Random Matrices and the Melting Polar Ice Caps

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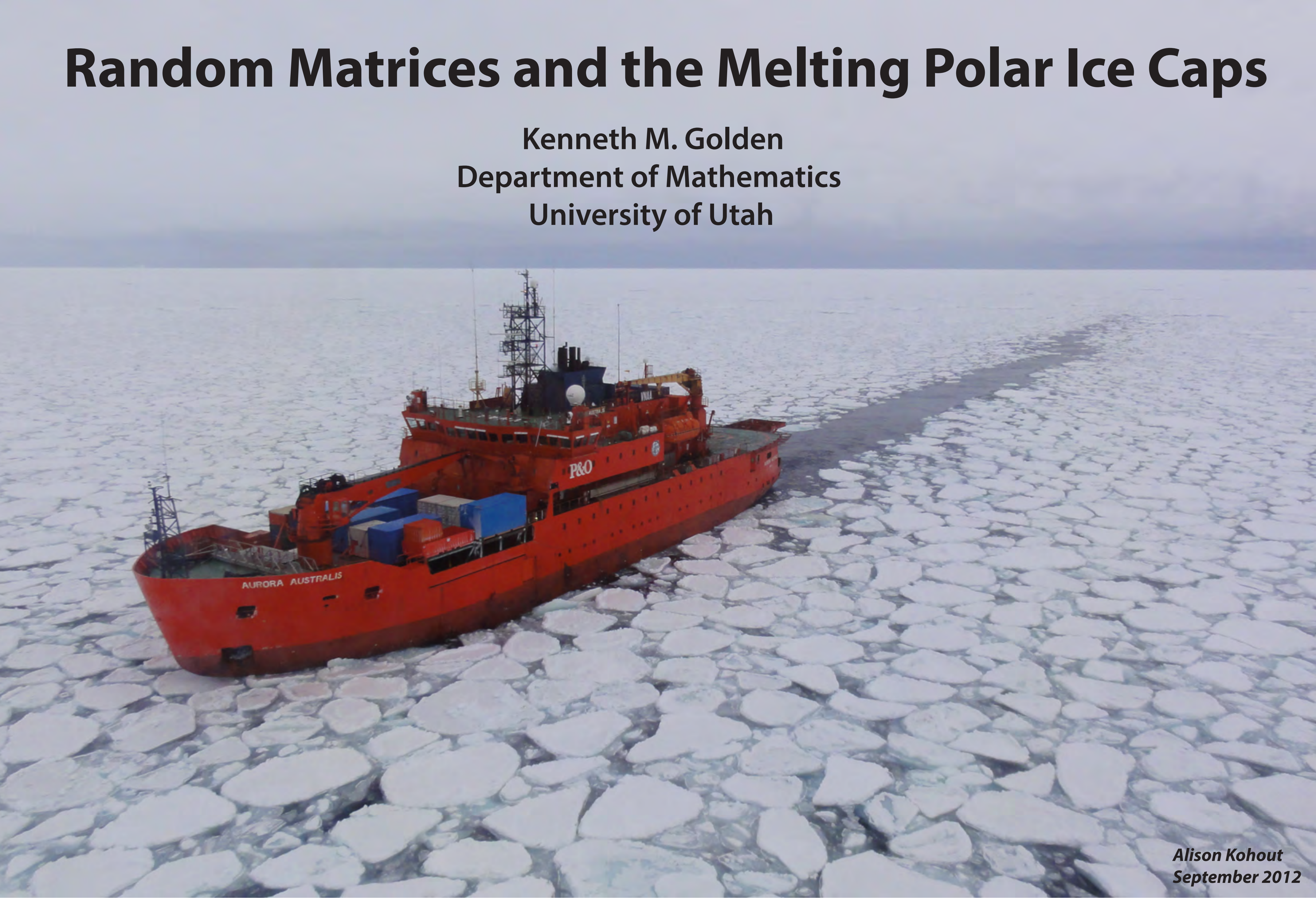


Alison Kohout
September 2012

Workshop on Random Matrices   Montevideo   15 December 2014

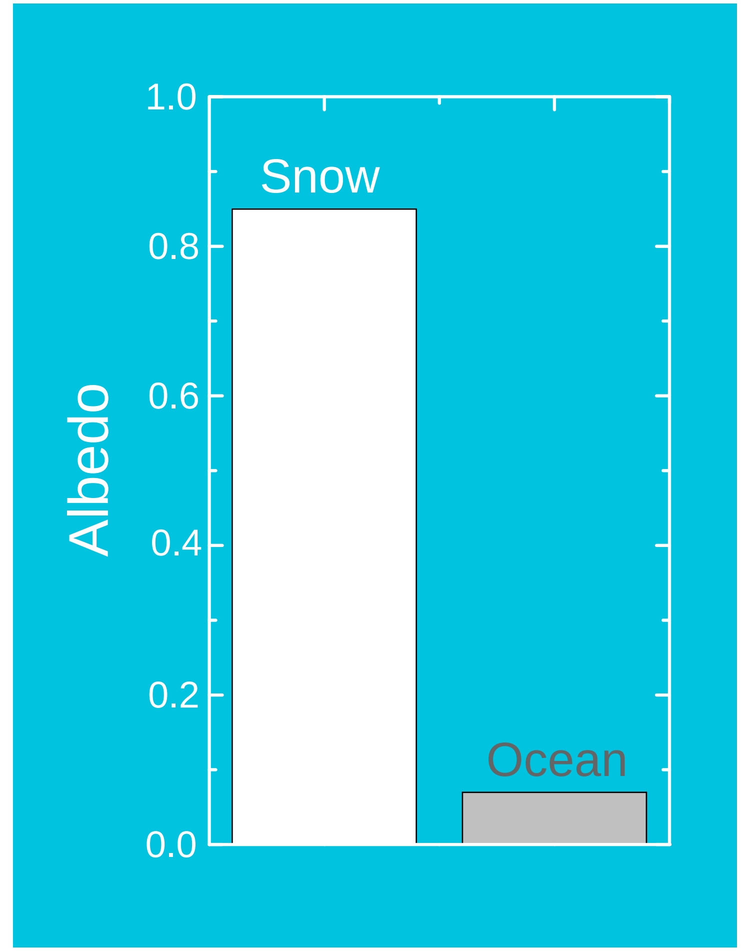
Foundations of Computational Mathematics Conference

SEA ICE covers 7 - 10% of earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

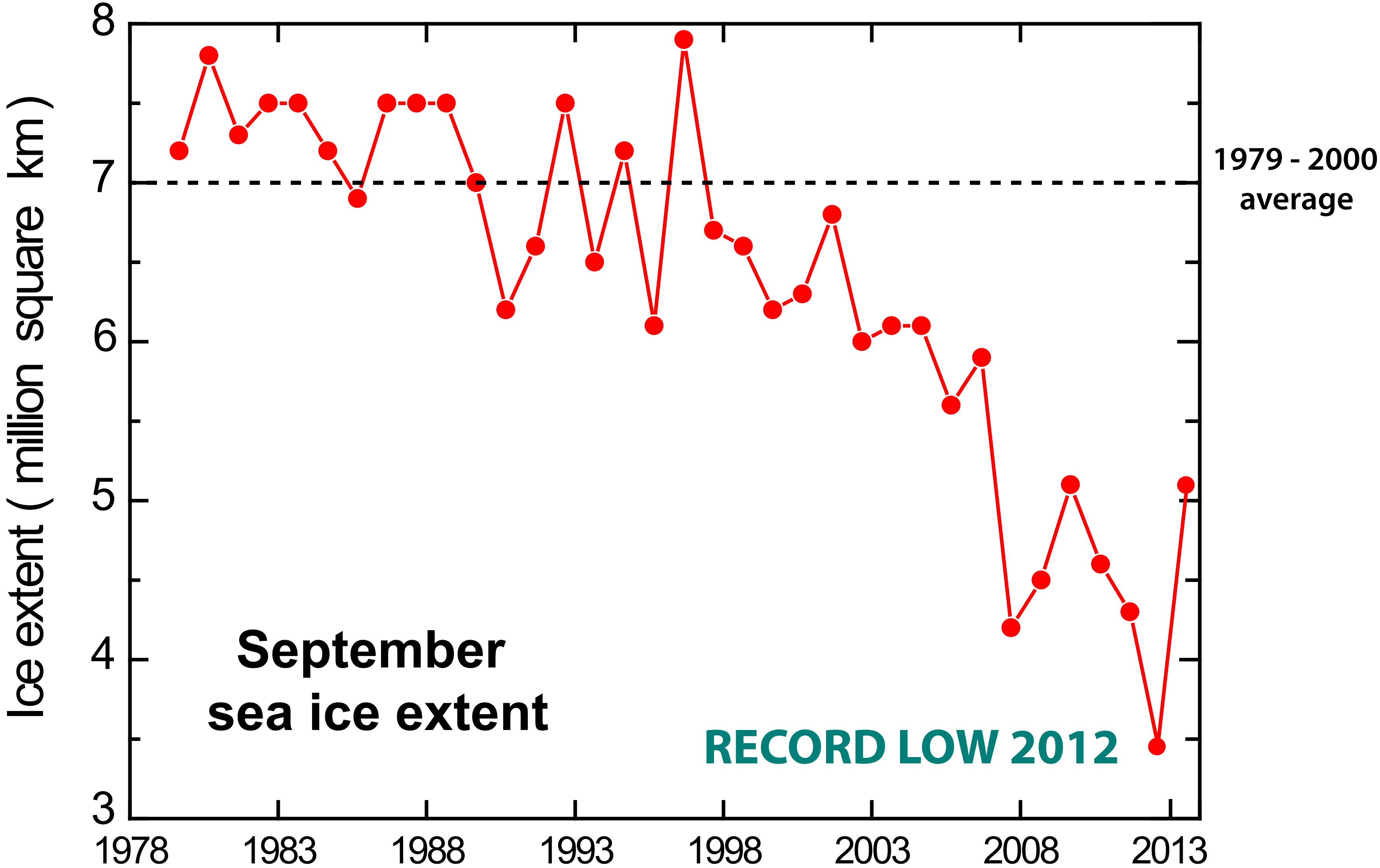
white snow and ice reflect



dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

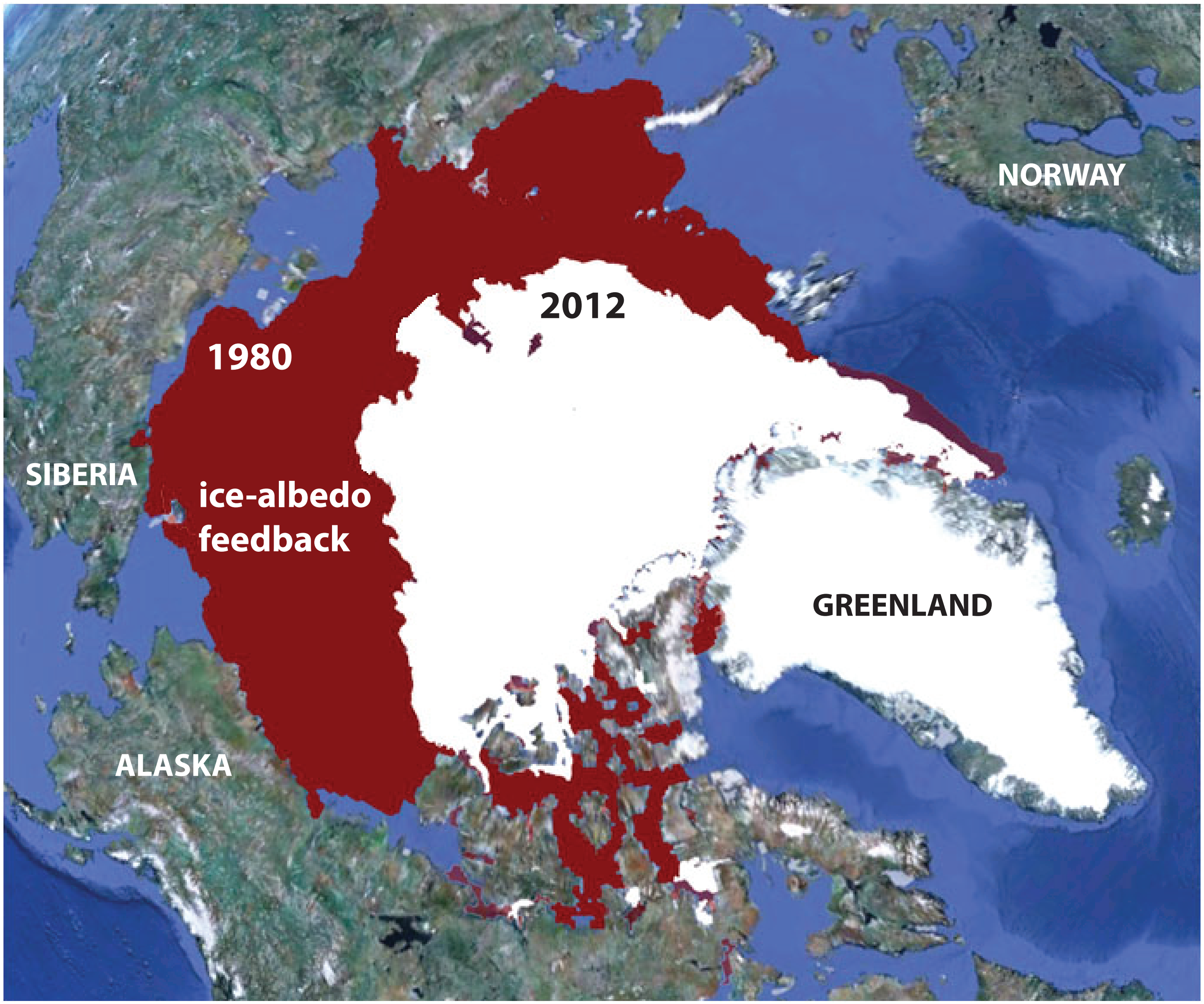
the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

Change in Arctic Sea Ice Extent

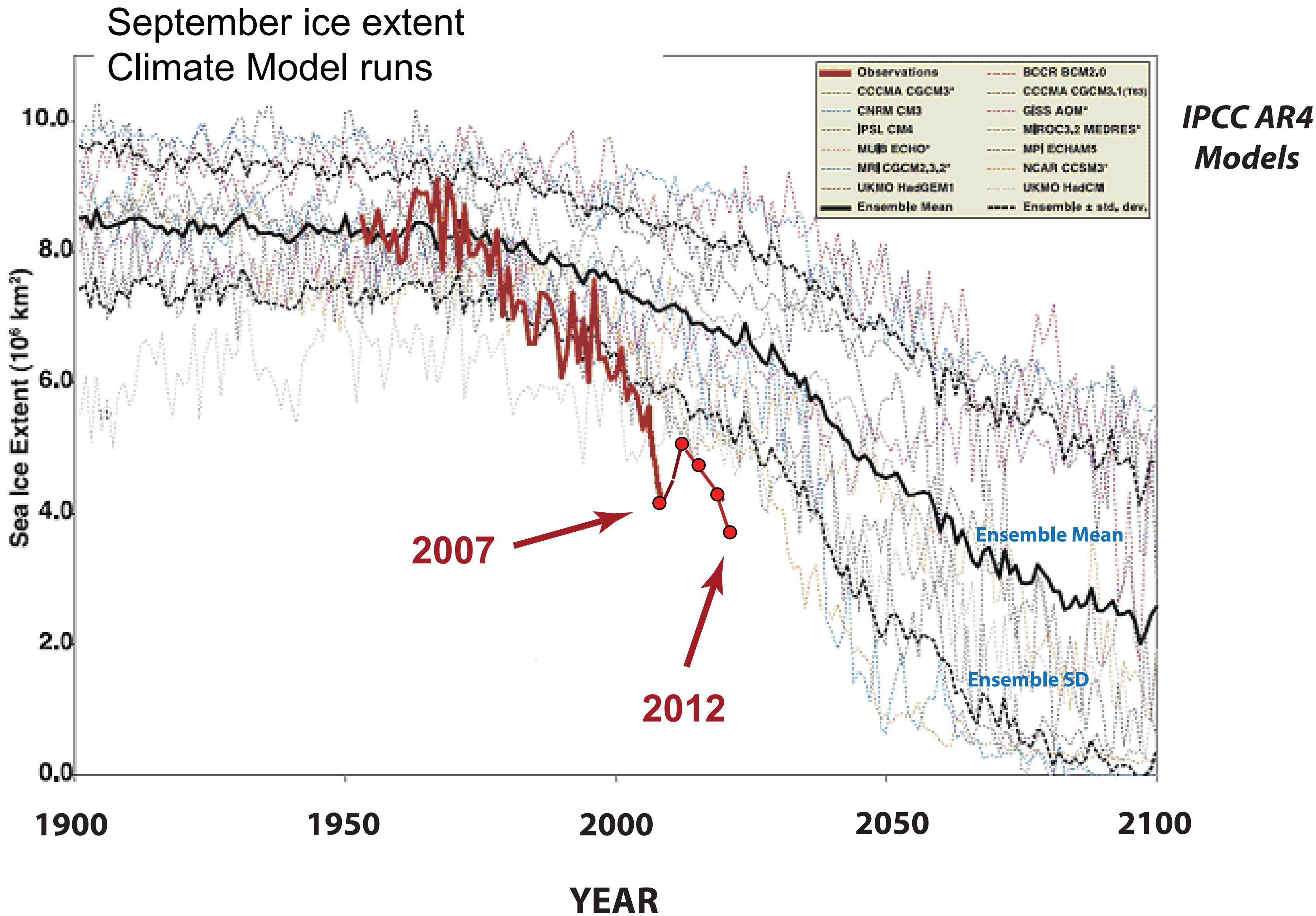
September 1980 -- 7.8 million square kilometers
September 2012 -- 3.4 million square kilometers



Perovich

Arctic sea ice decline - faster than predicted by climate models

Stroeve et al., GRL, 2007



Perovich

challenge

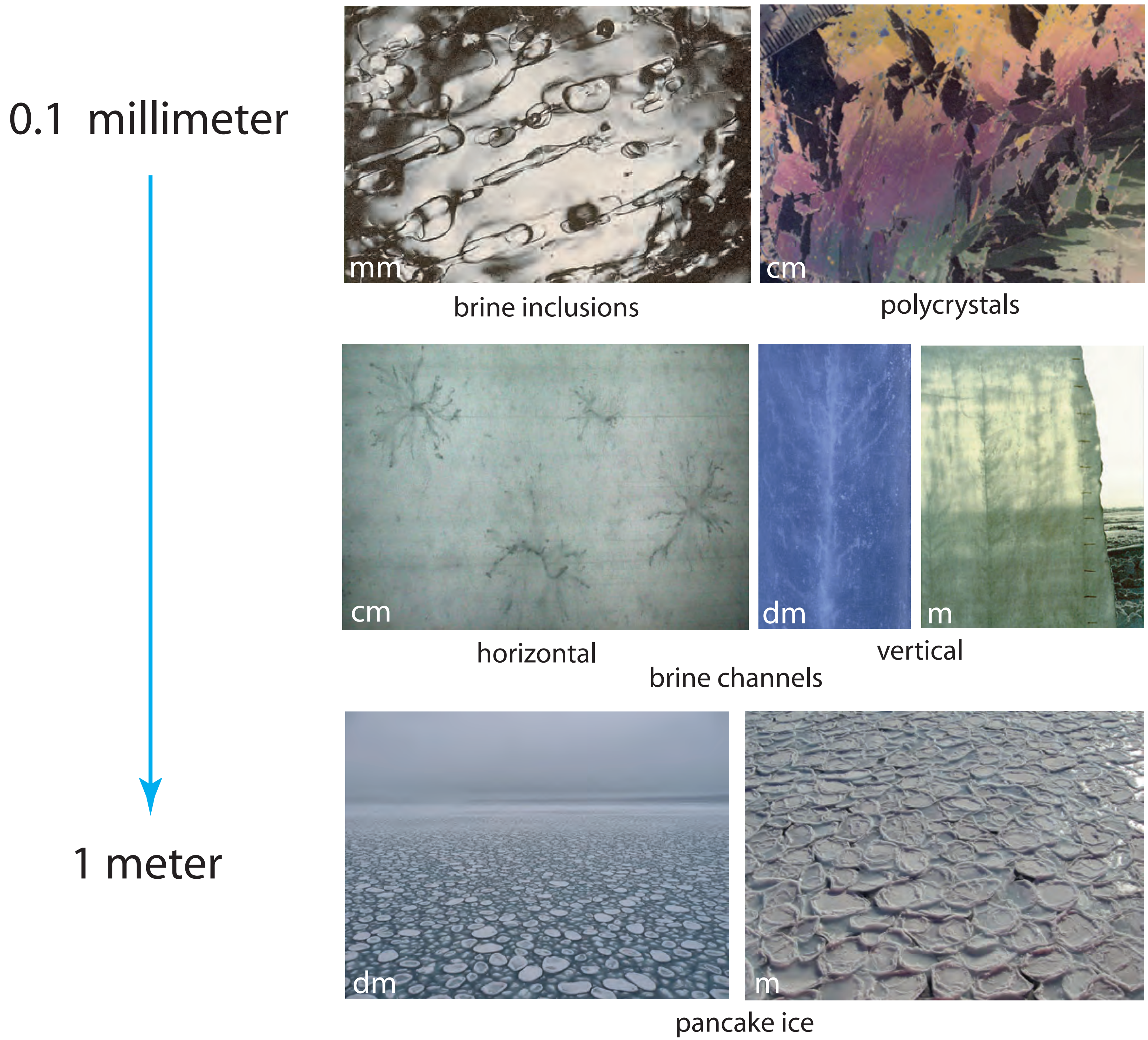
represent sea ice more rigorously in climate models

