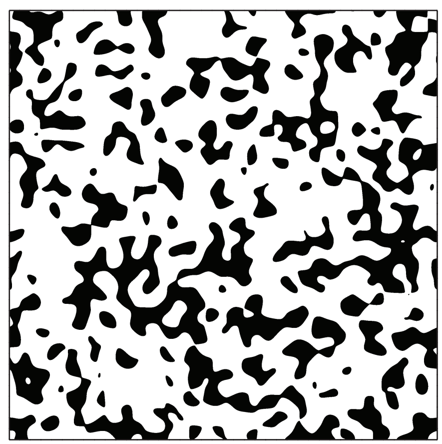


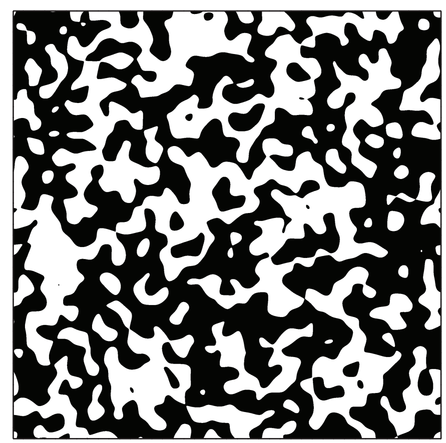
random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds







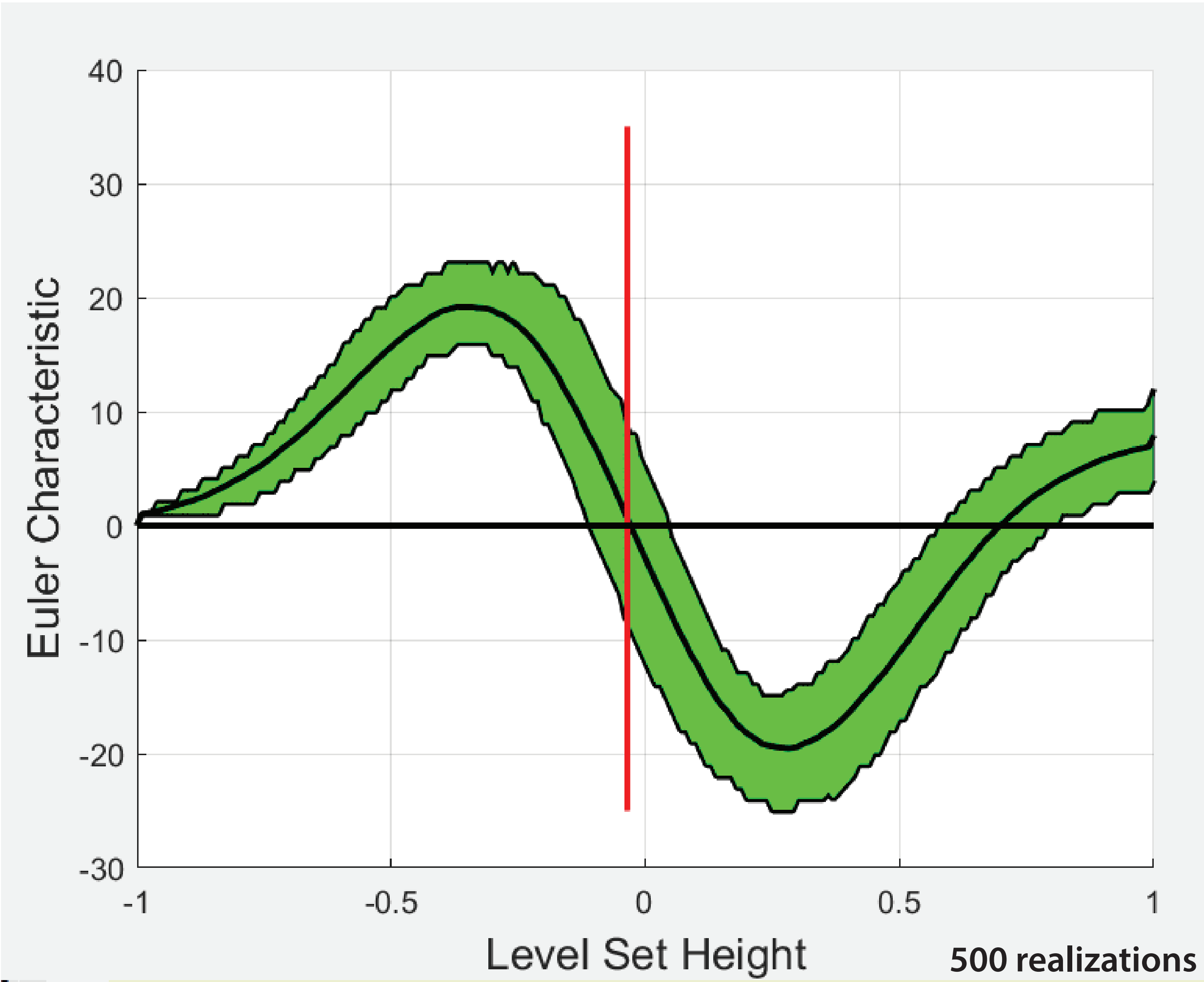
electronic transport in disordered media *diffusion in turbulent plasmas* *Isichenko, Rev. Mod. Phys., 1992*

