

On Thinning Ice: Modeling Sea Ice in a Warming Climate

Kenneth M. Golden
Department of Mathematics
University of Utah

UPA Rotary Club Environmental Impact Forum, 6 May 2021

Frey

ANTARCTICA

southern cryosphere



**Weddell
Sea**

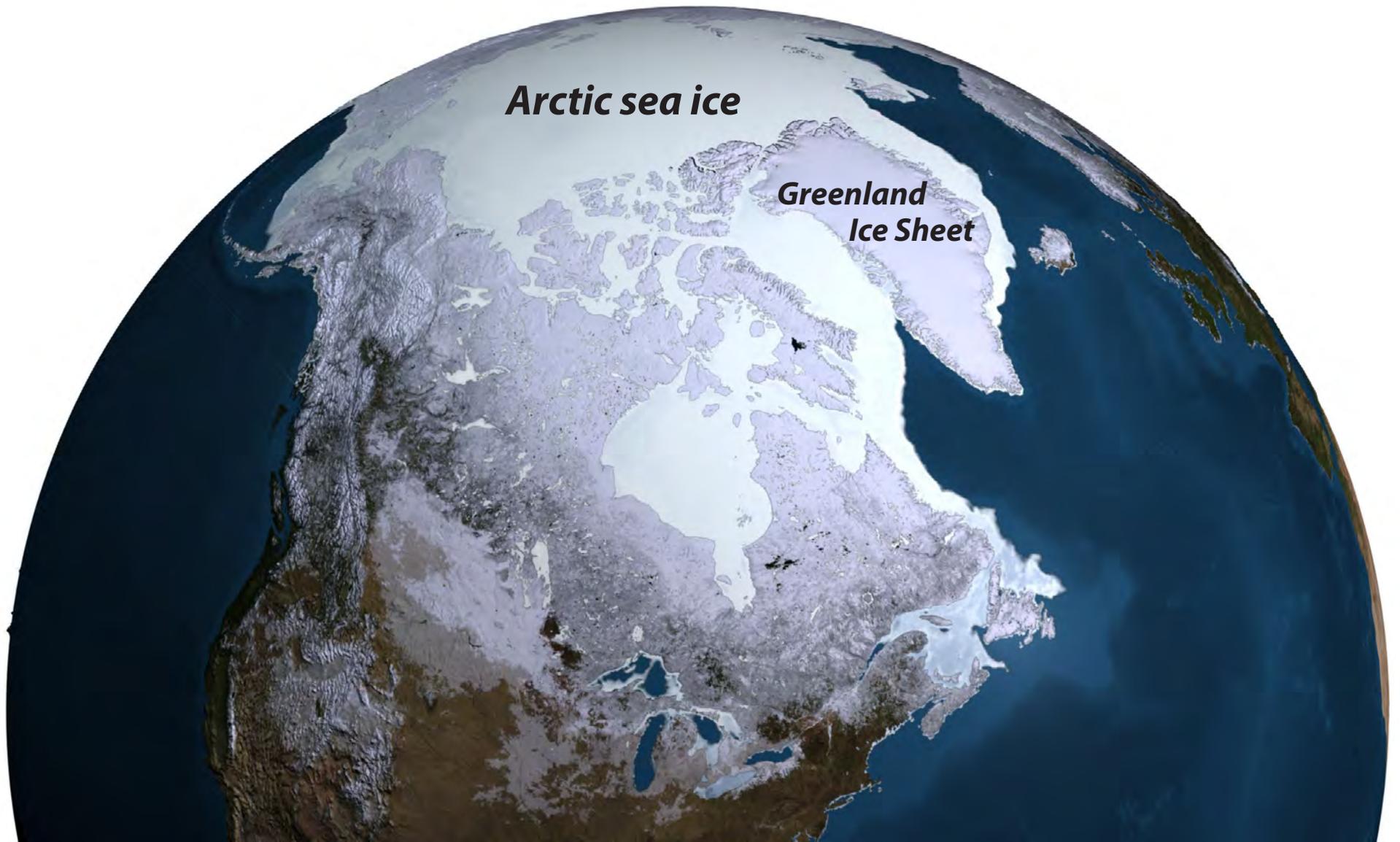
***East Antarctic
Ice Sheet***

***West Antarctic
Ice Sheet***

Ross Sea

sea ice

northern cryosphere



Arctic sea ice

***Greenland
Ice Sheet***

SEA ICE covers ~12% of Earth's ocean surface

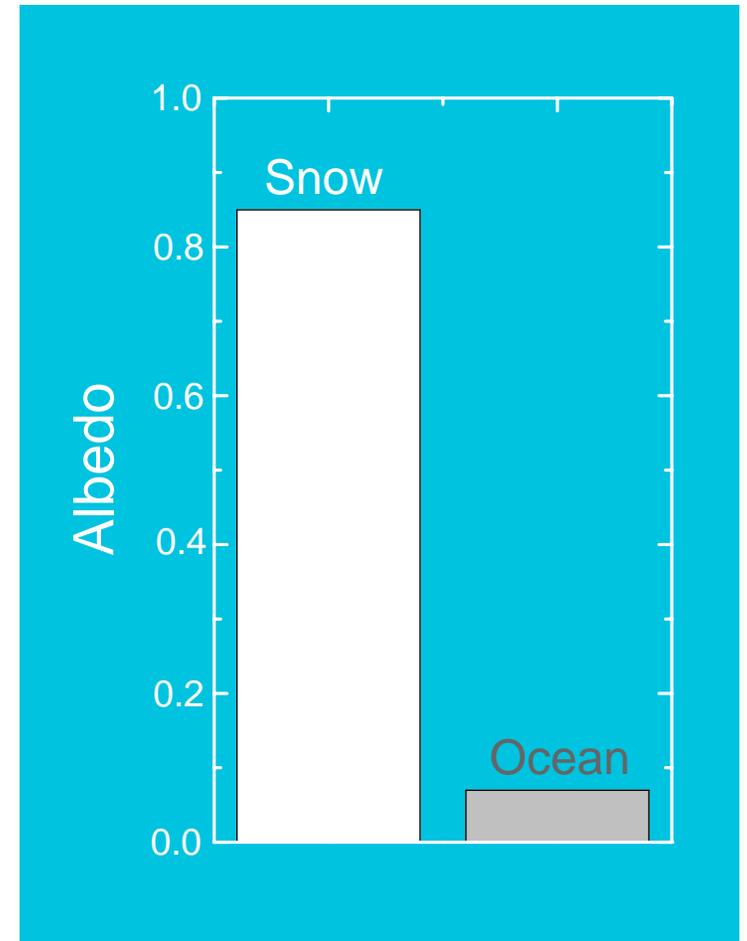
- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of **climate change**



