Name Solution

Student ID # ____________________

Class (circle one) 9:40 10:45

Math 1210
Fall 2009
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EXAM I

Friday, September 18, 2009

Problem	Points	Score
1.	10	
2.	15	
3.	20	
4.	30	
5.	10	
6.	15	
	TOTAL	

(10 points) 1. (a) Find the equation of the line containing the two points $P = (-1, 2)$ and $Q = (1, 1)$ in the form $y = mx + b$.

The slope of the line is given by:

$$\frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{1 - (-1)} = -\frac{1}{2}.$$

We know the line passes through the point $Q = (1, 1)$, so when $x = 1$ we have $y = 1$.
Using this fact to solve for b in the equation $y = mx + b$ we get:

$$1 = -\tfrac{1}{2}(1) + b$$
$$\tfrac{3}{2} = b$$

Thus our final equation for the line is $\mathbf{y} = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{x} + \frac{\mathbf{3}}{\mathbf{2}}$.

(b) Find the derivative $\frac{dy}{dx}$ of the expression for $y(x)$ you found in (a).

The derivative of a line is a constant equal to the line's slope. In this case, the derivative will be $-\frac{\mathbf{1}}{\mathbf{2}}$.

(15 points) 2. Find the following limits.

(a) $\lim_{\Delta x \to 0} (2 + 5\Delta x)$

$$\lim_{\Delta x \to 0} (2 + 5\Delta x) = 2 + 5 \lim_{\Delta x \to 0} \Delta x = 2 + 0 = \mathbf{2}.$$

(b) $\lim_{h \to 0} \dfrac{4xh + 5h^3}{h}$

$$\lim_{h \to 0} \frac{4xh + 5h^3}{h} = \lim_{h \to 0} (4x + 5h^2) = 4x + 5(0) = \mathbf{4x}.$$

(c) $\lim_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

$$\begin{aligned}
&\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\
&= \lim_{h \to 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right) \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) \\
&= \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\
&= \frac{1}{\sqrt{x} + \sqrt{x}} \\
&= \frac{\mathbf{1}}{\mathbf{2\sqrt{x}}}
\end{aligned}$$

(20 points) 3. (a) Let $f(x) = x^2 + 5$. Using the *definition* of the derivative, calculate $f'(x)$. Be sure to show all of your work.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 5 - x^2 - 5}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$f'(x) = \lim_{h \to 0} 2x + h$$

$$f'(x) = \mathbf{2x}$$

(b) Using your result from (a), find the equation of the line tangent to the graph of $f(x) = x^2 + 5$ at $x = 2$.

The slope of the tangent line is given by $f'(x) = 2x$. So, at $x = 2$ we have $f'(2) = 4$. So, the slope of the tangent line is 4.

We also require that the tangent line pass through the point $(2, f(2))$ with $f(2) = 2^2 + 5 = 9$.

So, our line is given by $y = 4x + b$, where $9 = 4(2) + b$ and therefore $b = 1$.

So, our line equation is $\mathbf{y = 4x + 1}$.

(30 points) 4. Find the following.

(a) $\frac{d}{dx}\left(3x^5 + x^4 - 2x^2 + 6\right)$

$$\frac{d}{dx}\left(3x^5 + x^4 - 2x^2 + 6\right) = \mathbf{15x^4 + 4x^3 - 4x}.$$

(b) $\int (3x+2)(x-1)\,dx$

$$\int (3x+2)(x-1)dx = \int (3x^2 - x - 2)\,dx$$

$= \mathbf{x^3 - \frac{x^2}{2} - 2x + C}$.

(c) $v(t)$, where $x(t) = -16t^2 + 4t + 10$ and $v(t) = \frac{dx}{dt}$

$\mathbf{v(t) = -32t + 4}$.

(d) $\int_0^2 (x^2+1)\,dx$

$$\int_0^2 (x^2+1)dx = \frac{x^3}{3} + x|_0^2 = \frac{2^3}{3} + 2 - \frac{0^3}{3} - 0 = \frac{8}{3} + 2 = \mathbf{\frac{14}{3}}.$$

(e) $x(t)$, where $\frac{dx}{dt} = -32t + 64$ and $x(0) = 50$.

$\frac{dx}{dt} = -32t + 64 \to x(t) = -16t^2 + 64t + C$

Given $x(0) = 50$ we get $x(0) = -16(0^2) + 64(0) + C = 50$ $\to C = 50$.

So, our final equation is $\mathbf{x(t) = -16t^2 + 64t + 50}$.

(10 points) 5. A pebble is dropped from a height of 32 feet. Its height above the ground at time t is given by $x(t) = -16t^2 + 32$. Find t^*, the time when the pebble hits the ground. Find the velocity of the pebble $v(t^*) = x'(t^*)$ when it strikes the ground.

When the pebble hits the ground $x(t^*) = 0$. Solving $x(t) = -16t^2 + 32$ for this value we get:

$0 = -16(t^*)^2 + 32 \rightarrow 16(t^*)^2 = 32 \rightarrow (t^*)^2 = 2 \rightarrow \mathbf{t}^* = \sqrt{\mathbf{2}}$.

To find the velocity we must calculate the derivative of position:

$v(t) = \frac{d}{dt}(-16t^2 + 32) = -32t$. For $t^* = \sqrt{2}$ we get $v(t^*) = -\mathbf{32}\sqrt{\mathbf{2}}$.

(15 points) 6. (a) Find the antiderivative of $3x^2 - 2x + 2$ that has the value 10 when $x = 2$.

$\int (3x^2 - 2x + 2)\, dx = x^3 - x^2 + 2x + C$.

We know $x^3 - x^2 + 2x + C = 10$ when $x = 2$, so we can use this to solve for C.

$2^3 - 2^2 + 2 \cdot 2 + C = 10 \rightarrow C = 2$.

So, the antiderivative is: $\mathbf{x^3 - x^2 + 2x + 2}$.

(b) Find the position $x(t)$ satisfying Newton's law

$$\frac{d^2x}{dt^2} = -32$$

such that $x(0) = 0$ and $x'(0) = v(0) = 32$. Be sure to show all of your work.

$\frac{d^2x}{dt^2} = -32 \rightarrow \frac{dx}{dt} = -32t + C_1 \rightarrow x(t) = -16t^2 + C_1 t + C_2$. We're

given $x(0) = 0$, so $x(0) = -16 \cdot 0 + C_1 \cdot 0 + C_2$, and thus $C_2 = 0$.

We're given $x'(0) = 32$, so $32 = -32 \cdot 0 + C_1$, and thus $C_1 = 32$.

Therefore, $\mathbf{x(t) = -16t^2 + 32t}$.