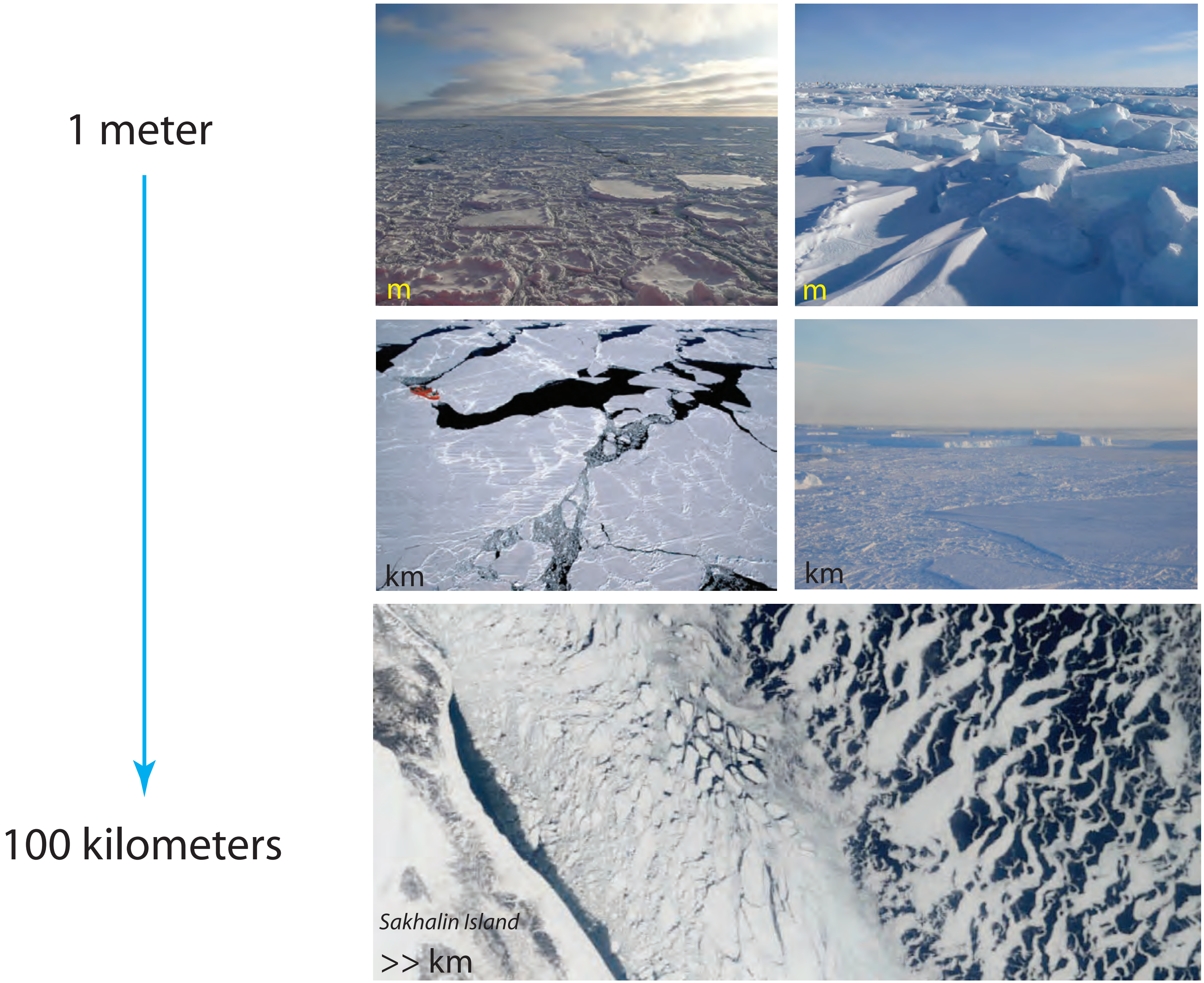
incorporate key processes

fundamental problem -- linkage of scales

sub-grid scale processes

sea ice displays ***multiscale*** structure over 10 orders of magnitude




1 meter
100 kilometers
m
m
km
km
Sakhalin Island
>> km

What is this talk about?

Using the mathematics of composite materials and statistical physics to study sea ice structures and processes ... to improve projections of climate change.

tour through random matrices describing sea ice structures

1. ***Fluid flow through sea ice - percolation***

   homogenzation for composite materials

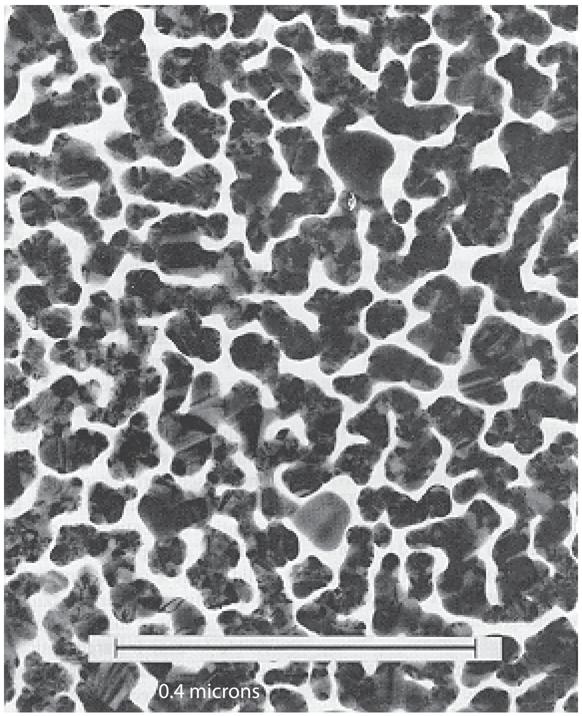
2. ***Electromagnetic monitoring of sea ice***

   homogenzation for larger scale structures

3. ***Fractal geometry of Arctic melt ponds***

thin silver film

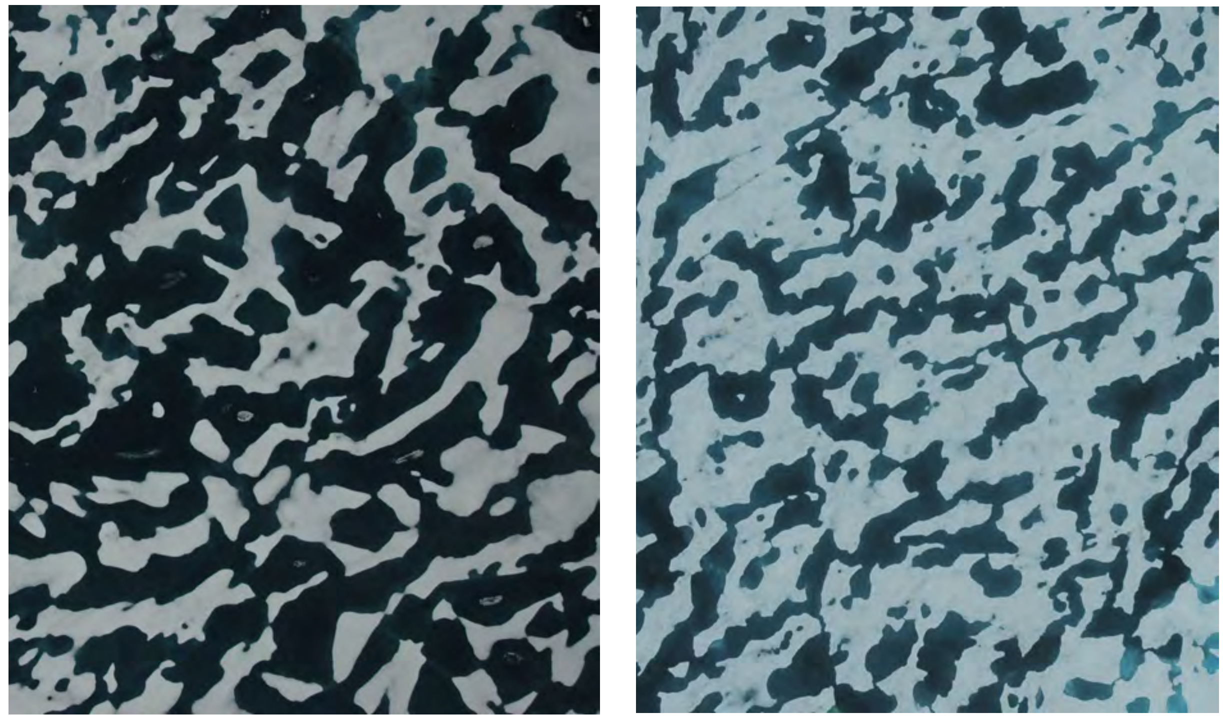
microns



(Davis, McKenzie, McPhedran, 1991)

Arctic melt ponds

kilometers



(Perovich, 2005)

optical properties

composite geometry -- area fraction of phases, connectedness, necks

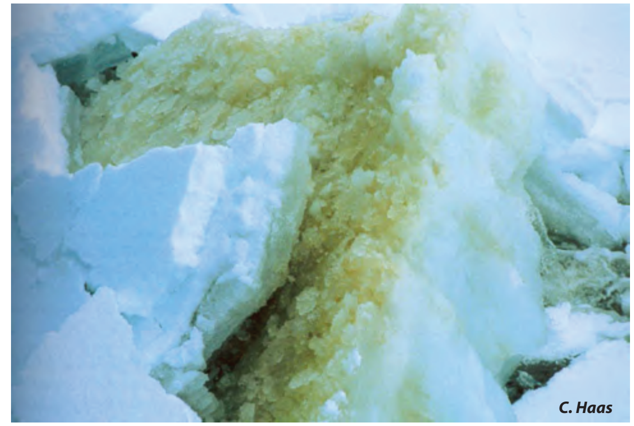
sea ice microphysics

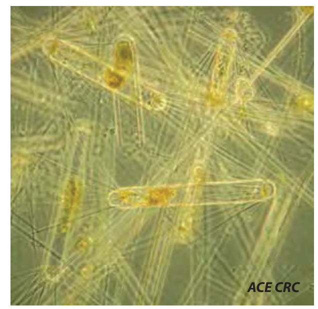
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

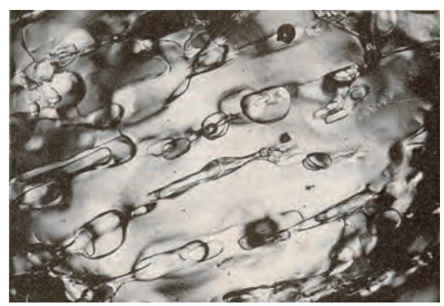
nutrient flux for algal communities





- *drainage of brine and melt water*
- *ocean-ice-air exchanges of heat, CO_2*
- *Antarctic surface flooding and snow-ice formation*
- *evolution of salinity profiles*

↕ linkage of scales



Darcy's Law for slow viscous flow in a porous medium

averaged fluid velocity

pressure gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta}\,\nabla p$$

viscosity

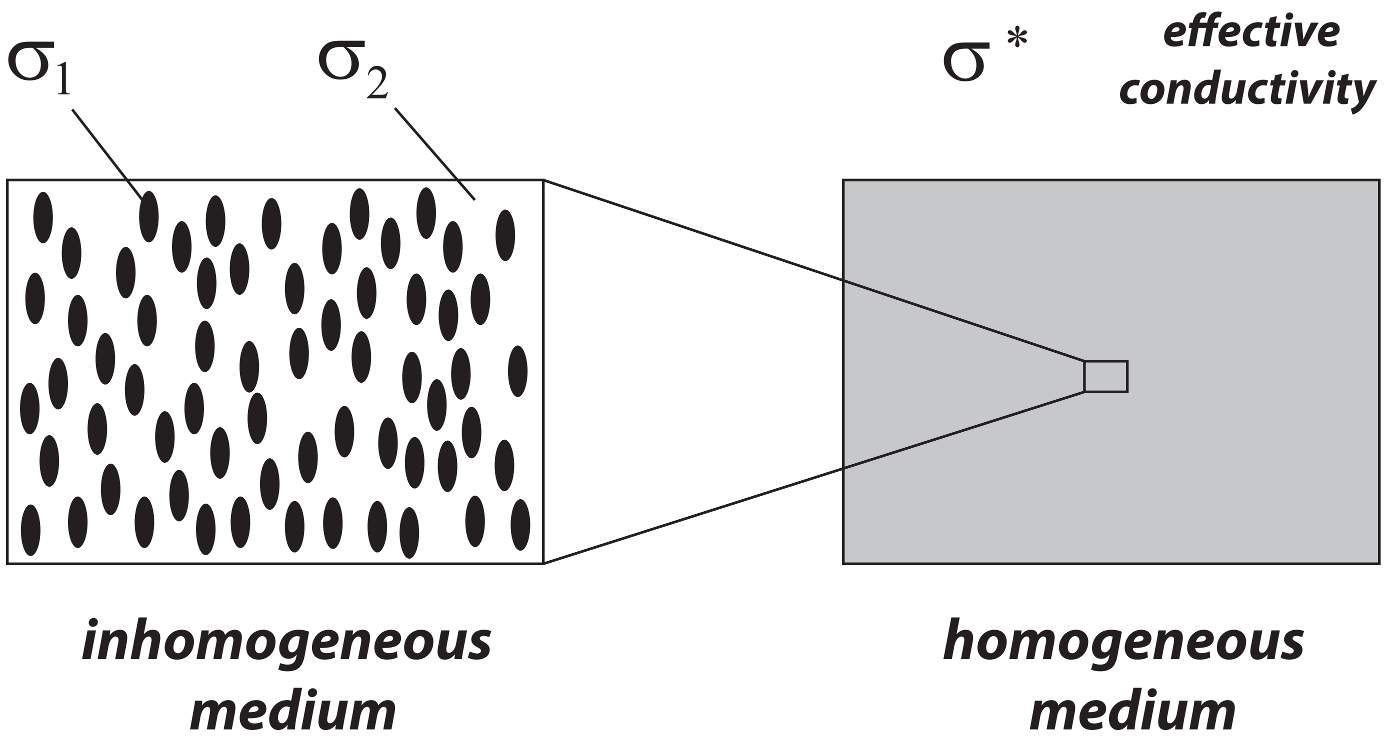
$\mathbf{k}$ = fluid permeability tensor

example of *homogenization*

mathematics for analyzing effective behavior of heterogeneous systems

e.g. transport properties of composites - electrical conductivity, thermal conductivity, etc.

HOMOGENIZATION



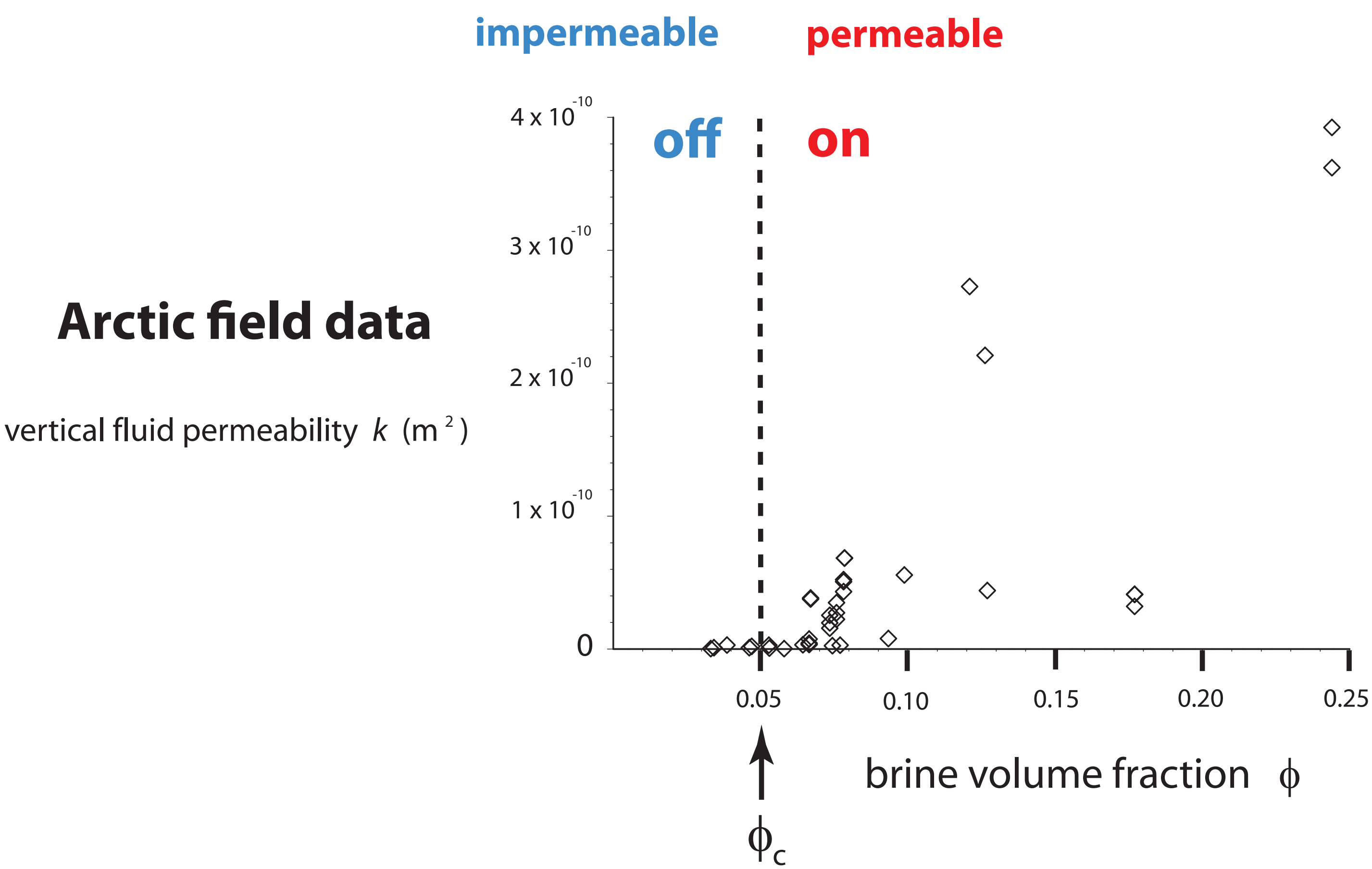
find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

Critical behavior of fluid transport in sea ice

Arctic field data



on - off switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ C,\ S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998

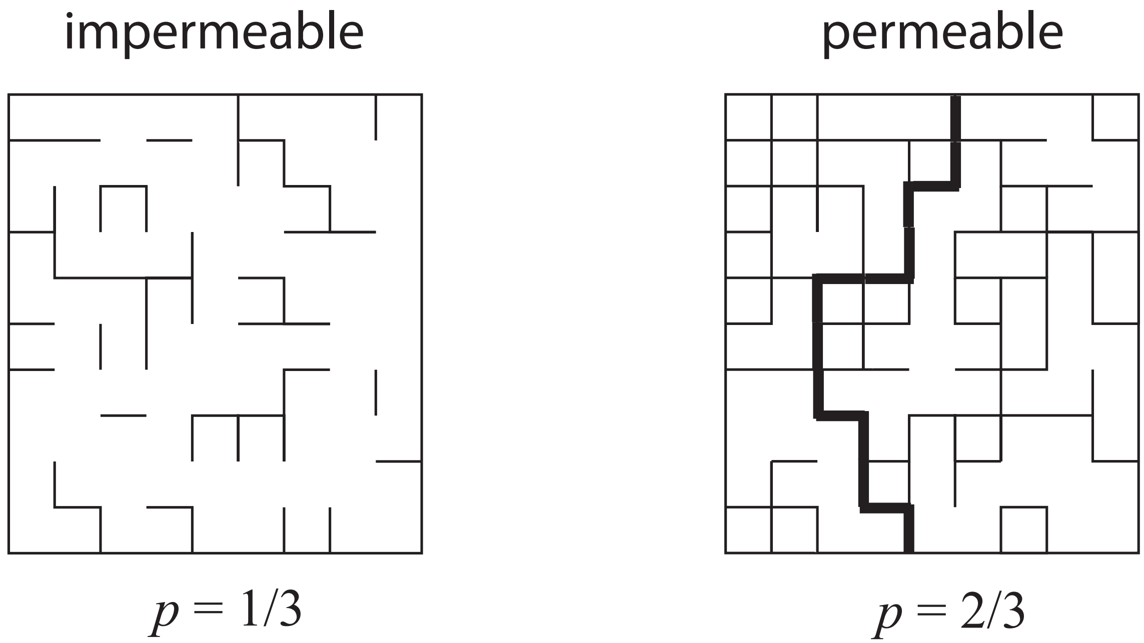
Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

Why is the rule of fives true?

percolation theory

mathematical theory of connectedness



bond → *open* with probability *p*
closed with probability *1-p*

percolation threshold

$$p_c = 1/2 \quad \text{for} \quad d = 2$$

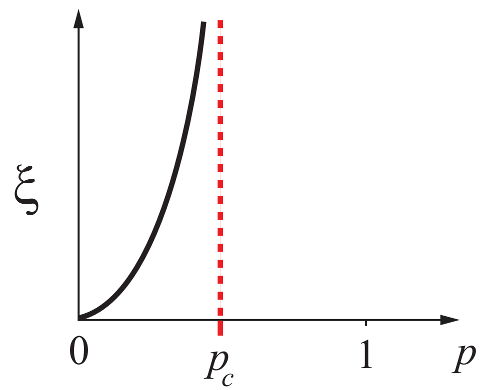
first appearance of infinite cluster ***"tipping point" for connectivity***

order parameters in percolation theory

geometry

correlation length

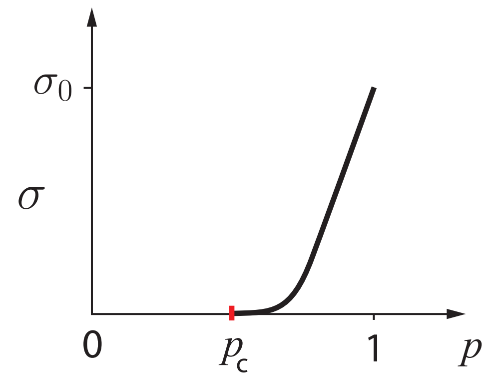
(characteristic scale of connectedness)



$$\xi(p) \sim |p - p_c|^{-\nu}$$

transport

effective conductivity
or fluid permeability



$$\sigma(p) \sim \sigma_0 \, (p - p_c)^t$$

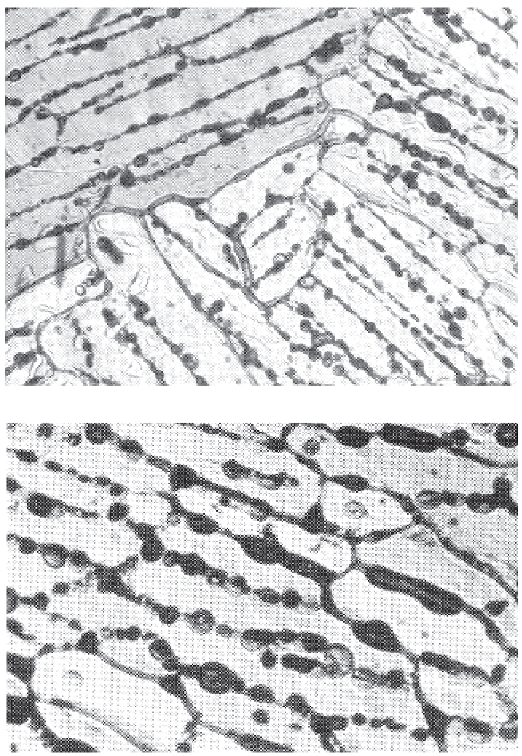
UNIVERSAL critical exponents for lattices -- depend only on dimension