polar ice caps critical to global climate in reflecting incoming solar radiation



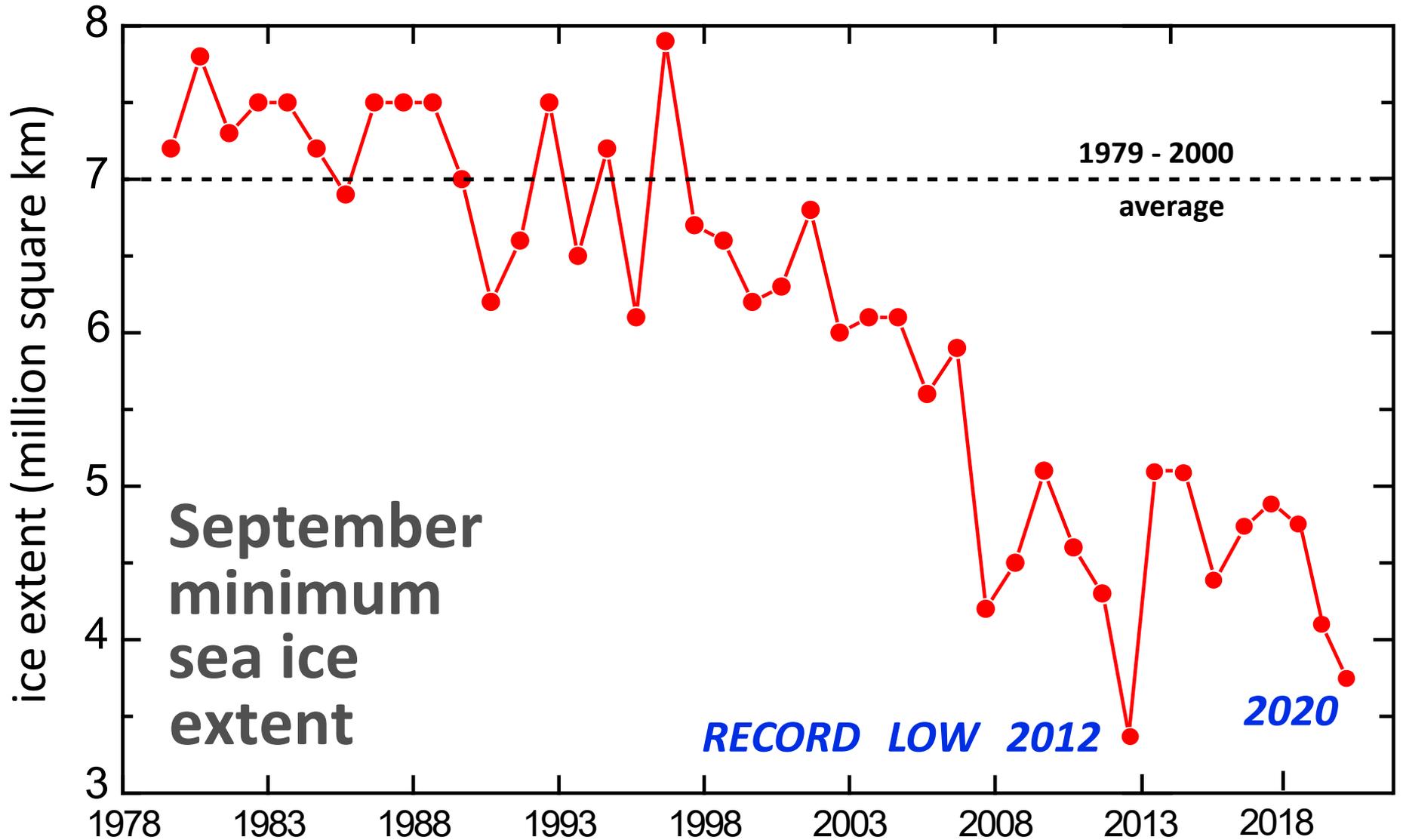
white snow and ice
reflect



dark water and land
absorb

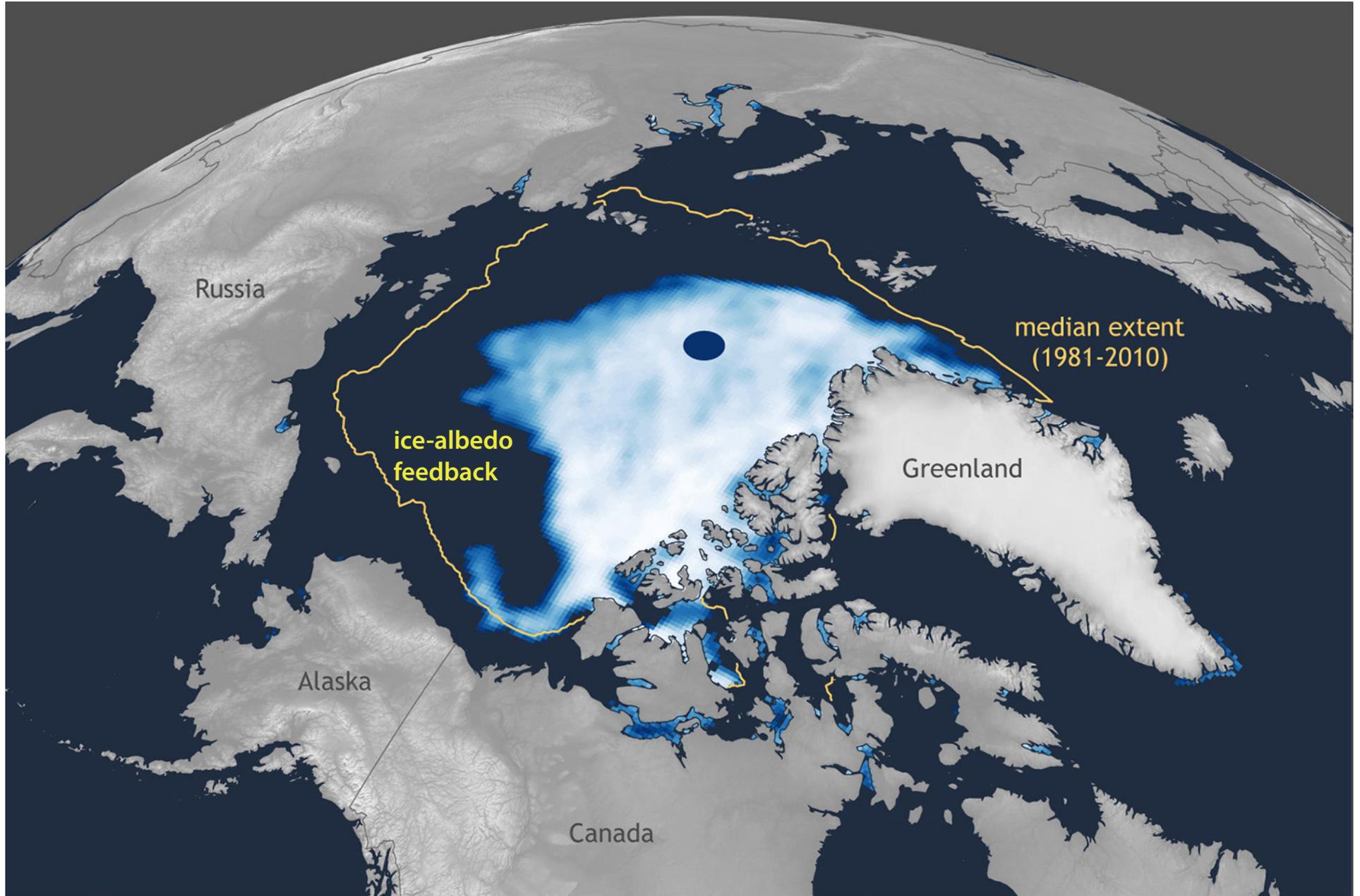
$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

