TECHNICAL PROPOSAL

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ONR Arctic & Global Prediction Program
and
ONR Applied Computational Analysis Program

Multiscale Models of Melting Arctic Sea Ice

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PROJECT SUMMARY

Multiscale Models of Melting Arctic Sea Ice

During the Arctic melt season, the sea ice surface undergoes a remarkable transformation from vast expanses of snow covered ice to beautiful mosaics of ice and melt ponds. Small, disconnected ponds on the ice surface grow and coalesce to form much larger connected structures with complex boundaries. Melt pond area fraction ϕ can undergo a critical transition with a rapid rise from 0% to more than 70% in just a few days.

Sea ice albedo, a key parameter in climate modeling, is determined by the evolution of melt pond and ice floe configurations. Ice-albedo feedback has played a major role in the recent declines of the summer Arctic sea ice pack. However, understanding the evolution of melt ponds and sea ice albedo remains a significant challenge to improving climate models. In fact, not including the significant effects of melt ponds in the last generation of these models is a key factor in explaining why the observed losses outpaced the projections.

Viewed from high above, the sea ice surface can be thought of as a two phase composite of ice and melt water. The boundaries between the two phases evolve with increasing complexity and a rapid onset of large scale connectivity, or percolation of the melt phase. Pond fractal dimension transitions from about 1 to 2 around a critical length scale of 100 square meters in area. This type of behavior is similar to what is observed in phase transformations in statistical physics and geometrical transitions in composite materials.

We propose here to study the evolution of melt pond geometry on Arctic sea ice, and to develop multiscale models of pond structure which can provide fundamental new insights into the melting process and the evolution of sea ice albedo. We will explore how mathematical models arising in statistical mechanics, composite materials, and pattern formation can be used to efficiently characterize and quantify melt pond structure. Such systems often exhibit universal behavior described by critical exponents depending only on dimension and not on the details of the system. Developing such theories for melt ponds will provide novel ways of looking at the evolution of Arctic melting and ice albedo, as well as a rigorous framework for finding large scale properties from local information. Key components of our proposed research include:

- We will investigate the transition in fractal geometry of melt ponds, and develop mathematical models which capture this striking behavior. Related studies will focus on the melt pond percolation threshold and the onset of large scale connectedness.
- Models of statistical physics and composite materials will be used to explore universal properties of melt pond evolution. Theories of *homogenization* and *upscaling* provide a framework for the calculation of effective properties relevant to climate models.
- Mathematical findings and tools developed in this project will be exploited to improve the representation of melt ponds in high-resolution sea ice models.
- We will gather as much melt pond imagery as possible from a wide range of sources, and develop image analysis techniques to support the mathematical investigations, such as mapping complex melt ponds onto networks of nodes and edges.

Contents

1 Technical Approach 1

1.1 Introduction…………………………………………………………….……….1

1.2 Objectives and Goals…………………………………………………………4

1.3 Mathematical Models of Composites and Phase Transitions……………………4

1.4 Principal Scientific Investigations……………………………………………….7

1.4.1 Melt pond percolation…………………………………….……………7

1.4.2 Transition in the fractal geometry of Arctic melt ponds………………….8

1.4.3 Ising model for melt pond formation………………………………….10

1.4.4 Homogenization theory and spectral measures…………………………..12

1.4.5 Calculating sea ice albedo in climate models…………………………….12

1.5 Image Analysis………………………………………………………………13

1.6 Linkages to Other Programs…………………………………………………15

2 References……………………………………………………………………...16

3 Project Schedule and Milestones………………………………………………21

4 Management Approach……………………………………………………...22

5 Current and Pending Projects and Proposals……………………….……..23

6 Qualifications..27

7 Budget…………………………………………………………………………29

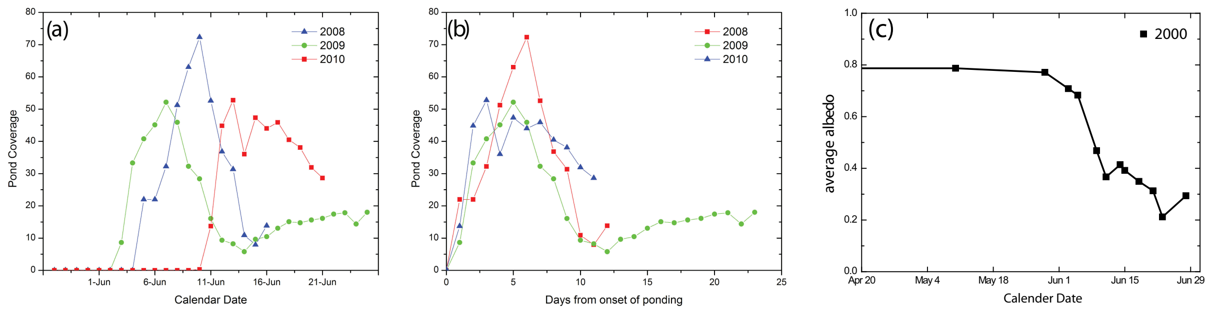


Figure 1: Arctic melt pond coverage vs. calendar date near Barrow, AK in (a) and melt pond coverage vs. days since onset of pond formation in (b), for the years 2008–2010 [51]. Average albedo vs. calender date in 2000 near Barrow is shown in (c). Each data set exhibits critical behavior at the onset of melt pond formation, similar to the behavior of order parameters characterizing phase transitions in statistical physics and composite materials.

1 Technical Approach

1.1 Introduction

During the melt season the Arctic sea ice cover becomes a complex, evolving composite of ice, melt ponds, and open water. Melting is strongly influenced by the morphological characteristics of the ice cover, such as the size and shape of melt ponds and ice floes. Albedo, light transmission, and melting are closely connected to melt pond characteristics, while floe perimeter is a primary controlling parameter for lateral melting. We propose to develop multiscale mathematical descriptions of the melting sea ice pack, focusing on the geometry and evolution of surface melt ponds. Models of phase transitions in statistical physics and composite materials will be used to investigate melt pond structure. This work will improve our ability to model the partitioning of solar radiation between reflection, absorption in the ice, surface melting, bottom melting, lateral melting, and heat storage in the upper ocean.

While snow and ice reflect most incident sunlight, melt ponds and ocean absorb most of it. The overall reflectance or albedo of sea ice is determined by the evolution of melt pond geometry [48, 57, 51] and ice floe configurations [64]. As melting increases, so does solar absorption, which leads to more melting, and so on. This *ice–albedo feedback* has played a significant role in the decline of the summer Arctic ice pack [50], which most climate models have underestimated [58, 7]. Sea ice albedo is a significant source of uncertainty in climate projections and a fundamental problem in climate modeling [20, 57, 47, 51].

From the first appearance of visible pools of water, often in early June, the area fraction ϕ of sea ice covered by melt ponds can increase rapidly to over 70% in just a few days [51, 56], as demonstrated in Figure 1 (a) and (b). Moreover, the accumulation of water at the surface dramatically lowers the albedo where the ponds form. A corresponding critical drop-off in average albedo is displayed in Figure 1 (c). The resulting increase in solar absorption in the ice and upper ocean accelerates melting [49], possibly triggering *ice-albedo feedback*. Similarly, an increase in open water fraction ψ lowers albedo, thus increasing solar absorption and subsequent melting. The spatial coverage and distribution of melt ponds on the surface

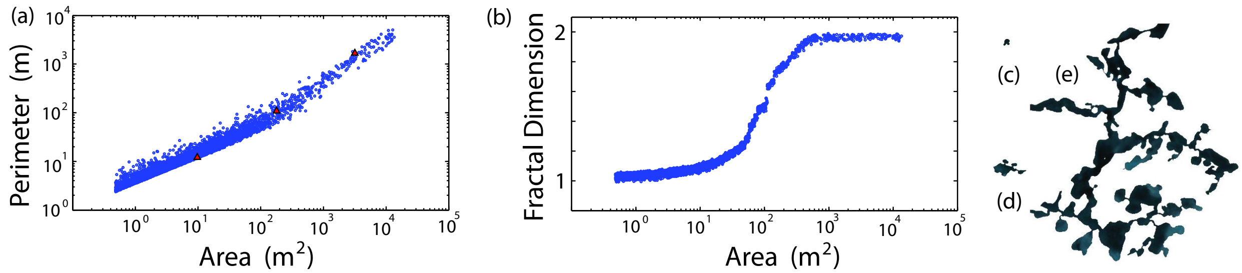


Figure 2: (a) Area – perimeter data for 5,269 Arctic melt ponds, plotted on logarithmic scales. The slope transitions from about 1 to 2 at a critical length scale of around 100 square meters. (b) Melt pond fractal dimension D as a function of area A, computed from the data in (a), showing the transition to complex ponds with increasing length scale. Ponds corresponding to the three red triangles in (a), from left to right, are shown in (c), (d), and (e), respectively. The transitional pond in (d) has horizontal scale of about 30 m.

of ice floes and the open water between the floes thus exerts primary control of ice pack albedo and the partitioning of solar energy in the ice-ocean system [16, 51].

While melt ponds form a key component of the Arctic marine environment, comprehensive observations or theories of their formation, coverage, and evolution remain relatively sparse. Available observations of melt ponds show that their areal coverage is highly variable, particularly for first year ice early in the melt season, with rates of change as high as 35% per day [56, 51]. Such variability, as well as the influence of many competing factors controlling melt pond and ice floe evolution, makes the incorporation of realistic treatments of albedo into climate models quite challenging [51]. Small and medium scale models of melt ponds which include some of these mechanisms have been developed [19, 59, 57], and melt pond parameterizations are being incorporated into global climate models [20, 38, 47].

The surface of an ice floe is viewed here as a two phase composite [45] of dark melt ponds and white snow or ice. The onset of ponding and the rapid increase in coverage beyond the initial threshold is similar to critical phenomena in statistical physics [13, 62] and composite materials [60]. Here we ask if the evolution of melt pond geometry exhibits universal characteristics which do not necessarily depend on the details of the driving mechanisms in numerical melt pond models. Fundamentally, the melting of Arctic sea ice is a phase transition phenomenon, where a solid turns to liquid, albeit on large regional scales and over a period of time which depends on environmental forcing and other factors. We thus look for features of melt pond evolution which are mathematically analogous to related phenomena in the theories of phase transitions and composite materials. As a first step in this direction, and a key finding which provides a principal route of investigation in the proposed work, we consider the evolution of complexity of Arctic melt ponds.

