FRACTAL GEOMETRY AND ITS APPLICATIONS

A project report submitted to Christ College (Autonomous) in partial fulfillment of requirement for the award of the B.Sc. Degree Programme in Mathematics

AUSTIN FRANCIS

Reg. No. CCASSMT025

DEPARTMENT OF MATHEMATICS

CHRIST COLLEGE (AUTONOMOUS)

Irinjalakuda

2021

CERTIFICATE

This is to certify that the project titled "**FRACTAL GEOMETRY AND ITS APPLICATIONS**" submitted to the Department of Mathematics in partial fulfillment of the requirement of the BSc, Degree Programme in Mathematics, is a bonafide record of the original research done by Mr. **AUSTIN FRANCIS** (**CCASSMT025**) during the period of her study in the department of Mathematics, Christ College (Autonomous), Irinjalakuda under my supervision and guidance during the year 2020-2021.

Fr. Dr. Vincent N S

Department of Mathematics

Christ College (Autonomous)

Irinjalakuda

Dr. Seena V

HoD in Charge

Department of Mathematics

Christ College (Autonomous)

Irinjalakuda.

External Examiner :

Sherin Jose T
Assistant Professor
Department of Mathematics
St. Joseph's College
Irinjalakkuda

Irinjalakuda

03 May 2021

DECLARATION

I hereby declare that the project work entitled submitted to the Department of Mathematics, Christ College (Autonomous), Irinjalakuda in partial fulfillment of the requirement for the award of the BSc. Degree programme in Mathematics is a record of original project work done by me during the period of my study in the Department of Mathematics, Christ College, Irinjalakuda.

Irinjalakuda AUSTIN FRANCIS

03 May 2021

ACKNOWLEDGEMENT

It is our proud privilege to release the feelings of our gratitude to several persons who helped us directly or indirectly to conduct this project work. Working on this project "**FRACTAL GEOMETRY AND ITS APPLICATIONS**" was a source of immense knowledge to us. We express our heartfelt indebtedness and owe and a deep sense of gratitude to our project guide Fr. Dr. Vincent N S , Assistant Professor, Christ College (Autonomous), Irinjalakuda for his sincere guidance and inspiration in completing this project. We also take this opportunity to express our sincere thanks to our beloved Principal Fr. Dr. Jolly Andrews CMI, who gave us a chance to do this project.

We take our word of gratitude to Dr. Seena V, Head of the department in charge and all other teachers of the department for providing us the necessary help to complete our project on time. We also think all faculty of library for their valuable support and enrichment. We would also like to thank our parents for their love and support to complete our project. We also thank all our friends for their contribution.

The study has indeed helped us to explore more knowledgeable avenues related to our topic and we are sure it will help in our future.

AUSTIN FRANCIS

CONTENTS

INTRODUCTION

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."

-Benoit Mandelbrot

Fractal geometry is a subject that has enjoyed substantial growth over the past decade or so, and has established connections with many areas of mathematics (including harmonic analysis, potential theory, partial differential equations, probability theory, operator algebra, number theory and dynamical systems).

It is also intrinsically a cross-disciplinary subject, with motivations from and applications to physics, biology, geology, economics, and even some artistic fields, like painting and music.

Most physical systems of nature and many human artefacts are not regular geometric shapes of the standard geometry derived from Euclid. Fractal geometry offers almost unlimited ways of describing, measuring and predicting these natural phenomena.

Many people are fascinated by beautiful images of fractals. Extending beyond the typical perception of mathematics as a body of complicated, boring formulas, fractal geometry mixes art with mathematics to demonstrate that equations are more than just a collection of numbers. What makes fractals even more interesting is that they are best existing mathematical descriptions of many natural forms, such as coastlines, mountains or parts of living organisms.

This project is a brief study about fractal geometry and its applications that is spanned over three major chapters. From a note of observations about the occurrence of fractals in nature, we come to the mathematical representation of fractals and basic concepts of fractal geometry. We also discuss a few examples like Cantor set, Sierpiński triangle, and Koch curve before analysing some applications of fractals.

CHAPTER 1

FRACTALS IN NATURE

Fractals are typically self-similar patterns that show up everywhere around us in nature and biology. The term "fractal" was first used by mathematician Benoit Mandelbrot in 1975 and used it to extend the concept of theoretical fractional dimensions to geometric patterns in nature.

A fractal is a pattern that the laws of nature repeat at different scales. Examples are everywhere in the forest. Trees are natural fractals, patterns that repeat smaller and smaller copies of themselves to create the biodiversity of a forest. Each tree branch, from the trunk to the tips, is a copy of the one that came before it. This is a basic principle that we see over and over again in the fractal structure of organic life forms throughout the natural world. Trees are perfect examples of fractals in nature. You will find fractals at every level of the forest ecosystem from seeds and pinecones, to branches and leaves, and to the self-similar replication of trees, ferns, and plants throughout the ecosystem. Something as simple as a leaf can have fractals in it, like the one below, a macro shot of the leaf shows the veins forming an irregular, recurrent pattern.

Fig 1.1 Fractal structure at different levels of forest

Fractals are found all over nature, spanning a huge range of scales. Self-similarity is ubiquitous in nature. We see self-similarity in a fern leaf, a snowflake, our lung structure, the path of a forest fire, villi in our intestines, the Internet, temporal processes such as music, social behaviors, and many other patterns in nature. Many of nature's patterns depend on the principle of self-similarity to function properly. A good example is the human lung. We find the same patterns again and again, from the tiny branching of our blood vessels and neurons to the branching of trees, lightning bolts, and river networks. Regardless of scale, these patterns are all formed by repeating a simple branching process. Fractal patterns are also observed on a cellular and sub-cellular level in the lungs and other tissues. The alveolar surface, including the cell membranes of individual cells, can be considered fractal; if these structures are examined at increasing magnification, increasing levels of detail and complexity can be observed. The same concept can also be applied to the membranes of sub-cellular organelles, such as those of the mitochondria, nucleus, and endoplasmic reticulum. The concept of fractional Brownian motion has been also applied to DNA sequence leading to the discovery of the long-range correlation in DNA sequence.

