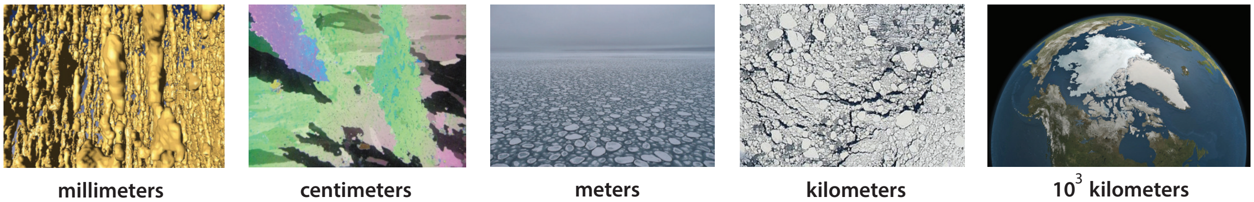

Figure 1: Multiscale structure of sea ice. From left to right: X-ray tomography image of the salty fluid inclusions (brine) in sea ice from the Arctic Ocean [9] Brine volume fraction, geometry, and connectedness depend strongly on temperature, and control fluid transport processes that are central to the role of sea ice in climate and polar ecosystems, such as the evolution of ponds on the surface of melting Arctic sea ice, and nutrient replenishment processes for algae and other microbes living in the brine inclusions; cross-polarized image of the polycrystalline microstructure of sea ice from the Ross Sea, Antarctica (Golden, Langhorne), where each individual crystal has complex fluid microstructure. Polycrystalline statistics and geometrical properties, and related fluid flow properties, depend strongly on conditions in the ocean during formation; pancake ice with a granular polycrystalline microstructure forming in a wave field in the Southern Ocean (Golden); the sea ice pack as a granular composite of ice floes in a sea water host, which displays large-scale fluid-like dynamics when viewed from space (NASA); satellite image of the Arctic Ocean with its sea ice cover (NASA). For perspective, rough image sizes from left to right are 1 cm, 5 cm, 50 m, 100 km, 10,000 km.

From Micro to Macro in the Fluid Dynamics of Sea Ice

Sea ice exhibits complex composite structure on length scales ranging over many orders of magnitude. From the millimeter scale brine inclusion and centimeter scale polycrystalline microstructures, to the evolution of melt ponds on the surface of Arctic sea ice and the ice pack itself, fluid dynamics is central to understanding sea ice behavior. A principal challenge in modelling sea ice and its role in climate and polar ecosystems is how to use information on smaller scale structure to find the effective or homogenized behaviour on larger scales relevant to climate and ecosystem models. In other words, how do we predict macroscopic behavior from microscopic laws and information? We'd like to give an overview of recent results on modelling macroscopic behaviour in the sea ice system, with a focus on novel mathematical approaches, and the central role that fluid behavior over a tremendous range of length and time scales plays in studying sea ice, and the climate system more broadly. Percolation theory for fluid flow, fractal geometry of the brine microstructure, and Stieltjes integral representations for homogenized parameters of two phase and polycrystalline composites, as well as for advection diffusion processes and ocean surface waves in the sea ice cover would be considered. Spectral analysis of these representations leads to a random matrix theory picture of connectedness processes in sea ice, with parallels to Anderson localization and semiconductor physics. Melt pond connectedness and complexification can also be viewed through the lens of Morse theory and persistent homology in topological data analysis, and the Euler characteristic curve as a function of pond water level in particular. Related heterogeneity in the parameters of nonlinear algal bloom models is addressed through polynomial chaos methods in uncertainty quantification, to analyse effective bloom dynamics when the local parameters are random variables, which is the case for the highly heterogeneous fluid microstructure of sea ice. Finally, we consider the application of homogenization ideas to the large scale dynamics of the marginal ice zone (MIZ), the transitional region between the dense inner core of pack ice and open ocean. We have developed a rather simple "mushy layer" model that quantitatively explains the dramatic annual cycle of MIZ width and location. Our work is helping to advance how sea ice is represented in global climate models, and improve projections of the fate of Earth's sea ice packs and the polar ecosystems they support.