Topological Data Analysis

Euler characteristic = # maxima + # minima - # saddles

topological invariant

filtration - sequence of nested topological spaces, indexed by water level



Expected
Euler Characteristic Curve (ECC)

tracks the evolution of the EC of the flooded surface as water rises

zero of ECC ~ percolation

percolation on a torus
creates a giant cycle

Bobrowski & Skraba, 2020

Carlsson, 2009

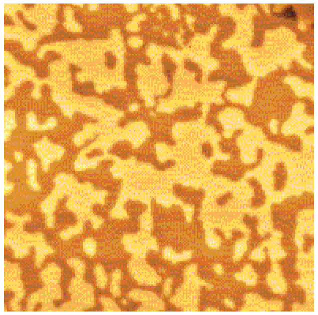
Vogel, 2002 GRF

image analysis
porous media
cosmology
brain activity

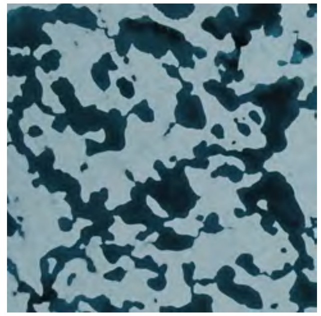
Physical Review Research (invited, in revision), R. Moore, J. Jones, D. Gollero, R. Hardenbrook, C. Strong, K. M. Golden 2024

From magnets to melt ponds

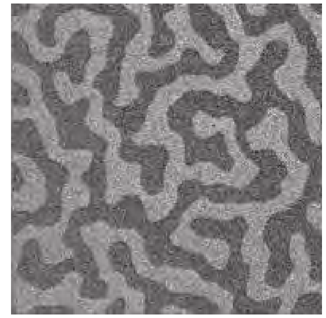
100 year old model for magnetic materials used to explain melt pond geometry



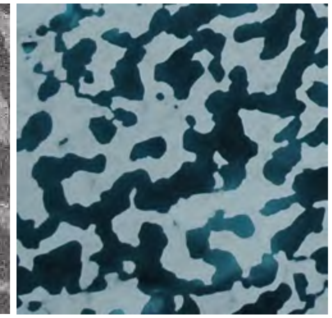
magnetic domains in cobalt



Arctic melt ponds

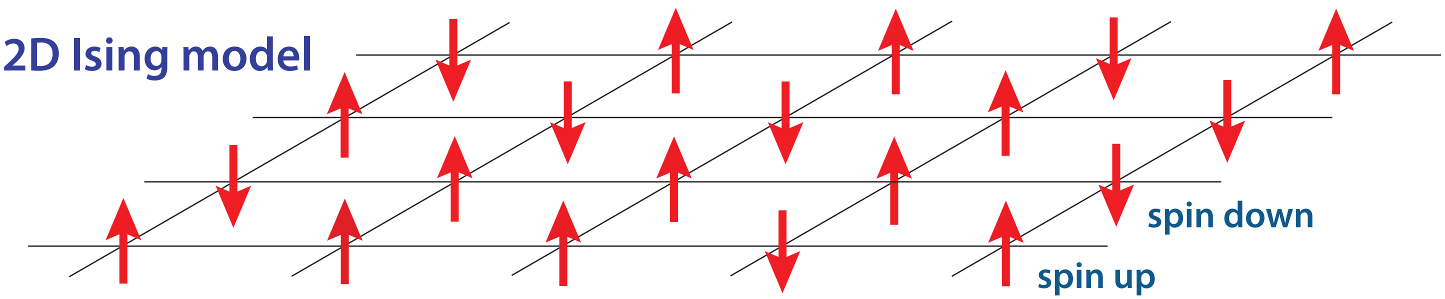


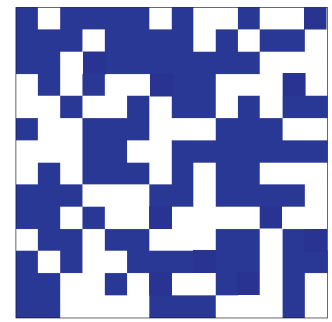
magnetic domains in cobalt-iron-boron



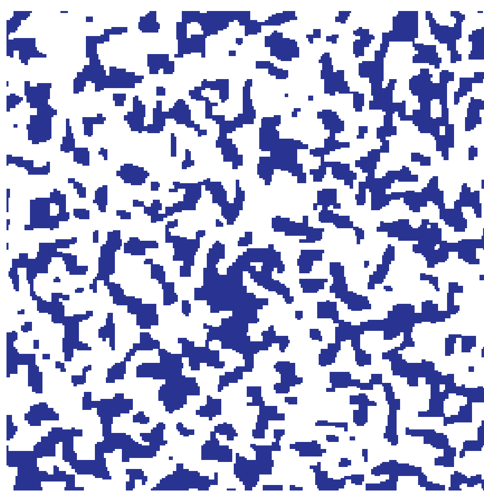
Arctic melt ponds

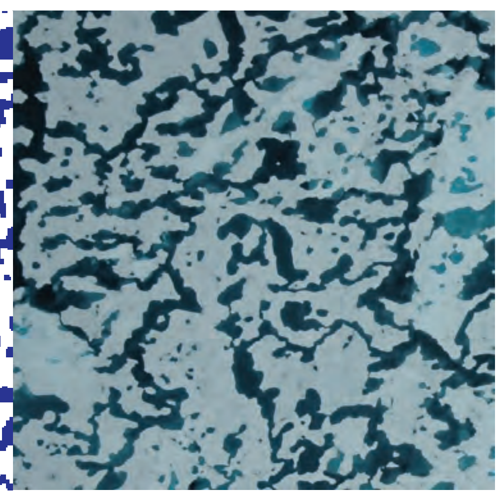
2D Ising model





model





real ponds (Perovich)