the summer Arctic sea ice pack is melting



Arctic sea ice extent

September 15, 2020



Sea ice concentration (percent)



NSIDC



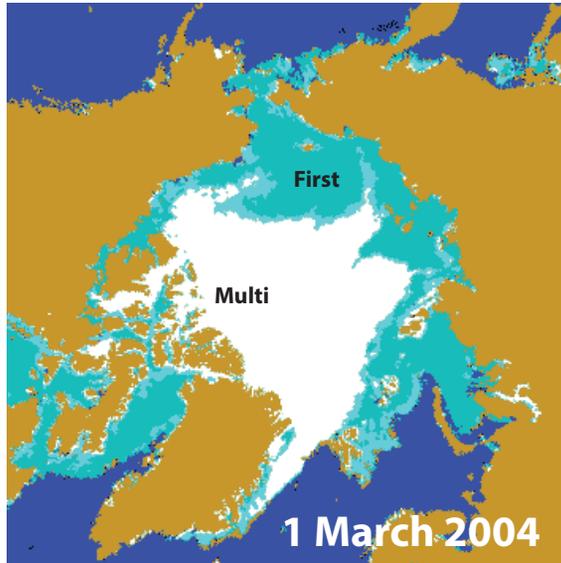
*recent losses
in comparison to
the United States*



