Mathematics 1210 PRACTICE EXAM III Fall 2001

1. Calculate the following:

   a. $\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^2 \tan x}$ b. $\lim_{x\to 0} \dfrac{x^{10/3}}{\sin x - x}$

   c. $\lim_{x\to \pi/2} \dfrac{\cos x}{x - \pi/2}$ d. $\lim_{x\to 0} \dfrac{\int_0^x (1-\cos t)\, dt}{x^3}$

   e. $\lim_{n\to\infty} \dfrac{1}{n} \sum_{i=1}^{n} \sin \dfrac{i}{n}$

2. Find the general solutions to the following differential equations.

   a. $\dfrac{dy}{dx} = \sqrt[3]{\dfrac{x}{y}}$ b. $\dfrac{d^2x}{dt^2} = -\omega^2 x$

   c. $\dfrac{d^2x}{dt^2} = -g$

3. section 5.2, #22,35

4. For each function below, determine whether or not it is Riemann integrable on the interval (0,1], and **fully** explain your result: (a) $f(x) = \sin(1/x)$, (b) $f(x) = 1/\sqrt{x^2 \sin x}$, (c) $f(x) = x$ for x rational and $= -x$ for x irrational.

5. Calculate $\int_1^2 (3x^2 - 2)\, dx$ from the definition of the integral, that is, using Riemann sums.

6. Calculate the following integrals:

   (a) $\int_0^4 \sqrt{x}\, dx$ (b) $\int_0^{\pi/2} \sin x\, dx$ (c) $\int_1^3 \dfrac{1-3x^3}{x^2}\, dx$

   (d) $\int_0^{\pi} \sin^2 x\, dx$ (e) $\int_0^{\pi} x\cos(x^2+\pi)\, dx$ (f) $\int_{-3}^{3} x^3\, dx$

7. Find $\dfrac{d}{dx} \int_0^{x^2} \tan\theta\, d\theta$.