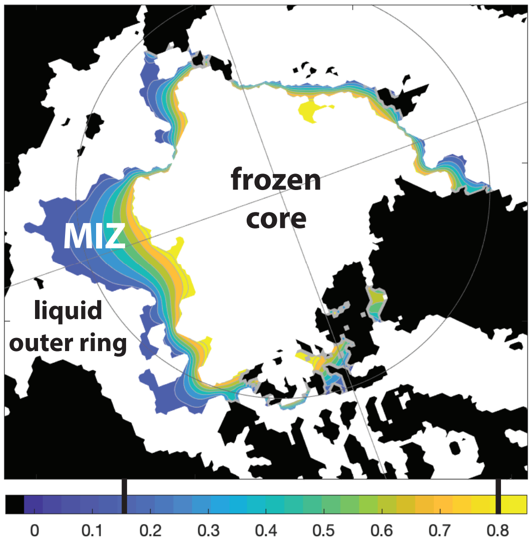
Ma, Sudakov, Strong, Golden, *New J. Phys.* 2019

Scientific American, EOS, PhysicsWorld, ...

macroscale

Model larger scale effective behavior with partial differential equations that *homogenize* complex local structure and dynamics.

Arctic MIZ



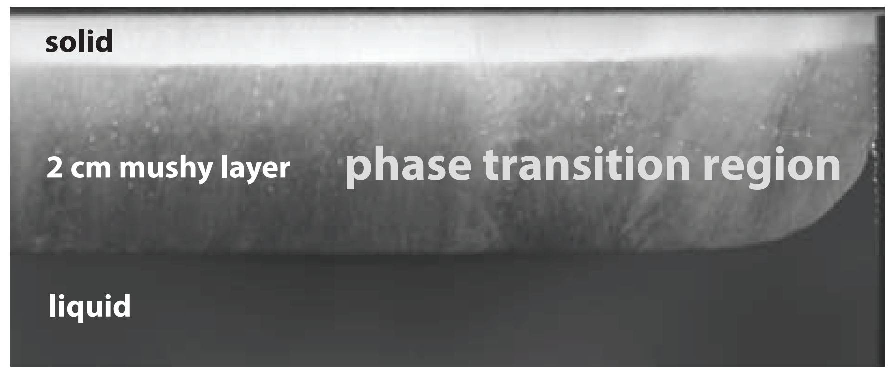
Partial differential equation models and deep learning for the sea ice concentration field, 2024

Delaney Mosier, Debdeep Bhattacharya, Court Strong, Jingyi Zhu, Bao Wang, Ken Golden

advection diffusion model

Predict MIZ width and location with basin-scale phase change model.

seasonal and long term trends



NaCl-H2O in lab
(Peppin et al., 2007;, J. Fluid Mech.)

Multiscale mushy layer model of Arctic marginal ice zone dynamics, 2024

C. Strong, E. Cherkaev, and K. M. Golden

northward 1600 km & widens by factor of 4

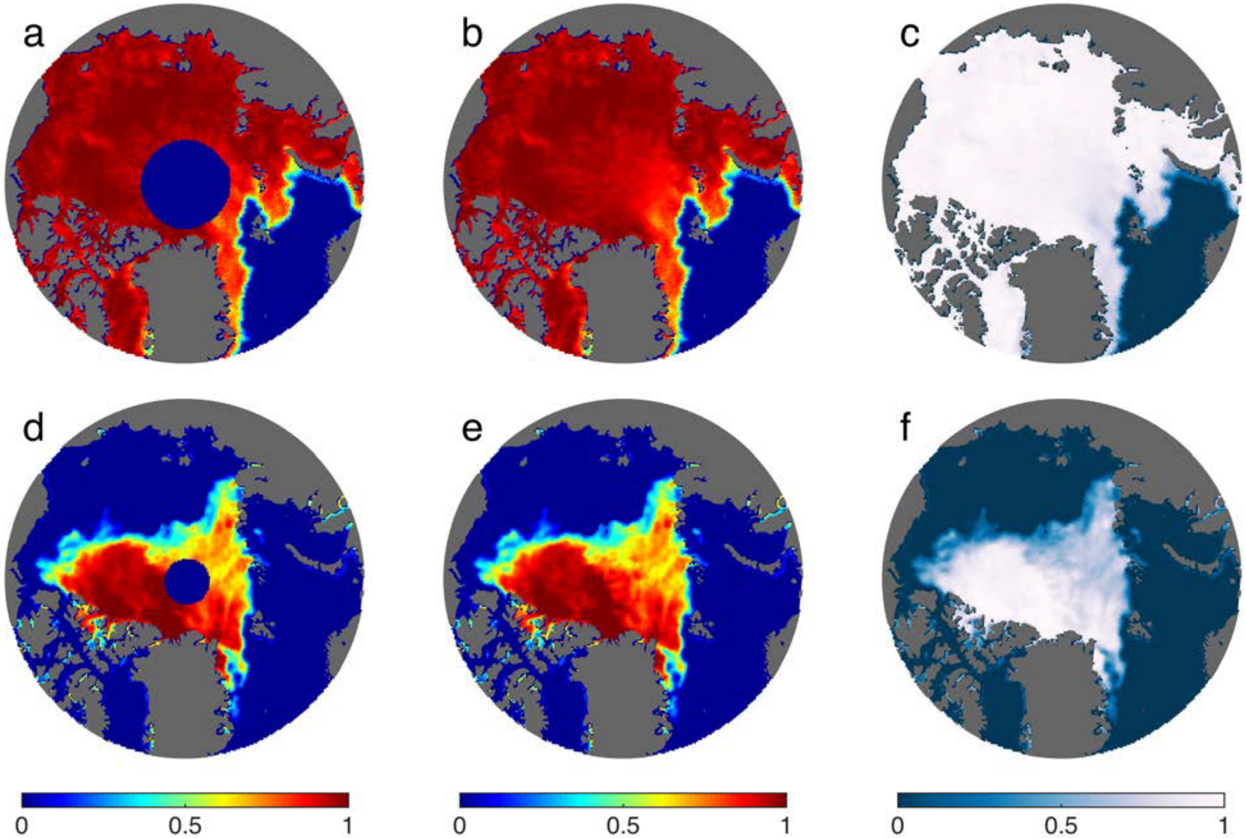
Filling the polar data gap with partial differential equations