(**1** $\leq$ ***t*** $\leq$ **2,** Golden, *Phys. Rev. Lett.* 1990 **;** *Comm. Math. Phys.* 1992)

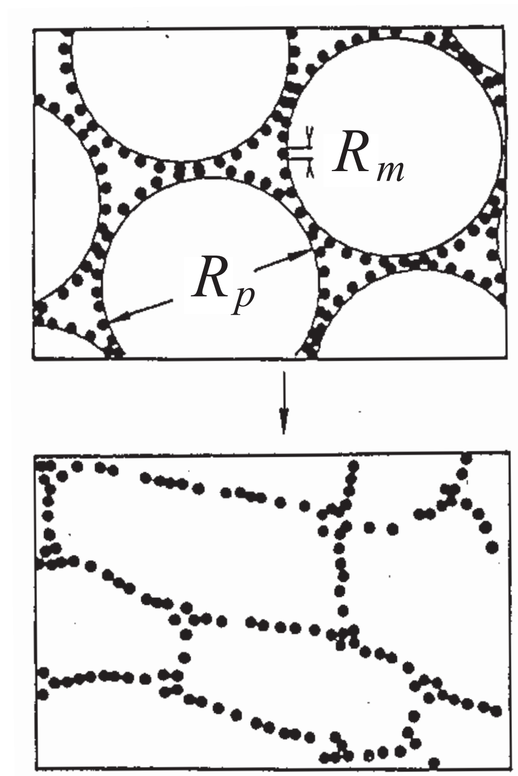
non-universal behavior in continuum

Continuum percolation model for *stealthy* materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

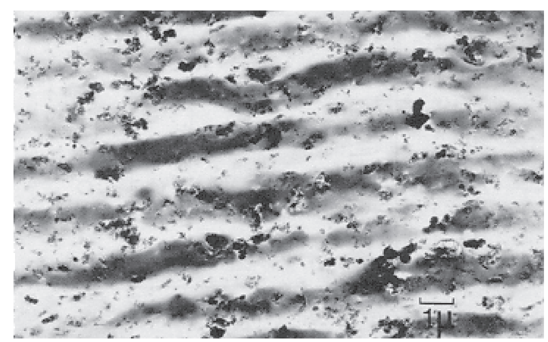
$\phi_c \approx 5\,\%$ Golden, Ackley, Lytle, *Science,* 1998



sea ice



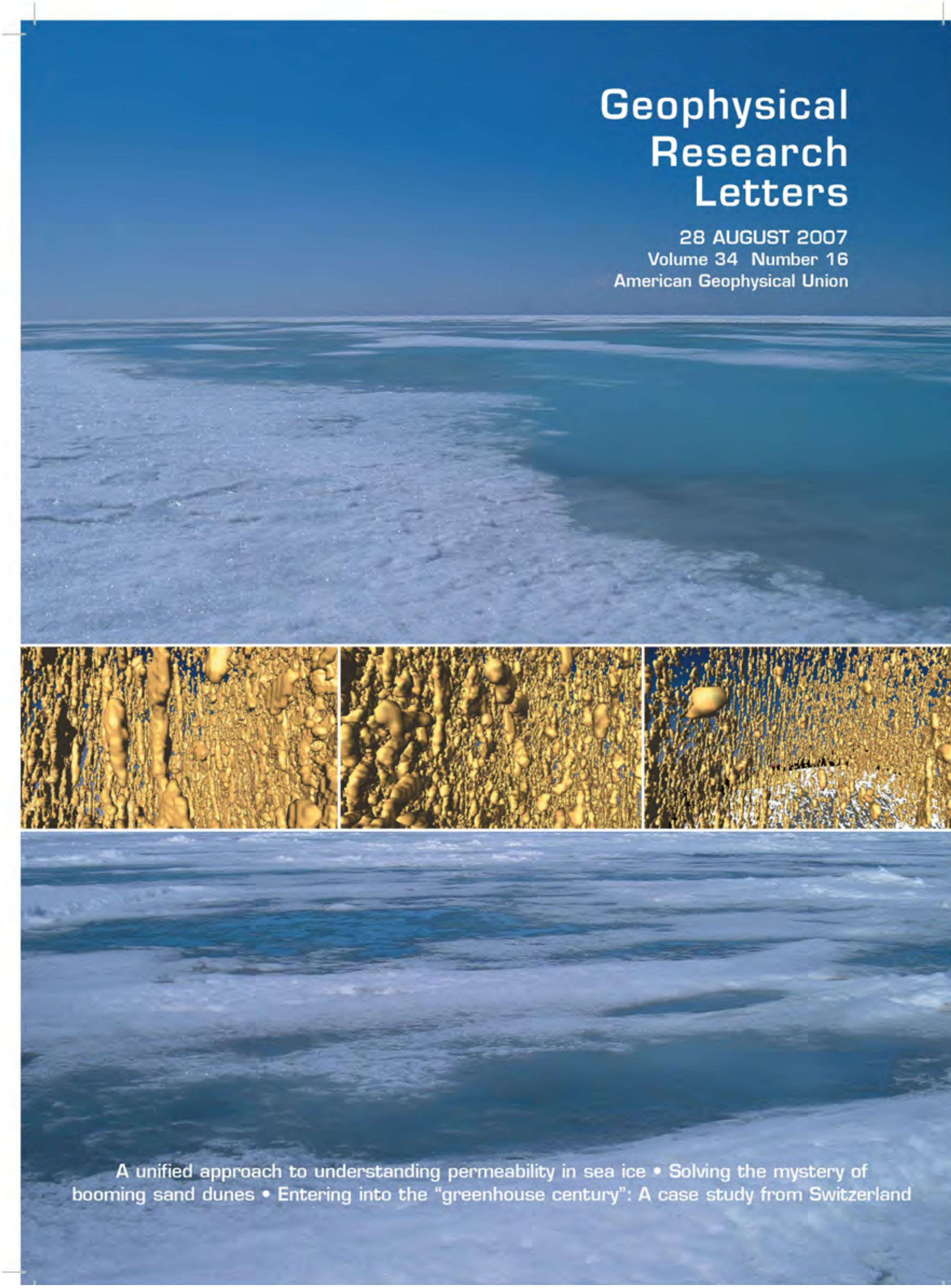
compressed powder



radar absorbing composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice **Golden, Eicken, Heaton, Miner, Pringle, Zhu**



rigorous bounds
percolation theory
hierarchical model
network model

field data

micro-scale

controls

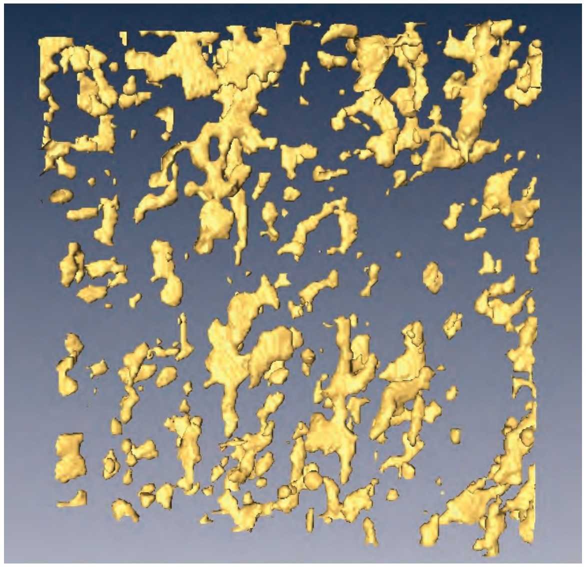
macro-scale processes

X-ray tomography for brine inclusions

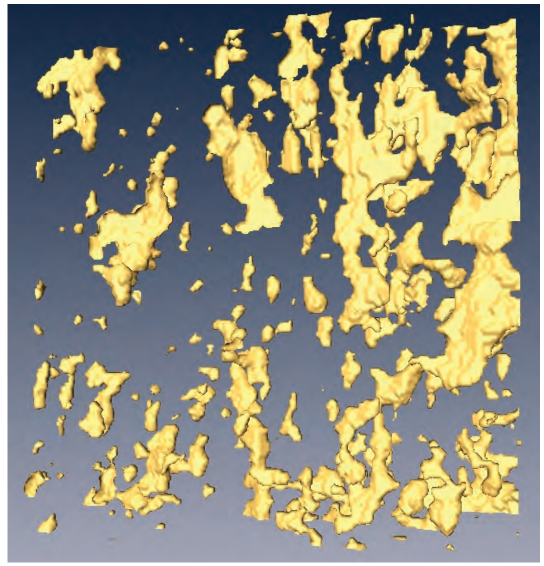
unprecedented look at thermal evolution of brine phase and its connectivity

brine connectivity (over cm scale)

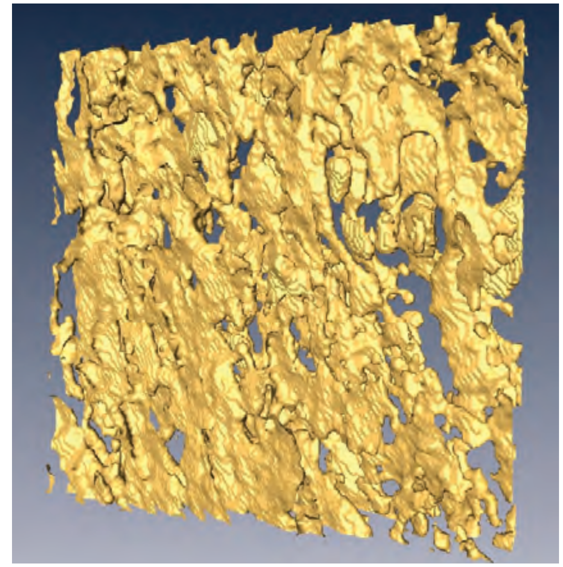
8 x 8 x 2 mm



-15 °C, $\phi = 0.033$



-6 °C, $\phi = 0.075$



-3 °C, $\phi = 0.143$

X-ray tomography confirms percolation threshold

3-D images
pores and throats

→

3-D graph
nodes and edges

analyze graph connectivity as function of temperature and sample size

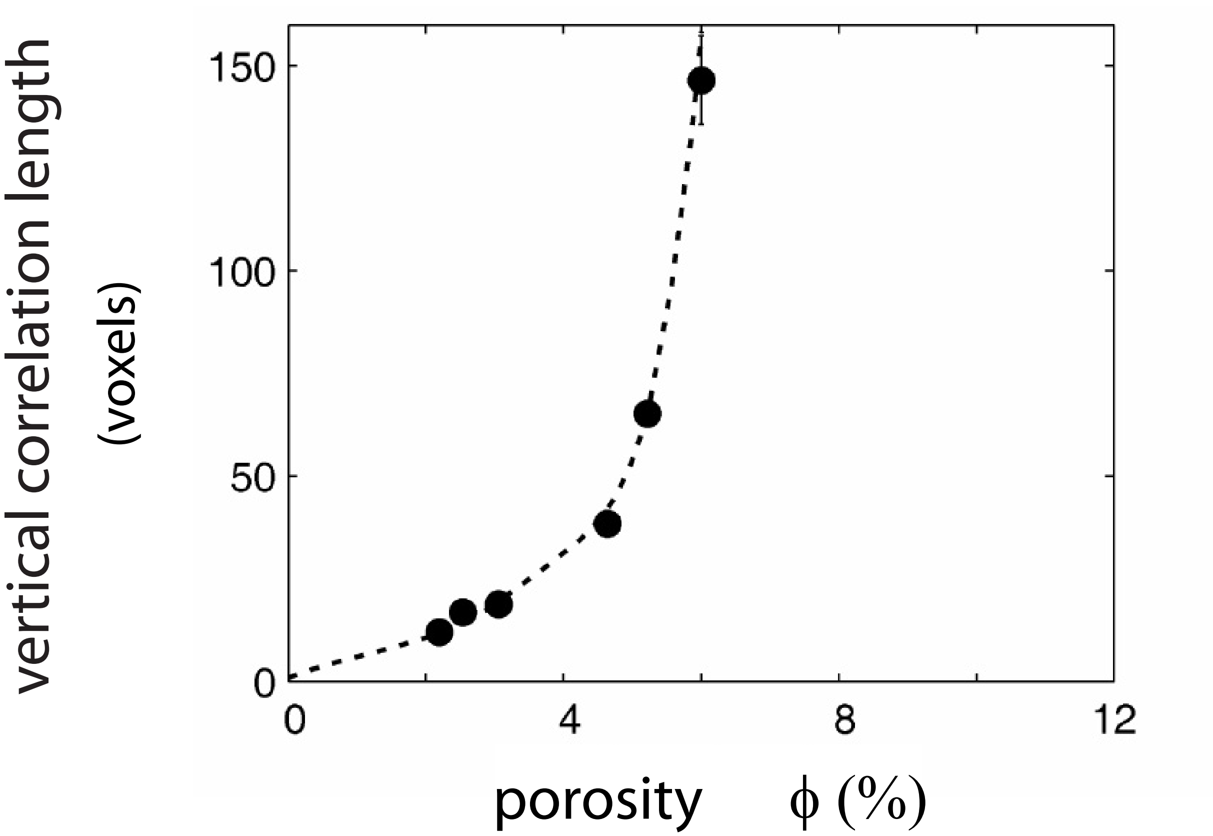
- ***use finite size scaling techniques to confirm rule of fives***
- ***order parameter data from a natural material***

Pringle, Miner, Eicken, Golden, J. Geophys. Res. 2009

The key connectivity functions of percolation theory have been computed extensively for many lattice models, but ***NOT*** for natural materials.

We have calculated them for sea ice single crystals and estimated anisotropic percolation thresholds.

Pringle, Miner, Eicken, Golden, JGR (Oceans) 2009



divergence of correlation length for single crystal data

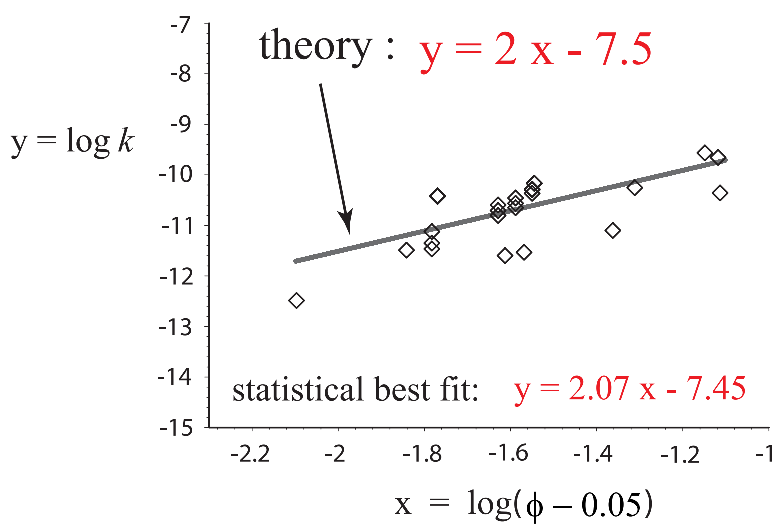
lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8}\ m^2$$

- exponent is UNIVERSAL lattice value $t \approx 2.0$
- sedimentary rocks like sandstones also exhibit universality
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0



Remote sensing of sea ice

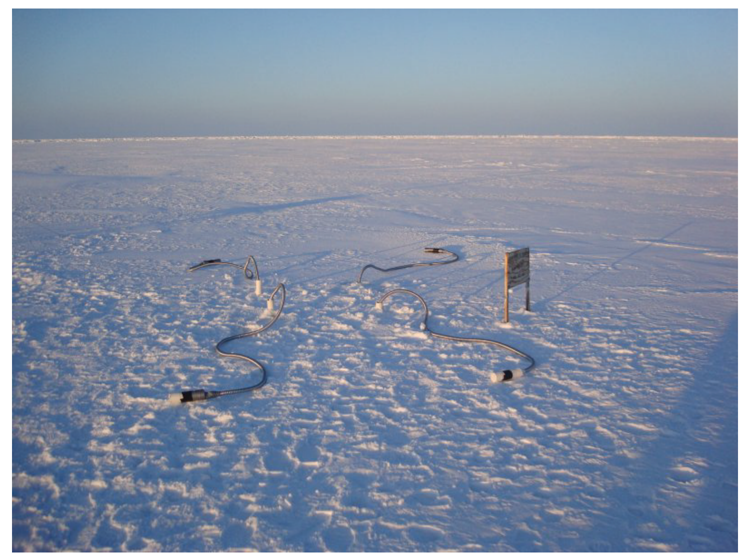
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

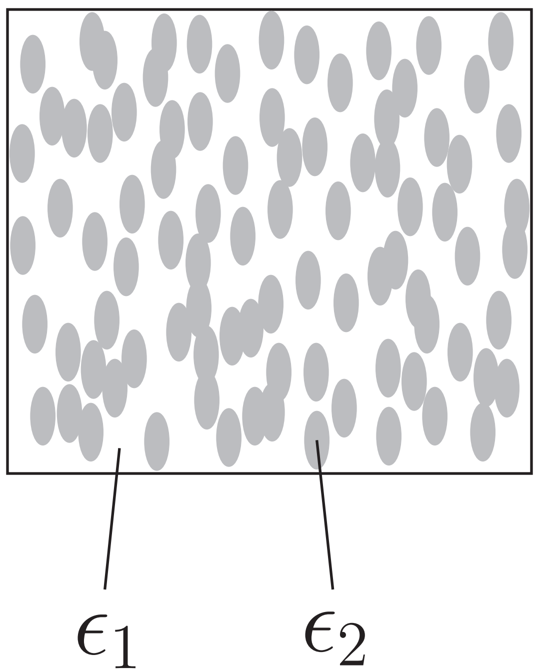
ε^*

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



$$\epsilon^*$$

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \text{ , composite geometry } \right)$$

ocean swells propagating through a vast field of pancake ice

HOMOGENIZATION: long wave sees an effective medium, not individual floes

Theory of Effective Electromagnetic Behavior of Composites

analytic continuation method

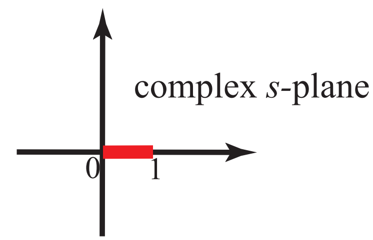
Forward Homogenization Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)

composite geometry (spectral measure μ) → ε^*

integral representations, rigorous bounds, approximations, etc.

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$



Inverse Homogenization Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001) (McPhedran, McKenzie, and Milton, 1982)

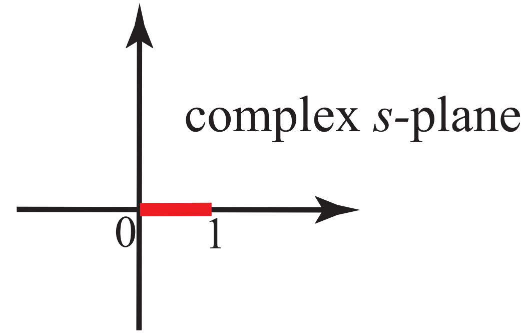
ε^* → **composite geometry (spectral measure μ)**

recover brine volume fraction, connectivity, etc.