By analyzing area−perimeter data from hundreds of thousands of melt ponds, we have discovered an unexpected separation of scales, where the pond fractal dimension D exhibits a transition from 1 to 2 around a critical length scale of 100 square meters in area [36], as shown in Figure 2. Small ponds with simple boundaries coalesce or percolate to form larger connected regions, as shown in Figure 3 (a). Pond complexity increases rapidly through the transition region and reaches a maximum for ponds larger than 1000 m^2 whose boundaries

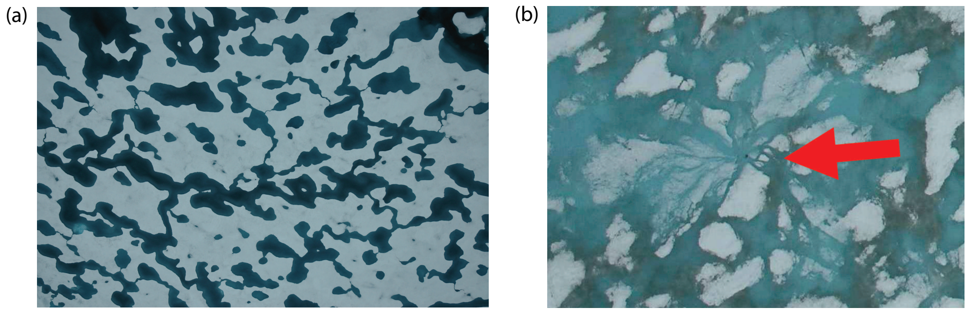


Figure 3: (a) The complex geometry of well developed Arctic melt ponds, illutrated in an aerial photo taken August 14, 2005 on the Healy-Oden Trans Arctic Expedition (HOTRAX), courtesy of Don Perovich. (b) A melt pond drains through a small seal hole (near the arrow tip). The length scale over which melt water drains through this hole is strongly influenced by the length scale of melt pond connectivity. Photo courtesy of Chris Polashenski.

resemble space filling curves [53] with $D \approx 2$. These configurations affect the complex radiation fields under melting sea ice, the heat balance of sea ice and the upper ocean [21], under-ice phytoplankton blooms [2], biological productivity, and biogeochemical processes.

Melt pond evolution also appears to exhibit a *percolation threshold*, where one phase in a composite becomes connected on macroscopic scales as some parameter exceeds a critical value [60, 13]. An important example of this phenomenon in the microphysics of sea ice, which is fundamental to the process of melt pond drainage, is the percolation transition exhibited by the brine phase in sea ice, known as the *rule of fives* [30, 31, 52]. When the brine volume fraction of columnar sea ice exceeds about 5%, the brine phase becomes macroscopically connected so that fluid pathways allow flow through the porous microstructure of the ice. Similarly, even casual inspection of aerial photos shows that the melt pond phase of sea ice undergoes a percolation transition where disconnected ponds evolve into much larger scale connected structures with complex boundaries. Connectivity of melt ponds promotes further melting and break-up of floes, as well as horizontal transport of meltwater and drainage through cracks, leads, and seal holes [56, 51], as illustrated in Figure 3 (b).

The rich behavior we observe for melt pond evolution is similar to phase transitions in statistical mechanics [13, 62, 14] and composite materials [60, 45]. Such systems often exhibit universal critical behavior, where order parameters like the effective conductivity of a composite or the magnetization of an Ising ferromagnet are described near their threshold by critical exponents depending only on dimension and not on the details of the system. Developing similar theories for melt ponds could provide fundamental new insights into the evolution of ice pack albedo and Arctic melting, as well as a rigorous framework for finding large scale, effective properties from local information, a fundamental issue in climate modeling. The application of modern theories of homogenization and statistical physics to melt pond and albedo evolution will lend insights into parameterizing sub-grid scale processes into sea ice and climate models. For example, the scale of connected fluid pathways on the surface influences melt pond drainage through cracks, seal holes, and the porous microstructure of sea ice [51]. Investigating critical behavior as a natural aspect of the polar marine environment will shed light on key questions such as the rapidity of the sea ice retreat and whether a so-called *tipping point* or critical transition has been passed in the decline [17]. It will also advance our ability to model the future trajectory of the Arctic sea ice pack.

1.2 Objectives and Goals

We propose to investigate the formation and evolution of Arctic melt ponds. Models and techniques of statistical physics and composite materials will be employed to provide new insights into melt pond structure, and to explore universal behavior characteristic of phase transition phenomena. We will also begin to investigate how such findings can impact the computation of ice pack albedo in sea ice numerical models.

1. Identify the critical melt pond area fraction ϕ_c, or percolation threshold, where the ponds become connected over large length scales. What is the critical exponent for the divergence of the correlation length? How does ϕ_c depend on local characteristics such as snow and ice topography, or whether the ice is first year or multiyear?
2. Investigate our finding that around a critical length scale, the fractal dimension of melt pond boundaries transitions from 1 (simple, Euclidean shapes) to about 2 (space filling curves) [36]. Explore the use of partial differential equations of pattern formation and phase change to model this striking phenomenon.
3. Use methods of homogenization and spectral measures to characterize melt pond and ice floe configurations, as well as critical transitions in their effective properties. This approach provides a rigorous framework for *upscaling* local morphological characteristics into larger scale models.
4. Develop stochastic lattice models of melt pond formation based on simple cellular automata and the Ising model for phase transitions in statistical mechanics.
5. Explore how our mathematical results can be exploited to improve the representation of melt ponds and albedo in global climate models and high-resolution sea ice models.
6. Develop methods of image analysis focused on addressing the questions about melt ponds raised above, such as tracking how features evolve in time, and mapping melt pond images to networks of nodes and edges. These techniques will likely be useful to other ONR investigators studying melt ponds.

1.3 Mathematical Models of Composites and Phase Transitions

Here we give a brief overview of some of the mathematical models and techniques that we will use in studying critical behavior of melting in the sea ice pack.

Percolation models. Consider the $d-$dimensional integer lattice $\mathbb{Z}^d$, and the square or cubic network of bonds joining nearest neighbor lattice sites. In the percolation model [8, 60, 33, 9], we assign to each bond a conductivity $\sigma_0 > 0$ with probability p, meaning it is open (black), and with probability $1 - p$ we assign a 0, meaning it is closed. Two examples of lattice configurations are shown in Figure 4, with $p = 1/3$ in (a) and $p = 2/3$ in (b). Groups of connected open bonds are called *open clusters*. In this model there is a critical probability p_c, $0 < p_c < 1$, called the *percolation threshold*, at which the average cluster size

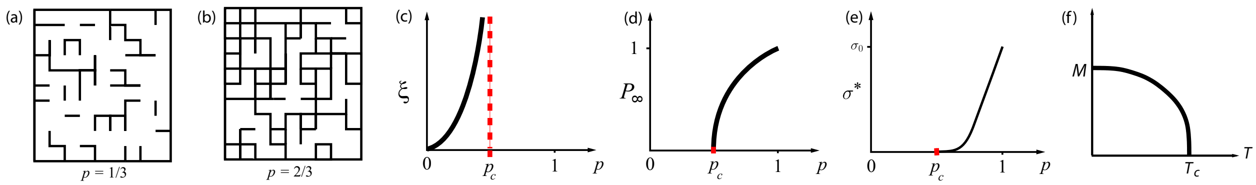


Figure 4: The two dimensional square lattice percolation model below its percolation threshold of $p_c = 1/2$ in (a) and above it in (b). (c) Divergence of the correlation length as p approaches p_c. The infinite cluster density of the percolation model is shown in (d), and the effective conductivity is shown in (e). (f) The magnetization of the Ising model.

diverges and an infinite cluster appears. For the two dimensional bond lattice $p_c = 1/2$. For $p < p_c$ the density of the infinite cluster $P_\infty(p)$ is 0, while for $p > p_c$, $P_\infty(p) > 0$ and near the threshold, $P_\infty(p) \sim (p - p_c)^\beta$ as $p \to p_c^+$, where β is a universal critical exponent, that is, it depends only on dimension and not on the details of the lattice. Let $x, y \in \mathbb{Z}^d$ and $\tau(x, y)$ be the probability that x and y belong to the same open cluster. Then for $p < p_c$, $\tau(x, y) \sim e^{-|x-y|/\xi(p)}$, and the correlation length $\xi(p) \sim (p_c - p)^{-\nu}$ diverges with a universal critical exponent ν as $p \to p_c^-$, as shown in Figure 4 (c).

The effective conductivity $\sigma^*(p)$ of the lattice, now viewed as a random resistor (or conductor) network, defined via Kirchoff's laws, vanishes for $p < p_c$ like $P_\infty(p)$ since there are no infinite pathways, as shown in Figure 4 (e). For $p > p_c$, $\sigma^*(p) > 0$, and near p_c, $\sigma^*(p) \sim \sigma_0(p - p_c)^t$, $p \to p_c^+$, where t is the conductivity critical exponent, with $1 \le t \le 2$ in $d = 2, 3$ [23, 24, 29], and numerical values $t \approx 1.3$ in $d = 2$ and $t \approx 2.0$ in $d = 3$ [60]. Consider a random pipe network with effective fluid permeability $\kappa^*(p)$ exhibiting similar behavior $\kappa^*(p) \sim \kappa_0(p - p_c)^e$, where e is the permeability critical exponent, with $e = t$ [12, 55, 29]. Both t and e are believed to be universal $-$ they depend only on dimension and not the lattice. Continuum models can exhibit nonuniversal behavior with exponents different from the lattice case and $e \neq t$ [34, 18, 60, 54, 40].

Homogenization and spectral measures. *Homogenization* denotes a field of applied mathematics where the goal is to find a homogeneous medium which behaves macroscopically the same as a given inhomogeneous medium. The methods are focused on finding the effective properties of inhomogeneous media such as composites. This discussion provides a framework for an effective albedo of the ice pack, as well as the effective horizontal flow of melt water on the sea ice surface, where the inhomogeneities are melt ponds or topography. We will see that the *spectral measure* provides a powerful tool for upscaling geometrical information about a composite into calculations of effective properties.

We now briefly describe the *analytic continuation method* for studying the effective properties of composite materials [6, 43, 25, 28]. This method has been used to obtain rigorous bounds on effective transport coefficients of composite materials from partial knowledge of the microstrucure, such as the relative volume fractions of the phases. For simplicity we choose the electrical conductivity of a two phase composite, although the method applies to any classical transport coefficient. This framework has been extended to finding the effective diffusivity of a passive tracer advected by a fluid velocity field [3, 4].