Perovich

thicker multiyear ice being replaced by thinner first year ice

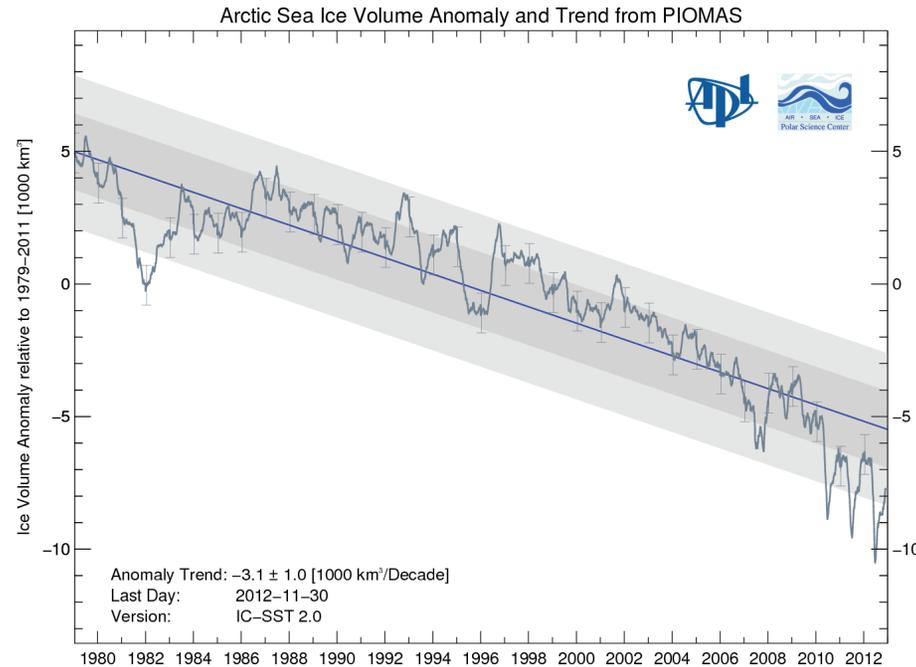
Winter 2004



Winter 2008

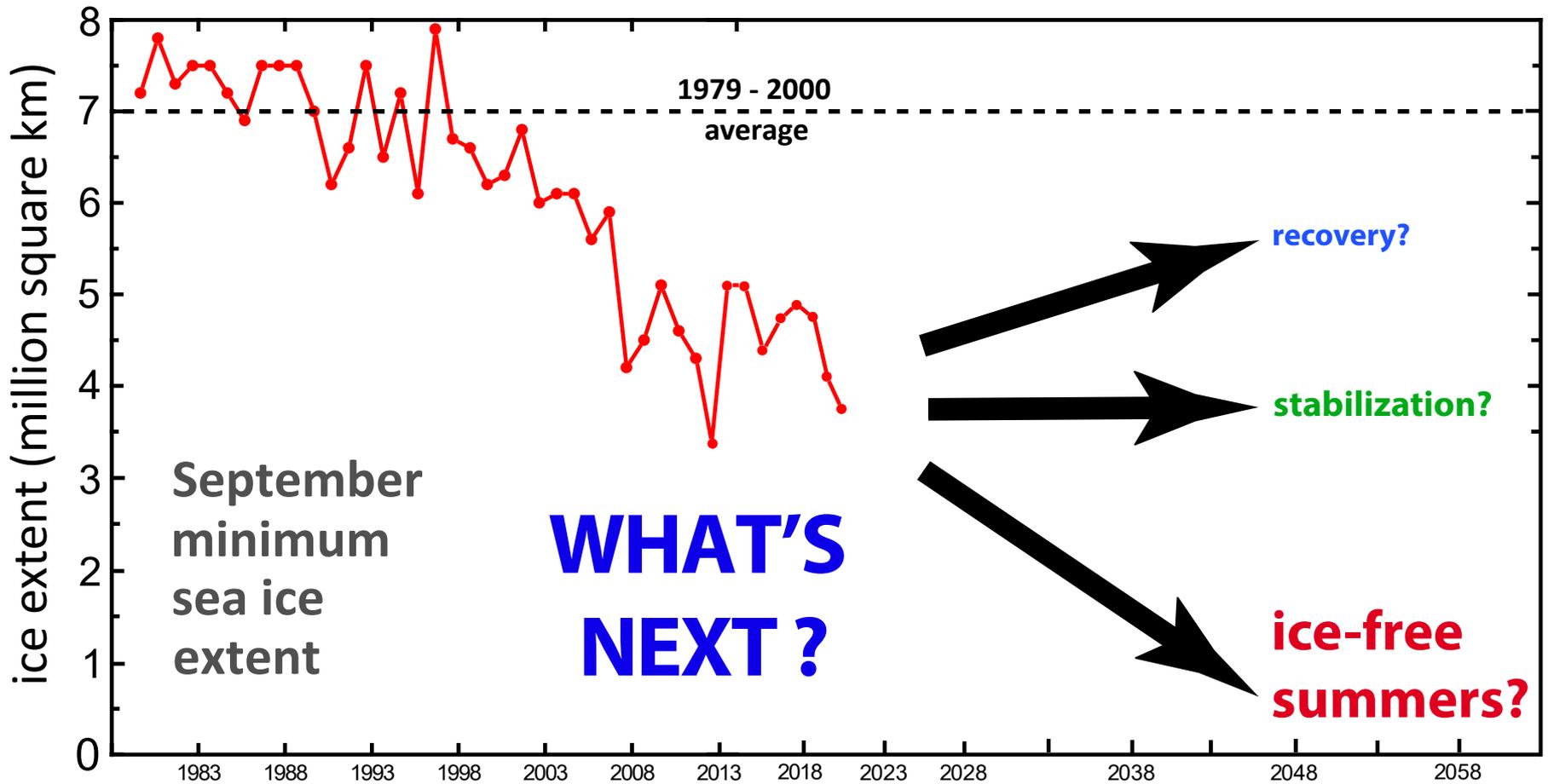


Son Nghiem



... and sea ice volume is declining

Predicting what may come next requires lots of math modeling.