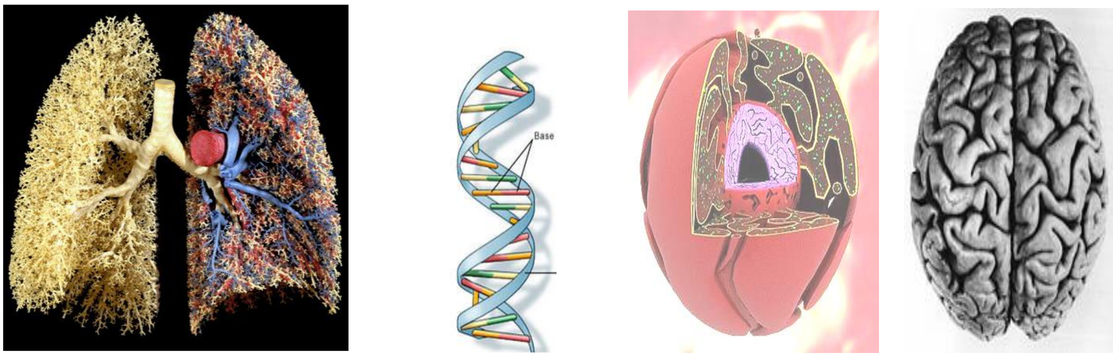


Fig 1.2 Some fractal patterns observed in biological sub-structures

Biologists have traditionally modeled nature using Euclidean representations of natural objects or series. They represented heartbeats as sine waves, conifer trees as cones, animal habitats as simple areas, and cell membranes as curves or simple surfaces. However, scientists have come to recognize that many natural constructs are better characterized using fractal geometry. Biological systems and processes are typically characterized by many levels of substructure, with the same general pattern repeated in an ever-decreasing cascade. Scientists discovered that the basic architecture of a chromosome is tree-like; every chromosome consists of many 'mini-chromosomes', and therefore can be treated as fractal.

Neurons from the human cortex: The branching of our brain cells creates the incredibly complex network that is responsible for all we perceive, imagine, and remember. Our lungs are branching fractals with a surface area. The similarity to a tree is significant, as lungs and trees both use their large surface areas to exchange oxygen and CO2.

Fractals can also be classified according to their self-similarity. There are three types of self-similarity found in fractals:

- **Exact self-similarity**: This is the strongest type of self-similarity; the fractal appears identical at different scales. Fractals defined by iterated function systems often display exact self-similarity.

- **Quasi-self-similarity**: This is a loose form of self-similarity; the fractal appears approximately (but not exactly) identical at different scales. Quasi-self-similar fractals contain small copies of the entire fractal in distorted and degenerate forms. Fractals defined by recurrence relations are usually quasi-self-similar but not exactly self-similar.
- **Statistical self-similarity**: This is the weakest type of self-similarity; the fractal has numerical or statistical measures which are preserved across scales. Most reasonable definitions of "fractal" trivially imply some form of statistical self-similarity. (Fractal dimension itself is a numerical measure which is preserved across scales.) Random fractals are examples of fractals which are statistically self-similar, but neither exactly nor quasi-self-similar.

Here are some examples of fractal patterns in nature:

1.1 River Deltas

Rivers flow downhill with their power derived from gravity. The direction can involve all directions of the compass and can be a complex meandering path.

Rivers flowing downhill, from river source to river mouth, do not necessarily take the shortest path. For alluvial streams, straight and braided rivers have very low sinuosity and flow directly downhill, while meandering rivers flow from side to side across a valley. Bedrock Rivers typically flow in either a fractal pattern, or a pattern that is determined by weaknesses in the bedrock, such as faults, fractures, or more erodible layers.


Fig 1.3 Fractal dimension of the river delta as branched structures

1.2 Flowers and Fruits

One of the beautiful creations of nature – flowers – can be fractals as well, and they add another dimension to the whole landscape. In a bunch of flowers, each bunch has the same pattern and so does every tiny flower. The repetitive patterns are also found in fruits and vegetables, and are often overlooked. Pineapple and Bitter Gourd are fine examples of Fractals. Broccoli is another vegetable displaying spiral patterns of fractal geometry. A geometric pattern that is repeated at ever smaller (or larger) scales to produce self-similar or irregular shapes and surfaces. Fractals are hyper-efficient in their construction and this allows plants to maximize their exposure to sunlight and also efficiently transport nutritious throughout their cellular structure. These fractal patterns of growth have a mathematical, as well as physical beauty.

Fig 1.4 Fractal patterns observed in Botanical Structures

1.3Animals and Birds

Fractal geometry techniques are particularly suitable for addressing intricate, complex and heterogeneous patterns. An interesting example of self-organization in nature is the problem of understanding how spots and stripes appear on the skins of some animals. Such patterns often serve as camouflage and so have definite survival value. Since leopard spots are not arranged in identical patterns from one individual to the next, there must be some amount of randomness involved, and yet the patterns of leopards are certainly distinguishable from those of tigers, so there must be some mechanism which differs across species. Leopards and ladybirds are spotted; angelfish and zebras are striped. The young leopards and ladybirds, inheriting genes that somehow create spottiness, survive.

Fig 1.5 Occurrence of fractal patterns as spots and stripes on animals

1.4 Mountains and Rivers

Mountains are the result of tectonic forces pushing them up and weathering breaking them down. Little surprise they are well-described by fractals. Rivers are also good examples of natural fractals, because of their tributary networks (branches off branches off branches) and their complicated winding paths.