Stieltjes integral representation

separation of geometry from parameters

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$



$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

μ

- spectral measure of self adjoint operator $\Gamma\chi$
- mass $= p_1$
- higher moments depend on n-point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$$

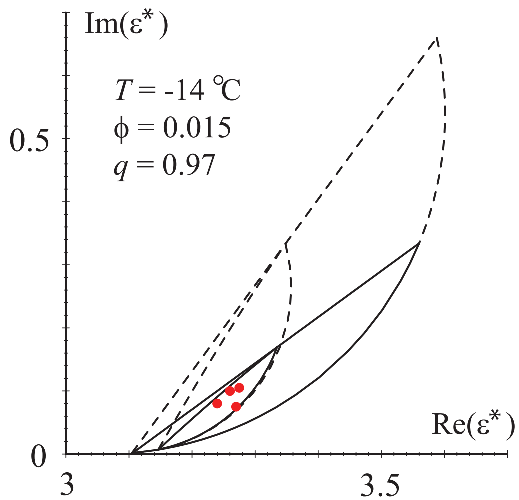
$\chi =$ characteristic function of the brine phase

$$E = (s + \Gamma\chi)^{-1} e_k$$

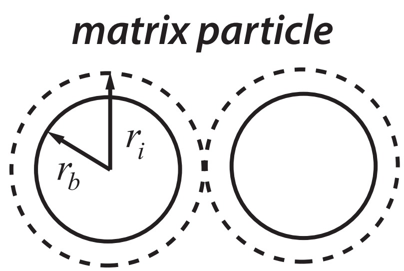
Golden and Papanicolaou, Comm. Math. Phys. 1983

forward and inverse bounds for sea ice

forward bounds



4.75 GHz data

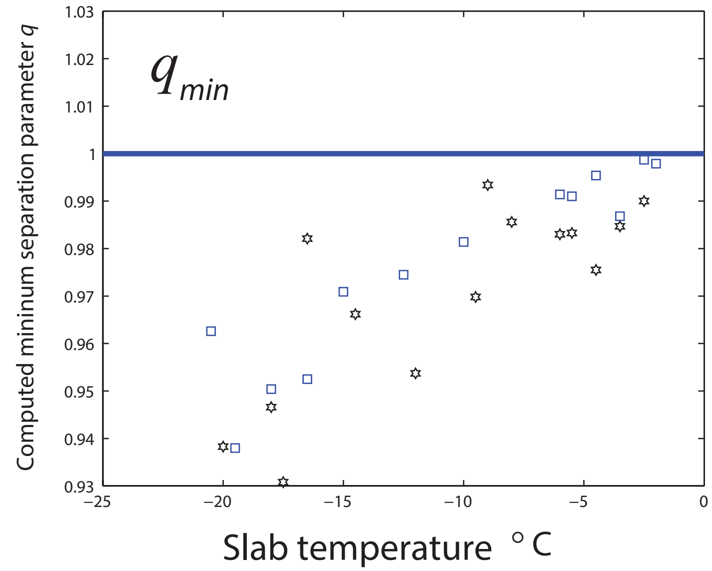


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

polycrystalline bounds two-scale homogenization

Gully, Lin, Cherkaev, Golden, 2014

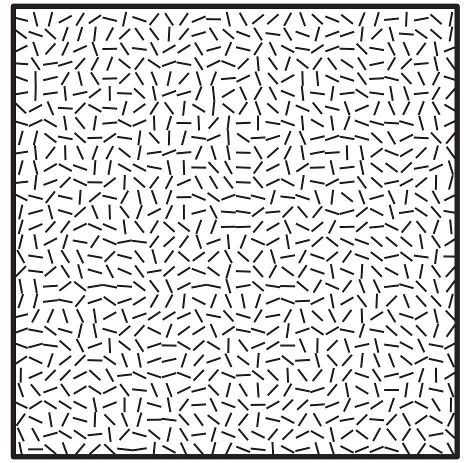
inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

Mathematical Formulation of the effective parameter problem for polycrystalline media



Quasi-static limit of *Maxwell's equations*

$$\vec{\nabla} \times \vec{E} = 0, \qquad \vec{\nabla} \cdot \vec{D} = 0, \qquad \vec{D} = \varepsilon \, \vec{E}$$

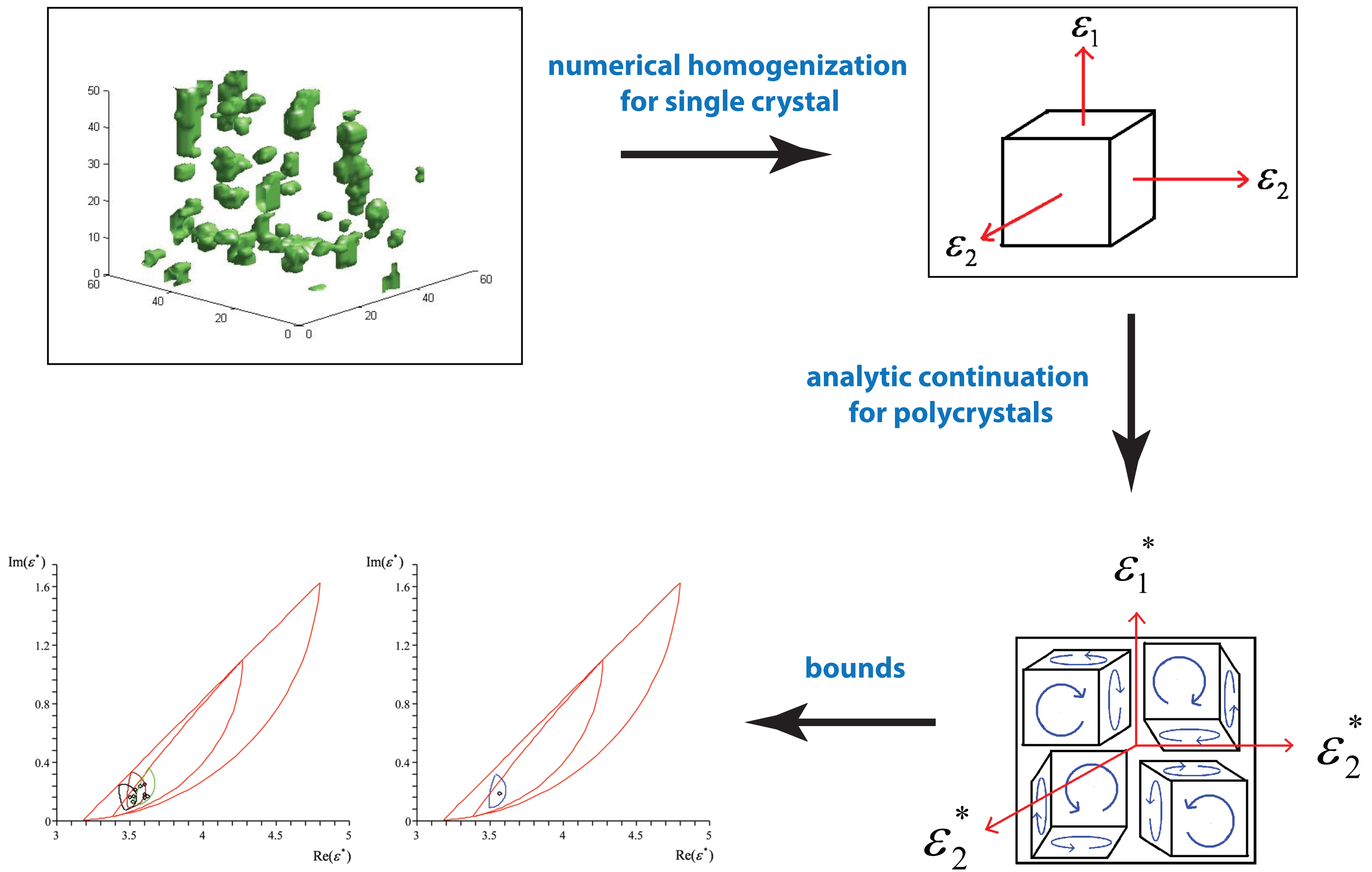
Uniaxial polycrystalline media

$$\varepsilon = R^T \text{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_2) R \longleftarrow \text{Random rotation matrix}$$

$$= \varepsilon_1 X_1 + \varepsilon_2 X_2, \quad X_2 = I - X_1, \quad X_i X_j = \delta_{ij} X_i$$

- A direct analog of the proof given by Golden and Papanicolaou for two-component composites yields an integral representaion for the effective permittivity tensor ε^* involving a ***spectral measure*** of a **random matrix**

two scale homogenization for polycrystalline sea ice



Gully, Lin, Cherkaev, Golden, Proc. Roy. Soc. A (and cover) 2014

Spectral analysis of multiscale sea ice structures

homogenization for brine inclusions, melt ponds, and sea ice pack

how to upscale information on "microstructure"
into effective behavior for larger scales

numerical computation of spectral measure μ

direct calculation of spectral measure

1. Discretization of composite microstructure gives lattice of 1's and 0's (random resistor network).

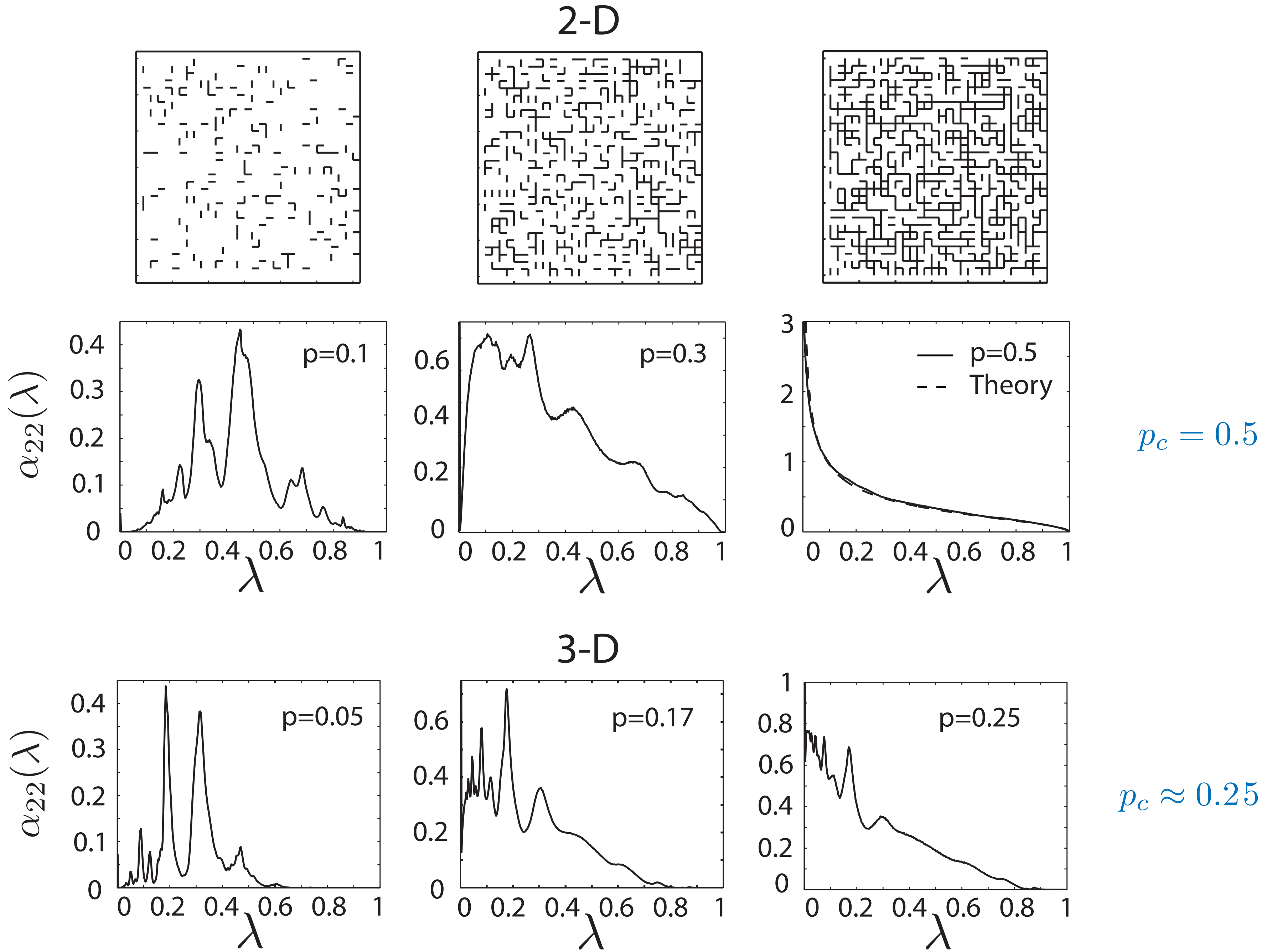
2. The fundamental operator $\Gamma\chi$ becomes a random matrix depending only on the composite geometry.

3. Compute the eigenvalues λ_i and eigenvectors of $\Gamma\chi$ with (length)^2 $= \alpha_i$

$$\mu(\lambda) = \sum_i \alpha_i \, \delta(\lambda - \lambda_i)$$

Dirac point measure (Dirac delta)

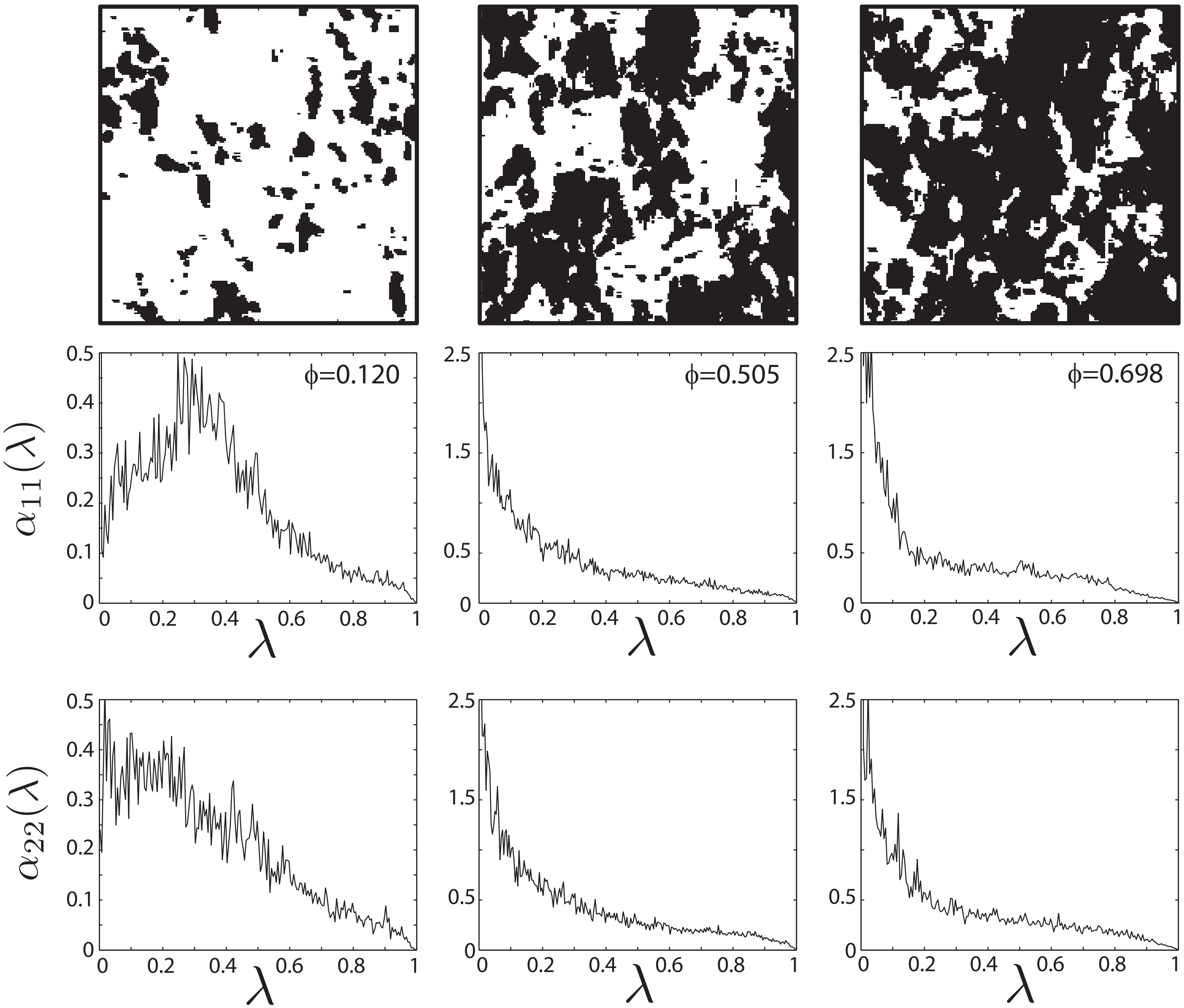
The Spectral Measures for Random Resistor Networks



spectral gaps collapse at the percolation transitions

Murphy and Golden, J. Math. Phys. (2012)

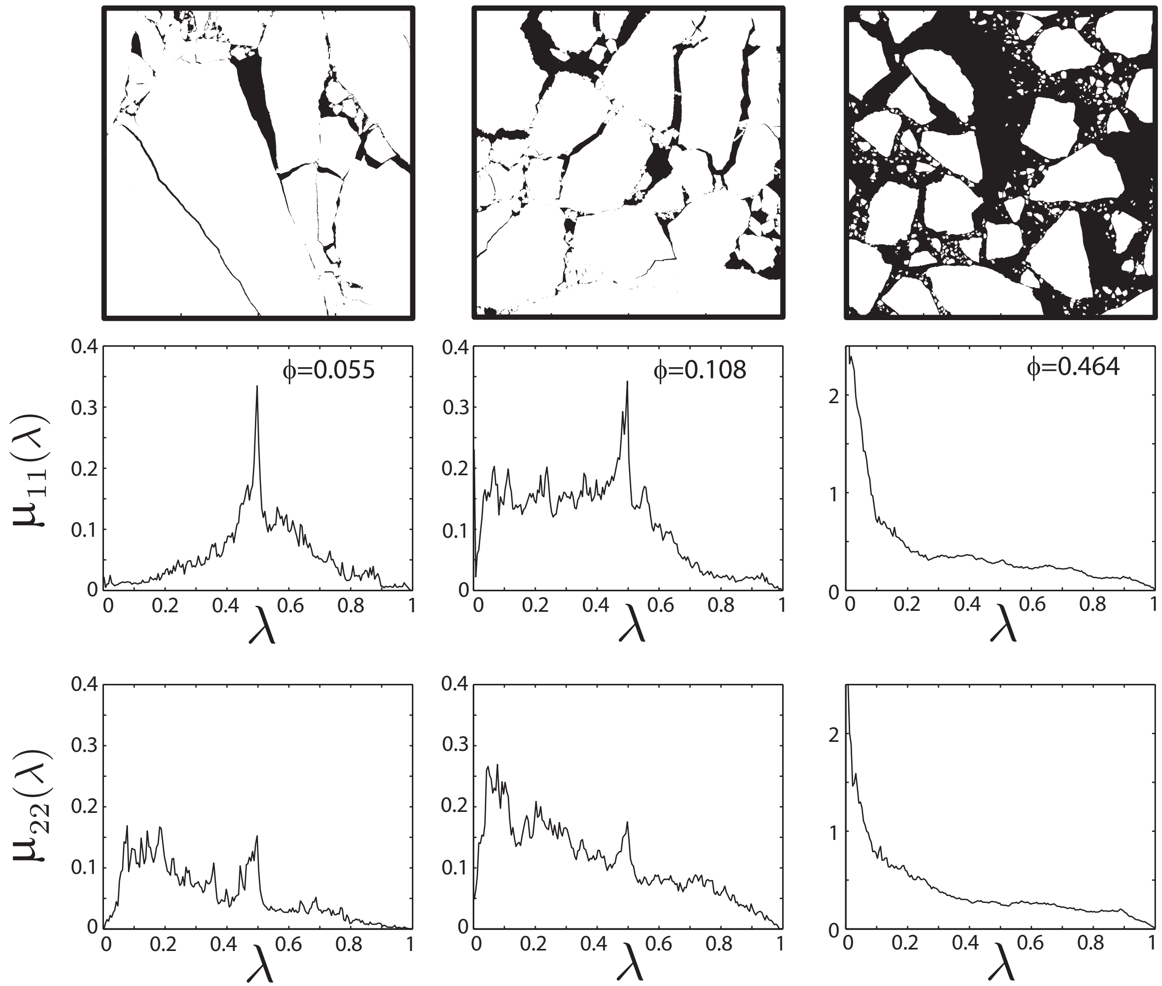
Spectral Measures for Sea Ice Structures: Brine Inclusions



N. B. Murphy, C. Hohenegger, C. S. Sampson, D. K. Perovich, H. Eicken, E. Cherkaev, B. Alali, and K. M. Golden

spectral measures provide a path toward rigorously incorporating "composite microstructure" into calculations of effective behavior on larger scales

spectral measures for the Arctic sea ice pack



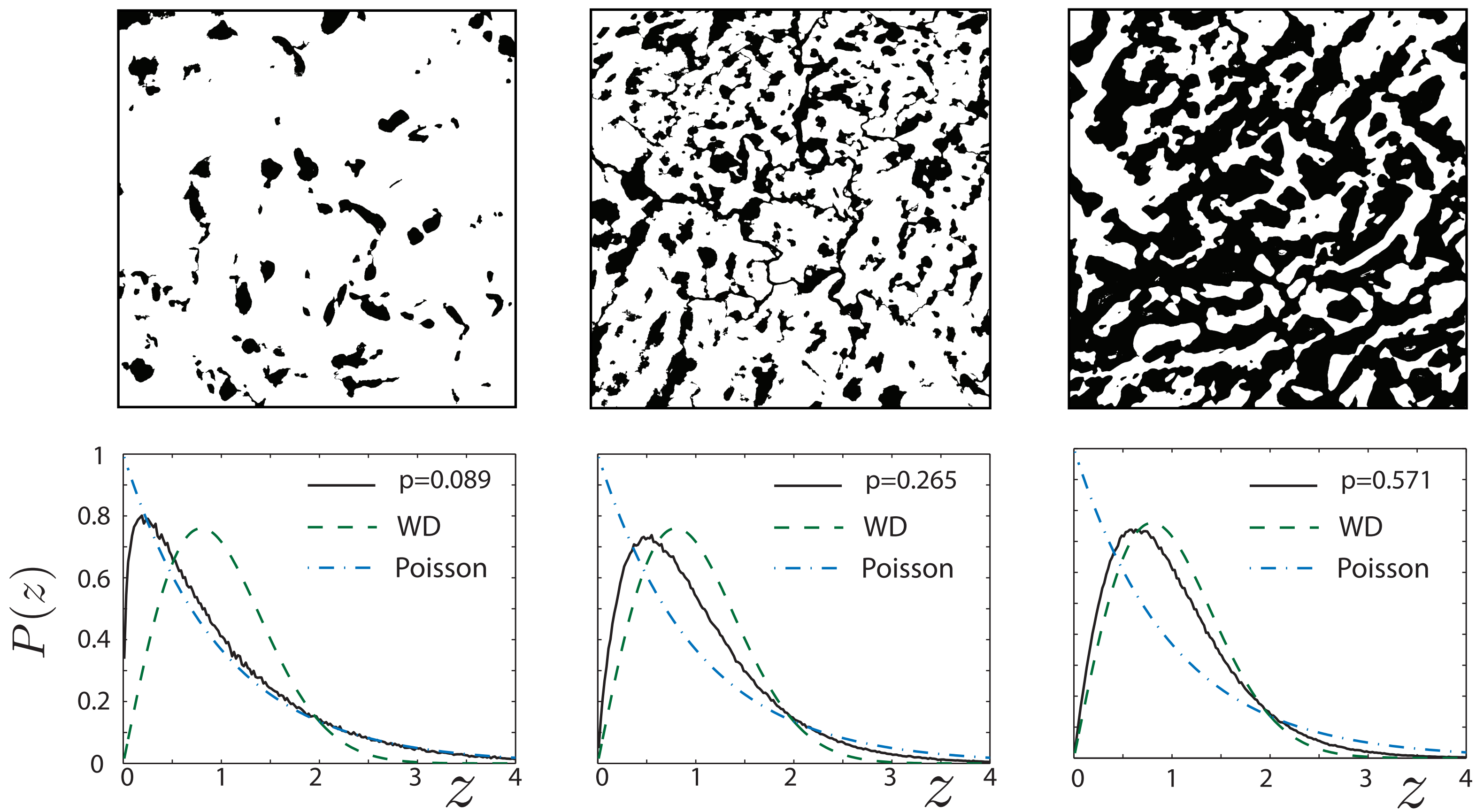
area under curve = φ = open water fraction

spectral gap closes as ocean phase becomes connected

N. B. Murphy, C. Hohenegger, C. S. Sampson, D. K. Perovich, H. Eicken, E. Cherkaev, B. Alali, and K. M. Golden, 2014 *random matrix characterization of connectedness transition -- discretization of* $\chi\Gamma\chi$