We consider a two-phase random medium with $\sigma(x, \omega)$ the local conductivity, a spatially stationary random field in $x \in \mathbb{R}^d$ and $\omega \in \Omega$, where Ω is the set of realizations of the medium. Assume $\sigma(x) = \sigma_1\chi_1 + \sigma_2\chi_2$, where $\chi_j(x,\omega)$ is the characteristic function of medium $j = 1, 2$, equaling 1 for $\omega \in \Omega$ with medium j at x, and 0 otherwise. Let $E(x)$ and $J(x)$ be the stationary random electric and current fields, related by $J = \sigma E$, satisfying $\nabla \cdot J = 0$ and $\nabla \times E = 0$, where the average $\langle E(x)\rangle = e_k$, and e_k is a unit vector in the k^{th} direction. The effective conductivity tensor $\boldsymbol{\sigma}^*$ is defined as $\langle J\rangle = \boldsymbol{\sigma}^*\langle E\rangle$. We focus on one diagonal coefficient $\sigma^* = \sigma^*_{kk}$, with $\sigma^* = \langle \sigma E_k\rangle$, and since σ^* depends on $h = \sigma_1/\sigma_2$, we define $m(h) = \sigma^*/\sigma_2$, which is a Stieltjes function. It is analytic off $(-\infty, 0]$ in the h–plane, and maps the upper half plane to the upper half plane [5, 25].

The key step [25, 5, 43, 45] is to obtain an integral representation for σ^*. Consider $F(s) = 1 - m(h)$, $s = 1/(1-h)$, which is analytic off $[0, 1]$ in the s–plane. Then [25]

$$F(s) = 1 - \frac{\sigma^*}{\sigma_2} = \int_0^1 \frac{d\mu(\lambda)}{s-\lambda}\,, \qquad s = \frac{1}{1-\sigma_1/\sigma_2}\,, \tag{1}$$

where μ is a positive measure on $[0, 1]$. This formula arises from the resolvent representation of the electric field $E = (s+\Gamma\chi_1)^{-1}e_k$, where $\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$ and $\Delta = \nabla^2$ is the Laplacian, yielding $F(s) = \langle \chi_1[(s+\Gamma\chi_1)^{-1}e_k]\cdot e_k\rangle$. In the Hilbert space $L^2(\Omega, P)$ with weight χ_1 in the inner product, $\Gamma\chi_1$ is a bounded self adjoint operator. Formula (1) is the spectral representation of the resolvent, and μ is a spectral measure of $\Gamma\chi_1$, in the e_k state. Formula (1) separates the component parameters in s from the geometrical information in μ. (Extensions to multicomponent media involve several complex variables [26, 22, 46, 44, 15].) Information about the geometry enters through the moments $\mu_n = \int_0^1 z^n d\mu(z) = (-1)^n\langle \chi_1[(\Gamma\chi_1)^n e_k]\cdot e_k\rangle$. Then $\mu_0 = \phi$, where ϕ is the volume or area fraction of phase 1, such as the melt pond coverage, and $\mu_1 = \phi(1-\phi)/d$ if the material is statistically isotropic. In general, μ_n depends on the $(n+1)$–point correlation function of the medium.

Computing the spectral measure μ for a given composite microstructure first involves discretizing a two phase image of the composite into a square lattice filled with 1's and 0's corresponding to the two phases. Then the key operator $\Gamma\chi_1$, which depends on the geometry of the network via χ_1, becomes a self adjoint matrix. The spectral measure may be calculated directly from the eigenvalues $\{\lambda_i\}$ and eigenvectors $\{v_i\}$ of this matrix via

$$d\mu(\lambda) = \sum_{i=1}^{n} m_i\delta(\lambda-\lambda_i)d\lambda, \quad m_i = \langle e_0^T v_i v_i^T e_0\rangle, \tag{2}$$

where $\delta(\lambda)$ is the Dirac delta function and e_0 is a vector of ones. In [32] we computed the spectral measure for samples of healthy and osteoporotic bone to distinguish them via the connectivity of the trabecular architecture.

As the system size N increases, the eigenvalues become increasingly dense in the spectral interval [0,1]. The presence or absence of gaps in the spectrum near the endpoints of [0,1], and the details of how large a gap is or how large the spectral values m_i are, give important information pertaining to the connectivity and effective transport properties of the system.

Phase transitions and the Ising model of a ferromagnet. The Ising model of a ferromagnet in a magnetic field H and at temperature T is perhaps the most studied example of a phase transition in statistical mechanics [62, 13, 27]. We consider a finite box $\Lambda \subset \mathbb{Z}^d$

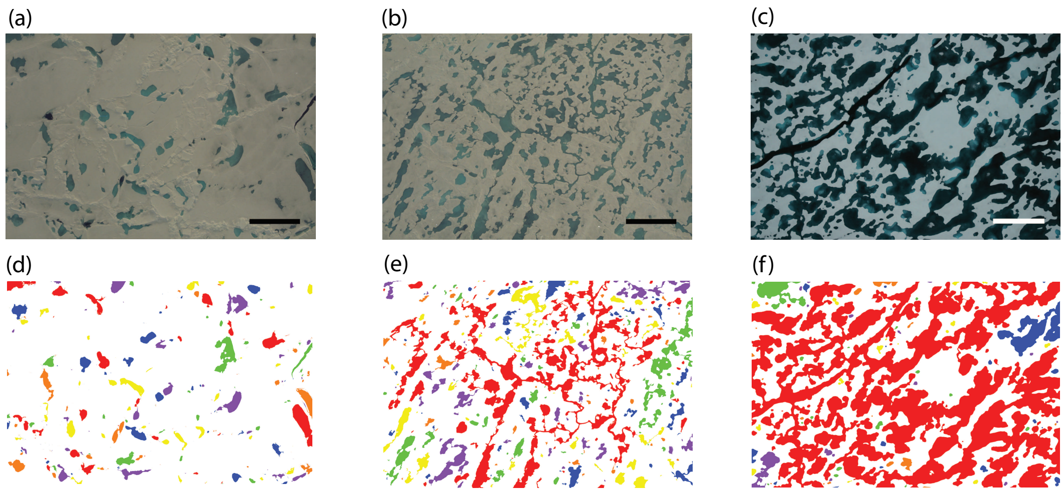


Figure 5: Evolution of melt pond connectivity. (a) Disconnected ponds, (b) transitional ponds, (c) fully connected melt ponds. The bottom row shows the color coded connected components for the corresponding image above: (d) no single color spans the image, (e) the red phase just spans the image, (f) the connected red phase dominates the image. The scale bars represent 200 m for (a) and (b), and 35 m for (c).

containing N sites. At each site there is a spin variable s_i which can take the values $+1$ or -1. We consider a Hamiltonian with ferromagnetic pair interaction $J \geq 0$ between nearest neighbor pairs $< i, j >$,

$$\mathcal{H}_\omega = -H \sum_i s_i \; - \; J \sum_{<i,j>} s_i s_j \; , \tag{3}$$

for any configuration $\omega \in \Omega = \{-1, 1\}^N$ of the spins. The average magnetization, which serves as the principal *order parameter* in the system, $M(T, H) = \lim_{N\to\infty} \frac{1}{N} \langle \sum_{i=1}^{N} s_i \rangle$, where $\langle \cdot \rangle$ in this context denotes averaging over $\omega \in \Omega$ with Gibbs weights, can be expressed in terms of the free energy (per unit site) f as $M(T, H) = -\frac{\partial f}{\partial H}$. The magnetic susceptibility $\chi(T, H)$, which is the analog of the effective conductivity in the models above, is given by $\chi(T, H) = \frac{\partial M}{\partial H} = -\frac{\partial^2 f}{\partial H^2} \geq 0$. When $H = 0$, $M(T) \sim (T_c - T)^\beta$ as $T \to {T_c}^-$, as shown in Figure 4 (f), and $\chi \sim (T - T_c)^{-\gamma}$ as $T \to {T_c}^+$. The universal exponent β here plays a similar role as β for the percolation model, but has a different numerical value. Below we will discuss how this framework can be used to model melt ponds.

1.4 Principal Scientific Investigations

1.4.1 Melt pond percolation

Melt pond connectivity, as shown in Figure 5, plays a fundamental role in the evolution of the melting Arctic sea ice pack. Such processes as horizontal melt water transport, illustrated in Figure 3 (b), facilitate drainage through macro-pores like seal holes, cracks, and leads. The break-up of ice floes as they melt is strongly influenced by the connectivity of the ponds. For example, as ponds deepen they weaken an ice floe, making it more susceptible to breakage and fracture. Connected ponds spanning an ice floe are more efficient at promoting fracture

(a) (b)

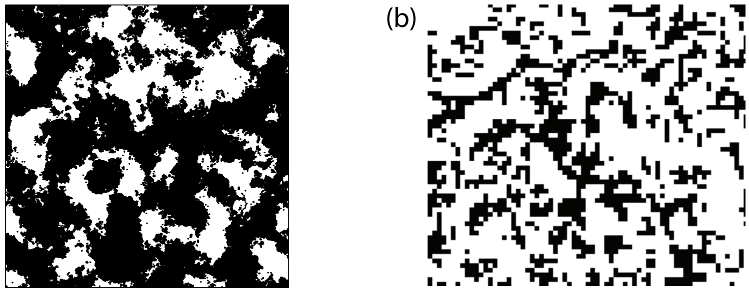


Figure 6: (a) Connected clusters in a continuum percolation model where a level set (water level) intersects a surface defined by a Gaussian random field with covariance determined by snow topography data [57]. The resulting configurations resemble melt ponds on first year sea ice. (b) A simple, stochastic lattice growth model yields fairly realistic "ponds," and also exhibits a transition in pond fractal dimension with length scale.

than a series of disconnected ponds. We therefore propose to examine the key issue of melt pond connectivity, as illustrated in Figure 5.

