Mathematics 1210 PRACTICE EXAM I Fall 2000

1. Calculate the following limits. If a particular limit does not exist, state this clearly and tell why.

   (a) $\lim_{x\to\sqrt{2}} 3x^2$ (b) $\lim_{\theta\to\pi/2} \tan\theta$ (c) $\lim_{x\to -1} \frac{x^2-x+2}{x+1}$ (d) $\lim_{x\to 0^+} \sqrt{x}\sin\left(\frac{1}{x^2}\right)$

   (e) $\lim_{x\to 0} \frac{\sin(x^2)}{x}$ (f) $\lim_{x\to +\infty} \frac{\sin x}{x}$ (g) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^3, & x \le 2 \\ x, & x > 2 \end{cases}$

   (h) $\lim_{x\to\pi} f(x)$, where $f(x) = \begin{cases} 0, & x \text{ irrational} \\ \sin\left(\frac{1}{q}\right), & x = \frac{p}{q} \text{ rational} \end{cases}$ (i) $\lim_{x\to +\infty} \sqrt[3]{\frac{8x^7+3x^5}{x^7+6x^2}}$

2. (a) Let $f(x) = \sqrt{x}$. Using the *definition* of the derivative, calculate $f'(x)$. Do the same for $g(x) = 1/x$.

   (b) Using your result from (a), find the equation of the line tangent to the graph of $f(x) = \sqrt{x}$ at $x = 1$. Do the same for $g(x) = 1/x$.

3. Let $f(x) = -x$ when $x \le 0, x \ne -1$; 2 when $x = -1$; $\sqrt{x}$ when $0 < x < 1$; $\sqrt[3]{3-x}$ when $x \ge 1$. Sketch the graph of $f(x)$.

   (a) For which points c does $\lim_{x\to c} f(x)$ exist? (b) For which points is f continuous?

   (c) For which points is f differentiable?

4. Let $f(x) = x + 2$ when $x \le 0$; $-\frac{1}{2}x + 2$ when $0 < x \le 2$; $\sqrt{x-2} + 1$ when $x > 2$. Sketch the graph of $f(x)$, and then using your result sketch the graph of $f'(x)$.

5. Find the derivative and antiderivative of (a) $f(x) = 12x^5 + 5x^4 + x^2 + 2x + 1$, (b) $f(x) = (x+1)^3$, (c) $f(x) = (3x^2 - 2x + 1)(x - 1)$.

6. Let the position $x(t)$ of a particle at time t be given by $x(t) = 3t^2 - 2t + 1$. Find the instantaneous velocity $v(t)$ of the particle for any time t. Where is the particle when its velocity is zero?

7. A clever tick falls strategically from the top of a 22 foot tree onto the top of the head of a 6 foot tall hiker. How long does it take the tick to hit the hiker's head (neglecting air friction), and what is the tick's velocity when it hits?

8. On earth, the acceleration $a(t)$ due to gravity is essentially constant in time, with $a(t) = -g$, where $g = 32\ f/s^2$. On nearby planet Ψ, scientists have discovered how to vary their planet's gravitational force with time. If the acceleration due to gravity on Ψ is $a(t) = -t$, find the analog of the earth formula $x(t) = -16t^2 + v_0 t + x_0$ for planet Ψ. That is, find $x(t)$ for $a(t) = -t$ with initial velocity v_0 and position x_0. Using your expression for $x(t)$, find how long it will take for a ball thrown upward from the ground on Ψ at $t = 0$ with initial velocity $6\ f/s$ to hit the ground.