Basics of climate modeling and planetary warming



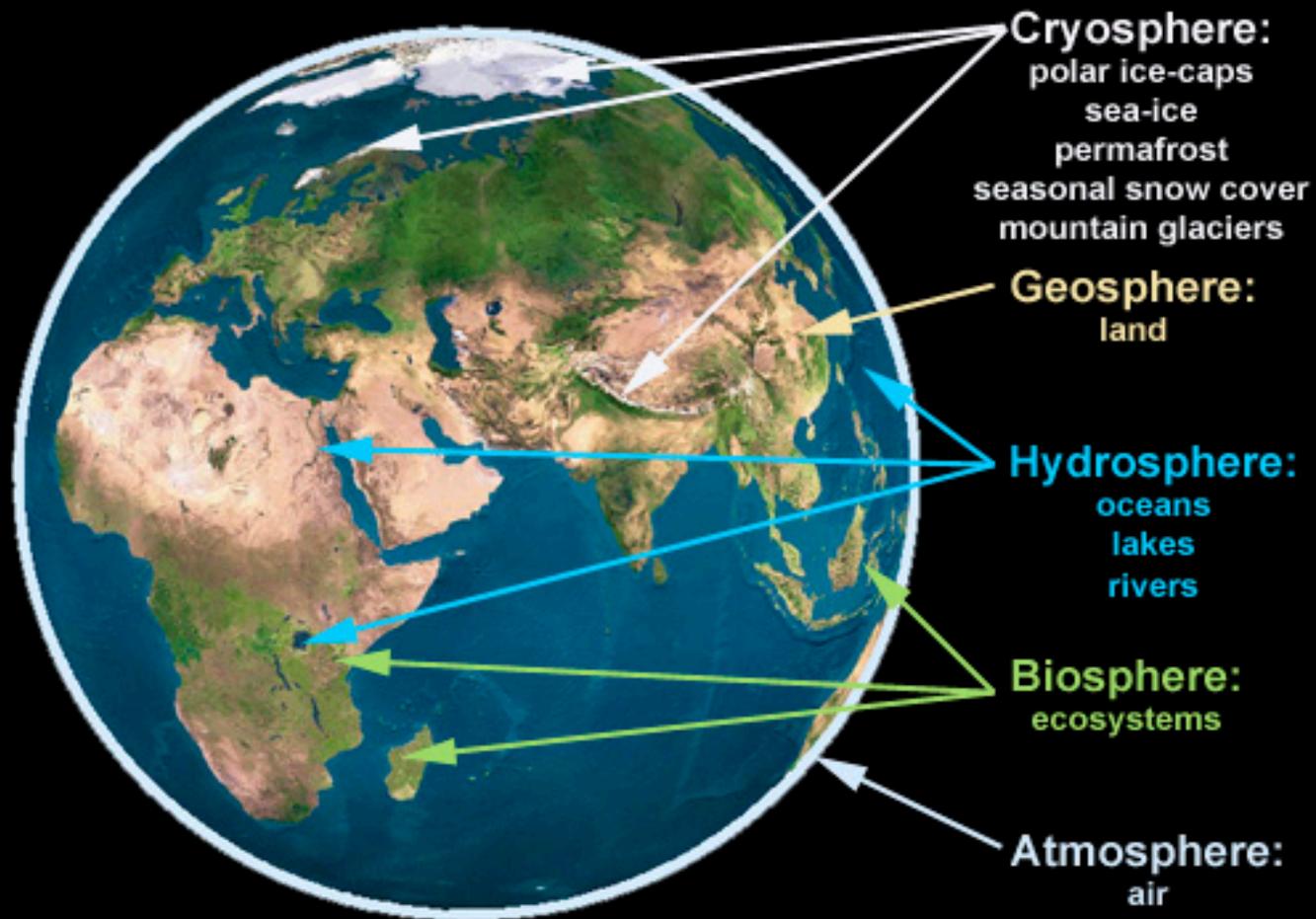
Earthrise

December 24, 1968

William Anders

NASA

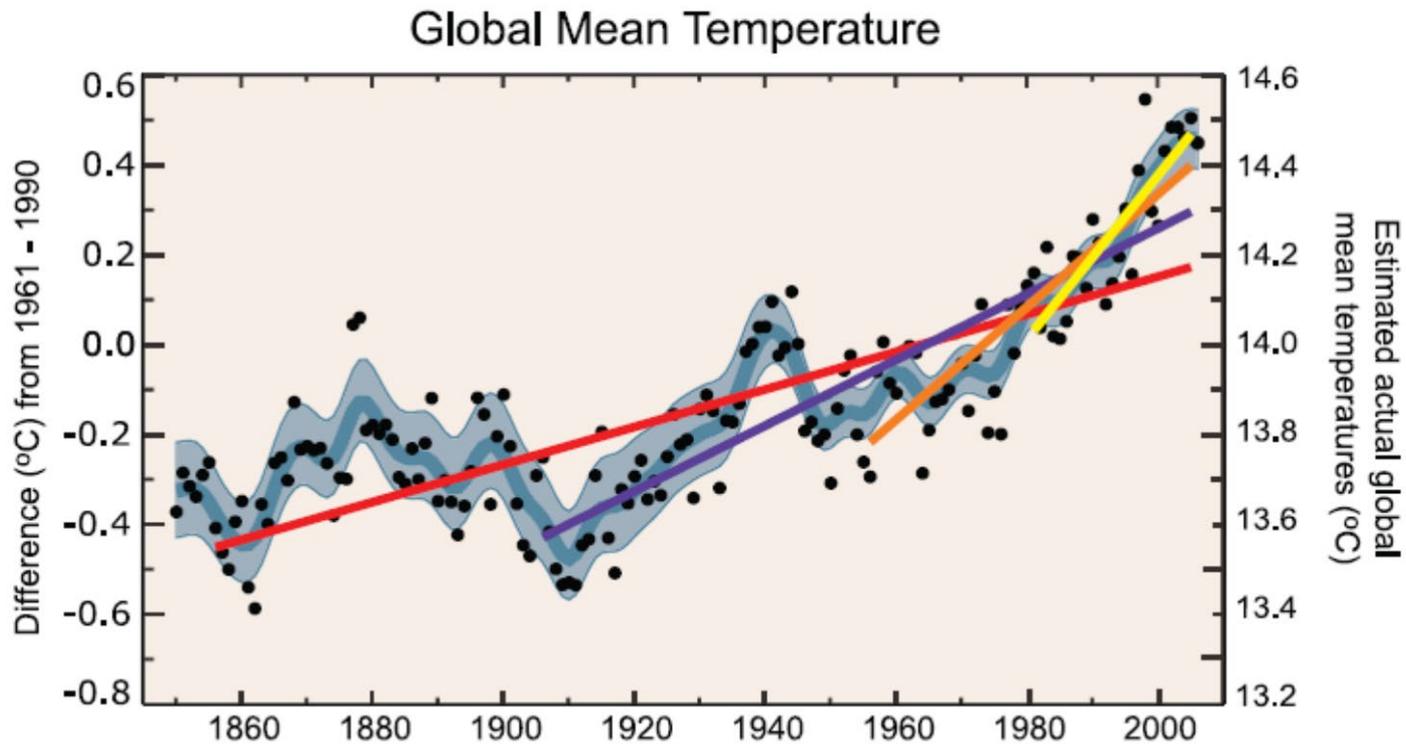
The components of Earth's Climate System