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007



$$\Delta\psi=0$$

fill = harmonic function satisfying satellite BC's plus learned stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

Global Sea Ice Concentration Climate Data Records, 2022
Lavergne, Sorensen, et al., Norwegian Met. Inst., ... OSI SAF

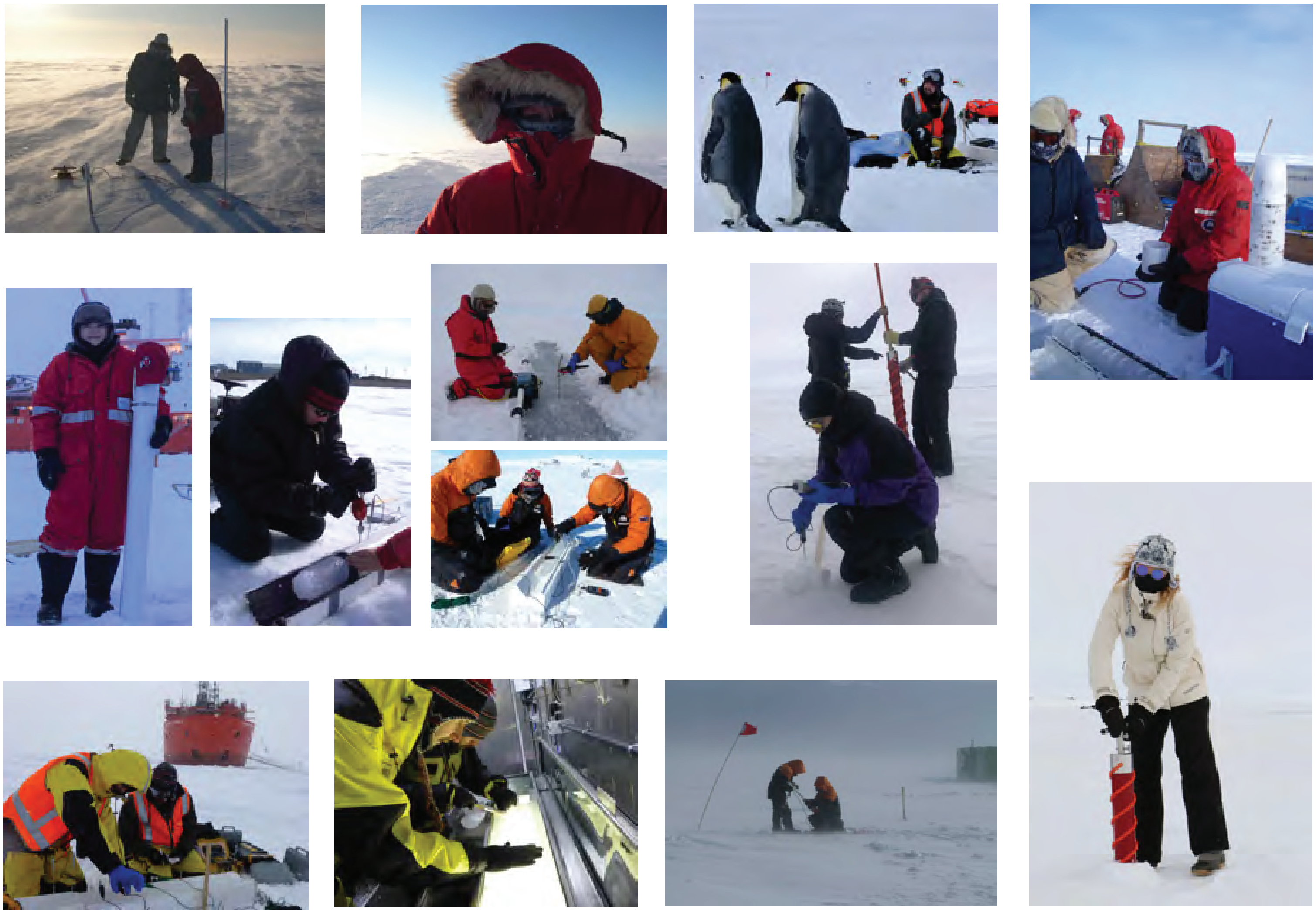
Conclusions

Our research is helping to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

Mathematics for sea ice advances the theory of composites, inverse problems, and other areas of science and engineering.

Modeling sea ice leads to unexpected areas of math and physics.

Thank you to so many postdocs, graduate students, undergraduates, high school students and colleagues who contributed to this work!



U. of Utah students in the Arctic and Antarctic (2003-2022): closing the gap between theory and observation - making math models come alive and experiencing climate change firsthand.

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Notices

of the American Mathematical Society

November 2020

Volume 67, Number 10

AMERICAN MATHEMATICAL SOCIETY

Advancing research. Creating connections.

The cover is based on "Modeling Sea Ice," page 1535.

NSF Research Training Grant (RTG) with 15 Applied Math faculty:

optimization and inverse problems

July 2022 - June 2027

Overall goal: Build an advanced, competitive U.S. STEM workforce.

- Strengthen our graduate and postdoctoral programs in applied math to attract top students in the nation, and place them in top jobs.
- Provide transformative experiences that draw students into math.

