Mathematics 1210 PRACTICE EXAM II Fall 2001

ANSWER KEY

1. a. $x''(t) = -\omega^2 A\sin(\omega t - \phi) = -\omega^2 x(t)$

   b. $f'(x) = -3\left(\dfrac{x-2}{x-\pi}\right)^{-4} \dfrac{(2-\pi)}{(x-\pi)^2}$

   c. $y' = -(2 + y\sin xy)/(2y + x\sin xy)$

   d. 2 (use $\sin x \sim x,\ \tan x \sim x,\ \cos x \sim 1 - x^2/2$ as $x \to 0$)

   e. $f'(x) = \cos\sqrt{\dfrac{\tan x}{1+x^2}}\ \dfrac{1}{2}\left(\dfrac{\tan x}{1+x^2}\right)^{-1/2}\ \dfrac{(1+x^2)\sec^2 x - 2x\tan x}{(1+x^2)^2}$

   f. $f'(x) = 2x\sin^2(x^3) + 6x^4\sin(x^3)\cos(x^3)$

2. $x(t) = A\sin(\omega t - \phi)$ or $x(t) = B\cos(\omega t - \psi)$. For $x(t) = 2\sin(t - \pi/4)$, velocity is extremized when $x(t) = 0$, or $t = n\pi + \pi/4$, position is extremized when $v(t) = x'(t) = 0$, or $t = (n + \frac{1}{2})\pi + \pi/4$. Figure on other side.

3. $\dfrac{dr}{dt} = 0.02\ in/sec,\ A = \pi r^2,\ \dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} = 1.018\ in^2/sec.$

4. The positions of the tips of the hands (so that 3:00 corresponds to $t = 0$ hours), are given by $(x_1(t), y_1(t)) = (4\cos\omega_1 t, -4\sin\omega_1 t)$ for the hour hand, and $(x_2(t), y_2(t)) = (5\sin\omega_2 t, 5\cos\omega_2 t)$ for the minute hand, where $\omega_1 = 2\pi f = 2\pi$ and $\omega_2 = 2\pi f = 2\pi 60$. The rate of change of the distance separating the tips of the hands at 3:00 is changing at the rate $r'(t)$ at $t = 0$, where $r(t) = \sqrt{(x_1(t) - x_2(t))^2 + (y_1(t) - y_2(t))^2}$.

5. $8\frac{1}{8}$, $\pi/100$, $\dfrac{m(0.92c) - m(0.9c)}{m(0.9c)} \approx 9.5\%$, where $m(0.9c + 0.02c) \approx m(0.9c) + m'(0.9c)0.02c$, and $m(0.9c) = m_0 2.29$, $m'(0.9c)0.02c = m_0 0.217$.

6. Figure on other side. For $f(x) = x^3 - 12x + 1$, see Sec. 4.2, #19, and for $f(x) = \dfrac{x}{1+x^2}$, see example 4, p. 170, section 4.2.

7. $A = 2xy = 2x(12 - x^2)$. $\dfrac{dA}{dx} = 2(12 - 3x^2) = 0, x = 2, y = 8$, dimensions 4x8.

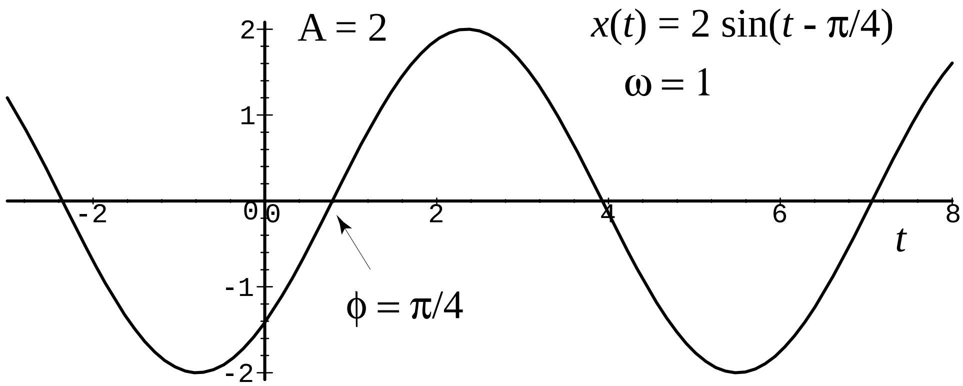

Figure 1: Graph of $f(x)$ for #2.

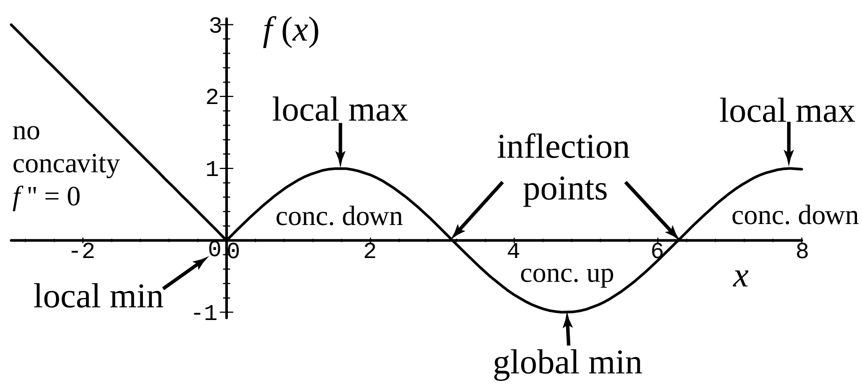

Figure 2: Graph of $f(x)$ for #6.