Fig 1.6 Fractal patterns visible in mountains

Another type of fractal pattern we see in nature is the spiral: spirals in some types of mollusk shells, octopus, spirals in star formations and the shapes of galaxies, and hurricanes are spiral-shaped. Each chamber of its shell is an approximate copy of the next one, scaled by a constant factor and arranged in a logarithmic spiral. A growth spiral can be seen as a special case of self-similarity. All fractals are formed by simple repetition, and combining expansion and rotation is enough to generate the ubiquitous spiral.

Plant spirals can be seen in phyllotaxis, the arrangement of leaves on a stem, and in the arrangement of other parts as in composite flower heads and seed heads like the sunflower or fruit structures like the pineapple and snake fruit, as well as in the pattern of scales in pine cones, where multiple spirals run both clockwise and anticlockwise. In disc phyllotaxis as in the sunflower and daisy, the florets are arranged in Fermat's spiral with Fibonacci numbering; at least when the flower head is mature so all the elements are the same size. Ammonites are extinct relatives of the nautilus. The sutures where the internal chamber walls meet the outer shell are fractal curves. A spiral galaxy is the largest natural spiral comprising hundreds of billions of stars. A hurricane is a self-organizing spiral in the atmosphere, driven by the evaporation and condensation of sea water.

Fig 1.7 Fractals as spirals

Fractal geometry is a product of fractal theory, a mathematical approach that describes the way space is filled by figures or objects. Every pattern visible in nature reflects the fractal pattern in different scales. Apart from these examples; on analysing each and every pattern in nature, it eventually tends to follow a fractal pattern. The fractal patterns are found all over the nature spanning a wide range of scales.

CHAPTER 2

FRACTAL GEOMETRY

The word fractal from the Latin word ‘Frangere’ which means to break, was coined by Benoit Mandelbrot in 1975.

“Fractal objects contain structures nested within one another. Each smaller structure is a miniature, though not necessarily identical, version of the larger form (Peterson,1988,pp.114-115).” In other words, one part of the object is a scaled down version of the entire object. The Koch curve and the Sierpiński gasket are classic, yet simple, examples of self-similar objects.

2.1 Geometry of Fractal

- Most of the fractals are self-similar geometrical objects.
- Several parts of a fractal look similar as the entire image.
- It is possible to copy the fractal several times on itself.
- Examples are clouds, forests, galaxies, leaves, feathers, carpets, bricks etc.
- They are complex at your first sight, while in fact can be described by a simple algorithm.
- They can be generated by repeated self copy or partial self copy.
- Therefore, the redundancy is very high.