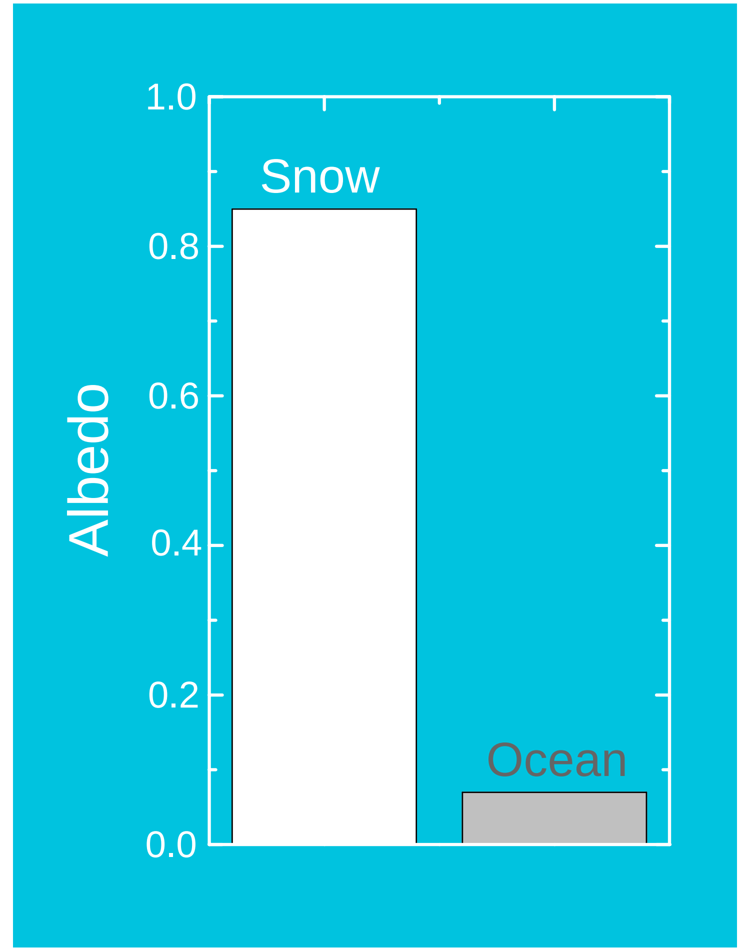
Arctic Mathpeditions - May 2024 & 2026

OPEN POSITIONS:
Postdoctoral, Ph.D., Undergraduate

Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