Unfolded Eigenvalue Spacing Distribution

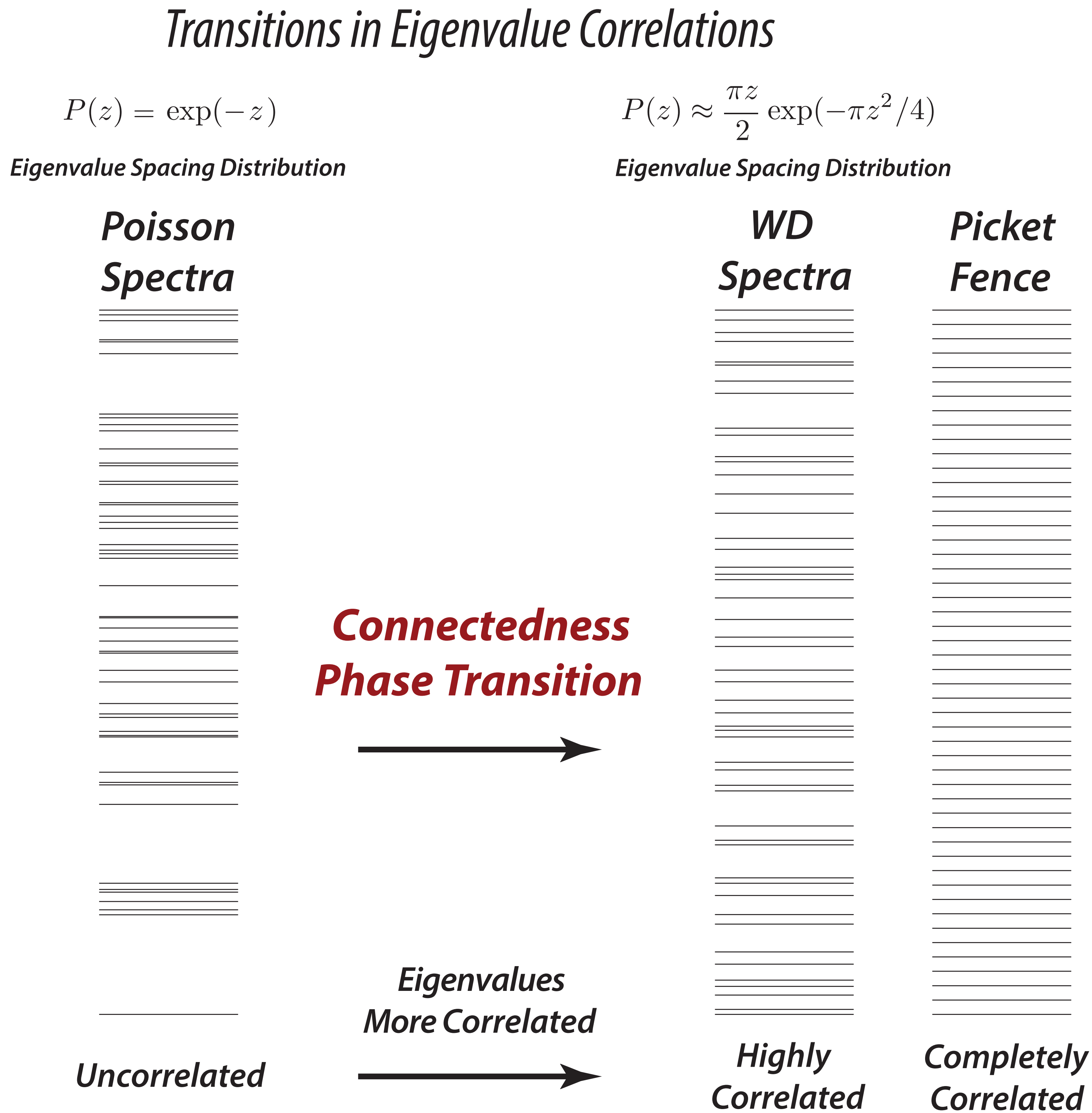
ARCTIC MELT PONDS



eigenvalue statistics for transport tend toward the UNIVERSAL Wigner-Dyson distribution as the "conducting" phase becomes connected over large scales

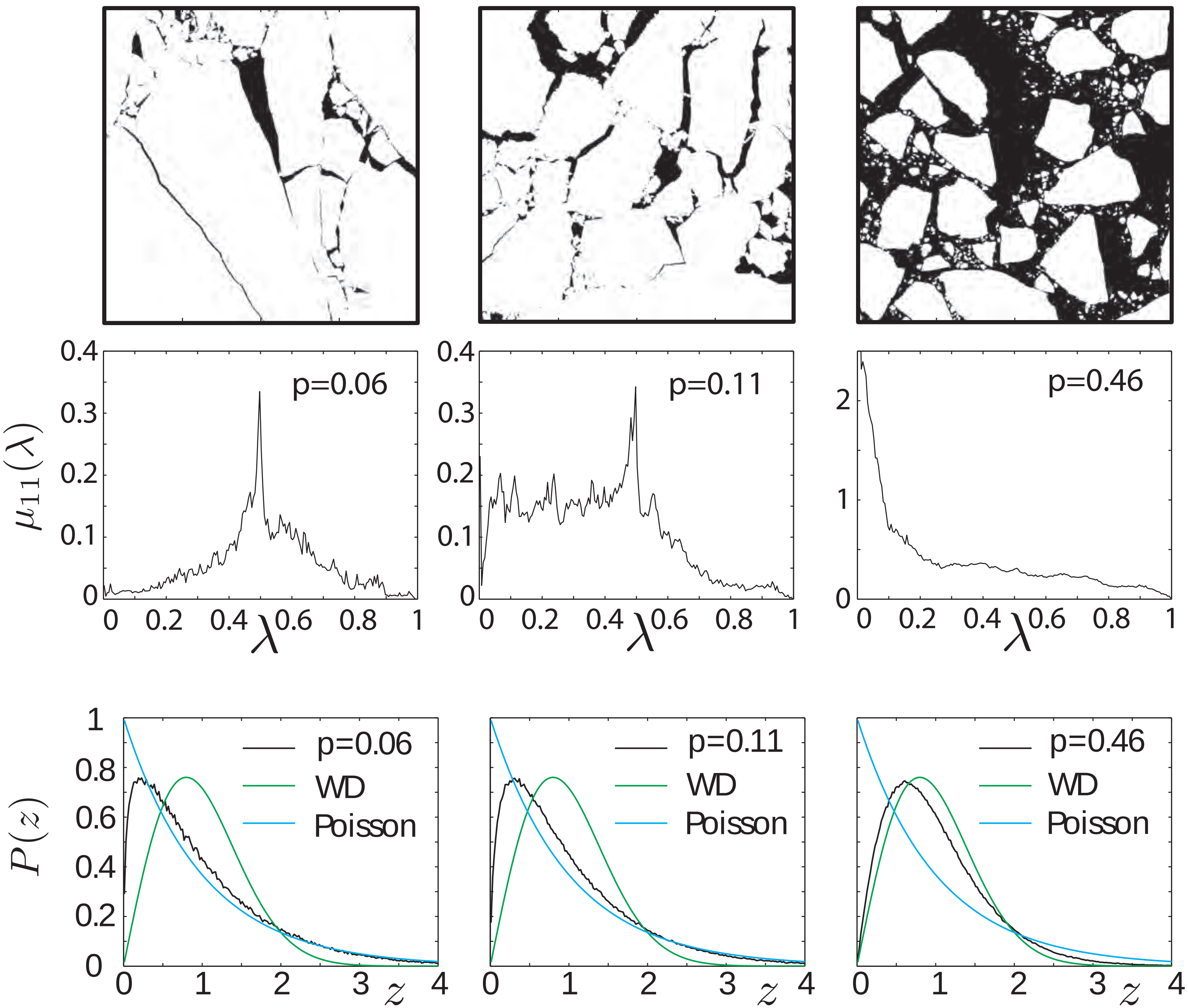
uncorrelated → *"level repulsion"*

N. B. Murphy, K. M. Golden 2014

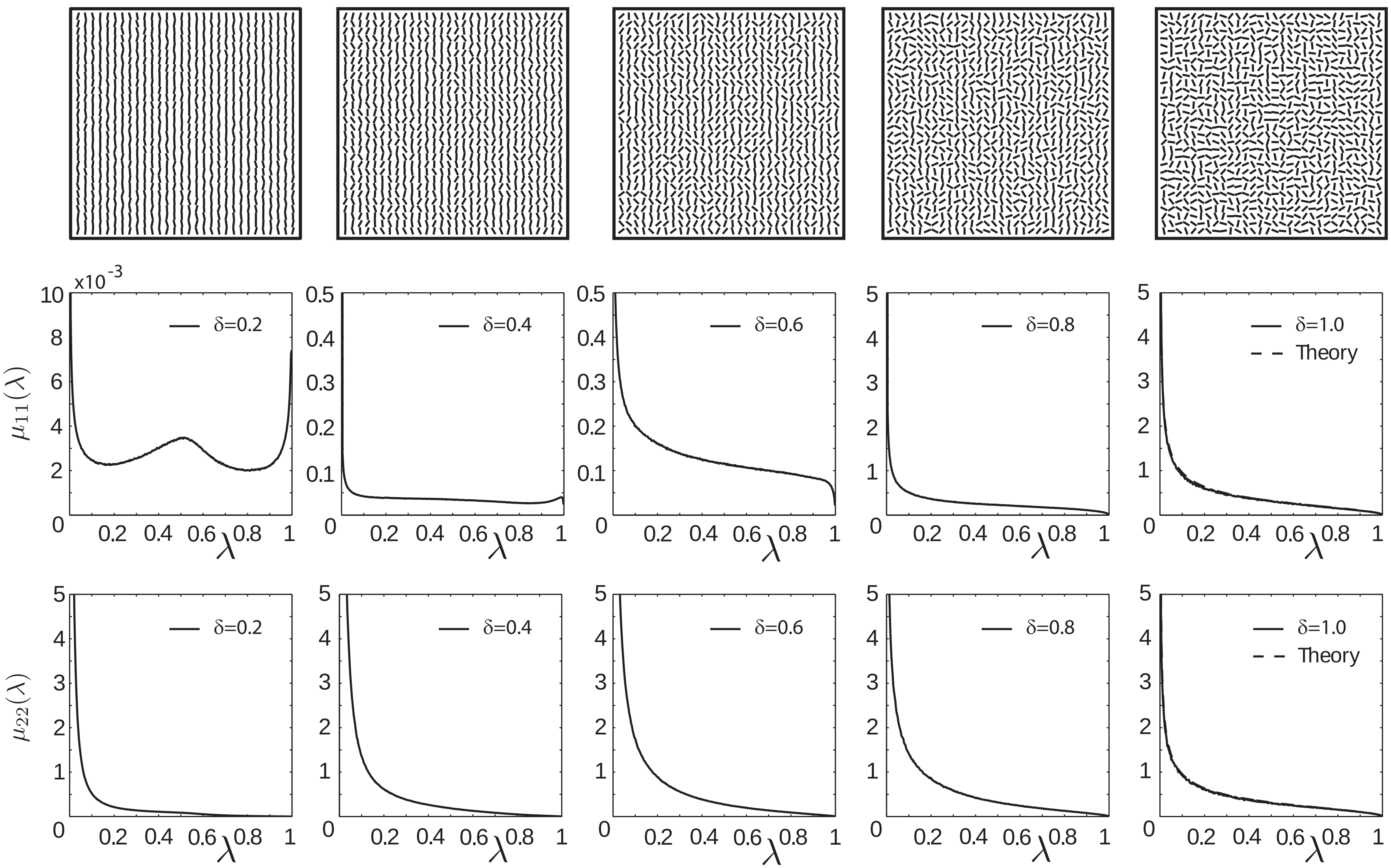


Transitions in Eigenvalue Correlations
$P(z) = \exp(-z)$
Eigenvalue Spacing Distribution
$P(z) \approx \frac{\pi z}{2} \exp(-\pi z^2/4)$
Eigenvalue Spacing Distribution
Poisson Spectra
WD Spectra
Picket Fence
Connectedness Phase Transition
Eigenvalues More Correlated
Uncorrelated
Highly Correlated
Completely Correlated

Spectral computations for Arctic sea ice pack

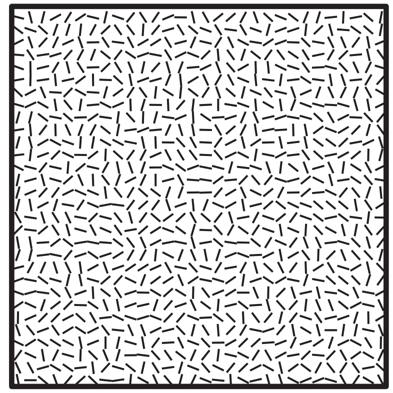


Spectral measures for uniaxial polycrystalline media



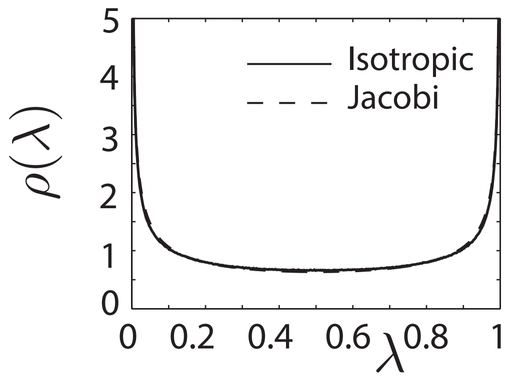
- Random crystallographic orientations angles θ measured from the vertical direction, uniformly distributed $\theta \sim U(-\delta\pi/2, \delta\pi/2)$

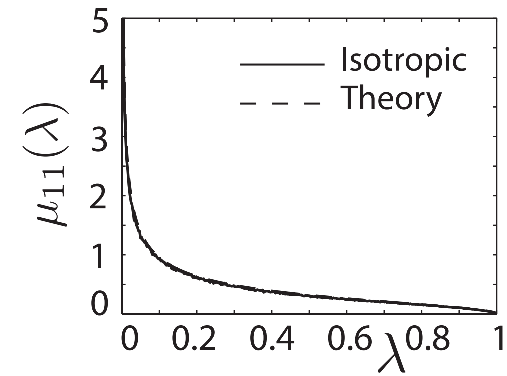
spectral data for isotropic 2-D polycrystalline materials



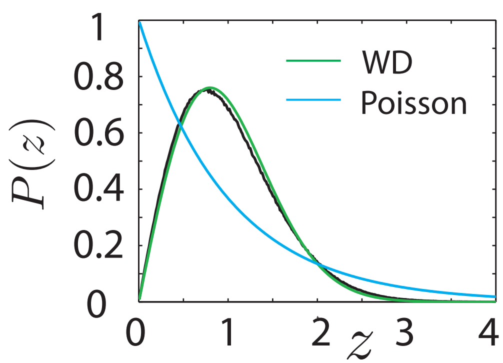
Murphy, Cherkaev, Golden

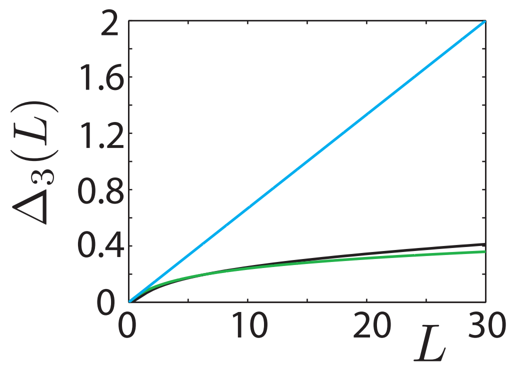
spectral density

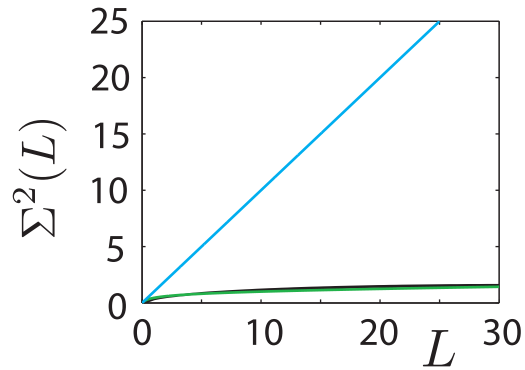




spectral measure







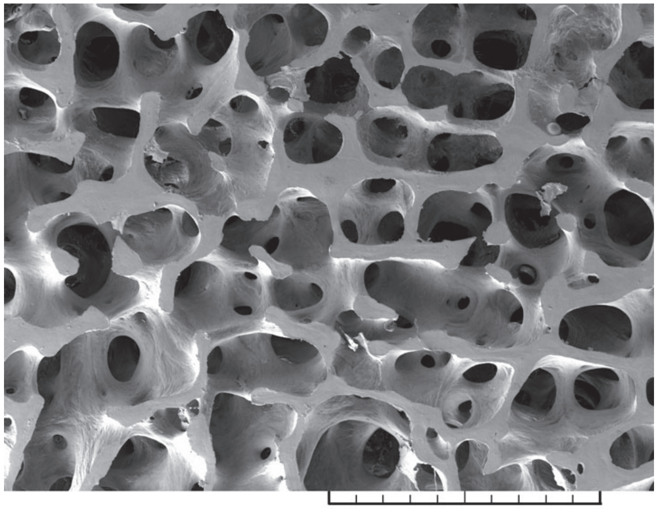
eigenvalue spacings

long range correlations

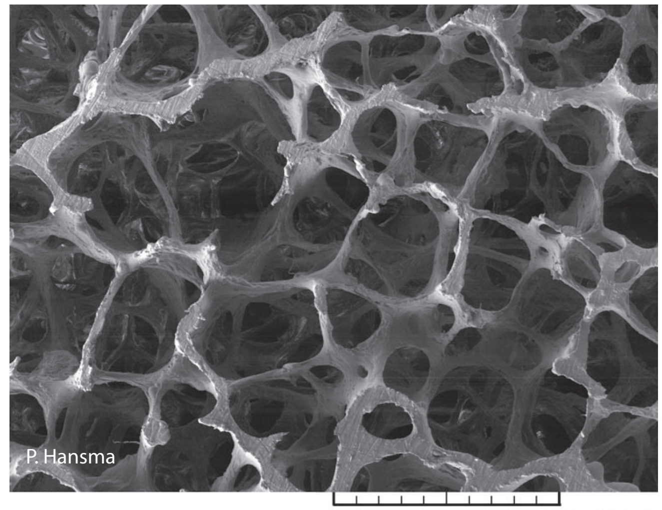
spectral characterization of porous microstructures in bone

Golden, Murphy, Cherkaev, J. Biomechanics 2011

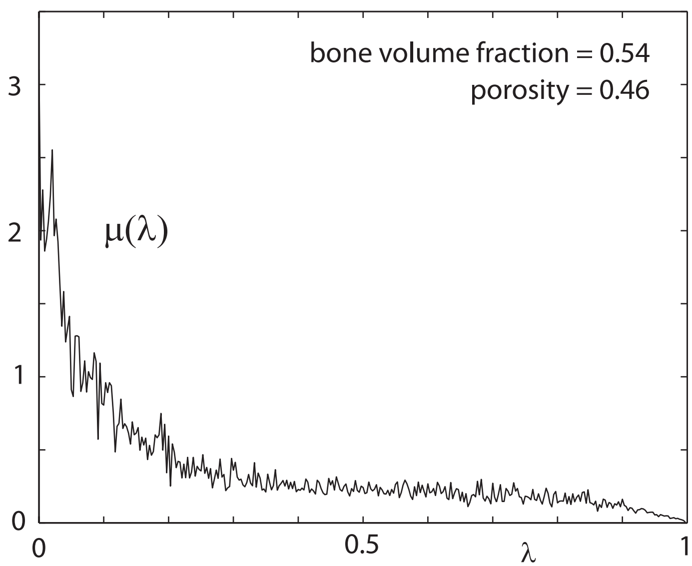
(a) young healthy trabecular bone



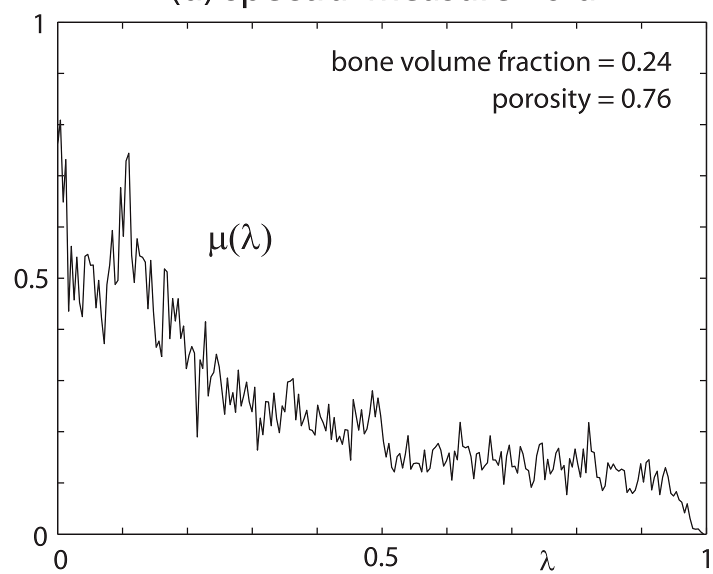
(b) old osteoporotic trabecular bone



(c) spectral measure - young



(d) spectral measure - old



+

reconstruction of spectral measures from complex permittivity data

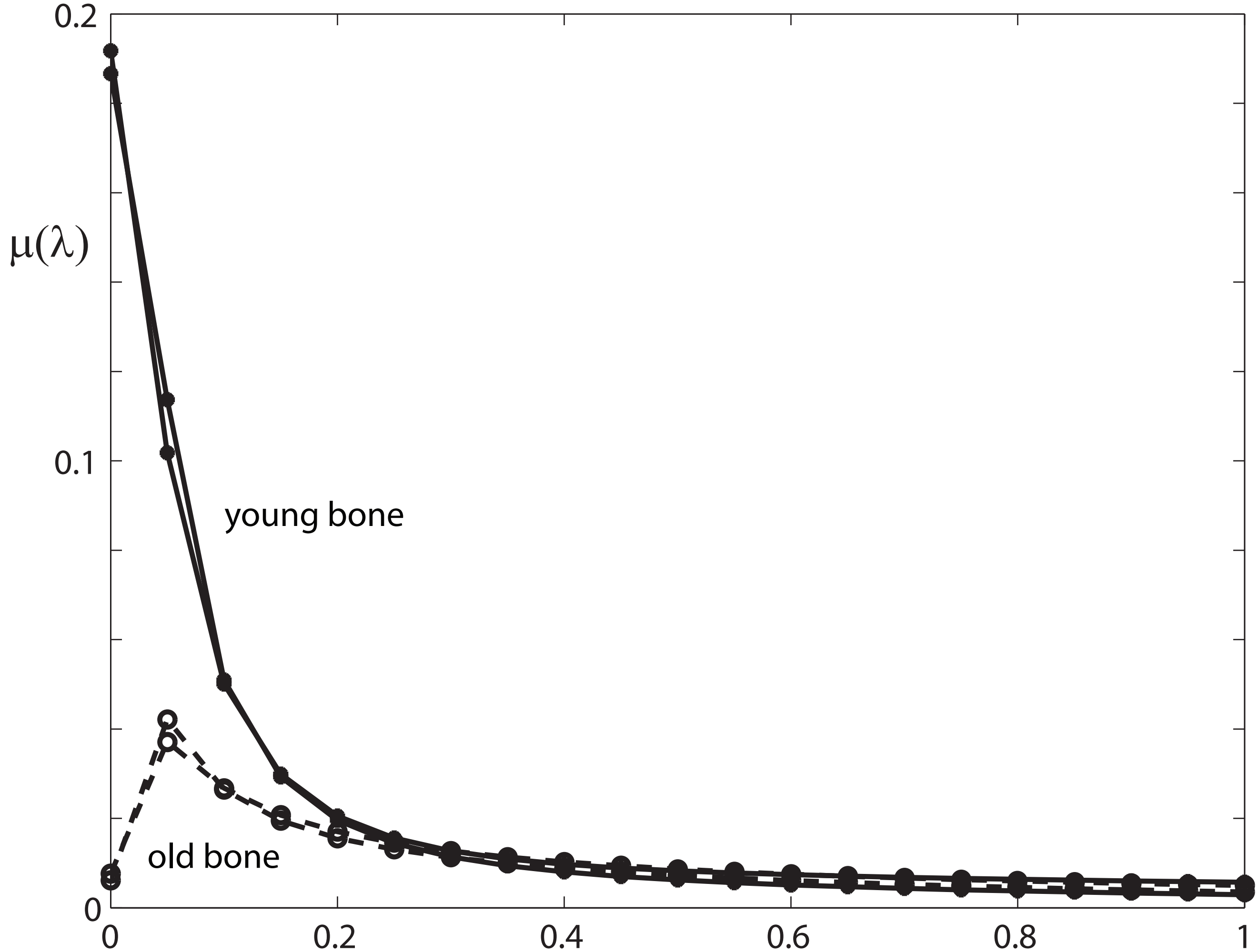
using regularized inversion scheme

EM monitoring of osteoporosis

loss of bone connectivity

the math doesn't care if it's sea ice or bone!