2.2 Examples Of fractals

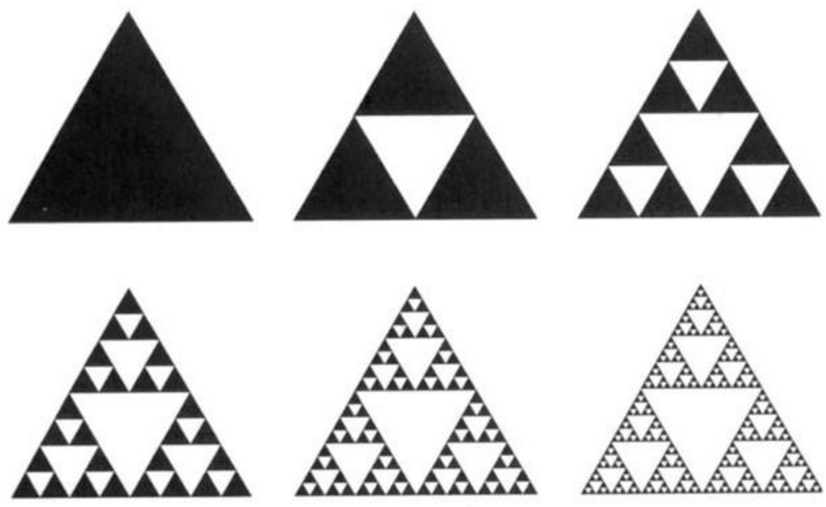


Fig 2.1: Sierpiński Triangle

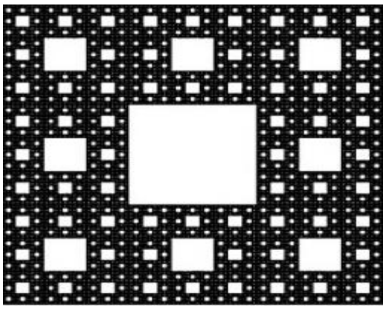


Fig 2.2 Sierpiński Carpet

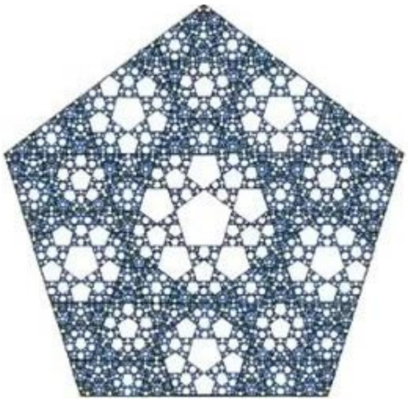


Fig 2.3 Pentagonal Carpet

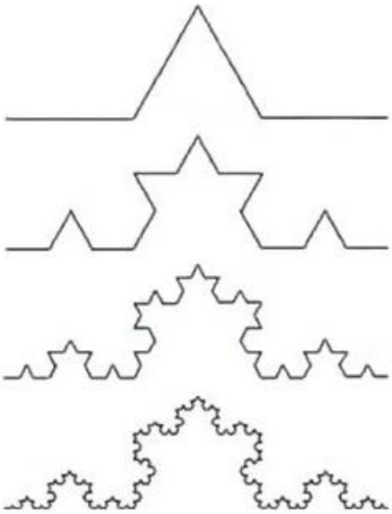


Fig 2.4 Koch curve

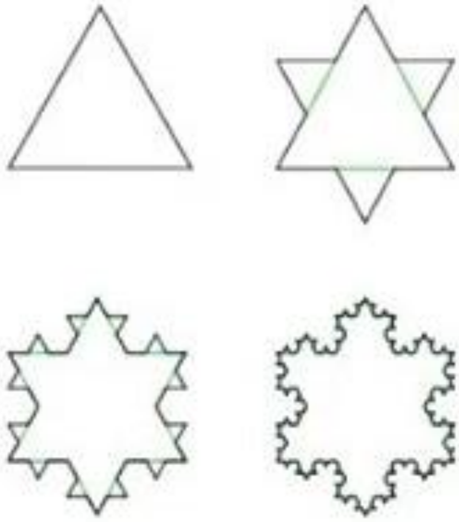


Fig 2.5 Koch Snowflake

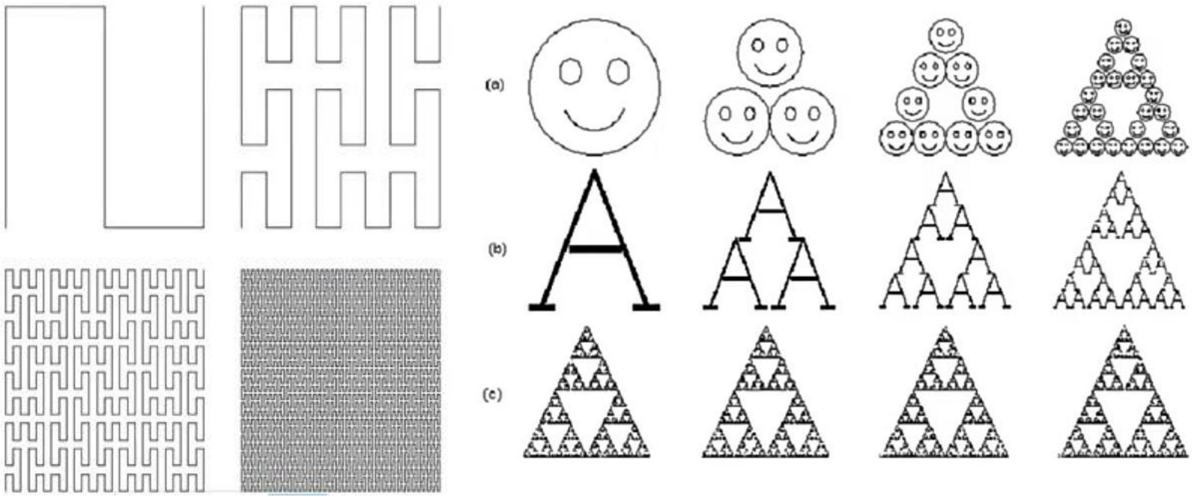


Fig 2.6 Some more examples

2.3 Transformation between Fractals

- Imagine a special type of photocopying machine that reduces the image to be copied by a half and reproduces it three times on the copy.

Fig 2.7 Transformation

- All the copies seem to be converging to the same final Image.
- We call the final image the attractor of the copy machine.
- Because the copying machine reduces the input image, any initial image will be reduced to a point as we repeated run the machine.
- Thus, the initial image placed on the copying machine does not affect the Final Attractor.
- In fact, it is only the position and the orientation of the copies that determines what the final image will look like.
- We only describe these transformations.
- Different transformations lead to different Attractors.
- The transformations must be Contractive.
- In practice, Affine transformations are rich enough and yield interesting set of Attractors.

$$\mathbf{t_i} \begin{matrix} \boldsymbol{x} \\ \boldsymbol{y} \end{matrix} = \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix} \begin{matrix} \boldsymbol{x} \\ \boldsymbol{y} \end{matrix} + \begin{matrix} e_i \\ f_i \end{matrix}$$

Each Affine transformation can skew, stretch, scale and translate an input image.

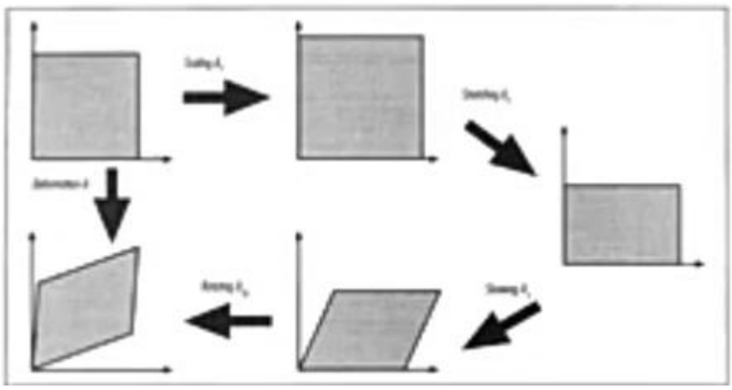


Fig 2.8 Example by Affine Transformation

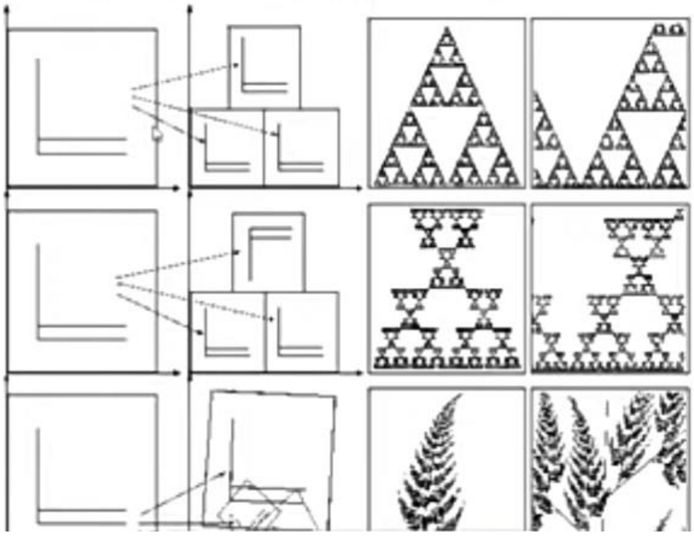


Fig 2.9 Examples by Affine Transformation

- Each Affine transformation t_i is defined by 6 numbers a_i, b_i, d_i, e_i and f_i.
- Storing images as collections of transformation leads to image.