polar ice caps critical to global climate in reflecting incoming solar radiation

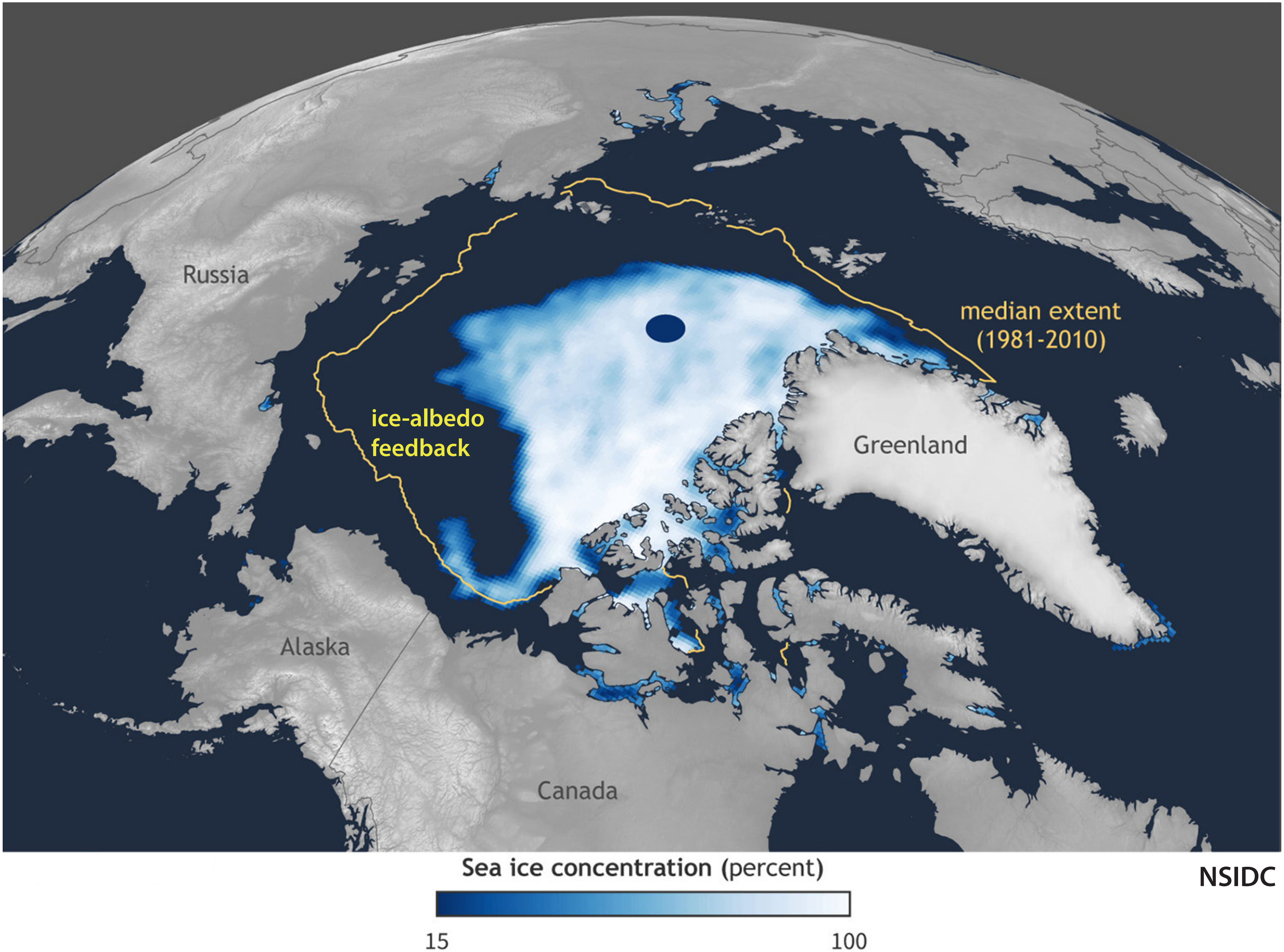
white snow and ice reflect



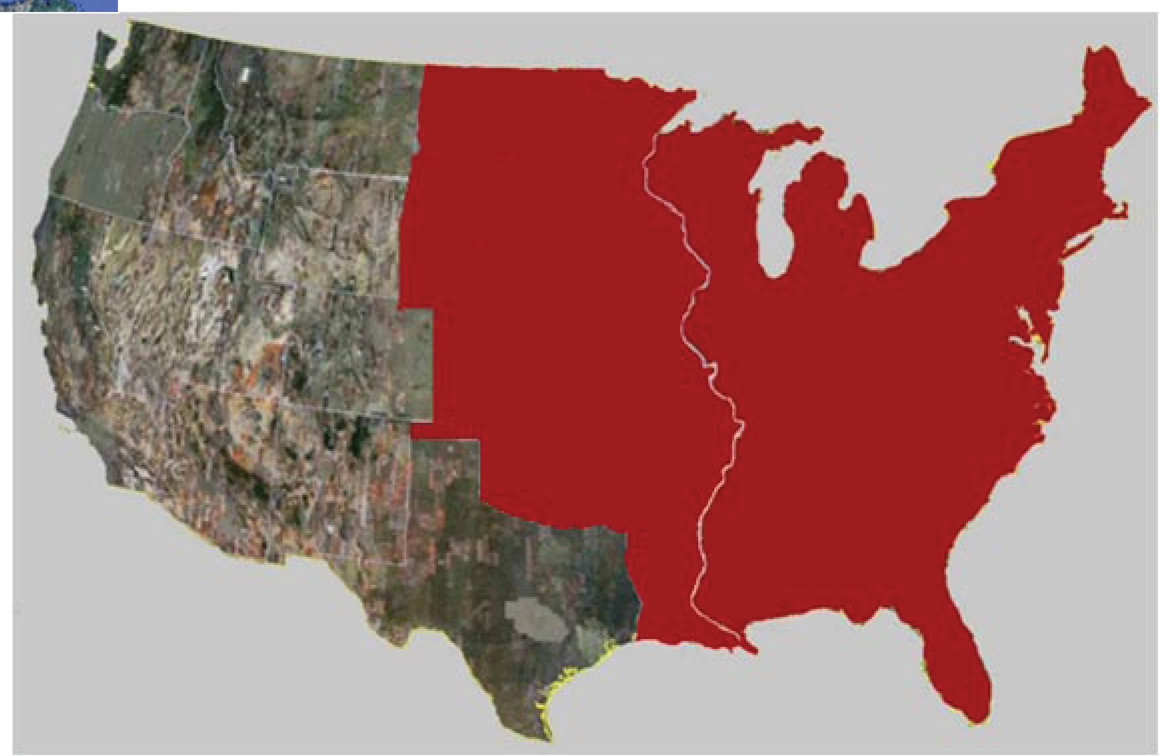
dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