For melt ponds, we ask: what is the percolation threshold for melt pond evolution. More precisely, if ϕ is the area fraction of melt pond coverage on a large floe, what is the critical threshold ϕ_c for large scale connectivity? This can be addressed by using image analysis to study how the correlation length $\xi(\phi)$ grows as the ponds evolve. As outlined above, the percolation threshold can be characterized by where $\xi(\phi)$ "diverges," or in the case of large floes, where the scale of the correlation length spans the floe size. Does this threshold value depend on floe topography or the age of the floe? What is the variability of ϕ_c from floe to floe? In mathematical percolation theory, the threshold value depends on the details of the lattice model, such as square vs. triangular lattice. However, the critical exponent ν characterizing the divergence of ξ is universal, depending only on dimension. Does the value of ν for melt ponds match the universal lattice value of 4/3 in two dimensions?

We also expect that our investigations of melt pond connectivity will be aided by continuum percolation models based on the intersection of a level set with a random surface [39]. Here, the level set represents the melt water level and the random surface the ice or snow topography. A configuration for a Gaussian model is shown in Figure 6 (a). Questions of percolation thresholds and critical exponents have been studied in this context, and we expect that such studies will lend insights into the melt pond problem.

Since one aerial photo generally does not span a large floe, to address the above questions we must develop techniques for stitching together many images to create a mosaic. We are also interested in the time evolution of melt ponds and their geometry during the melt season, so being able to track the growth of individual ponds and their connectivity will drive some of the imaging work. Moreover, mapping melt ponds onto networks (graphs) of vertices and edges facilitates analysis of pond connectivity and the correlation length.

1.4.2 Transition in the fractal geometry of Arctic melt ponds

For simple objects like circles and polygons, the perimeter P scales like the square root of the area A, that is, $P \sim \sqrt{A}$. However, for complex planar regions with fractal curves as their boundaries,

$$P \sim \sqrt{A}^{\,D} \tag{4}$$

where the exponent D is the fractal dimension of the boundary curve. By analyzing area-perimeter data, Lovejoy [41] found that clouds have a fractal dimension of $D \approx 1.35$.

Viewing images of complex melt ponds, such as in Figures 3 and 5, suggests similar fractal behavior. However, after developing automated techniques for obtaining data on the area A and perimeter P for melt ponds from helicopter photos, and analyzing the data from thousands of melt ponds, we found something unexpected. In particular, we discovered a bend in the P *vs.* A data displayed on logarithmic scales around a critical length scale of about 100 square meters in terms of area, as shown in Figure 2 (a). We then devised an algorithm to statistically compute the derivative of this data set, yielding the graph for $D(A)$ in Figure 2 (b), the fractal dimension D as a function of melt pond length scale, as measured by the area A. These data showed that melt ponds exhibit a transition in fractal dimension from about 1 for A less than 10 m^2 to about 2 for A greater than 1000 m^2. We remark that $D = 2$ is the upper bound for the fractal exponent, since it corresponds to curves which fill two dimensional space, such as the famous Peano curve or Brownian motion.

A principal goal of the proposed research is to develop mathematical models of melt pond evolution. We will explore deterministic models involving partial differential equations (PDE) as well as stochastic models. In particular, we will look for models which help explain the transition in fractal dimension we observe in actual melt ponds, and the value of the critical length scale. In [36] we proposed an argument which relates the critical length scale to the onset of self-similarity, as illustrated in Figure 2. However, we seek a quantitative understanding of how this scale is set through appropriate mathematical models.

One natural approach is to explore the use of phase field equations, such as the Ginzburg–Landau model [10]. It was originally developed in the context of superconductivity, yet has been used in the study of many phase transition and pattern formation problems. The two main functions of interest are the order parameter $\psi = \psi(x, y, t)$ (not to be confused with open water fraction) and the temperature $u = u(x, y, t)$, for $(x, y) \in \mathcal{R}$ for some region $\mathcal{R} \subset \mathbb{R}^2$, and $t \geq 0$. In the case of melt ponds, we have two extreme states, ice and water. The frozen state is characterized by $\psi = -1$ and the melted state by $\psi = +1$. We use the following coupled equations for ψ and u,

$$u_t = K\Delta u - \frac{b}{2}\psi_t, \tag{5}$$

$$\alpha\xi^2\psi_t = \xi^2\Delta\psi + a^{-1}g(\psi) + 2(u - \theta), \tag{6}$$

where $g = c(\psi - \psi^3)$, θ is the melting temperature, K is the coefficient of thermal conductivity, the parameters a, b, c, α, and ξ are determined from the specific physics of the problem, and appropriate thermal boundary conditions are applied at the phase interface. We propose to explore how this system applies to the melt pond problem, by studying the evolution of the interface separating the melted and frozen states, and incorporating relevant physics, such as snow topography. Interestingly, using asymptotic methods, the classical Stefan and Hele-Shaw systems can be obtained from the Ginzburg–Landau model in the limit of a sharp interface [10], which we believe should be quite relevant for the melt pond problem.

We will also develop simple stochastic models on lattices to simulate the evolution of melt ponds. In Figure 6 (b) we show the results of one such very simple growth model. Here we

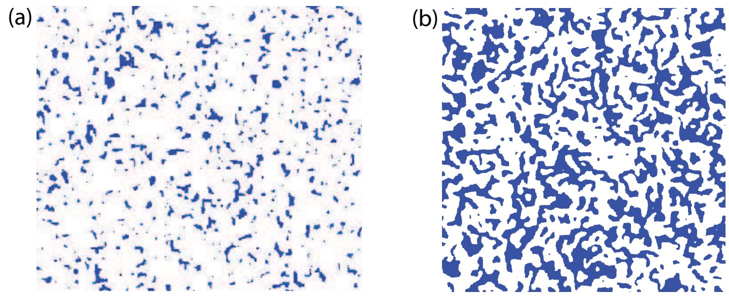


Figure 7: Islands of like spins in the Ising model of a ferromagnet. The configurations resemble melt ponds. (a) Early melt ponds. In this simulation the heat content $H > 0$ is small. (b) Well-developed melt ponds. In this simulation the heat content $H > 0$ is large.

specifically wanted to begin to understand why the fractal dimension undergoes a transition. We made the following simple observation, that adding a melted square connected only by a corner to an existing pond increases the perimeter more than a square connected along an edge. The probability of this occurring is significantly increased for pond sizes above some critical size, and in fact, we see a transition in the fractal dimension around that length scale, with data quite similar to Figure 2 (a). We plan to build on this initial success to construct more realistic models which develop a transition in fractal geometry on their own, now that we begin to understand one possible mechanism.

1.4.3 Ising model for melt pond formation

We consider an Ising model for melt ponds where the "spins" s_i are assigned either -1 for ice or $+1$ for melt water. We will explore various Hamiltonians that model the energy associated with melt pond configurations. The Ising model of statistical mechanics is a feasible model for melt pond evolution because it possesses the following two properties: 1. Disconnected small scale same-phase regions tend to cluster and favor the formation of larger regions when the system is not dominated by random fluctuations. 2. The system favors same-phase regions to grow or shrink their boundaries as opposed to, for example, shifting or being deformed.

The role of the applied magnetic field in Equation (3) can be played by the external forcing of the sea ice layer principally by net solar radiation and air temperature [1]. Then H can be thought of as external heating of the system. For $H > 0$, the system is driven toward more widespread melting. For $H < 0$, the system is driven toward less melting. The average magnetization $M(H)$ is then closely related to the sea ice albedo. Two realizations of the model configurations are shown in Figure 7, which bear striking resemblances to actual melt ponds. Moreover, the model yields an average albedo displaying the critical behavior shown in Figure 4 (f), which is similar to the albedo in Figure 1.

The model can be enhanced further by assuming an initial surface topography for the ice cover. For a given topography, represented by a height function h, we can define the interaction field $J = J_h(i, j)$ in such a way that the system favors water moving in the direction of steepest topographical descent. The surface topography used to initialize the model can ideally be obtained from measurements by our colleagues (C. Polashenski and D. Perovich), or calculated using topography models; see for example [57].

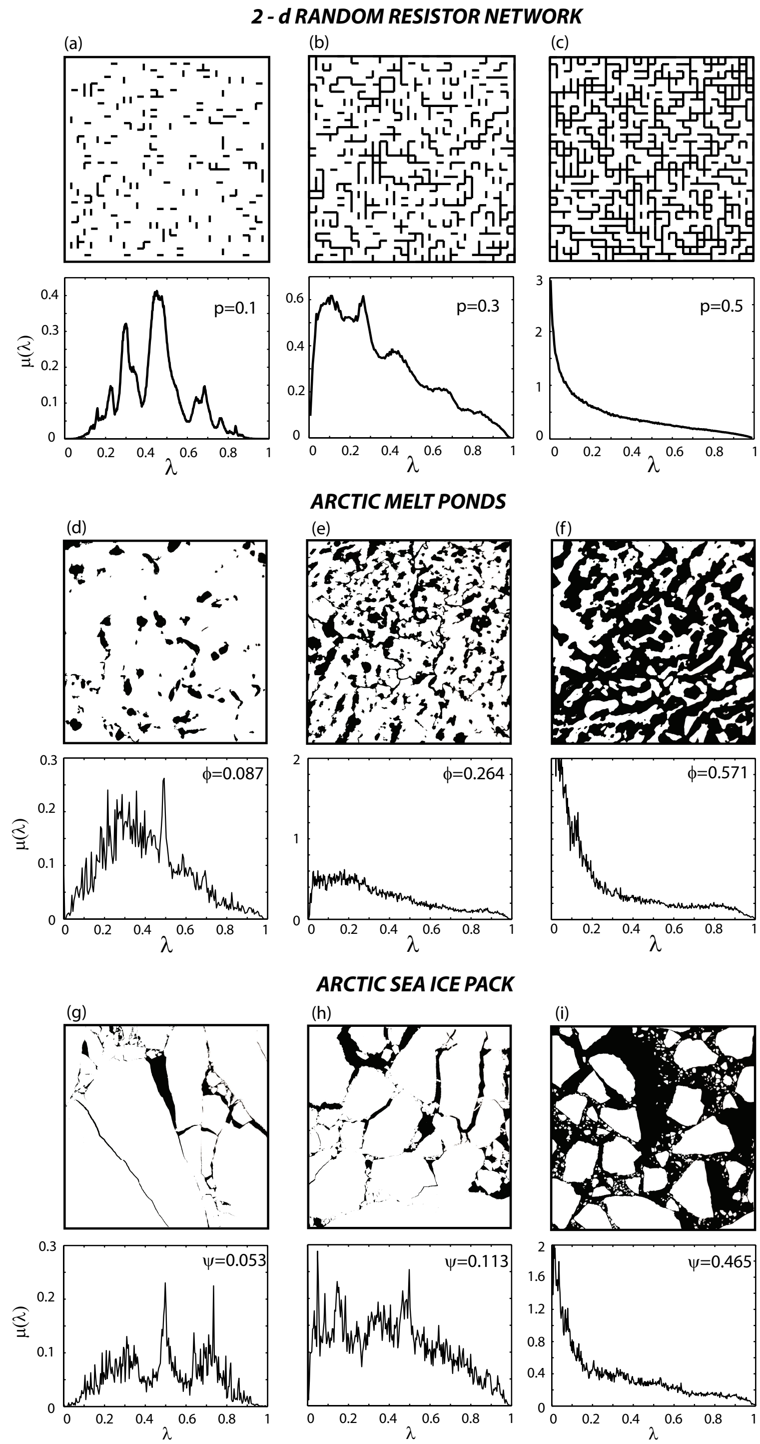


Figure 8: *Spectral analysis of sea ice structures.* Increasingly connected configurations of the percolation model (a)-(c), melt ponds (d)-(f), and leads in the ice pack (g)-(i), with corresponding spectral measures below. The area under each spectral function in (d)-(f) is the melt pond area fraction ϕ, and the open water fraction ψ in (g)-(i). For the network, the width of the gap in the spectrum near $\lambda = 1$ for $p = 0.1$ shrinks to 0 with increasing connectedness as the percolation threshold $p = 0.5$ is approached, with similar behavior for melt ponds and the ice pack.