2.4 Contractive Affine Transformation

A transformation f is said to be Contractive if for any two points p_1, p_2, the distance

$$\mathbf{d\,(f\,(p_1),\, f(p_2)) < s\ d\,(p_1,\, p_2),\ for\ some\ s<1}$$

where $d\,(p_1, p_2) = \sqrt{(\boldsymbol{x_1} - \boldsymbol{x_2})^2 + (\boldsymbol{y_1} - \boldsymbol{y_2})^2}$

Let p_f be the fixed point of contractive transformations t.

Then for any input point p_t,

$$\lim_{\boldsymbol{n} \to \infty} \boldsymbol{t^n}(\boldsymbol{p}) = \boldsymbol{p_f}$$

- Consider the grey-scale images

  $\{ (x_1, y_1, z_1) \mid z_1 = f(x_i, y_i)$ is the grey-level at position$(x_i, y_i) \}$

  $$\mathbf{t_i} \begin{array}{c} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{array} = \begin{array}{ccc} a_i & b_i & \mathbf{0} \\ c_i & d_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & s_i \end{array} \quad \begin{array}{c} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{array} + \begin{array}{c} e_i \\ f_i \\ 0_i \end{array}$$

  where s_i controls the contrast and 0_i the brightness of the transformation.
- Contractive transformations can send any input points to a particular fixed-point property of contractive transformation.

2.5 Dimensions

Dimension is a property of a mathematical object that refers to the extent it occupies the space in which it is embedded. There are many formal definitions of dimension, if the definition allows non-integer values (a fraction), it is a Fractal dimension. The box-counting dimension are defined over a vector space, but there is also the packing dimension, compass dimension etc.

2.5.1 Regular dimensions

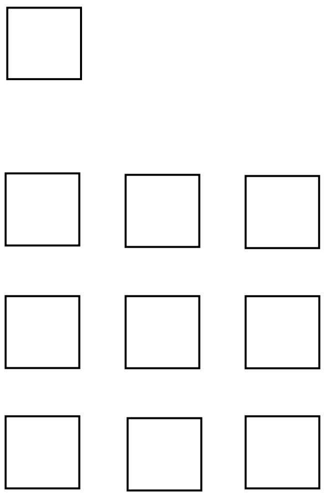
D=1

Magnify by R =2

Get N= 2 copies

$N=R^1$

2D (D=2)



Magnify by R = 3

Get N = 9 copies

$N= 3^2 = R^2$

2.5.2 General rule for Dimensions

- A figure is in D dimensions
- If I magnify the length by R, then I would get R^D copies
- $N = R^D$

2.5.3 The dimension formula

Define R as the magnifying factor,

Define N as the number of identical(“self-duplicating”) copies.

Then the dimension of a figure is:

$$D = \frac{\log(N)}{\log(R)}$$

2.6 Fractal

A Fractal is a never- ending patters. Fractals are infinitely complex patterns that are self-similar across different scales.

Fractals are considered to be important because they define images that are otherwise cannot be defined by Euclidean geometry.

2.6.1 Self-Similar Objects and Fractal Dimensions

- Fractal dimension is a measure of how “complicated” a self-similar figure is.
- i.e., to measure the fractal Dimension, the picture must be self-similar.
- Self-similar regular shapes: Line, Plane, Cube
- Self-similar irregular shapes: cauliflower, Galaxy, Coast Line.

2.6.2 Scaling Factor

We can divide the object in N self-similar pieces then, how to get original object from size of these N pieces?

Scaling Factor: If we want to get original object from any part of self-similar then we have to scale the object using scaling factor.
For example:
If we divide the line in 2 equal pieces then SF is 4
If we divide the plane in 4 equal pieces then SF is 2

Mandelbrot defined a fractal to be a set with Hausdroff dimension strictly greater than its topological dimension. (The topological dimension of a set is always an integer and is 0 if it is totally disconnected. If each point has arbitrarily small neighbourhoods with boundary of dimension 0 and so on.)

The **Hausdroff dimension**, more specifically, is a further dimensional number associated with a given set, where the distances between all members of that set are defined. Such a set is termed a metric space. The dimension is drawn from the extended real numbers, R, as opposed to more intuitive notion of dimension, which is not associated to general metric spaces, and only takes values in the non-negative integers.

2.7 Fractal Dimension (Or Non-integer dimension)

Input:

- No of self-similar pieces

- Scaling Factor

Fractal Dimension=log (No. Self-Similar Object)/log (Scaling Factor)

Dimension for the plane=2

Dimensions for the cube=3

While the Hausdroff dimension of a single point is zero, that of a line segment is 1,of a square is 2,and of a cube is 3,for fractals such as this, the object can have a non-integer dimension.

Example of non-integer dimensions:

Division of certain sets into four parts. The parts are similar to the whole with ratios:

1. $\frac{1}{4}$ for line segment
2. $\frac{1}{2}$ for square
3. $\frac{1}{9}$ for middle third Cantor set
4. $\frac{1}{3}$ for von Koch curve

These dimensions indicate how they reflect scaling properties and self-similarity.