reconstruction of spectral measures from simulated complex permittivity data



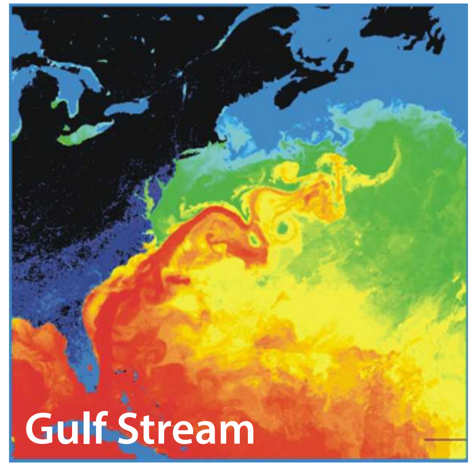
regularized inversion scheme

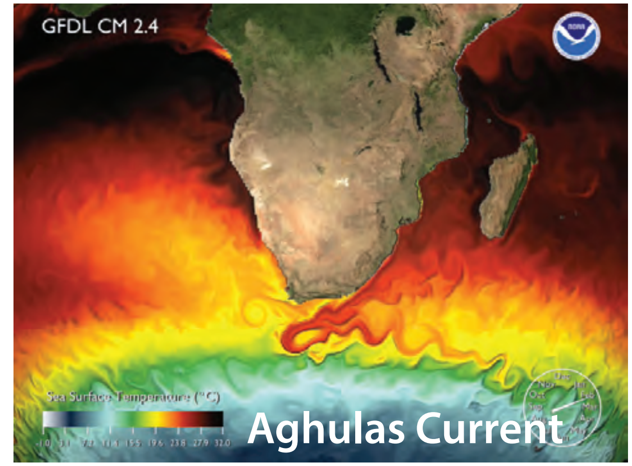
Golden, Murphy, Cherkaev, J. Biomech. 2011

advection enhanced diffusion

effective diffusivity

tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere
salt and heat transport in ocean





advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

↓ ***homogenize***

$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

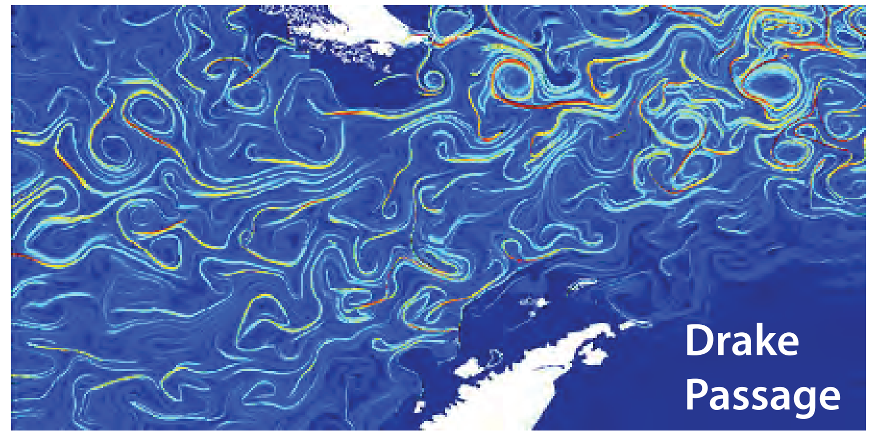
κ^* ***effective diffusivity***

analytic function of Péclet number

Stieltjes integral for κ^* ***with spectral measure***

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Zhu, Golden 2014



Stieltjes integral for κ^* with spectral measure

composites

Golden and Papanicolaou, CMP 1983

$$\frac{\epsilon^*}{\epsilon_2} = 1 - \int_0^1 \frac{d\mu(\lambda)}{s-\lambda}$$

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

advection diffusion

Avellaneda and Majda, PRL 89, CMP 91

$$\frac{\kappa^*}{\kappa_0} = 1 - \int_0^\infty \frac{d\rho(z)}{t-z}$$

$t = -1/\xi^2, \ \xi$ = Péclet number

- computations of spectral measures and effective diffusivity for model flows

$$\mathrm{i}\Gamma\mathbf{H}\Gamma \qquad \vec{u} = \kappa_0\, \xi\, \vec{\nabla} \cdot \mathbf{H}$$

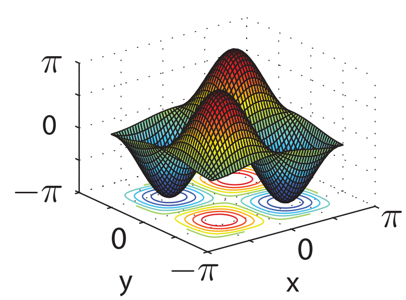
$\mathbf{H}$ antisymmetric vector potential

Murphy, Zhu, Golden 2014

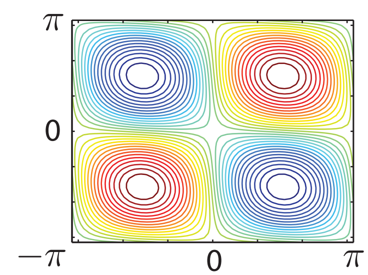
- rigorous bounds and computations on convection enhanced thermal conductivity of sea ice

Wang, Liu, Zhu, Golden 2014

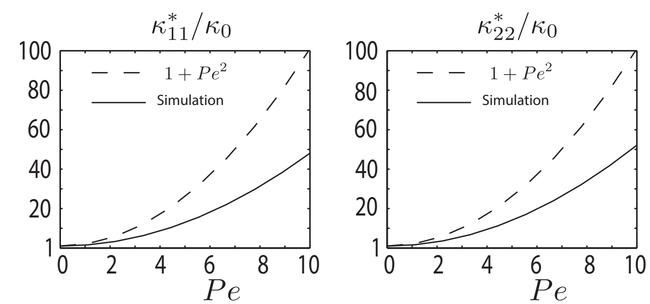
stream function



streamlines



effective diffusivities



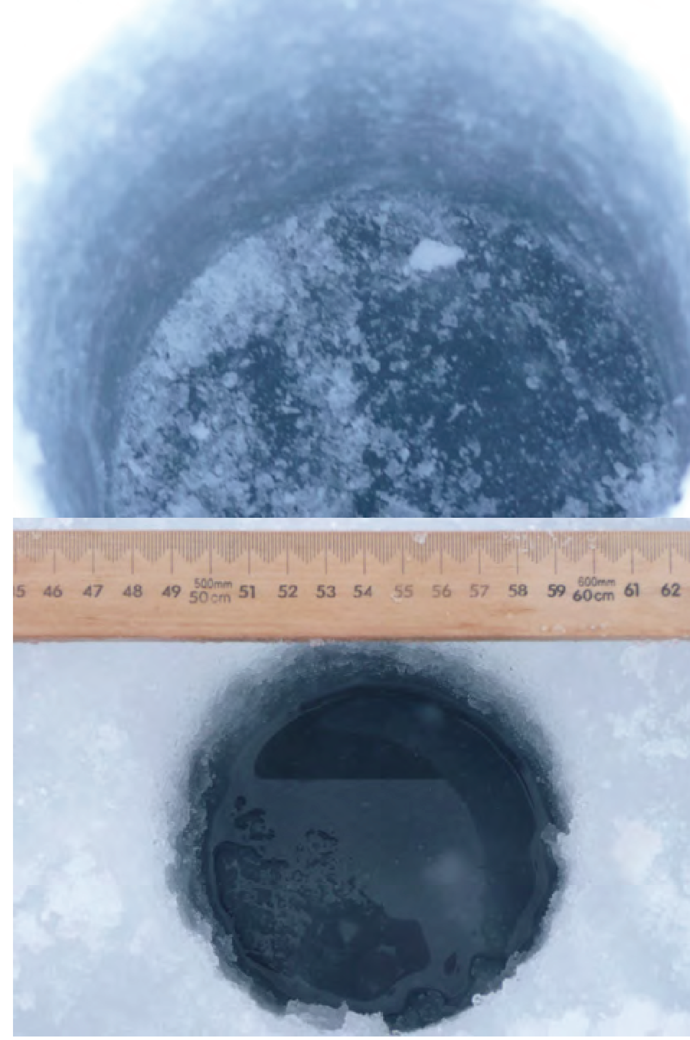
Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	*Antarctic*	*SIPEX*
2010	*Antarctic*	*McMurdo Sound*
2011	*Arctic*	*Barrow AK*
2012	*Arctic*	*Barrow AK*
2012	*Antarctic*	*SIPEX II*
2013	*Arctic*	*Barrow AK*

photo by Jan Lieser



measuring fluid permeability of Antarctic sea ice

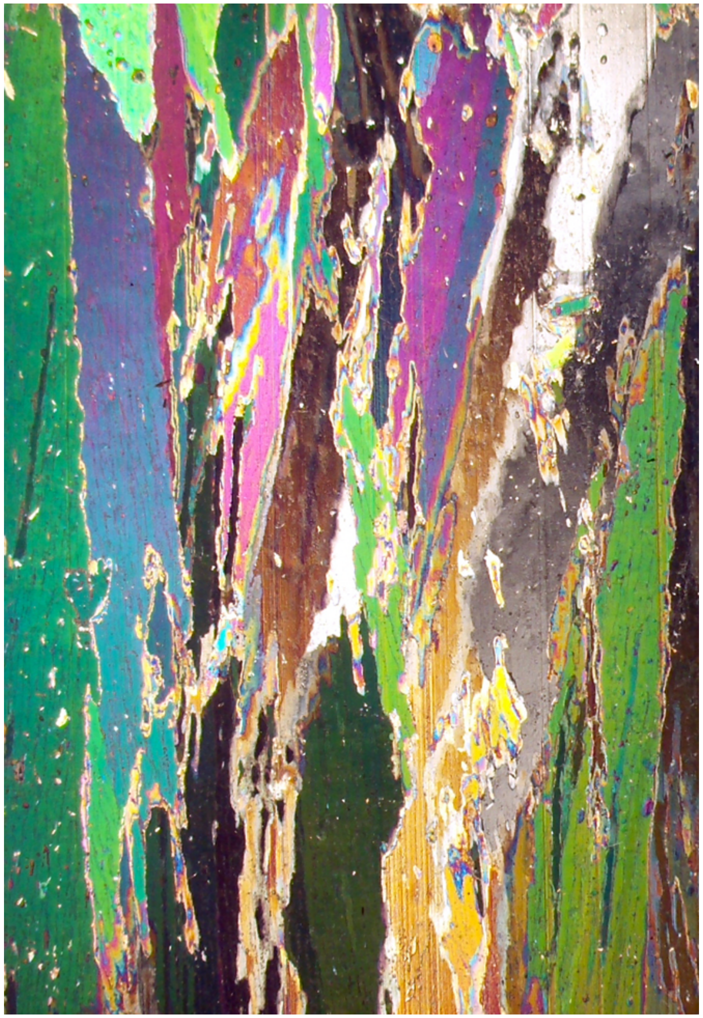
SIPEX 2007

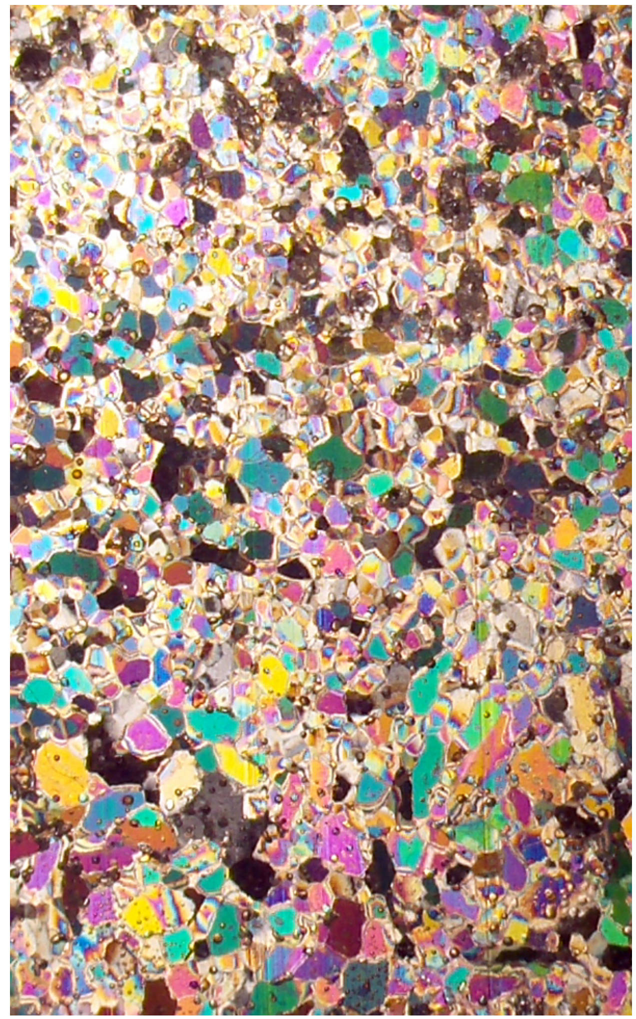
higher threshold for fluid flow in Antarctic granular sea ice

columnar

granular

5%

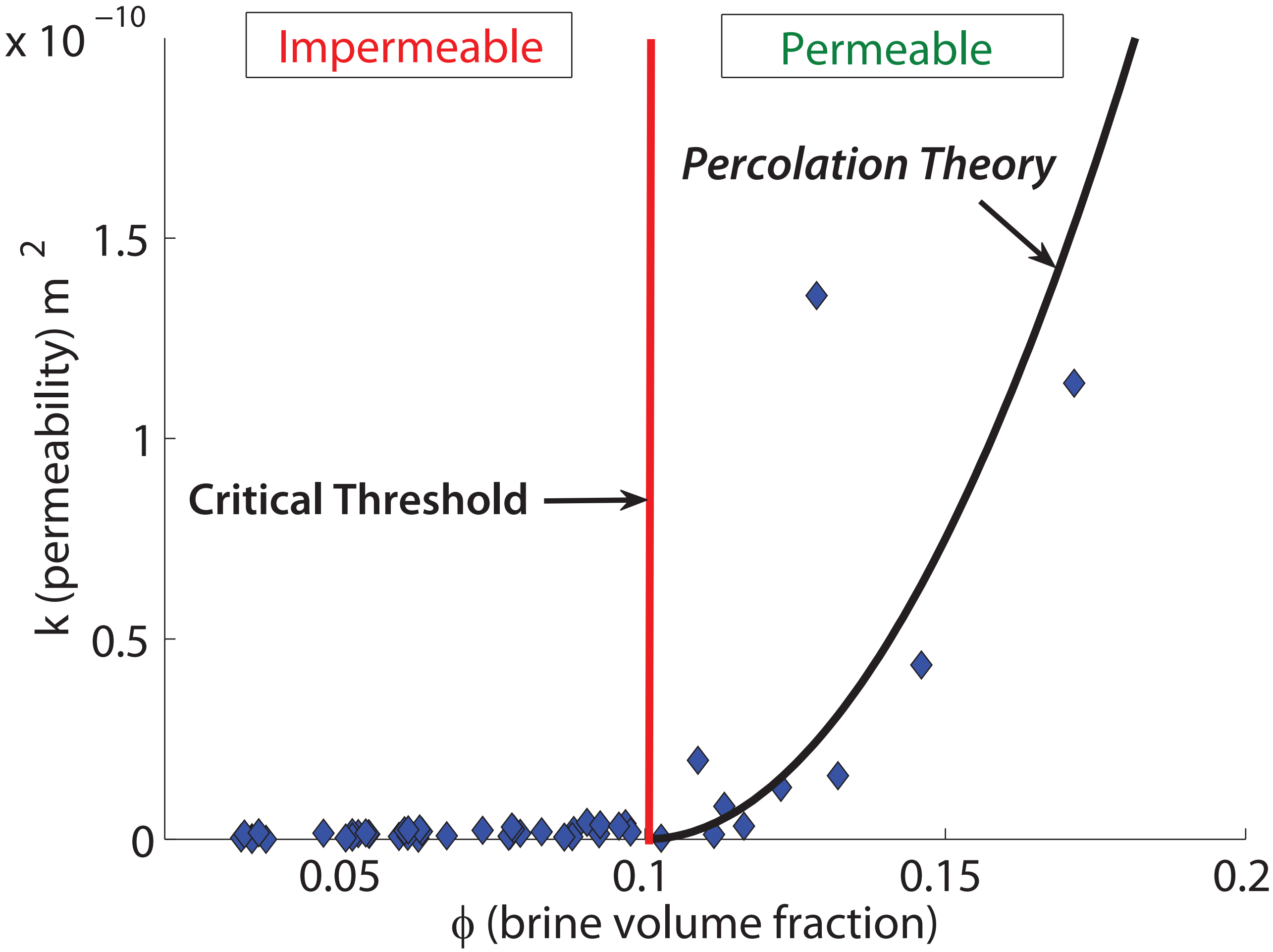




10%

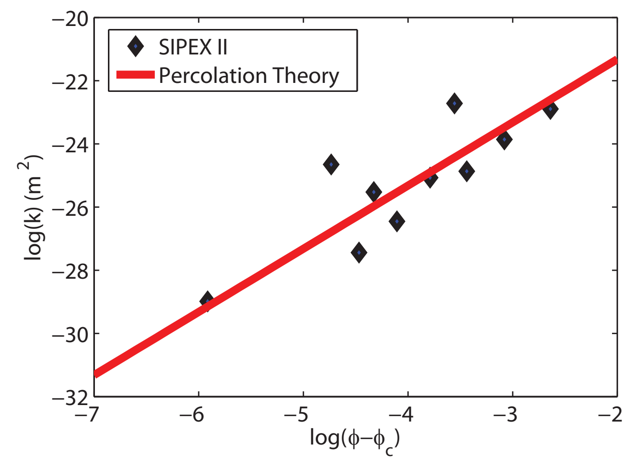
Golden, Gully, Lubbers, Sampson, Tison 2014

SIPEX II vertical permeability data



same universal critical exponent as lattice models

data above threshold

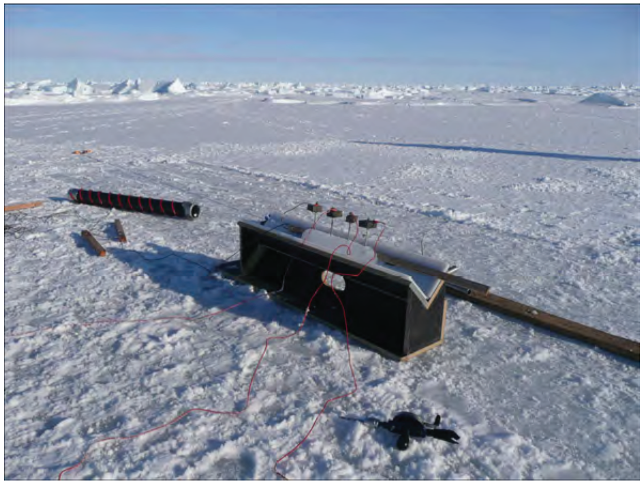


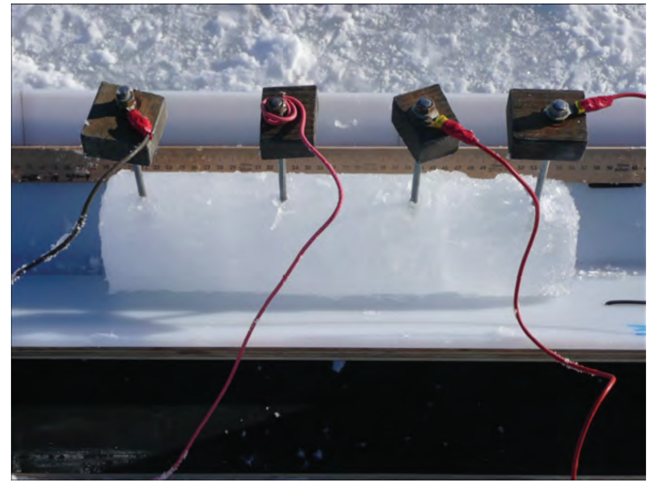
higher threshold in granular ice predicted with percolation theory by Golden, et al. (Science, 1998)