1.4.4 Homogenization theory and spectral measures

As discussed above, homogenization theory [45, 63] provides a mathematical framework to analyze the effective transport and rheological properties of complex, heterogeneous systems. The analytic continuation method [25], in particular, is based on powerful integral representations for effective properties, and involves the *spectral measures* which depend on the composite geometry, such as the volume or area fractions of the phases, as well as the connectivity and percolation properties. The moments of the spectral measure are related to the correlation functions of the microstructure. We propose to use this framework to *upscale* the local geometry of the ice pack, such as melt pond and ice floe configurations, into calculations of their effective properties. In Figure 8 we show a series of spectral measures computed from discretizations of melt pond and ice pack images, which raises the following questions. Does the evolution of the melt pond spectral measures obey universal laws of critical behavior, such as the collapse of a spectral gap with the onset of large scale connectivity? Can critical behavior in the local geometry be related to critical behavior in effective rheological and transport properties? In view of Figure 3 (b), which illustrates the importance of horizontal flow of meltwater on the sea ice surface, we observe that the calculation of melt pond spectral measures immediately gives a rigorous formula for the effective behavior of such flows, such as the large scale, horizontal surface fluid permeability considered in the next section. Sinks representing seal holes and cracks, which are relevant to regional sea ice models and albedo evolution, can be incorporated into homogenized PDE models.

1.4.5 Calculating sea ice albedo in climate models

Modern global climate models (GCM's) account for melt pond albedo effects by explicitly calculating meltwater volume from sea ice or snow and then parameterizing the associated pond fractional coverage ϕ and depth h_p [47, 37]. The various parameterizations in use for ϕ and h_p are based on observed relationships [49], curves fit to output from high-resolution mass balance simulations [42], or the assumption that meltwater travels horizontally to cover the ice of lowest surface height [19, 20]. In higher-resolution frameworks with explicit sea ice topography, spatial patterns of ϕ and h_p are simulated by transporting meltwater horizontally according to Darcy's Law for flow through porous media [42].

We will investigate the potential for applying the mathematical models and techniques developed in this proposal to the simulation of melt ponds in modeling frameworks including GCM's. Recall that GCM's with physically based melt pond schemes calculate meltwater volume V_p, and then the albedo calculation involves specifying the associated pond fractional coverage ϕ. Pond depth h_p is often assumed constant on a given sea ice thickness class, so the governing equation

$$V_p = \phi h_p \tag{7}$$

is typically closed by assuming that ϕ is a known function of h_p [47, 37].

We expect that the critical transition in pond complexity discovered using pond perimeter and area [36] will project strongly into the GCM variable space (V_p, ϕ, h_p), and that this projection can be exploited to formulate a closure for (7) that obeys the observed universal scaling. For example, when meltwater volumes reach levels conducive to complex networking of ponds, it may be more natural to specify ϕ from V_p rather than from h_p. To enable

Region	Type	Year	Reference	Comment
Beaufort Sea	Aerial	1998	Perovich et al., 2002	Time series May–October
Trans-Arctic	Aerial	2005	Perovich et al., 2005	Spatial survey in August–September
Several years, several sites	Satellite	Several	Fetterer and Untersteiner, 2002	National Technical means
Chukchi, Beaufort Seas	Satellite	2011	Frey, personal communication	Images from Quickbird
Chukchi, Beaufort Seas	Satellite	2013, 2014	Results from Polashenski et al. proposal	Images from Quickbird
Arctic Ocean, north of Svalbard	Aerial	2012, 2013	Results from Granskog and Pedersen	Spatial survey in July-August

Table 1: Sources of images for melt pond analysis.

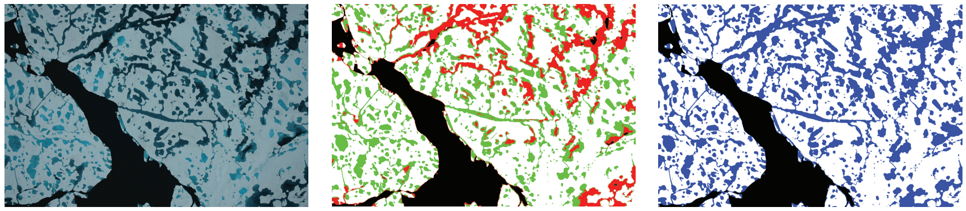
the required multivariate analyses, extraction of ϕ will be a straightforward addition to the perimeter-area analysis, and more sophisticated image processing will be used to make color-based estimates of h_p and hence V_p [36]. The required algorithms can build on the *color–gradient* technique developed in [36]. A validation and sensitivity analysis of the new specification of ϕ will be performed using the Los Alamos CICE model [38].

Finally, we will also explore developing a predictive modeling framework that captures the observed critical transition in melt pond complexity. We will begin with the high-resolution two-dimensional model introduced by Luthje and collaborators [42]. This framework's horizontal transport scheme is based on Darcy's Law for flow through porous media, and thus does not simulate more rapid horizontal movements of water across the surface of the ice in "outflow pathways" [51]. We will replace this scheme with a coupled framework that uses Darcy's Law for the porous subsurface flow and uses Navier Stokes equations for the surface flow [11]. Incorporating an explicit mechanism for surface outflow pathways will enable the model to develop more realistic networks of interconnected ponds and reproduce observed critical transitions in pond complexity [36]. The spectral calculations discussed above for the effective flow behavior based on melt pond geometry and connectivity will be employed here to make the computations more rigorous and efficient.

1.5 Image Analysis

We will first compile and then analyze a library of aerial and satellite images of sea ice during the melt season. Table 1 summarizes some of the sources for the image library. The spatial resolution of the images is approximately 1 m. In many cases a preliminary processing of these images has occurred that we will extend.

For the analysis in [36] we had access to image sets captured during two measurement campaigns: Healy Oden TRans Arctic EXpedition (HOTRAX, 2005) and Surface Heat Budget of the Arctic Ocean (SHEBA,1998). Helicopter photographic survey flights gave extensive high resolution imagery of the polar marine environment consisting of ice, melt ponds and open water. To obtain melt pond area–perimeter and connectivity data, it was



(a) original image (b) four color image (c) segmented image

Figure 9: Image segmentation for August 14th (HOTRAX): (a) original image; (b) initial four color threshold with ice, shallow ponds, deep ponds and ocean/deep ponds colored as white, green, red and black, respectively; (c) segmented image with ice, ponds and ocean colored as white, blue and black, respectively.

Figure 10: A series of image tiles corresponding to a horizontal pass of the helicopter over the ice aligned into a strip using *ir-tweak*. The result is shown after 10× downsampling.

necessary to develop *segmentation algorithms* capable of distinguishing between ponds and open water. We developed a method based on computing the directional derivative in color space, where ponds have at least slight color gradients while open water generally does not. The segmenation algorithm is illustrated in Figure 9.

In our investigations of large scale connectedness and area–perimeter relations, individual ponds can be much larger than one image, thus it is critical to be able to *stitch* images together. Tasdizen *et al.* [61] have developed a suite of automatic and interactive software tools for mosaicking of microscopy image tiles. More specifically, *ir-tweak* is an interactive graphical user interface for image alignment. It allows users to drop and drag fiducial control points on either the moving or the stationary image. As the user moves the control points, the resulting warping of the moving image is shown in real time to the user allowing for a smooth interaction. The user utilizes information in overlapping areas of the moving and stationary images to bring them into alignment. Currently, *ir-tweak* does not support rigid rotation because it was designed to handle microscopy images which typically require deformable maps. In this project, we will add a rigid rotation capability into *ir-tweak* which is needed for better alignment of the ice images into strips.

Hogrebe *et al.* [35] have developed an automatic landmark based algorithm to align light microscopy sections of the retina with deformable transformations. In this project, we will modify the Hogrebe algorithm to track melt ponds in time. After segmentation of individual melt ponds, we will use their properties such as color, size, location and other shape properties to match them across time using bipartite matching. This matching will drive a deformable warping of the image at time $t+1$ to the image at time t. This calculated warp, iterated until convergence, allows refinement of the landmark matching.

As discussed above, a key tool which will facilitate the structural analysis of melt ponds is mapping them onto graphs, or networks of vertices and edges. In general, a melt pond consists of simpler round ponds, which correspond to vertices of a graph, connected through thin elongated channels, which correspond to its edges. For each melt pond, the thin elongated channels can be separated from the round convex parts by applying appropriate mathematical morphological operations. A graph which captures the essential geometrical and connectedness characteristics of a melt pond can be obtained using such methods. Viewing melt ponds as graphs allows for their classification as complex, simple, and transitional, as in Figure 2, and is essential to any studies of percolation in melt pond evolution.