Dimension for Cantor set $D = \frac{log2}{log3} = 0.631$

Dimension for the Koch curve $D = \frac{log4}{log3} = 1.26$

Dimension for Sierpiński triangle $D = \frac{log3}{log2} = 1.585$

2.8 Cantor Set

The Cantor ternary set is created by iteratively deleting the open middle third from a set of line segments.

Choose a particular portion say between two points 0 and 1.

Let $F_0 = [0,1]$.

We first remove the open middle third segment $(\frac{1}{3}, \frac{2}{3})$ of [0,1]. Then define F_1 as

$F_1 = [0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$

Next, we remove the open middle third of each of the two closed intervals in F_1 to obtain the set F_2

$F_2 = [0, \frac{1}{9}] \cup [\frac{2}{9}, \frac{3}{9}] \cup [\frac{6}{9}, \frac{7}{9}] \cup [\frac{8}{9}, 1]$

We see that F_2 is the union of $2^2 = 4$ closed intervals each of which is of the form $[k / 3^2, (k+1) / 3^2]$ each having length $1/3^2$.

Next, we remove the open middle thirds of each of the sets to get F_3. Then F_3 is the union of $2^3 = 8$ closed intervals of length $1/3^3$.

Continuing this way, we obtain a sequence of closed sets F_n such that

- $F_1 \supset F_2 \supset F_3 \supset \dots$
- F_n is the union of 2^n interval of the form $[k/ 3^n, (k+1) / 3^n]$ each of length $1/3^n$
- F_{n+1} is obtained from F_n by removing the open middle third of each of the intervals in F_n.

The set $F = \cap_{n \in N} F_n$ is called the Cantor set. The Cantor ternary set contains all points in the interval [0,1] that are not deleted at any step in this infinite process.

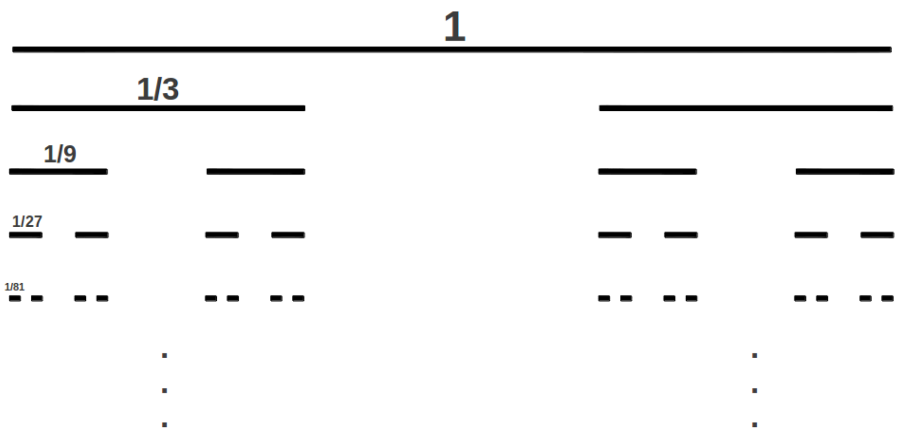


Fig 2.10 Cantor Set

2.9 The Sierpiński triangle

The Sierpiński triangle is a fractal described in 1915 by Waclaw Sierpiński. It is a self-similar structure that occurs at different levels of iterations, or magnifications. It is one of the simplest fractal shapes in existence.

2.9.1 Construction

In the Sierpiński triangle a pattern is begun by finding the midpoints of the line segments of the largest triangle. Then, by connecting these midpoints smaller triangles are created. This pattern is then repeated for the smaller triangles, and essentially has infinitely many possible iterations.

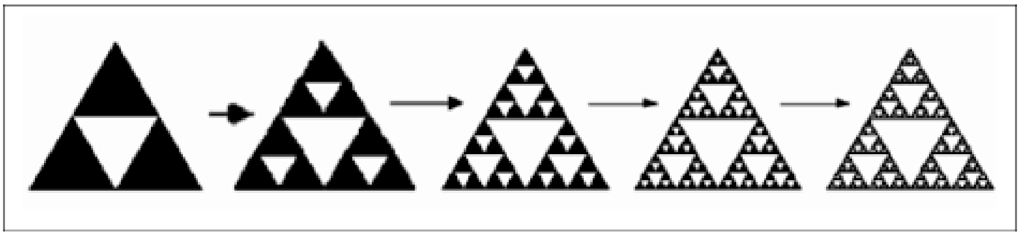


Fig 2.11: First five iterations of Sierpiński triangle.

2.9.2 Non – Integer Dimension

Using this fractal as an example, we can prove that the fractal dimensions is not an integer. Looking at the picture of the first step in building the Sierpiński Triangle, we can notice that if the linear dimension of the basis triangle is doubled, then the area of the whole fractal (black triangles) increases by a factor of three.

Using the pattern given above, we can calculate a dimension for the Sierpiński Triangle.

$$D = \frac{log3}{log2} = 1.585$$

The result of this calculation proves the non-integer fractal dimension.

The number of triangles in the Sierpiński Triangle can be calculated with the formula:

$N = 3^{k}$

Where N is the number of triangles and k is the number of iterations.

2.10 Von Koch Curve

The Koch snowflake or the Koch Curve is a fractal curve and one of the earliest fractals to have been discovered. Koch constructed his curve in 1904 as an example of a non-differentiable curve, that is, a continuous curve that does not have a tangent at *any* of its points.

2.10.1 Construction

Begin with a straight line. Divide it into three equal segments and replace the middle segment by the two sides of an equilateral triangle of the same length as the segment being removed. Now repeat, taking each of the four resulting segments, dividing them into three equal parts and replacing each of the middle segments by two sides of an equilateral triangle. Continue this construction.

The Koch curve is the limiting curve obtained by applying this construction an infinite number of times.