Evidence of a Changing Climate

Intergovernmental Panel on Climate Change (IPCC):
Warming is “unequivocal”



- Dots: yearly average
- Curve: decadal average
- Blue: uncertainty interval



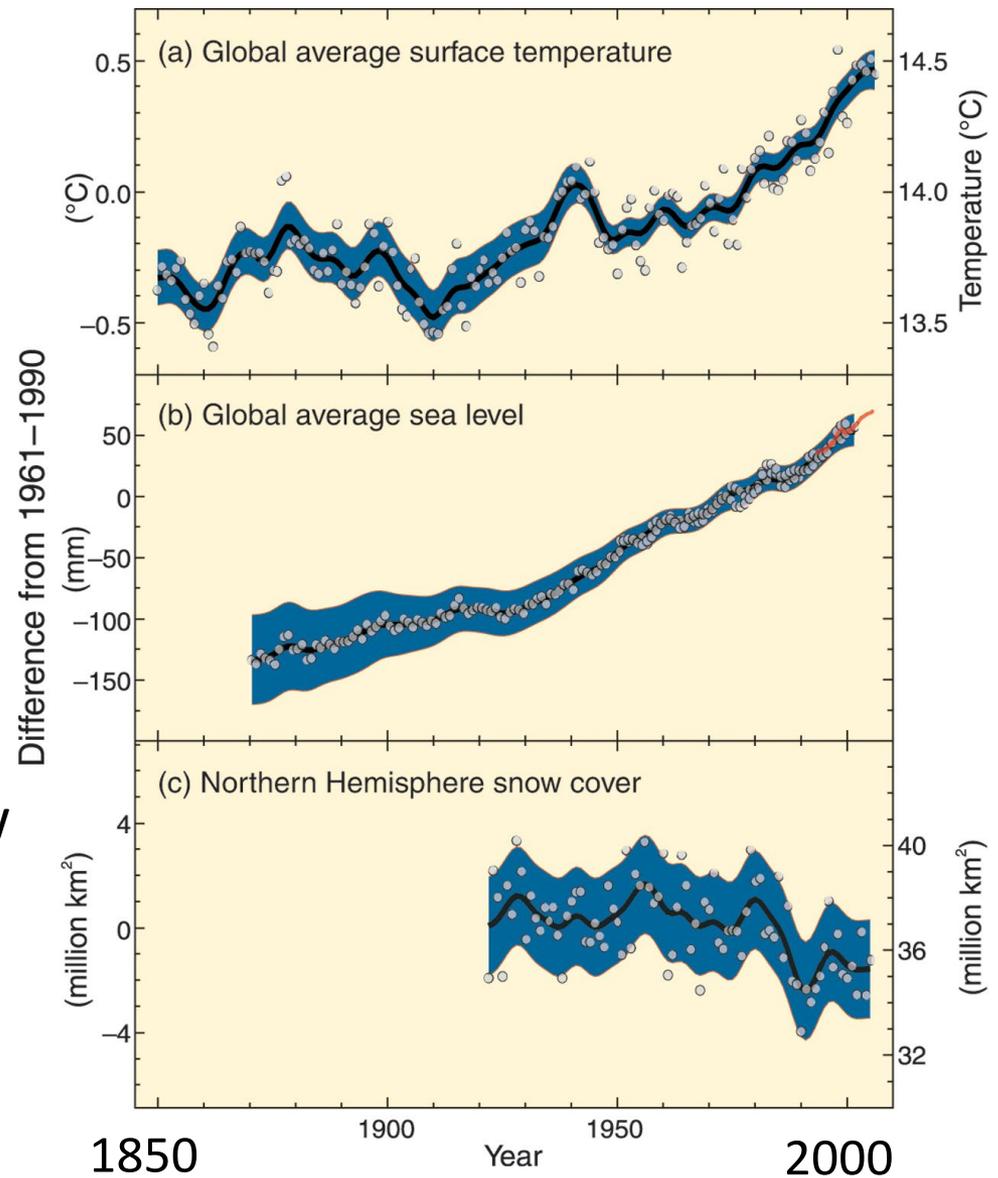
IPCC: Warming is “unequivocal”