1.6 Linkages to Other Programs

1. ONR Melt Pond Proposals. Following the suggestion of Dr. Jeffries, the Principal Investigators from the proposals Sunlight, sea ice, and the ice albedo feedback in a changing Arctic sea ice cover (Perovich and Light), Multiscale models of melting Arctic sea ice (Golden et al.), and Developing remote sensing capabilities for meter-scale sea ice properties (Polashenski et al.) have had several discussions about our projects and how best to integrate them and reduce costs. Describing melt ponds is a major focus in all three proposals. We believe that the proposals nicely complement each other and are synergistic. For example, the analysis of the high-resolution images proposed by Polashenski et al. could be implemented into the Golden et al. mathematical pond description. It could also be applied as input data for the Perovich and Light solar partitioning calculations. The results of these calculations would in turn provide context for the Polashenski work. Floe size distributions from Polashenski et al. could be used in the lateral melting computations of Perovich and Light, and in fractal analyses of Golden et al. Through these discussions, we identified several areas of collaboration where our efforts dovetail together. We also found a few places where there was some overlap. By eliminating the overlap, we were able to reduce our budgets.

Our project is more theoretically oriented, however, after developing mathematical descriptions of pond evolution, we will relate them to the physical processes that govern ponds, and to the other ONR melt pond projects. Melt pond formation and evolution is related to surface topography, melt rates, and ice permeability [51]. We will determine how changes in sea ice conditions, such as melt onset and ice permeability, relate to changes in pond fractal dimension. We will examine whether the variation in topography between first year and multiyear ice is manifested in the mathematical properties of ponds in these two ice types.

Ultimately, we will apply the mathematical description of melt ponds to examine the heat and mass balance of the ponds. The pond area affects the partitioning of incident solar radiation on the ice cover. The pond perimeter to area ratio influences the total amount of melting along the pond edges.

2. NSF Math Climate Research Network (MCRN). Golden is a Co-PI on a 5 year NSF DMS funded project to bring young mathematicians into climate research, through a network of 12 hub institutions, including the Math Department at the University of Utah. The MCRN grant is structured to facilitate interaction between core research programs, and the project proposed here would dovetail perfectly with the objectives of the MCRN. This linkage will broaden the expertise available to us in this project. For example the network would bring in additional expertise in dynamical systems, ice–albedo feedback, bifurcation theory, data assimilation, time series analysis, and climate modeling. It will also broaden the impact of our results, and speed the dissemination of the results of our work.

2 References

[1] E. L. Andreas and S. F. Ackley. On the differences in ablation seasons of Arctic and Antarctic sea ice. *J. Atmos. Sci.*, 39:440–447, 1982.

[2] K. Arrigo, D. K. Perovich, R. S. Pickart, Z. W. Brown, G. L. van Dijken, K. E. Lowry, M. M. Mills, M. A. Palmer, W. M. Balch, F. Bahr, N. R. Bates, C. Benitez-Nelson, B. Bowler, E. Brownlee, J. K. Ehn, K. E. Frey, R. Garley, S. R. Laney, L. Lubelczyk, J. Mathis, A. Matsuoka, B. G. Mitchell, G. W. K. Moore, E. Ortega-Retuerta, S. Pal, C. M. Polashenski, R. A. Reynolds, B. Scheiber, H. M. Sosik, M. Stephens, and J. H. Swift. Massive phytoplankton blooms under Arctic sea ice. *Science*, 336(6087):1408, 2012.

[3] M. Avellaneda and A. Majda. Stieltjes integral representation and effective diffusivity bounds for turbulent transport. *Phys. Rev. Lett.*, 62:753–755, 1989.

[4] M. Avellaneda and A. Majda. An integral representation and bounds on the effective diffusivity in passive advection by laminar and turbulent flow. *Comm. Math. Phys.*, 138:339–391, 1991.

[5] D. J. Bergman. The dielectric constant of a composite material – A problem in classical physics. *Phys. Rep. C*, 43(9):377–407, 1978.

[6] D. J. Bergman. Exactly solvable microscopic geometries and rigorous bounds for the complex dielectric constant of a two-component composite material. *Phys. Rev. Lett.*, 44:1285–1287, 1980.

[7] J. Boé, A. Hall, and X. Qu. September sea-ice cover in the Arctic Ocean projected to vanish by 2100. *Nature Geoscience*, 2(doi: 10.1038/NGEO467):341–343, 2009.

[8] S. R. Broadbent and J. M. Hammersley. Percolation processes I. Crystals and mazes. *Proc. Cambridge Philos. Soc.*, 53:629–641, 1957.

[9] A. Bunde and S. Havlin, editors. *Fractals and Disordered Systems*. Springer-Verlag, New York, 1991.

[10] G. Caginalp. Stefan and Hele–Shaw type models as asymptotic limits of the phase-field equations. *Phys. Rev. A*, 39(11):5887–5896, 1989.

[11] A. Cesmelioglu and B. Riviere. Analysis of time-dependent navier-stokes flow coupled with darcy flow. *Journal of Numerical Mathematics*, 16(4):249–280, 2008.

[12] J. T. Chayes and L. Chayes. Bulk transport properties and exponent inequalities for random resistor and flow networks. *Comm. Math. Phys.*, 105:133–152, 1986.

[13] K. Christensen and N. R. Moloney. *Complexity and Criticality*. Imperial College Press, London, 2005.

[14] A. Coniglio. Fractal structure of Ising and Potts clusters: Exact results. *Phys. Rev. Lett.*, 62(26):3054–3057, 1989.

[15] G. F. Dell'Antonio and V. Nesi. A general representation for the effective dielectric constant of a composite. *J. Math. Phys.*, 29:2688, 1988.

[16] H. Eicken, T. C. Grenfell, D. K. Perovich, J. A. Richter-Menge, and K. Frey. Hydraulic controls of summer Arctic pack ice albedo. *J. Geophys. Res. (Oceans)*, 109(C18):C08007.1–C08007.12, 2004.

[17] I. Eisenman and J. S. Wettlaufer. Nonlinear threshold behavior during the loss of Arctic sea ice. *Proc. Natl. Acad. Sci.*, 106(1):28–32, 2009.

[18] S. Feng, B. I. Halperin, and P. N. Sen. Transport properties of continuum systems near the percolation threshold. *Phys. Rev. B*, 35:197–214, 1987.

[19] D. Flocco and D. L. Feltham. A continuum model of melt pond evolution on Arctic sea ice. *J. Geophys. Res.*, 112:C08016, doi:10.1029/2006JC003836, 2007.

[20] D. Flocco, D. L. Feltham, and A. K. Turner. Incorporation of a physically based melt pond scheme into the sea ice component of a climate model. *J. Geophys. Res.*, 115:C08012 (14 pp.), doi:10.1029/2009JC005568, 2010.

[21] K. E. Frey, D. K. Perovich, and B. Light. The spatial distribution of solar radiation under a melting Arctic sea ice cover. *Geophys. Res. Lett.*, 38:L22501, doi:10.1029/2011GL049421, 2011.

[22] K. Golden. Bounds on the complex permittivity of a multicomponent material. *J. Mech. Phys. Solids*, 34(4):333–358, 1986.

[23] K. Golden. Convexity and exponent inequalities for conduction near percolation. *Phys. Rev. Lett.*, 65(24):2923–2926, 1990.

[24] K. Golden. Exponent inequalities for the bulk conductivity of a hierarchical model. *Comm. Math. Phys.*, 43(3):467–499, 1992.

[25] K. Golden and G. Papanicolaou. Bounds for effective parameters of heterogeneous media by analytic continuation. *Comm. Math. Phys.*, 90:473–491, 1983.

[26] K. Golden and G. Papanicolaou. Bounds for effective parameters of multicomponent media by analytic continuation. *J. Stat. Phys.*, 40(5/6):655–667, 1985.

[27] K. M. Golden. Statistical mechanics of conducting phase transitions. *J. Math. Phys.*, 36(10):5627–5642, 1995.

[28] K. M. Golden. The interaction of microwaves with sea ice. In G. Papanicolaou, editor, *Wave Propagation in Complex Media, IMA Volumes in Mathematics and its Applications, Vol. 96*, pages 75 – 94. Springer – Verlag, 1997.

[29] K. M. Golden. Percolation models for porous media. In U. Hornung, editor, *Homogenization and Porous Media*, pages 27 – 43. Springer – Verlag, 1997.

[30] K. M. Golden, S. F. Ackley, and V. I. Lytle. The percolation phase transition in sea ice. *Science*, 282:2238–2241, 1998.

[31] K. M. Golden, H. Eicken, A. L. Heaton, J. Miner, D. Pringle, and J. Zhu. Thermal evolution of permeability and microstructure in sea ice. *Geophys. Res. Lett.*, 34:L16501 (6 pages and issue cover), doi:10.1029/2007GL030447, 2007.

[32] K. M. Golden, N. B. Murphy, and E. Cherkaev. Spectral analysis and connectivity of porous microstructures in bone. *J. Biomech.*, 44:337–344, 2011.

[33] G. Grimmett. *Percolation*. Springer-Verlag, New York, 1989.

[34] B. I. Halperin, S. Feng, and P. N. Sen. Differences between lattice and continuum percolation transport exponents. *Phys. Rev. Lett.*, 54(22):2391–2394, 1985.

[35] L. Hogrebe, A. Paiva, E. Jurrus, C. Christensen, M. Bridge, J.R. Korenberg, and T. Tasdizen. Trace driven registration of neuron confocal microscopy stacks. In *IEEE International Symposium on Biomedical Imaging (ISBI)*, 2011.

[36] C. Hohenegger, B. Alali, K. R. Steffen, D. K. Perovich, and K. M. Golden. Transition in the fractal geometry of Arctic melt ponds. *The Cryosphere Discussions (in submission to The Cryosphere)*, 6:2161–2177, 2012.

[37] M. M. Holland, D. A. Bailey, B. P. Briegleb, B. Light, and E. Hunke. Improved Sea Ice Shortwave Radiation Physics in CCSM4: The Impact of Melt Ponds and Aerosols on Arctic Sea Ice. *J. Climate*, 25(5):1413–1430, September 2011.

[38] E. C. Hunke and W. H. Lipscomb. CICE: the Los Alamos Sea Ice Model Documentation and Software Users Manual Version 4.1 LA-CC-06-012. T-3 Fluid Dynamics Group, Los Alamos National Laboratory, 2010.

[39] M. B. Isichenko. Percolation, statistical topography, and transport in random media. *Rev. Mod. Phys.*, 64(4):961–1043, 1992.