Arctic sea ice extent September 15, 2020



recent losses
in comparison to
the United States

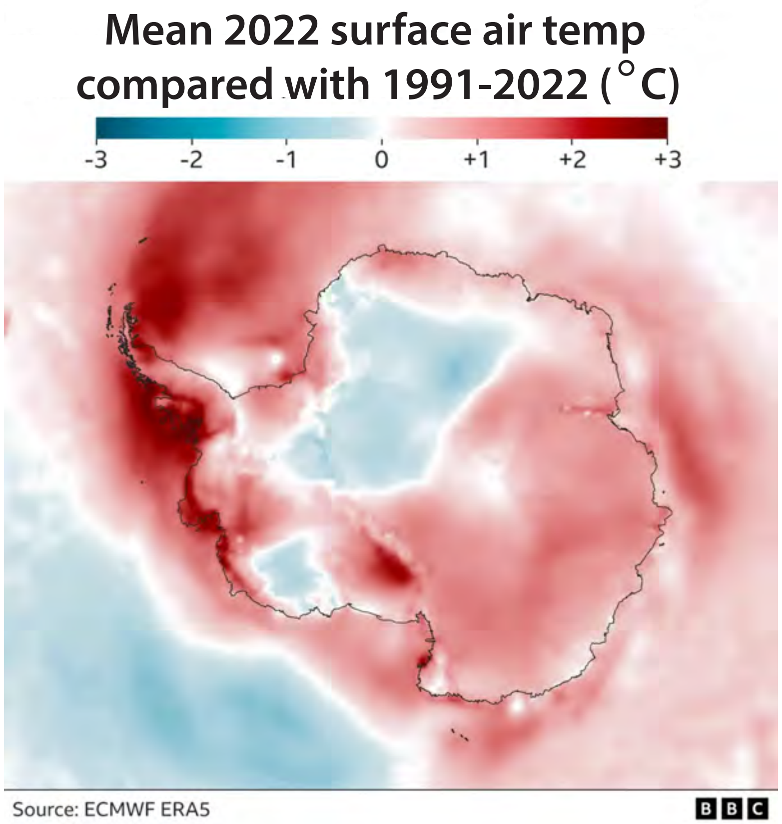


Perovich

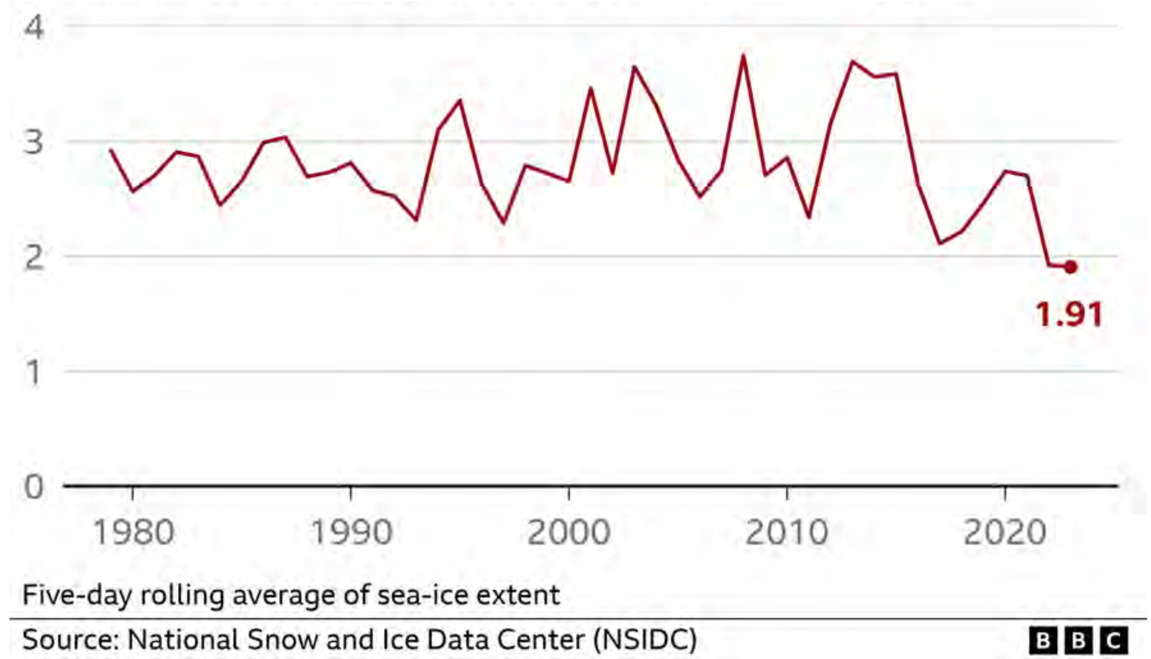
New Record Low for Antarctic Sea Ice

February 13, 2023

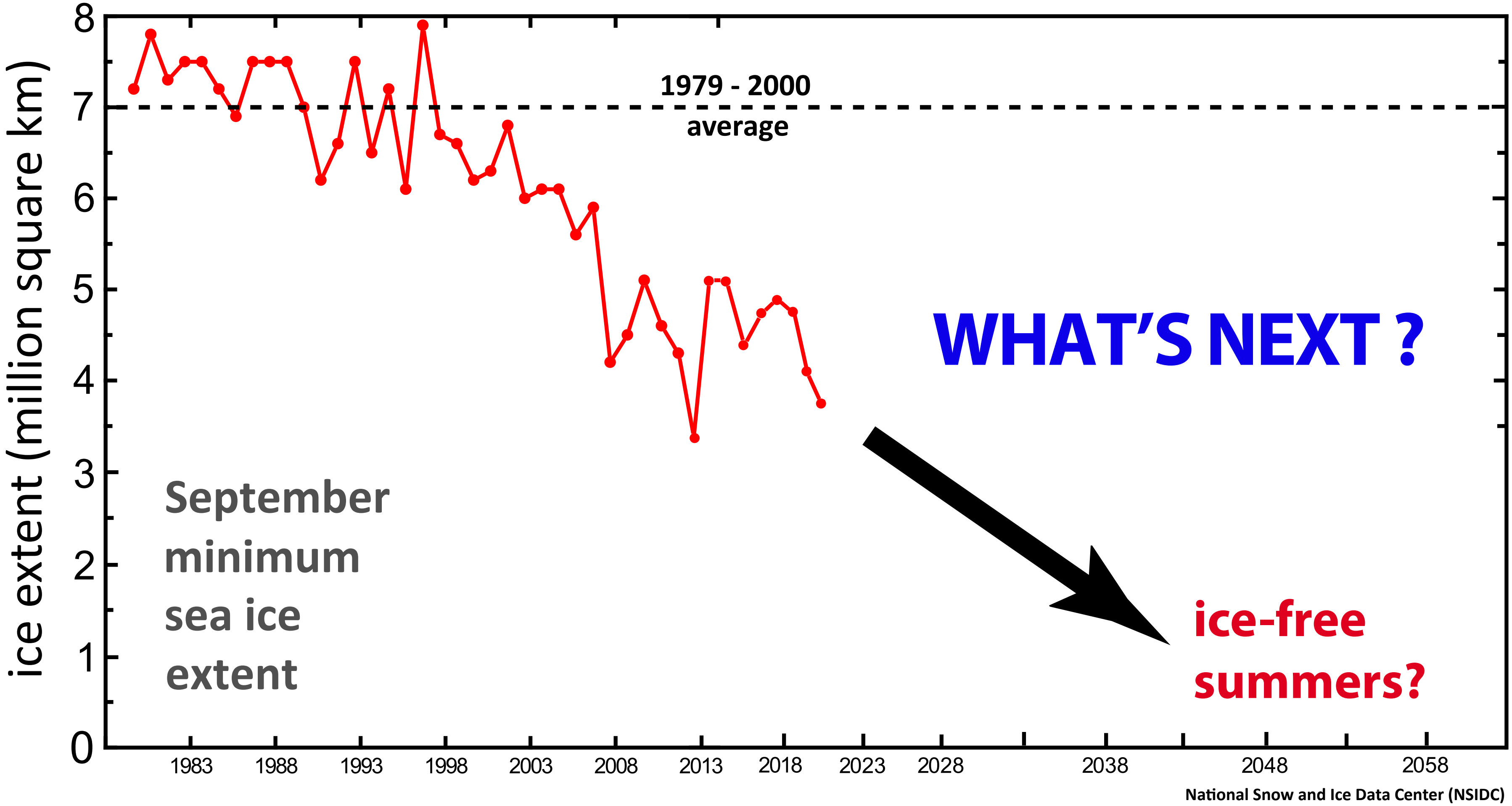
Much of Antarctica warmer than average



Minimum extent 1979-2023 (million sq km)