Global mean surface temp

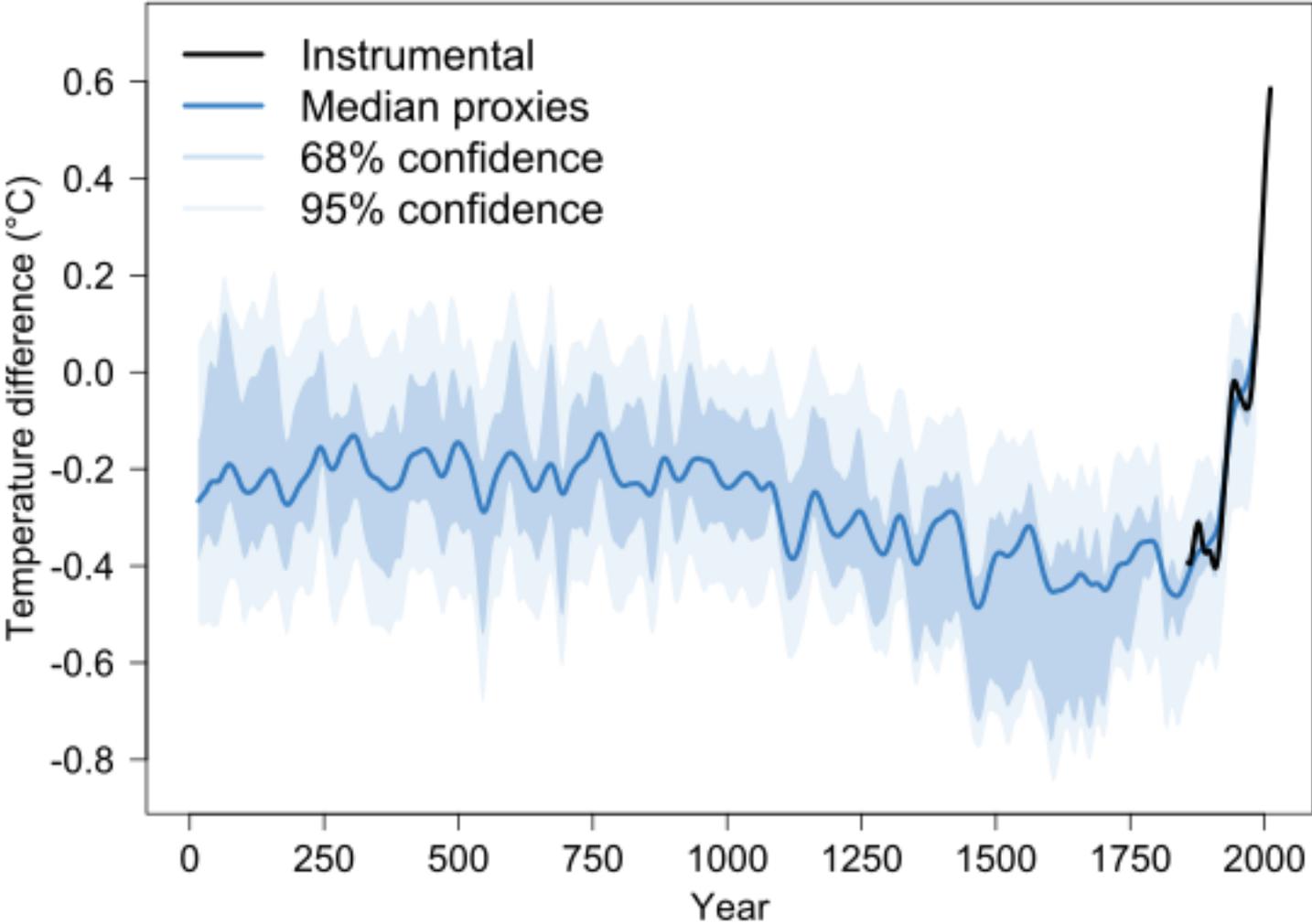
Global mean sea level

Northern Hemisphere snow cover (March-April)

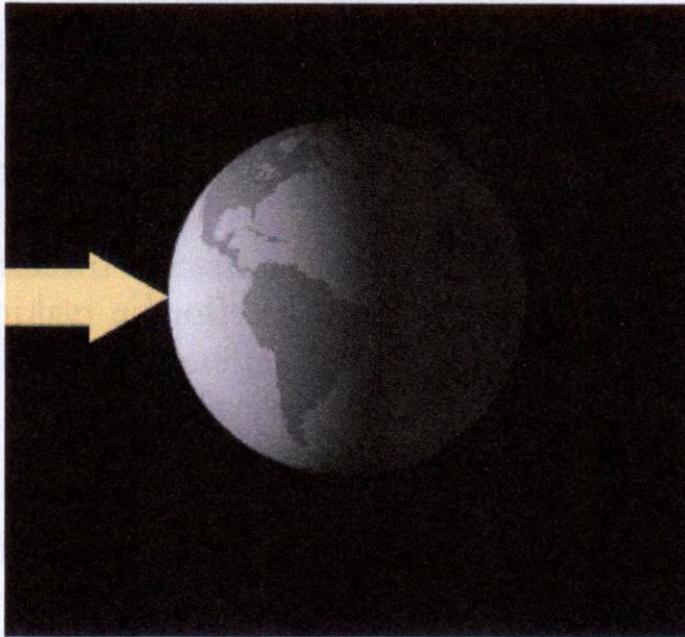
Dots: yearly average
Curve: decadal average
Blue: uncertainty interval