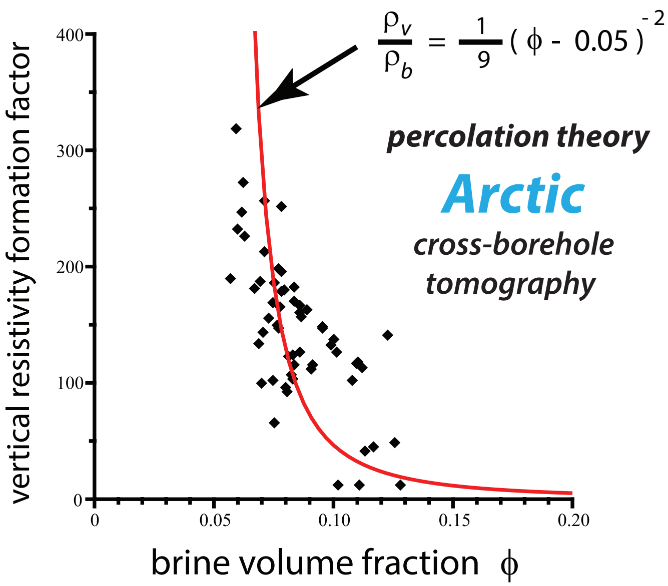
not confirmed experimentally until SIPEX I (2007) and SIPEX II (2012)

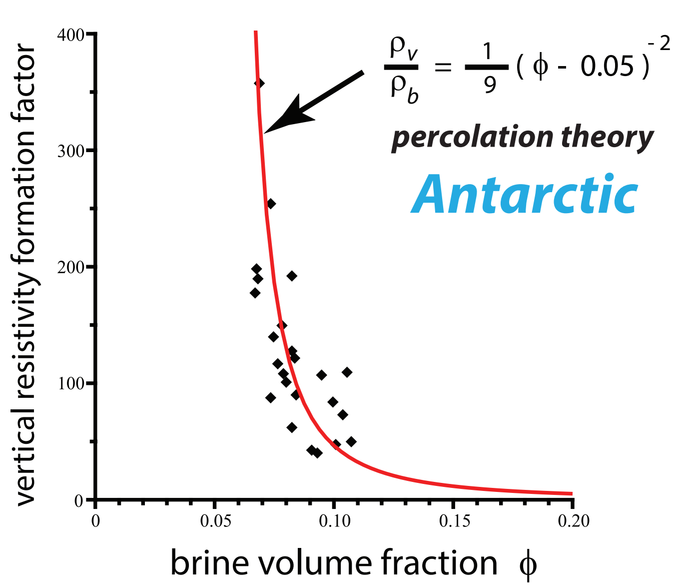
critical behavior of electrical transport in sea ice

electrical signature of the on-off switch for fluid flow









cross-borehole tomography - electrical classification of sea ice layers

Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2014

melt ponds on the surface of Arctic sea ice

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

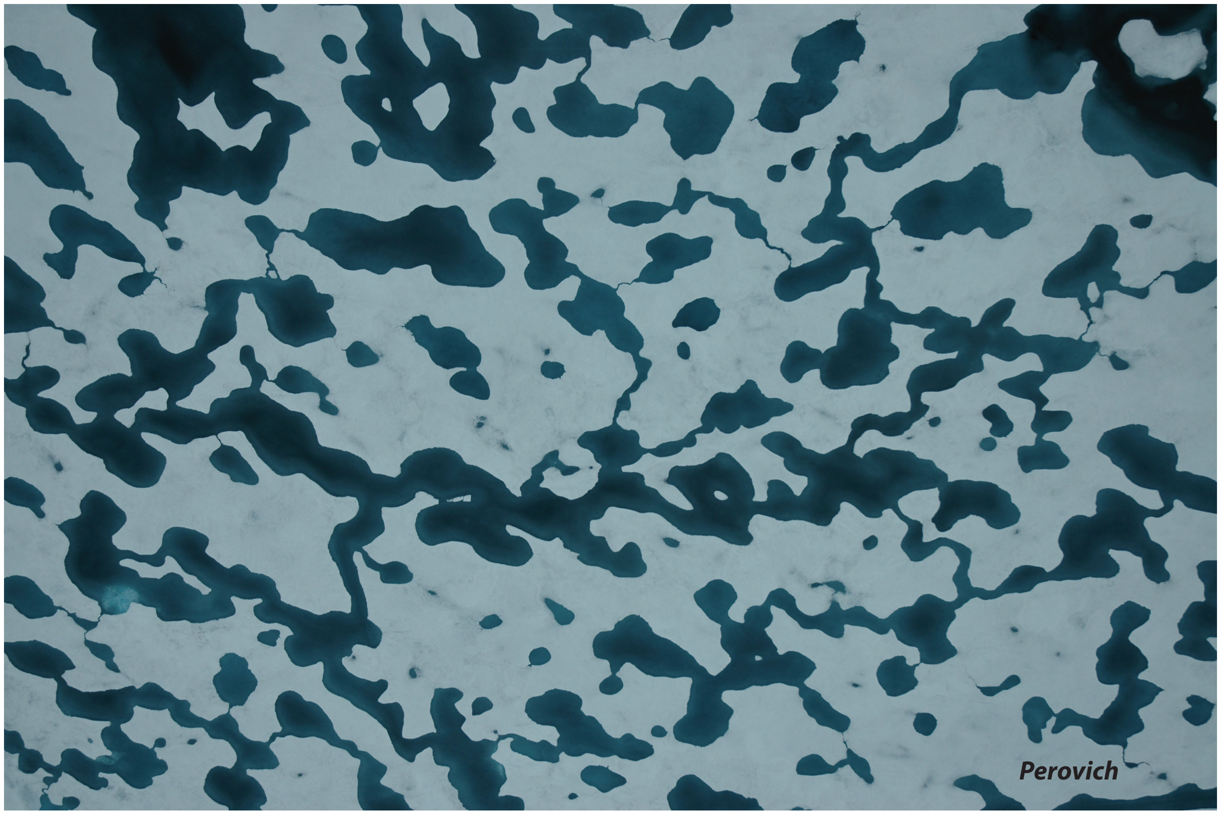
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

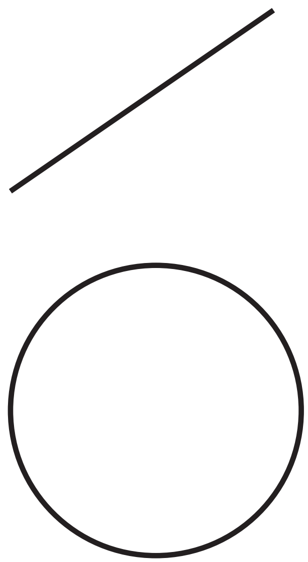
Flocco, Feltham, Hunke 2012

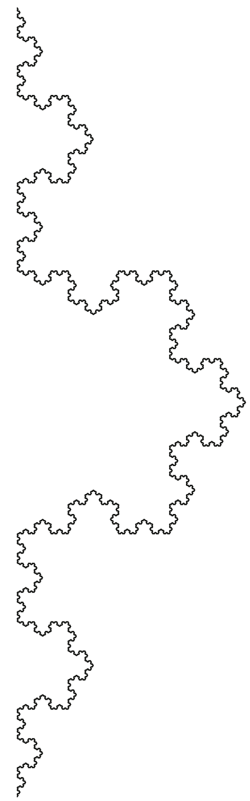


Are there universal features of the evolution similar to phase transitions in statistical physics?

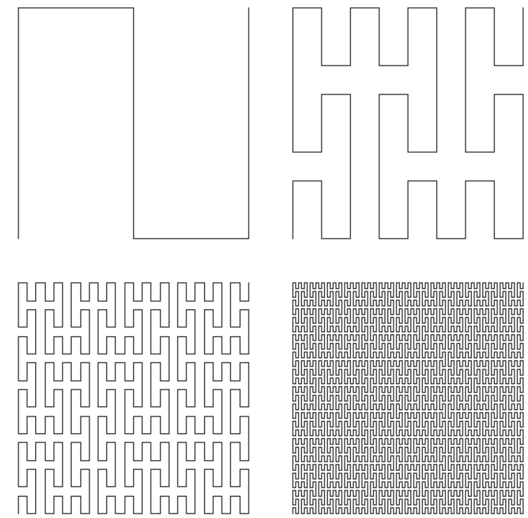
fractal curves in the plane

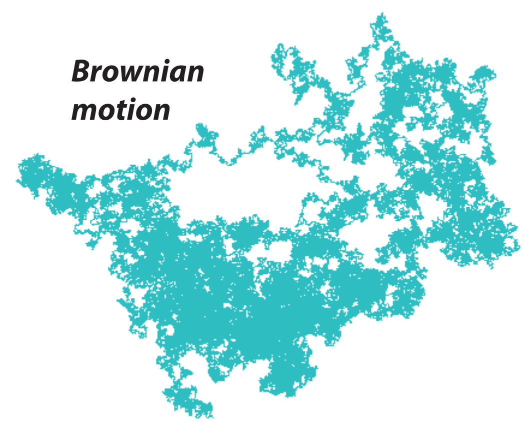
they wiggle so much that their dimension is >1





Peano curve





simple curves

D = 1

Koch snowflake

D = 1.26

space filling curves

D = 2

clouds exhibit fractal behavior from 1 to 1000 km

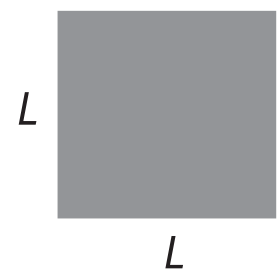
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

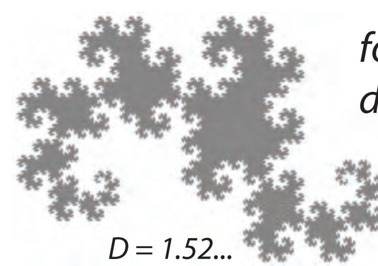
simple shapes



$A = L^2$

$P = 4L = 4\sqrt{A}$

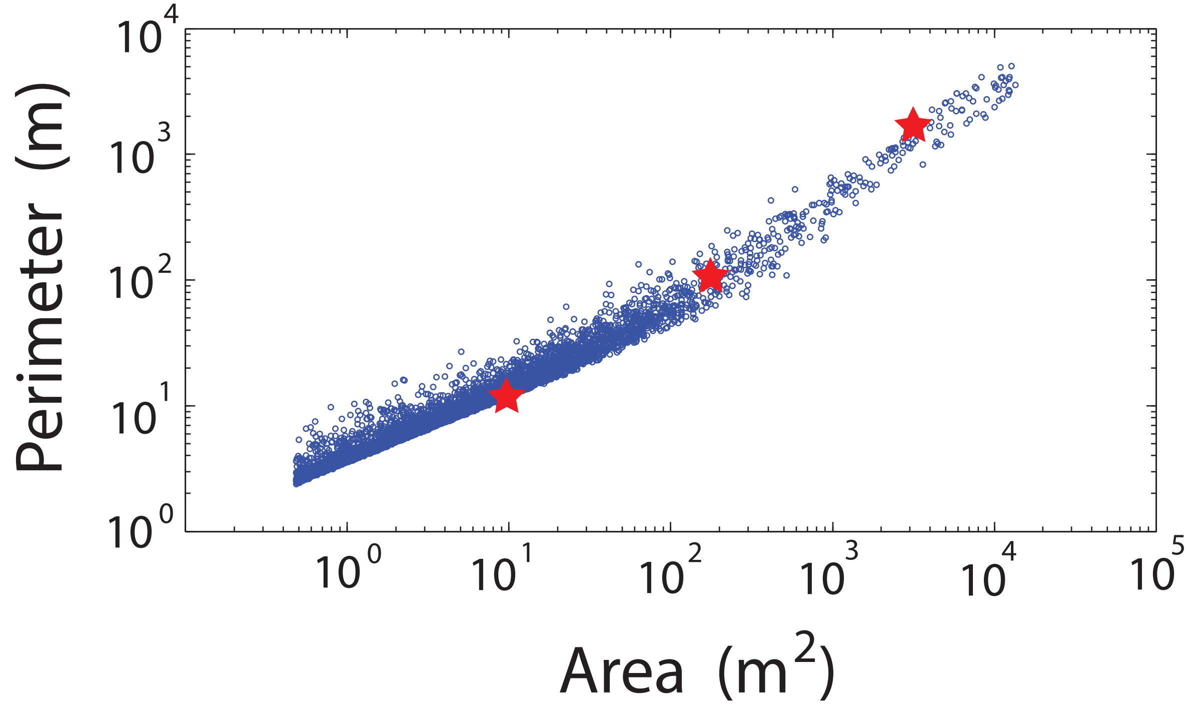
$$P \sim \sqrt{A}^{\,D}$$



for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds *The Cryosphere, 2012*

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

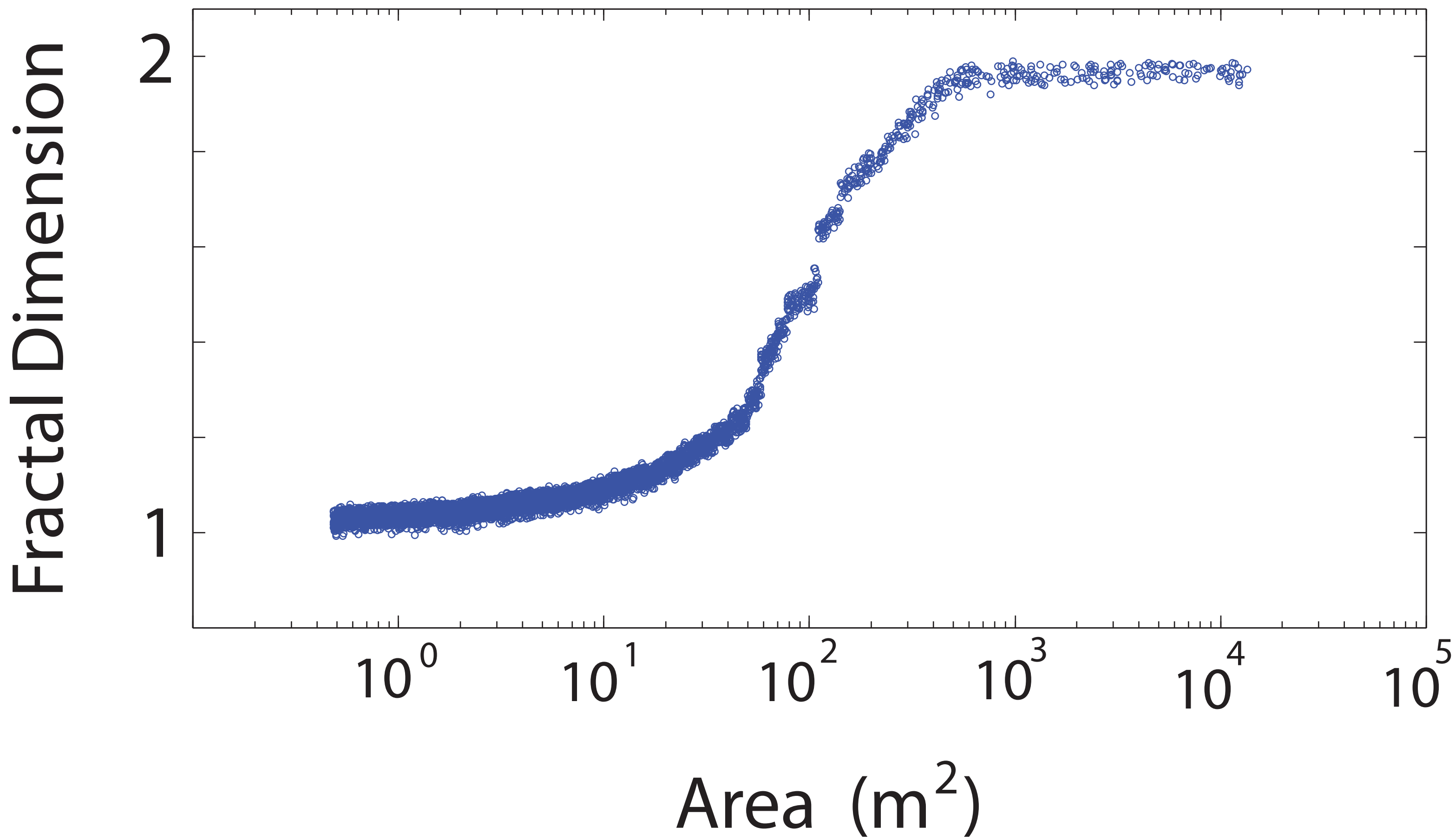


data from 5269 Arctic melt ponds

simple pond ***transitional pond*** ***complex pond***

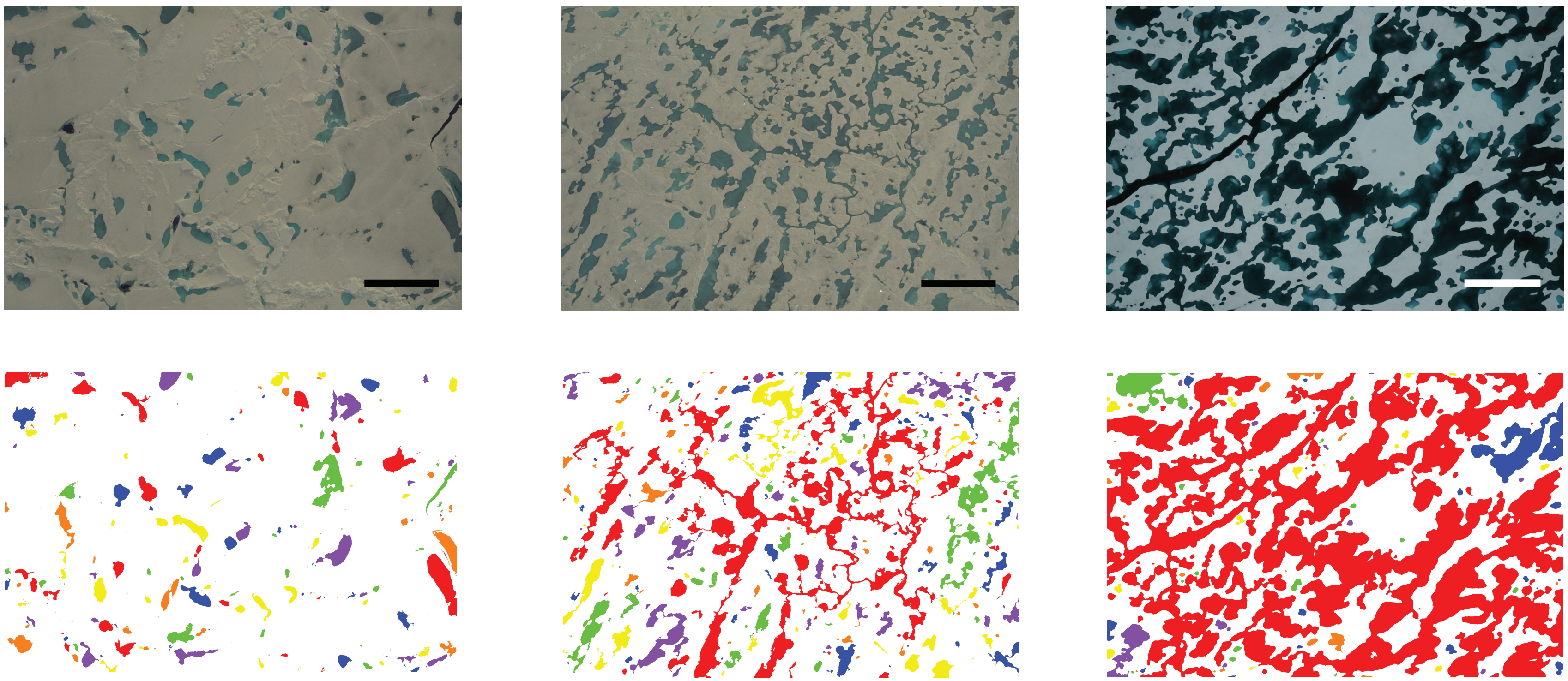
transition in the fractal dimension

complexity grows with length scale



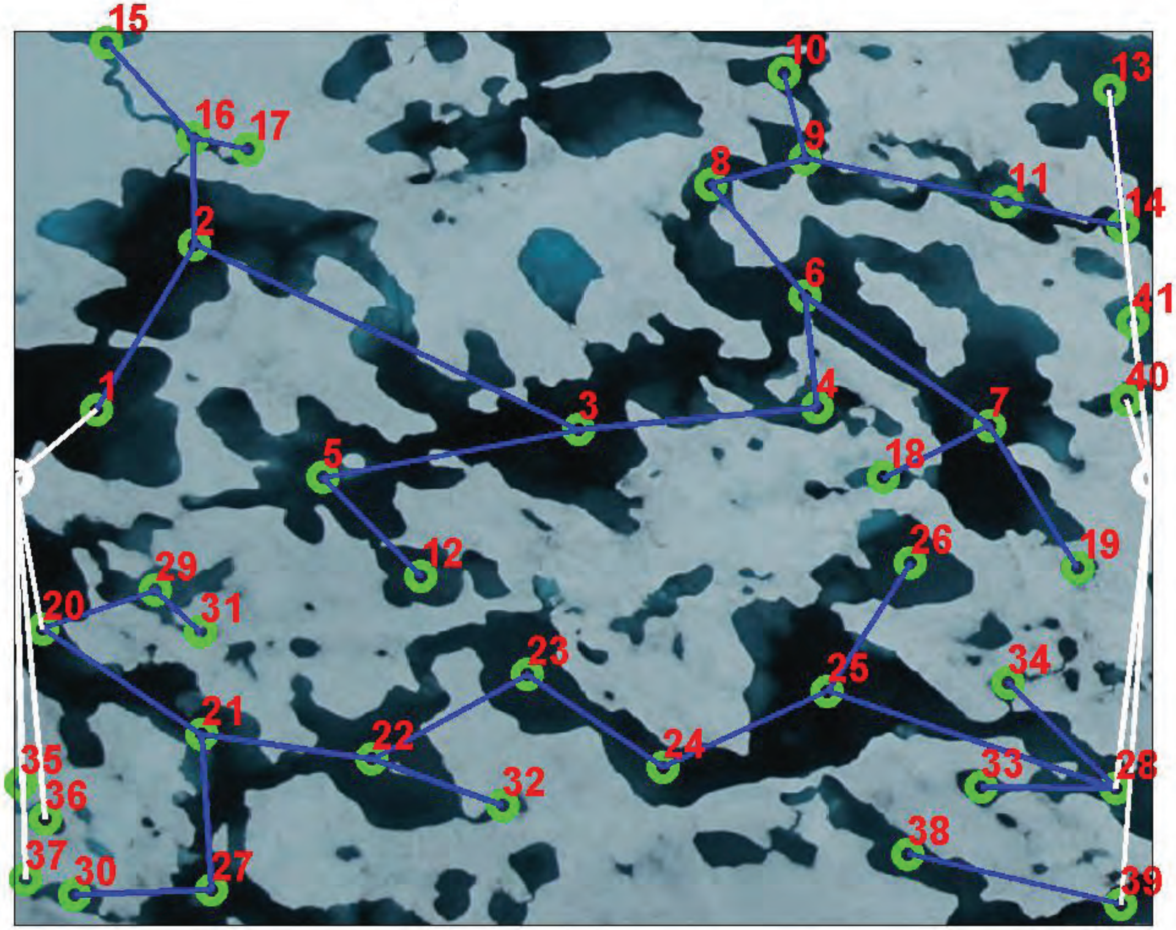
compute "derivative" of area - perimeter data

small simple ponds coalesce to form
large connected structures with complex boundaries



melt pond percolation

map melt pond configurations onto resistor networks
compute horizontal fluid permeability