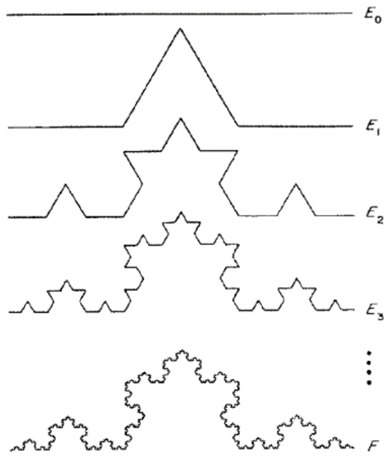


Fig 2.12 The Koch curve or Koch snowflake

2.10.2 Properties of Koch Edge

The Von Koch Curve clearly shows the self-similarity of fractal. The same pattern appears everywhere along the curve in different scale, from visible to infinitesimal. Ideally, the iteration process should go on indefinitely.

The length of the intermediate curve at the nth iteration of the construction is $(4/3)^n$, where $n = 0$ denotes the original straight line segment. Therefore, the length of the Koch curve is infinite.

Moreover, the length of the curve between any two points on the curve is also infinite since there is a copy of the Koch curve between any two points.

Three copies of the Koch curve placed outward around the three sides of an equilateral triangle form a simple closed curve that forms the boundary of the Kock snowflake. Three copes of the Koch curve placed so that they point inside the equilateral triangle create a simple closed curve that forms the boundary of the Koch anti-snowflake.

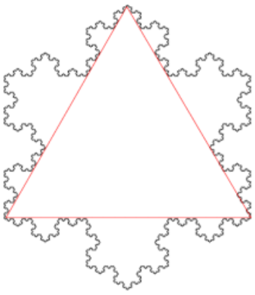


Fig 2.13 Koch snowflake

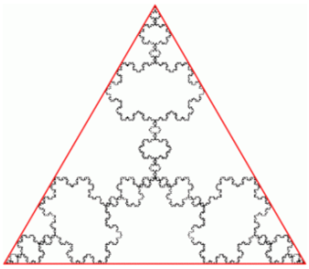


Fig 2.14 Koch Anti-snowflake

CHAPTER 3

APPLICATIONS OF FRACTAL GEOMETRY

The facts that fractals are abundant in nature and natural phenomena, is itself a testimony to the potential applicability and design efficiency of these shapes. Fraction shapes capture the fine details and organic irregularity of natural forms like clouds, cost lines and land shapes.

Fractals have variety of applications in science. Because it's properly of self-similarity exists everywhere. They can be used to model plants, blood vessels, nerves, explosions, clouds, mountains, turbulence, etc. Fractal geometry models natural objects more closely than does other geometries.

Engineers have begun designing and constructing fractals in order to solve partial engineering problems. Fractals are also used in computer graphics are even in composing music.

Fractal geometry has permeated many areas of science. Such as astrophysics, biological science, and has become one of the most important techniques in computer graphics. Architects are using fractal geometries to create more impressive buildings. Digital artists use fractal geometries to create interesting art work which engages views at variable scales Game designers are always seeking to create natural organic environments. Which do not seem to be constructed and synthetic. Fractal geometry can be applied in such environments to include random elements which can enrich user experience.

Fractals are also used to generate natural patterns which can create effective camouflage and preclude artificial repetitive motifs. Fractals have been used by seismologists to understand earthquake phenomena and gain deeper understanding of the earth physical constitution. As well as the distribution pattern of earthquakes. Financial theorists have even applied fractals to understand and forecast stock market patterns.

3.1 Fractals in Computer Graphics

The biggest usage of fractals in everyday life is in computer science. Many images compression schemes use fractal algorithms to compress computer graphics files to less than a quarter of their original size.

Computer graphics artists uses many fractals forms to create text termed landscapes and other intricate models.

It's possible to create all sorts up realistic "Fractal forgesies" images of natural scene, such a lunar landscape, mountain ranges and coastlines. We can see them it may special effects in Hollywood movies and also in television ads. The "genesis effect" in the films "star trek II". "The worth of khan" was created using fractal used to create the geography of a moon. and to draw the outline of dreaded "death star". But fractal signals can be used to model natural objectives. Allowing up to define mathematically our environment with a higher accuracy than ever before.

3.2 Fractals in Biological Science

Biological scientists have traditionally model nature using Euclidean representations of natural object or series. They represented heartbeats as sine waves. Conifer trees as cones, animals habit a simple area, and cell membranes as curves or simple surfaces however scientists have come to recognize that many natural constructs are better characteristic using fractal geometry. Biological systems and processes are typically characterized by many levels of substructure with some general pattern repeated in an ever-decreasing cascade.

Scientists discovered that basic architecture of a chromosome in tree like: every chromosome consist of many "mini chromosomes" and therefore can be treated as fractal for a human chromosome, for in theory one can argue that everything existent on this world is fractal: -

- The branching of tracheal tubes
- The leaves in trees
- The veins in hand
- Water swirling and twisting out of a tap
- A puffy cumulus clouds
- Tiny oxygen molecules or the DNA molecules
- The stocks market

All of these are fractals from people ancient civilizations to the marker of star trek II: The worth of khan scientists. Mathematicians and artists alike have been captivated by fractal and have utilized them in their work.

3.3 Fractals in Film Industry

One of the more trivial applications of fractals is their visual effect. Not only do fractals have a stunning aesthetic value that is, they are remarkably pleasing to the eye, but they also have a way to trick the mind. Fractals have been used commercially in the film industry. Fractal images are used as an alternative to costly elaborate sets to produce fantasy land scape.

3.4 Fractals in Astrophysics

Nobody really how many stars actually glitter in our skies, but have you ever wondered how they were formed and ultimately found their home in the world? Astrophysicist believe that the key to this problem in the fractal nature of interstellar gas. Distributions are hierarchical, like smoke trails or billow cloud in the sky and the clouds in space. Giving them an irregular but repetitive pattern that would be impossible to describe without the help of fractal geometry.

