Mathematics 1210 PRACTICE EXAM II Fall 2001

1. Calculate the following:

   a. $\dfrac{d^2x}{dt^2}$, $x(t) = A\sin(\omega t - \phi)$

   b. $\dfrac{df}{dx}$, $f(x) = \left(\dfrac{x-2}{x-\pi}\right)^{-3}$

   c. $\dfrac{dy}{dx}$, $\cos xy = y^2 + 2x$

   d. $\lim\limits_{x\to 0} \dfrac{\sin x \tan x}{1-\cos x}$

   e. $\dfrac{df}{dx}$, $f(x) = \sin\sqrt{\dfrac{\tan x}{1+x^2}}$

   f. $\dfrac{df}{dx}$, $f(x) = x^2\sin^2(x^3)$

2. Find all solutions to the differential equation for a mass on a spring, $\dfrac{d^2x}{dt^2} = -\omega^2 x(t)$, where $x(t)$ is the position as a function of time t, $\omega = \sqrt{k/m}$, k is the spring constant and m is the mass. (Hint: see problem 1(a) above.) Choose a set of parameters characterizing a particular solution, and graph it. Also find in this case for which times the position is maximized, and for which times the velocity is maximized.

3. A metal disk expands during heating. If its radius increases at the rate of 0.02 inch per second, how fast is the area of one of its faces increasing when its radius is 8.1 inches?

4. The hands on a clock are of length 5 inches (minute hand) and 4 inches (hour hand). How fast is the distance between the tips of the hands changing at 3:00?

5. Approximate $\sqrt{66}$ and $\sin\left(\dfrac{\pi}{100}\right)$ using linear approximation (i.e., the differential). Also do problem #35 on page 155, section 3.10.

6. Consider $f(x) = \begin{cases} |x|, & x < 0, \\ \sin x, & x \geq 0. \end{cases}$ Find all local maxima and minima of f, where f is increasing and decreasing, where f is concave up and concave down, and all inflection points. Does f have a global maximum or a global minimum? Sketch the graph of $f(x)$. Do the same for $f(x) = x^3 - 12x + 1$ and $f(x) = \dfrac{x}{1+x^2}$.

7. A rectangle has two corners on the $x-$axis and the other two on the parabola $y = 12 - x^2$, with $y \geq 0$. What are the dimensions of the rectangle of this type with maximum area?

8. Problem #29 on Snell's Law, page 186, section 4.4.