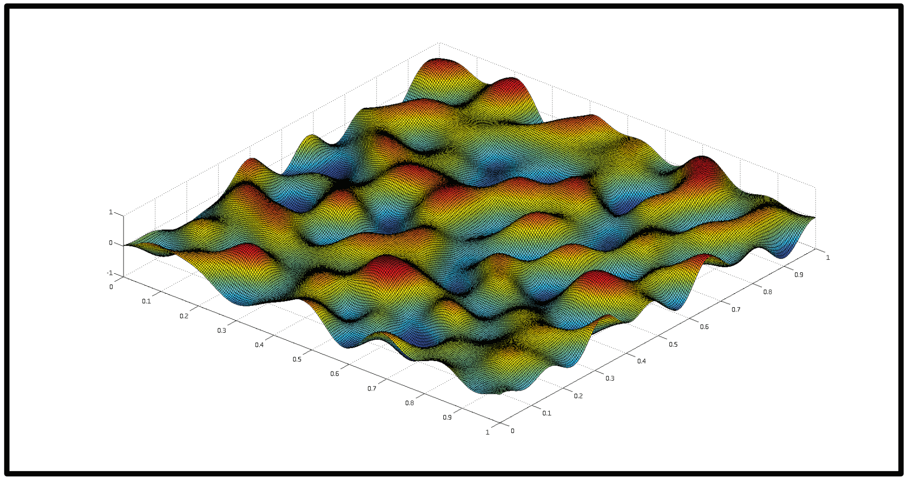


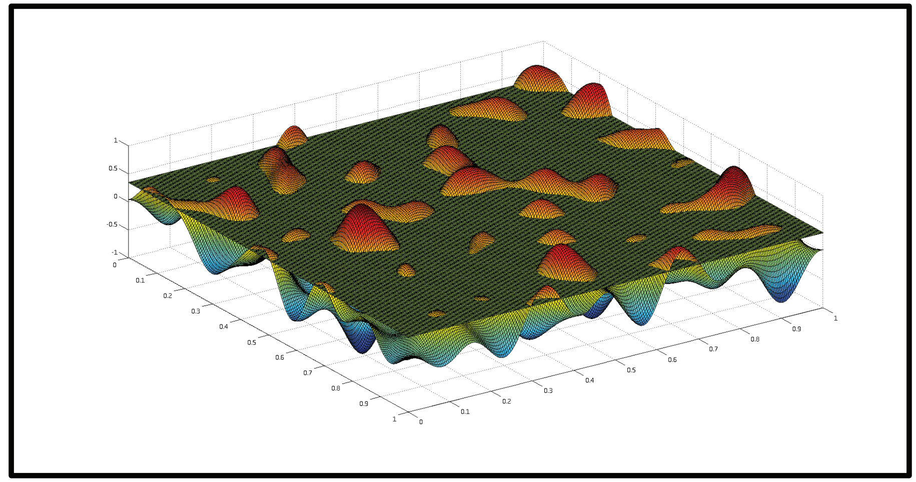
4 August 2005, Healy–Oden Trans Arctic Expedition (HOTRAX)

Barjatia, Song, Tasdizen, Golden 2014

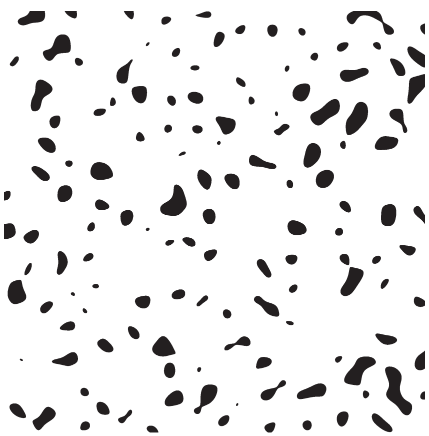
Continuum percolation model for melt pond evolution

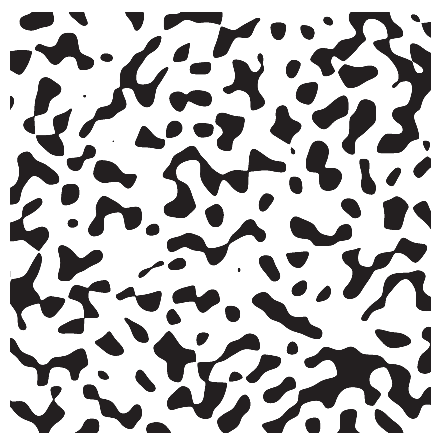
(Brady Bowen and Ken Golden, 2013)

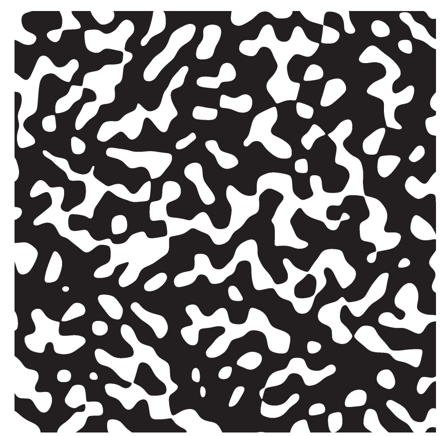




intersections of a plane with the surface define melt ponds





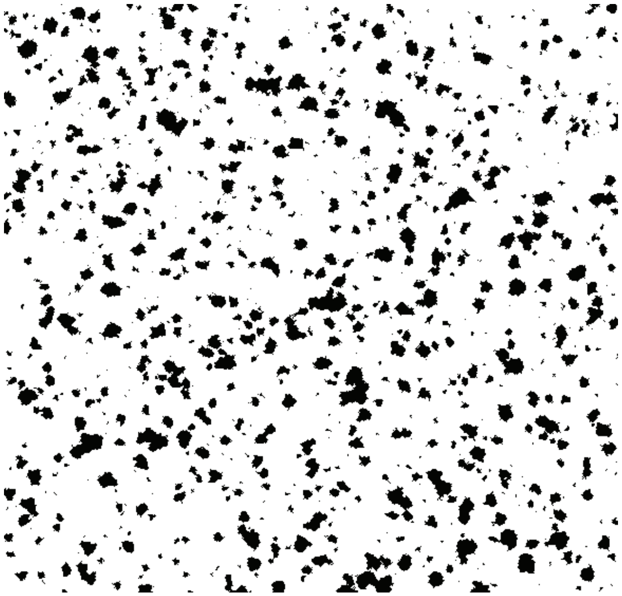


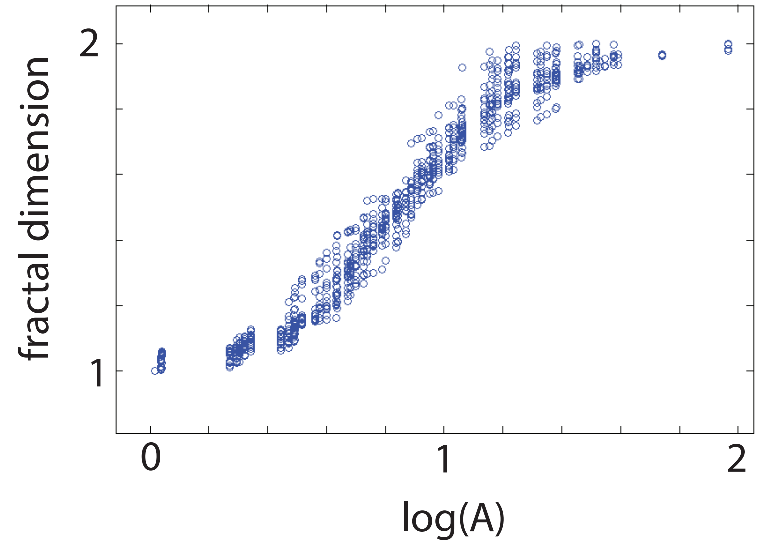
electronic transport in disordered media

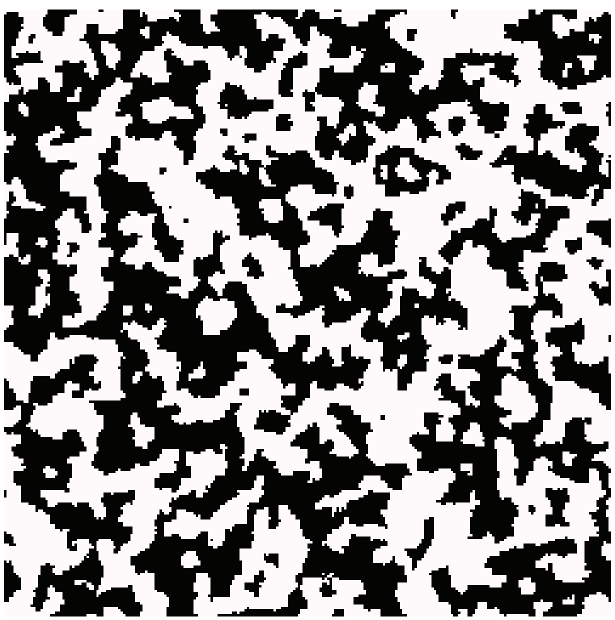
diffusion in turbulent plasmas

(Isichenko, Rev. Mod. Phys., 1992)

simple stochastic growth model of melt pond evolution







voter model

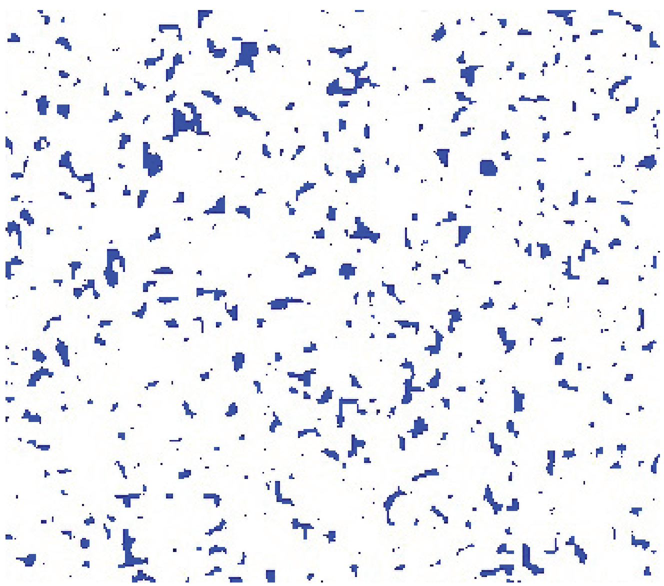
a square is more likely to melt if its neighbors have melted

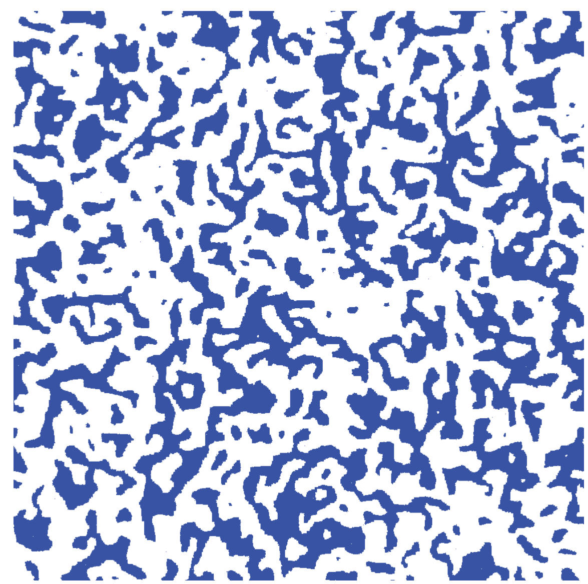
Rebecca Nickerson (West HS, Salt Lake City) and Ken Golden

Ising model for ferromagnets → Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow & -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

magnetization $M = \lim_{N\to\infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$ **pond coverage** $\frac{(M+1)}{2}$



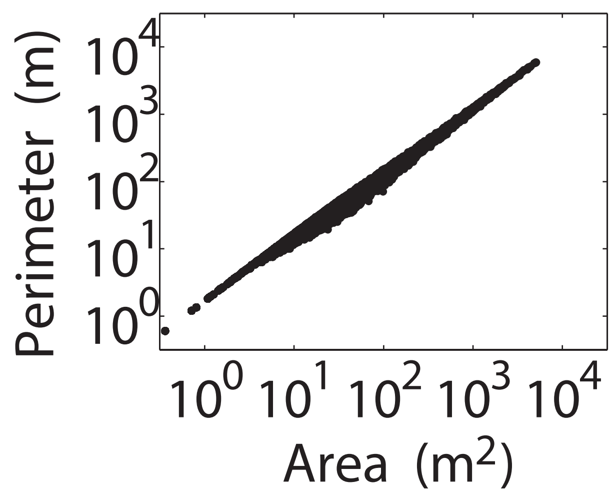


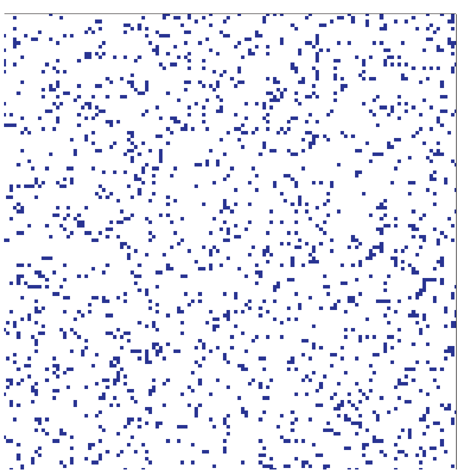
"melt ponds" are clusters of magnetic spins that align with the applied field

Ma, Sudakov, Golden 2014
(Thekkedath, Alali, Strong, Golden)

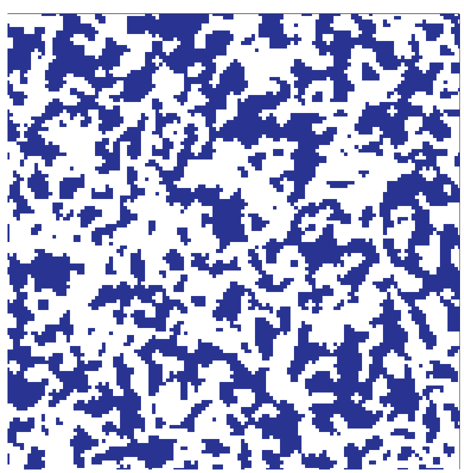
Melt Pond Ising Model

- Minimize an Ising Hamiltonian
  random magnetic field represents the initial ice topography
  interaction term represents horizontal heat transfer
- Ice-albedo feedback incorporated by taking coupling constant in interaction term to be proportional to the pond coverage





predicted length scale of fractal transition agrees well with data



Ising model for ferromagnets ⟶ Ising model for melt ponds

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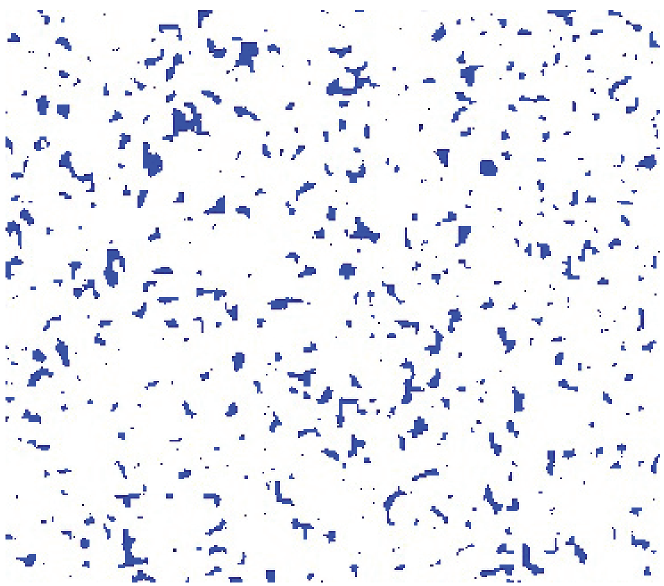

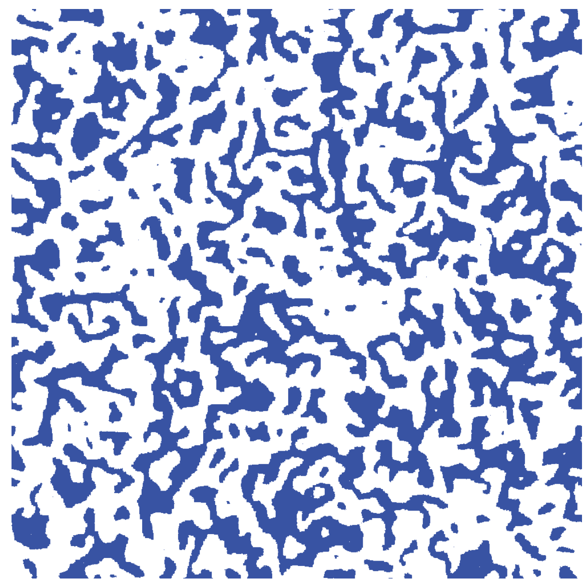

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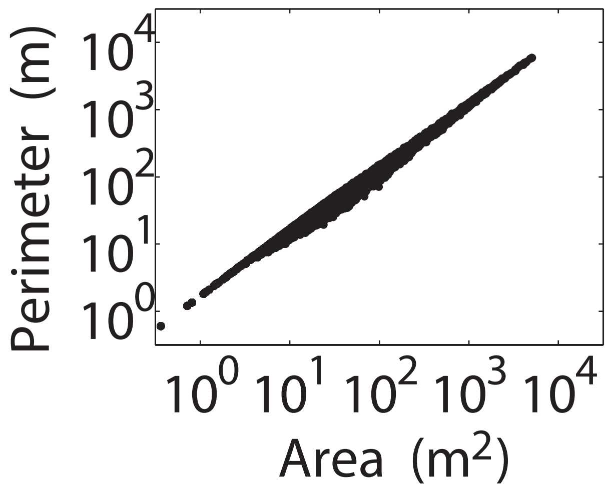

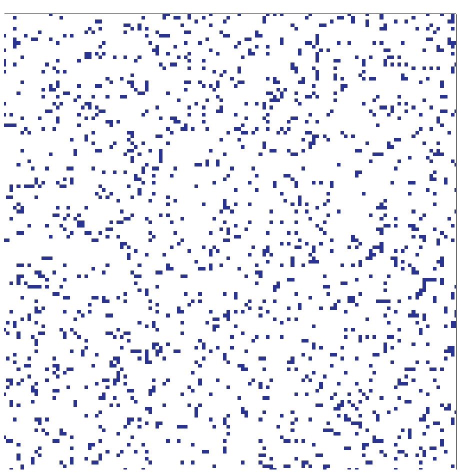

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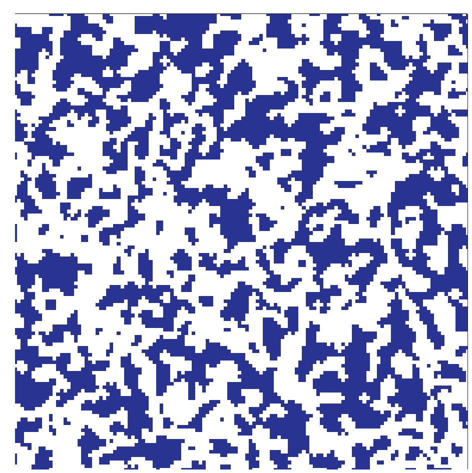

Conclusions

1. Summer Arctic sea ice is melting rapidly.

2. Fluid flow through sea ice mediates many processes of importance to understanding climate change and the response of polar ecosystems.

3. Mathematics of composite materials, statistical physics and dynamical systems help us understand sea ice, and suggest rigorous frameworks for representing sea ice in climate models .

4. Random matrices arise naturally and frequently in sea ice studies.

5. This research will help to improve projections of climate change and the fate of Earth's sea ice packs and their ecosystems.

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999

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