3.5 Fractals in Image Compression

Most use full application of fractals and Fractal geometry in image compression it is also one of the more controversial ideas. The basic concept behind of fractal image compression is to take an image and express it as an it rated system of functions the image can be quickly displayed, and at any magnification with infinite levels of fractal details. The largest problems behind its ideas is deriving the system of functions which describe an image.

3.6 Fractals in Fluid Mechanics

The study of turbulence in flows is very adapted to fractals. Turbulent flows are chaotic and very difficult to model correctly. A fractal representation of them helps engineers and physicists to better understand complex flows. Flames can also be simulated. Porous media have a very complex geometry and are well represented by fractal. This is actually used in petroleum science.

3.7 Fractals in Medicine

Biosensor interactions can be studied by using fractals

3.8 Fractals in Astronomy

Fractals will may be revolutionize the way that the universe is seen. Cosmologists usually assume that matter is spread uniformity across space. But observations show that is not true. Astronomers agree with that assumptions on “small” scales. But most of them think that the universe is smooth at very large scales. However, a dissident group of scientists claims that the structure of the universe is fractal at all scales. If this new theory is proved to be correct, even the big bung models should be adapted. Some years ago, we proposed a new approach for the analysis of galaxy and cluster correlations abused on the concepts and methods of modern statistical physics. This led to the surprising result that galaxy correlations are fractal and not homogeneous up to the limits of the available catalogues. In the meantime, many more redshifts have been measured and we have extended our method also to the analysis of number counts and angular catalogues. The result is that galaxy structures are highly irregular and self-similar. The usual statistical method, based on the assumption of homogeneity, are therefore inconsistent for all the length scales probed until now. A new move general conceptual frame work is necessary to identity the real physical properties of these structures. But present cosmologists need more data about the matter distribution in the universe to prove (or not) that we are living in a fractal universe

3.9 Fractals in Telecommunications

A new application is fractal- shaped antennae that reduce greatly the size and the weight of the antennae. Fractenna is the company which sells these antennae. The benefits depend on the fractal applied, frequency of interest, and so on. In general, the fractal parts produce ‘fractal loading’ and makes the antenna smaller for given frequency of use. Practical shrinkage of 2-4 times is realizable for acceptable performance. Surprisingly high performance is attained.

3.10 Fractal Antenna

A fractal antenna is an antenna that uses a fractal, self-similar design to maximize the length, or increase the perimeter (on inside section or the outer structure), of material that can receive or transmit electromagnetic radiation within a given total surface area or volume. cohen use this concept of fractal antenna. And it is theoretically it is proved that fractal design in the only design which receives multiple signals.

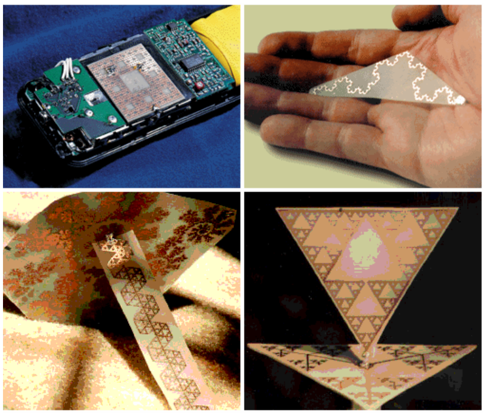


Fig 3.1 A fractal antenna

3.11 Fractals in Surface Physics

Fractals are used to describe the roughness of surface is characterized by a combination of two different fractals.

CONCLUSION

By now, we have talked about what fractals are, and we used some famous fractals to illustrate how we can create a fractal image. However, fractals are much more than that.

Many scientists have found that fractal geometry is a powerful tool for uncovering secrets from a wide variety of systems and solving important problems in applied science. The list of known physical fractal systems is long and growing rapidly.

Fractals improved our precision in describing and classifying “random” or organic objects, but maybe they are not perfect. Maybe they are just closer to our natural world, not the same as it is. Some scientists still believe that true randomness does exist, and no mathematical equation will ever describe it perfectly. So far, there is no way to say who is right and who is wrong.

Perhaps for many people fractals will never represent anything more than beautiful pictures.

Fractals are, without a doubt, foreign to a great many Mathematics students. It is precisely because of the newness of the science and the unfamiliarity with the concepts that students should study fractal geometry. They could benefit from an introduction to an area of mathematical research. They could read about new discoveries in the field in current periodicals. They could see applications of the science in popular culture. They could see Mathematics as a study of dynamic system other than one that has remained static for centuries.

We believe that students would benefit by learning different types of mathematics. One that could lead students to a sense of mathematical discovery. One that could show students that there is a way to do some mathematical experimentation using current technology. It is strongly felt that most high school mathematics students benefit by learning new branches of mathematics. An introduction to fractal geometry fulfil all these objectives.

BIBLIOGRAPHY

[1]B B. Mandelbrot, 'The Fractal Geometry of Nature', San Francisco, W. H. Freeman and company, 1982.

[2]B B. Mandelbrot, 'The Fractal Geometry of Nature' 1981 yearbook of science and future, Encyclopedia Britannica, Chicago, 1980.

Websites

1. https://youtu.be/v273ElS1TRU
2. Dr. Farhana Shaheen, FRACTALS
   https://slideplayer.com/amp/7690675/
3. Fractal- Wikipedia
   https://en.wikipedia.org/wiki/Fractal
4. Fractal antenna
   https://slideplayer.com/slide/10852710/39/images/4/WHAT+IS+A+FRACTAL+ANTENNA.jpg?hl=en_IN
   https://en.wikipedia.org/wiki/Fractal_antenna
5. https://fractalfoundation.org