[40] A. R. Kerstein. Equivalence of the void percolation problem for overlapping spheres and a network problem. *J. Phys. A*, 16:3071–3075, 1983.

[41] S. Lovejoy. Area-perimeter relation for rain and cloud areas. *Science*, 216(4542):185–187, 1982.

[42] M. Luthje, D. L. Feltham, P. D. Taylor, and M. G. Worster. Modeling the summertime evolution of sea-ice melt ponds. *J. Geophys. Res.*, 111(C2):C02001, 17 pp., February 2006.

[43] G. W. Milton. Bounds on the complex dielectric constant of a composite material. *Appl. Phys. Lett.*, 37:300–302, 1980.

[44] G. W. Milton. Multicomponent composites, electrical networks and new types of continued fractions I, II. *Comm. Math. Phys.*, 111:281–327, 329–372, 1987.

[45] G. W. Milton. *Theory of Composites.* Cambridge University Press, Cambridge, 2002.

[46] G. W. Milton and K. Golden. Representations for the conductivity functions of multicomponent composites. *Comm. Pure. Appl. Math.*, 43:647–671, 1990.

[47] C. A. Pedersen, E. Roeckner, M. Lüthje, and J. Winther. A new sea ice albedo scheme including melt ponds for ECHAM5 general circulation model. *J. Geophys. Res.*, 114:D08101, doi:10.1029/2008JD010440, 2009.

[48] D. K. Perovich, T. C. Grenfell, B. Light, and P. V. Hobbs. Seasonal evolution of the albedo of multiyear Arctic sea ice. *J. Geophys. Res. (Oceans)*, 107(C10):8044, doi:10.1029/2000JC000438, 2002.

[49] D. K. Perovich, T. C. Grenfell, J. A. Richter-Menge, B. Light, W. B. Tucker III, and H. Eicken. Thin and thinner: Sea ice mass balance measurements during SHEBA. *J. Geophys. Res. (Oceans)*, 108(C3):8050–8071, doi:10.1029/2001JC001079, 2003.

[50] D. K. Perovich, J. A. Richter-Menge, K. F. Jones, and B. Light. Sunlight, water, and ice: Extreme Arctic sea ice melt during the summer of 2007. *Geophys. Res. Lett.*, 35:L11501, doi:10.1029/2008GL034007, 2008.

[51] C. Polashenski, D. Perovich, and Z. Courville. The mechanisms of sea ice melt pond formation and evolution. *J. Geophys. Res. C (Oceans)*, 117:C01001 (23 pp.), doi:10.1029/2011JC007231, 2012.

[52] D. J. Pringle, J. E. Miner, H. Eicken, and K. M. Golden. Pore-space percolation in sea ice single crystals. *J. Geophys. Res. (Oceans)*, 114:C12017, 12 pp., doi:10.1029/2008JC005145, 2009.

[53] H Sagan. *Space-Filling Curves.* Springer Verlag, New York, 1994.

[54] M. Sahimi. *Applications of Percolation Theory.* Taylor and Francis Ltd., London, 1994.

[55] M. Sahimi. *Flow and Transport in Porous Media and Fractured Rock.* VCH, Weinheim, 1995.

[56] R. K. Scharien and J. J. Yackel. Analysis of surface roughness and morphology of first-year sea ice melt ponds: Implications for microwave scattering. *IEEE Trans. Geosci. Rem. Sens.*, 43:2927, 2005.

[57] F. Scott and D. L. Feltham. A model of the three-dimensional evolution of Arctic melt ponds on first-year and multiyear sea ice. *J. Geophys. Res.*, 115:C12064, doi:10.1029/2010JC006156, 2010.

[58] M. C. Serreze, M. M. Holland, and J. Stroeve. Perspectives on the Arctic's shrinking sea-ice cover. *Science*, 315:1533–1536, 2007.

[59] E. D. Skyllingstad, C. A. Paulson, and D. K. Perovich. Simulation of melt pond evolution on level ice. *J. Geophys. Res.*, 114:C12019, doi:10.1029/2009JC005363, 2009.

[60] D. Stauffer and A. Aharony. *Introduction to Percolation Theory, Second Edition*. Taylor and Francis Ltd., London, 1992.

[61] T. Tasdizen, P. Koshevoy, B. Grimm, J. Anderson, B. Jones, C. Watt, R. Whitaker, and R. Marc. Automatic mosaicking and volume assembly for high-throughput serial-section transmission electron microscopy. *Journal of Neuroscience Methods*, 193:132–144, 2010.

[62] C. J. Thompson. *Classical Equilibrium Statistical Mechanics*. Oxford University Press, Oxford, 1988.

[63] S. Torquato. *Random Heterogeneous Materials: Microstructure and Macroscopic Properties*. Springer-Verlag, New York, 2002.

[64] T. Toyota, S. Takatsuji, and M. Nakayama. Characteristics of sea ice floe size distribution in the seasonal ice zone. *Geophys. Res. Lett.*, 33:L02616, doi:10.1029/2005GL024556, 2006.

3 Project Schedule and Milestones

Year 1: Collect melt pond images. Begin analysis of existing images using percolation theory and image processing techniques. Specifically address the issue of what determines the critical length scale where melt pond geometry changes from Euclidean to space filling behavior. Explore appropriate theories in statistical physics and composite materials to understand critical behavior of melt pond evolution. Begin development of image analysis techniques.

Year 2: Begin development of PDE models for melt pond evolution. Continue percolation and statistical mechanics investigations. Investigate how the mathematical models can aid in representing melt ponds in climate models. Begin publishing results on mathematical models and image analysis.

Year 3: Continue model development and tune the models with results from new images. Continue work on including our theoretical results in climate models. Continue data analysis, model development and synthesis with other investigators. Focus on producing group publications and disseminating results to the broader community.

Image software development: In years one and two, we will focus on developing the image processing software and testing it on the available data sets. By the end of the project, we plan to have a user-friendly software package for the analysis of melt pond and floe images. The software will be made available to other groups and we will pay particular attention to the packaging of the software including an easy to use graphical interface and help documentation.

4 Management Approach

University of Utah: The PI Golden will be responsible for the overall success of the project, for providing mathematical direction and undertaking investigations, for supervising the post-doc and students. The co-PI's Strong and Alali will be responsible for scientific computation and physical modeling of the systems of interest, and co-advising students. Tasdizen and Alali will be responsible for developing methods of image analysis.

ERDC–CRREL: Perovich and Polashenski will be responsible for obtaining melt pond imagery that satisfies the requirements of the principal modeling objectives. They will also be closely involved in relating the mathematical results to the geophysics of melt ponds, sea ice, and the upper ocean. Perovich will also contribute to formulating relationships between pond properties and melting on sides and bottoms of ponds.

5 Current and Pending Projects and Proposals

Title: Sea ice, sunlight, and biogeochemistry in the Chukchi and Beaufort Seas in a changing climate.
Supporting Agency: NASA
Prime Offerer: Don Perovich
Technical Contact: Tom Wagner 202-358-4682
Investigator Months per year: 2.5
Total Funding: $1,322,778
Duration: 10/1/09 – 09/30/13
Support: Current
Relation to proposed effort: Will use results from this study for optical properties of first year ice.

Title: Collaborative Research: The O-buoy network of chemical sensors in the Arctic Ocean
Supporting Agency: NSF
Prime Offerer: Don Perovich
Technical Contact: Erica Key 702-292-7434
Investigator Months per year: 1
Total Funding: $823,000
Duration: 10/1/10 – 9/30/15
Support: Current
Relation to proposed effort: No overlap

Title: Implications of Arctic sea ice reduction on tropospheric bromine, ozone, and mercury chemical processes, transport, and distribution
Supporting Agency: NASA
Prime Offerer: Don Perovich
Technical Contact: Tom Wagner 202-358-4682
Investigator Months per year: 1
Total Funding: $437,200
Duration: 12/1/10 – 10/31/14
Support: Current
Relation to proposed effort: No overlap

Title: Collaborative research on gas transfer through polar sea ice (GAPS)
Supporting Agency: NSF
Prime Offerer: Don Perovich
Technical Contact: Peter Milne 702-292-4714
Investigator Months per year: 0.25
Total Funding: $110,000
Duration: 9/15/10 – 9/14/12
Support: Current
Relation to proposed effort: No overlap

Title: Autonomous Ice Mass Blance Buoys for an Arctic Observing Network.

Supporting Agency: NSF
Prime Offerer: Jacqueline Richter-Menge
Technical Contact: Erica Key 702-292-7434
Investigator Months per year: 2
Total Funding: $$2,266,257
Duration: 7/1/09 – 6/30/14
Support: Current
Relation to proposed effort: Will use observations of mass balance as input to radiative transfer models

Title: Collaborative research on the state of the Arctic sea ice cover: Sustaining the integrated seasonal ice zone observing network (SIZONET)
Supporting Agency: NSF
Prime Offerer: Don Perovich
Technical Contact: Erica Key 702-292-7434
Investigator Months per year: 1.5
Total Funding: $603,615
Duration: 1/1/10 – 12/31/14
Support: Current
Relation to proposed effort: Will incorporate results from melt pond evolution studies.

Title: Collaborative studies of Seasonality of ice growth and melt
Supporting Agency: NSF
Prime Offerer: Bonnie Light
Technical Contact: Neil Swanberg 702-292-8029
Investigator Months per year: 2
Total Funding: $197,188
Duration: 8/1/09 – 7/31/13
Support: Current
Relation to proposed effort: The proposed work will incorporate some albedo observations from this study.