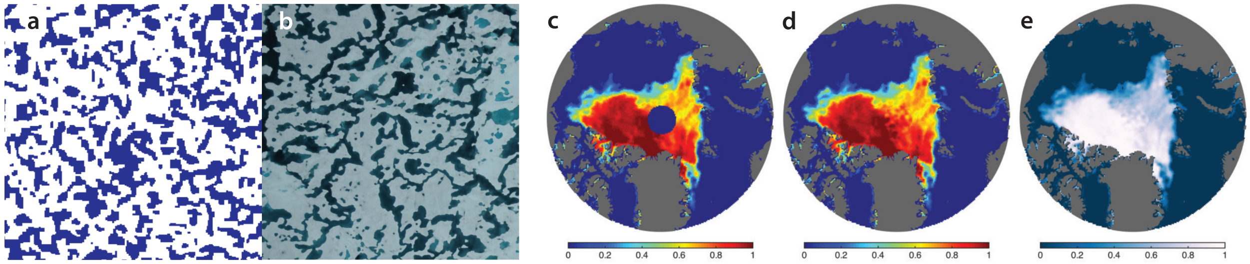

Figure 2: (a) Ising model simulation of melt ponds [7]; (b) photo of real melt ponds (Perovich); (c) example of the polar data gap (blue disc) on 30 August 2007 with shading indicating sea ice concentration; (d) and (e) show the data fill in [11], with the shading in (e) similar to that used by the National Snow and Ice Data Center (http://nsidc.org).

Selected References

1. K. M. Golden, L. G. Bennetts, E. Cherkaev, I. Eisenman, D. Feltham, C. Horvat, E. Hunke, C. Jones, D. Perovich, P. Ponte-Castañeda, C. Strong, D. Sulsky, A. Wells, Modeling sea ice (invited), *Notices of the American Mathematical Society* 67(10), pp. 1535-1555 and issue cover, 2020.

2. K. M. Golden, N. B. Murphy, and E. Cherkaev, Stieltjes functions and spectral analysis in the physics of sea ice (invited), Special Issue on “Interdisciplinary perspectives on climate sciences – highlighting past and current scientific achievements,” *Nonlinear Processes in Geophysics*, submitted, 2022.

3. J. R. Reimer, F. R. Adler, K. M. Golden, and A. Narayan. Uncertainty quantification for ecological models with random parameters. *Ecology Letters*, 25(10):2232–2244, 2022.

4. D. Morison, N. B. Murphy, E. Cherkaev, and K. M. Golden, Order to disorder in quasiperiodic composites, *Communications Physics* 5, 148 (9 pp.), 2022.

5. R. A. Moore, J. B. Jones, D. Gollero, C. Strong, and K. M. Golden, Topology of the sea ice surface and the fractal geometry of Arctic melt ponds (invited), *Physical Review Research*, in revision, 2022.

6. N. B. Murphy, E. Cherkaev, J. Zhu, J. Xin, and K. M. Golden, Spectral analysis and computation for homogenization of advection diffusion processes in steady flows, *Journal of Mathematical Physics*, Vol. 61, 013102, 34 pp., 2020.

7. Y. Ma, I. Sudakov, C. Strong, and K. M. Golden, Ising model for melt ponds on Arctic sea ice, *New Journal of Physics* 21, 063029, 9 pp., 2019.

8. K. R. Steffen, Y. Epshteyn, J. Zhu, M. J. Bowler, J. W. Deming, and K. M. Golden, Network modeling of fluid transport through sea ice with entrained exopolymeric substances, *Multiscale Modeling and Simulation*, Vol. 16, No. 1, pp. 106–124, 2018.

9. K. M. Golden, H. Eicken, A. L. Heaton, J. Miner, D. Pringle, and J. Zhu, Thermal evolution of permeability and microstructure in sea ice, *Geophysical Research Letters*, 34, L16501 (6 pages and issue cover), 2007.

10. K. M. Golden, Climate change and the mathematics of transport in sea ice, invited feature article for the *Notices of the American Mathematical Society*, Volume 56, Number 5, pages 562-584, (including issue cover), May 2009.

11. C. Strong and K. M. Golden, Filling the polar data gap in sea ice concentration fields using partial differential equations, invited, *Remote Sensing*, Vol. 8, No. 6, pp. 442-451, 2016.