ARCTIC summer sea ice loss



predictions require lots of math modeling

sea ice algal communities

D. Thomas 2004

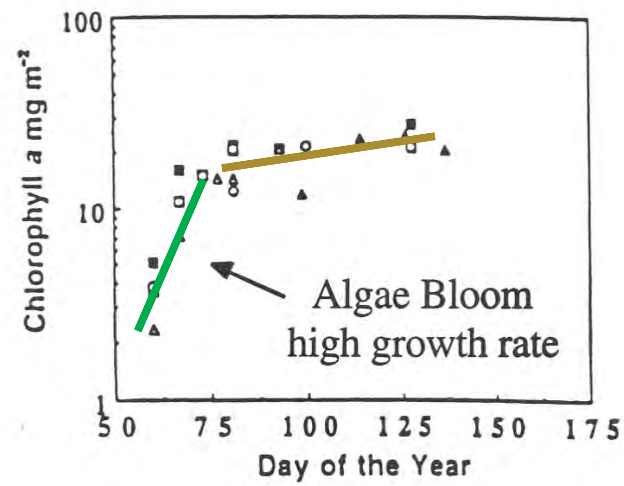
nutrient replenishment controlled by ice permeability

biological activity turns on or off according to
rule of fives

Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

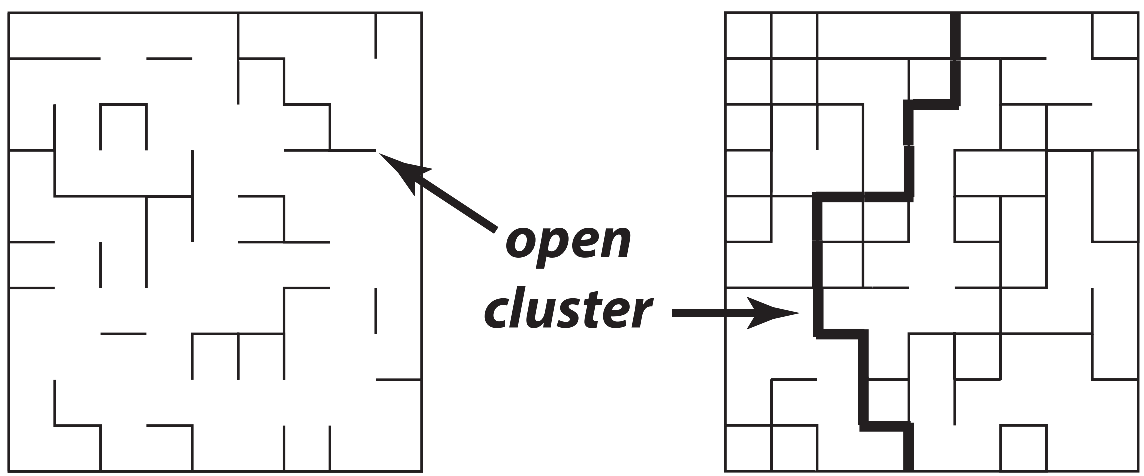
critical behavior of microbial activity

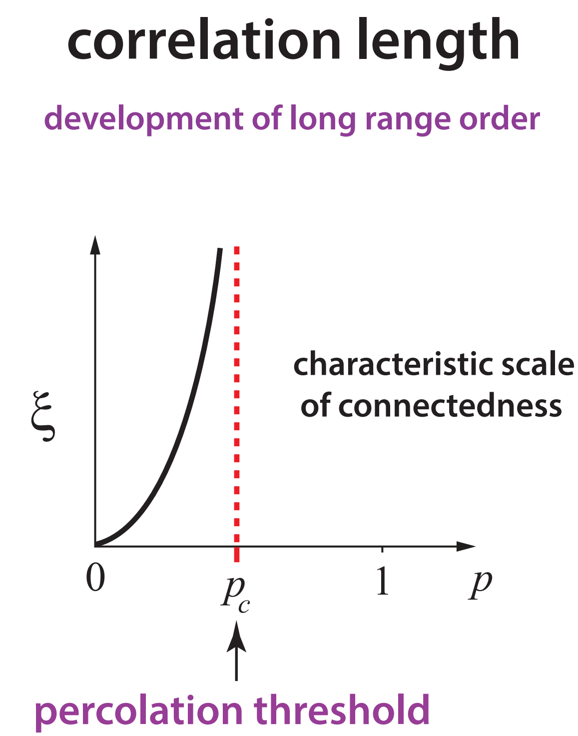


Convection-fueled algae bloom
Ice Station Weddell

percolation theory

probabilistic theory of connectedness





bond ⟶ ***open*** *with probability* p
closed *with probability* $1-p$

$$\xi(p) \sim |p - p_c|^{-\nu} \qquad p \to p_c$$

ν **universal: depends only on** ***d***

p_c depends on type of lattice and *d*

percolation threshold

$$p_c = 1/2 \quad \text{for} \;\; d = 2$$

smallest *p* for which there is an infinite open cluster