Title: A field campaign to promote integration between the sea ice field, remote sensing, and modeling communities
Supporting Agency: NASA
Prime Offerer: Don Perovich
Technical Contact: Tom Wagner 202-358-4682
Investigator Months per year: 0.75
Total Funding: $202,000
Duration: 10/1/12 – 9/30/13
Support: Under review
Relation to proposed effort: No overlap

Title: Multiscale models of the melting Arctic sea ice pack
Supporting Agency: ONR
Prime Offerer: Ken Golden
Technical Contact: Martin Jeffries 703-696-7825
Investigator Months per year: 1
Total Funding: $146,590
Duration: 12/1/12 – 11/30/15
Support: Under review
Relation to proposed effort: Close collaboration

Title: Sunlight, sea ice, and the ice albedo feedback in a changing Arctic sea ice cover
Supporting Agency: ONR
Prime Offerer: Don Perovich
Technical Contact: Martin Jeffries 703-696-7825
Investigator Months per year: 1
Total Funding: $254,137
Duration: 12/1/12 – 11/30/15
Support: Under review
Relation to proposed effort: Close collaboration

Title: Developing remote sensing capabilities for meter-scale sea ice properties
Supporting Agency: ONR
Prime Offerer: Chris Polashenski
Technical Contact: Martin Jeffries 703-696-7825
Investigator Months per year: 1
Total Funding: $584,000
Duration: 12/1/12 – 11/30/16
Support: Under review
Relation to proposed effort: Close collaboration

Current and Pending Support – Polashenski

Title: Spatial and Temporal Variability of Surface Albedo and Light Absorbing Chemical Species in Greenland
Source of Support: NSF
Investigator Months per year: 3
Total Funding: $1,000,000
Duration: 10/1/12 – 09/30/15
Support: Current

Title: Fluid Flow Through Porous Media Under Freezing Conditions
Source of Support: U.S. Army Basic Research Program
Investigator Months per year: 3
Total Funding: $486,000
Duration: 10/1/12 – 09/30/15
Support: Under Review

Title: Collaborative Research: Solar Partitioning in a Changing Arctic
Source of Support: NASA
Investigator Months per year: 3
Total Funding: $320,748
Duration: 10/1/12 – 09/30/15
Support: Under Review

6 Qualifications

Donald K. Perovich
ERDC – CRREL 72 Lyme Road Hanover, NH 03755
donald.k.perovich@usace.army.mil 603-646-4255

A. Professional Preparation

B.S. in Physics, Michigan State University, 1972
M.S. in Geophysics, University of Washington, 1979
Ph.D. in Geophysics, University of Washington, 1983
Postdoctoral Research Associate, Geophysics Program, U. Washington, 1983-1985.

B. Appointments

Research Geophysicist, ERDC – CRREL, 1986-present.
Adjunct Professor, Thayer School of Engineering, Dartmouth College, 2008 – present.

C. Selected relevant publications

Perovich, D.K. and C. Polashenski, Albedo evolution of seasonal Arctic sea ice, *Geophys. Res. Lett.,* 39, L08501, doi:10.1029/2012GL051432, 2012.

Arrigo, K., D.K. Perovich and others, Massive phytoplankton blooms under Arctic sea ice, *Science*, 336, 15 June 2012.

Perovich, D.K., B. Light, K.F. Jones, H. Eicken, J. Stroeve, and T. Markus, Solar partitioning in a changing Arctic sea ice cover, *Ann Glaciol.,* 52, 192-196, 2011.

Perovich, D. K., J. A. Richter-Menge, K. F. Jones, and B. Light, Sunlight, water, and ice: Extreme Arctic sea ice melt during the summer of 2007, *Geophys. Res. Lett*., 35, doi:10.1029/2008GL034007, 2008.

Light, B., T.C. Grenfell, and D.K. Perovich, Transmission and absorption of solar radiation by arctic sea ice during the melt season, *J. Geophys. Res.*, 113, 2008.

Perovich, D. K., B. Light, H. Eicken, K. F. Jones, K. Runciman, and S. V. Nghiem, Increasing solar heating of the Arctic Ocean and adjacent seas, 1979–2005: Attribution and role in the ice- albedo feedback, *Geophys. Res. Lett., 34,* doi:10.1029/2007GL031480, 2007.

Perovich, D.K., On the aggregate-scale partitioning of solar radiation in Arctic Sea Ice during the SHEBA field experiment, *J. Geophys. Res.*, 110,C03002 10.1029/2004JC002512, 2005.

Perovich, D.K., T.C. Grenfell, J.A. Richter-Menge, B. Light, W.B. Tucker III, H. Eicken, Thin and thinner: ice mass balance measurements during SHEBA, *J. Geophys. Res.*, 108, (C3), DOI 10.1029/2001JC001079, 26-1 – 26-21, 2003.

Perovich, D.K., T.C. Grenfell, B. Light, and P.V. Hobbs, The seasonal evolution of Arctic sea ice albedo, *J. Geophys. Res.*, 10.1029/2000JC000438, 2002.

Perovich, D.K., W.B. Tucker III, and K.A. Ligett, Aerial observations of the evolution of ice surface conditions during summer, *J. Geophys. Res.*, 107, doi:10.1029/2000JC000449, 2002.

D. Qualifications, capabilities, and experience

The central focus of my research is deceivingly simple to state: where does all the sunlight go? The interaction of solar radiation with sea ice is intimately interrelated with the ice albedo feedback, the ice mass balance, and the role of sea ice as an indicator and amplifier of climate change. I am very active in interdisciplinary efforts directed at understanding the role of sea ice in the Arctic system and in the global climate system. My field work includes ice camps and icebreaker cruises including a year-long drift (SHEBA 1997 – 1998) and a transArctic expedition (2005). Presently I am co-chief scientist of the NASA ICESCAPE program, and a member of the Arctic Icebreaker Coordinating Committee and the SEARCH Observing Change Panel. I have participated in numerous outreach activities including K – 12 education, science festivals, public lectures, television shows, and media interviews.

Christopher M. Polashenski
Research Geophysicist – ERDC – CRREL, 72 Lyme Road, Hanover, NH 03755
chris.polashenski@gmail.com (570) 956-6990

PROFESSIONAL PREPARATION

Dartmouth College, A.B. with High Honors	2007
Dartmouth College, Thayer School of Engineering, B.E. Env. and Materials Engineering	2007
Dartmouth College, Thayer School of Engineering, Ph.D., Materials Engineering	2011
Doctoral Thesis: *The Mechanisms of Summer Seasonal Ice Albedo Control*	
National Science Foundation IGERT Trainee in Polar Environmental Change	

APPOINTMENTS

Army Corps of Engineers, CRREL – *Research Geophysicist*	June 2011 **–** Present
Dartmouth College Arctic Studies Program – *NSF IGERT Traineeship*	Sept. 2009 – Jun. 2011
Army Corps of Engineers, CRREL – *SCEP Student Fellowship Program*	Sept. 2007 – Sept. 2009

PUBLICATIONS

Perovich, D. K. and **C. Polashenski** (2012), Albedo evolution of seasonal Arctic sea ice, *Geophys. Res. Lett.*, *39*, L08501, doi:10.1029/2012GL051432.

Polashenski, C., D. K. Perovich, and Z. Courville. (2011) The Mechanisms of Sea Ice Melt Pond Formation and Evolution. *J. Geophys. Res.*, 117, C1001.

Polashenski, C., D. K. Perovich, J. A. Richter-Menge, B. Elder, (2011) Seasonal Ice Mass-Balance Buoys: Adapting tools to the changing Arctic. *Ann. Glaciol.*, 52 (57).

Perovich, D., J. Richter-Menge, K. Jones, B. Light, B. Elder, **C. Polashenski,** D. Laroche, T. Markus, and R. Lindsay. (2011) Arctic sea-ice melt in 2008 and the role of solar heating. *Ann.Glaciol.*, 52 (57).

Arrigo, K., D. Perovich, R. Pickart, Z. Brown, G. van Dijken, K. Lowry, M. Mills, M. Palmer, W. Balch, F. Bahr, N. Bates, C. Benitez-Nelson, B. Bowler, E. Brownlee, J. Ehn, K. Frey, R. Garley, S. Laney, L. Lubelczyk, J. Mathis, A. Matsuoka, B. G. Mitchell, G. W. K. Moore, E. Ortega-Retuerta, S. Pal, **C. Polashenski,** R. Reynolds, Schieber, H. Sosik, M. Stephens, J. Swift (2012), Massive Phytoplankton Blooms Under Arctic Sea Ice, *Science*, 336/6087, p. 1408 DOI: 10.1126/science.1215065.

RESEARCH INTERESTS, QUALIFICATIONS, and EXPERIENCE

I am an early career polar researcher, having recieved my PhD in the spring of 2011. My research interests focus on how snow and ice physical properties control sea ice cover. The role of sea ice in the physical climate, ecological systems, and operational environment of the Arctic makes this work both intellectually interesting and of societal importance. My technical training examines snow and ice from a material science standpoint, though I have also broadened my research capabilities through inter-disciplinary training and collaboration. Participation in the NSF-IGERT program has developed my understanding of how the many disciplines of Arctic research connect around key, human-driven questions. Work on a range of research projects has given me experience with many cutting edge technologies including the use of LiDAR scanners, spectral optical sensors, isotopic analysis, surface exposure dating, and autonomous measurement platforms. An engineering background has allowed me to design and build new research instruments, including a patent pending ice mass balance buoy. I have done Arctic field work in Greenland, Canada, and Alaska, working from terrestrial stations, a variety of aircraft, and aboard an icebreaker. I am an active participant and collaborator in the research community. I have presented my work at twelve conferences, am an author on three peer reviewed publications, and have developed collaborations with researchers at many institutions including U. of Oregon, U. of Alaska Fairbanks, U. of Washington, and NCAR. I am active in teaching; having served as a TA for fifteen classes, received three teaching citations, and mentored several undergraduates. I enjoy outreach activities and have worked at schools, publicly lectured, blogged, and participated in radio and TV interviews.

7 CRREL Budget

Personnel:

Don Perovich (co-PI) – Don Perovich will participate in the various elements of the program focusing on i) providing aerial photographs from previous field campaigns, 2) interpreting the mathematical descriptions of melt ponds from an albedo perspective, and 3) examining pond melting.

Chris Polashenski (Senior personnel) – Polashenski will focus on providing a physical basis for the determined mathematical description of pond evolution.

Travel: Travel funds are requested each year for a trip to a conference or meeting.

Supplies: $1000 is requested in each year for miscellaneous supplies and page charges.

Cost reduction: Following the suggestion of Dr. Jeffries we found commonalities with the Golden et al. and Polashenski et al. proposals. This led to a cost savings of $24,504 in the CRREL budget for this proposal compared to the budget submitted in our planning letter.

	2013	**2014**	**2015**
Labor (months)			
Perovich (1 - 1 - 1)	$11,262	$11,262	$12,669
Polashenski (1 - 1 - 1)	$5,229	$5,438	$5,647
Benefits	$8,740	$8,851	$9,708
Travel	$1,100	$1,100	$1,300
Miscellaneous/Page charges	$1,000	$1,000	$2,000
Indirect costs	$19,090	$19,314	$21,880
Total	**$46,421**	**$46,965**	**$53,204**
Grand total	**$146,590**		