Mean Global Temperature over the past 2000 years

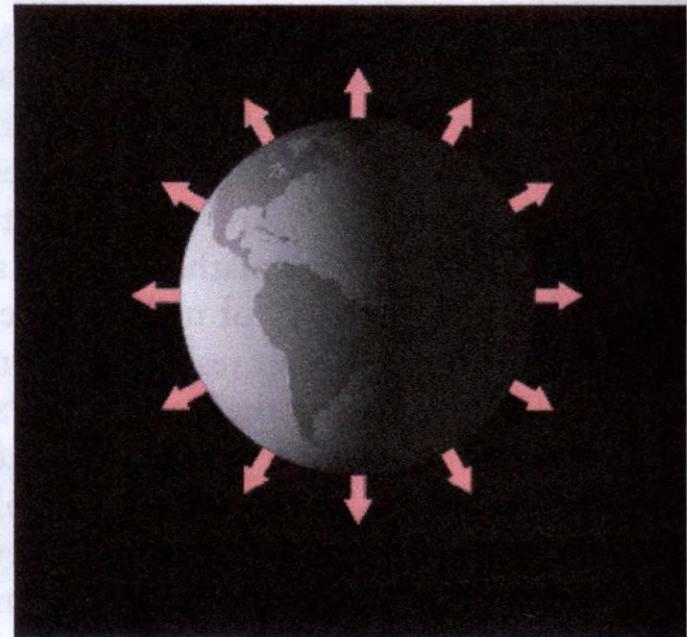


Earth's energy balance



E_{in}

incoming shortwave radiation
sunlight



E_{out}

outgoing longwave radiation
heat

if $E_{out} > E_{in}$ planet **cools**

if $E_{out} < E_{in}$ planet **warms**

albedo of Earth's surface

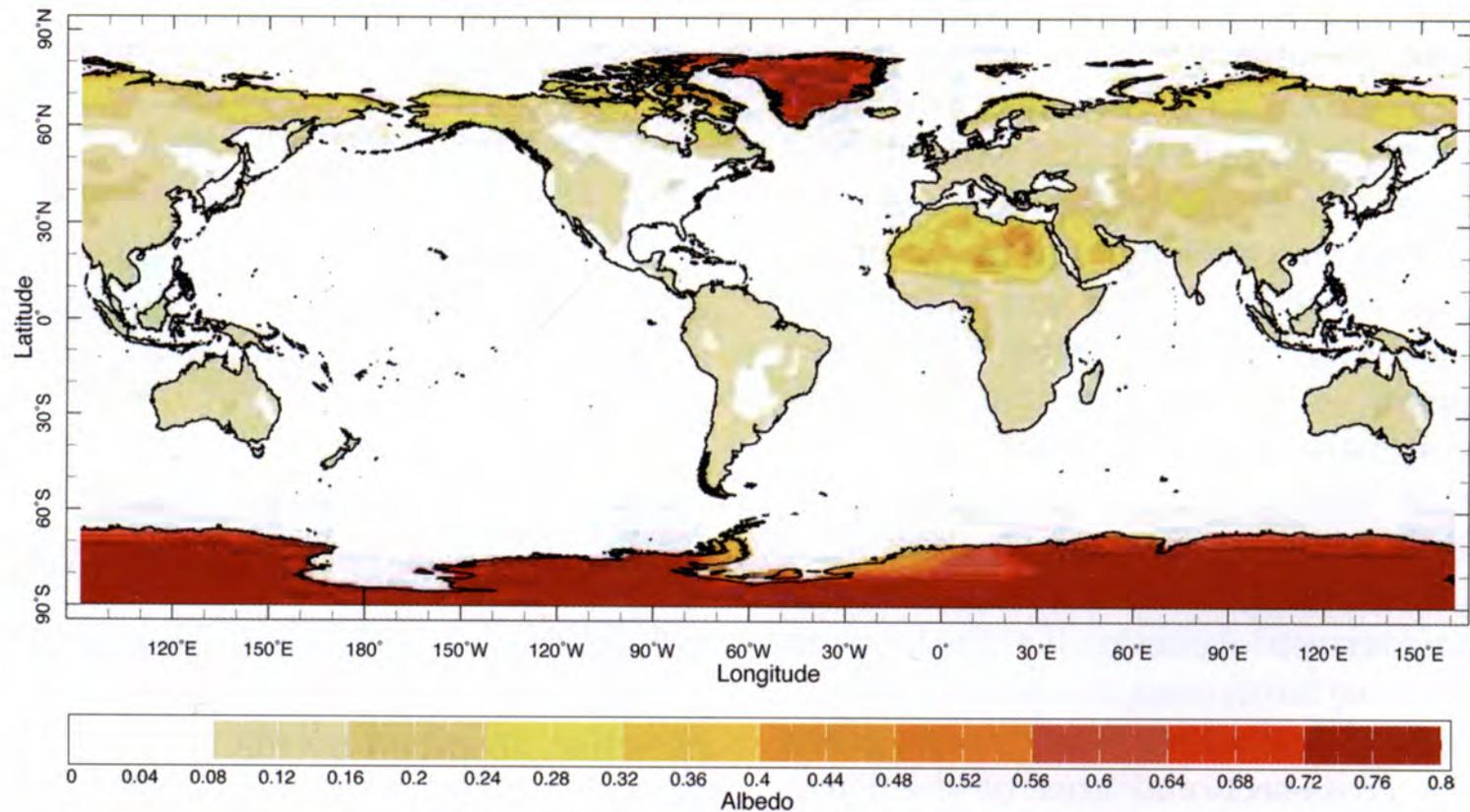


FIGURE 2.5. The albedo of the Earth's surface. Over the ocean the albedo is small (2–10%). It is larger over the land (typically 35–45% over desert regions) and is particularly high over snow and ice (~80%) (see Table 2.2).

average albedo ~ 0.3

Energy Balance Model

$$C \frac{dT}{dt} = E_{in} - E_{out}$$



***rate of change with time of
mean global temperature***

*C = heat capacity
= energy needed to
raise temperature
by one Kelvin*

$E_{in} = ?$

$E_{out} = ?$

Basics of global warming date back to the 1800's



1. Jean Baptiste Joseph Fourier (1768-1830), French mathematician and natural philosopher, did groundbreaking work in mathematics and the theory of heat. He was the first to propose that the Earth's atmosphere acts to raise the planet's temperature.

“As a dam built across a river causes a local deepening of the stream, so our atmosphere, thrown as a barrier across the terrestrial rays, produces a local heightening of the temperature at the Earth’s surface.”

**HEAT EQUATION
FOURIER SERIES**

atmosphere traps **heat** escaping
from Earth - acts like a **greenhouse**

Fourier, 1827

2. 1861 John Tyndall discovers in his laboratory that certain gases - water vapor and CO₂ are opaque to heat rays. He understood that such **gases high in the air help keep our planet warm by interfering with escaping radiation.**

3. 1896 Svante Arrhenius (**1903 Nobel Prize in Chemistry**) proposed:
relationship between atmospheric CO₂ concentrations and temperature;
global warming may result from fossil fuel combustion (burning coal).

He and Thomas Chamberlin calculated that human activities could warm the earth by adding carbon dioxide to the atmosphere.

In a nutshell:

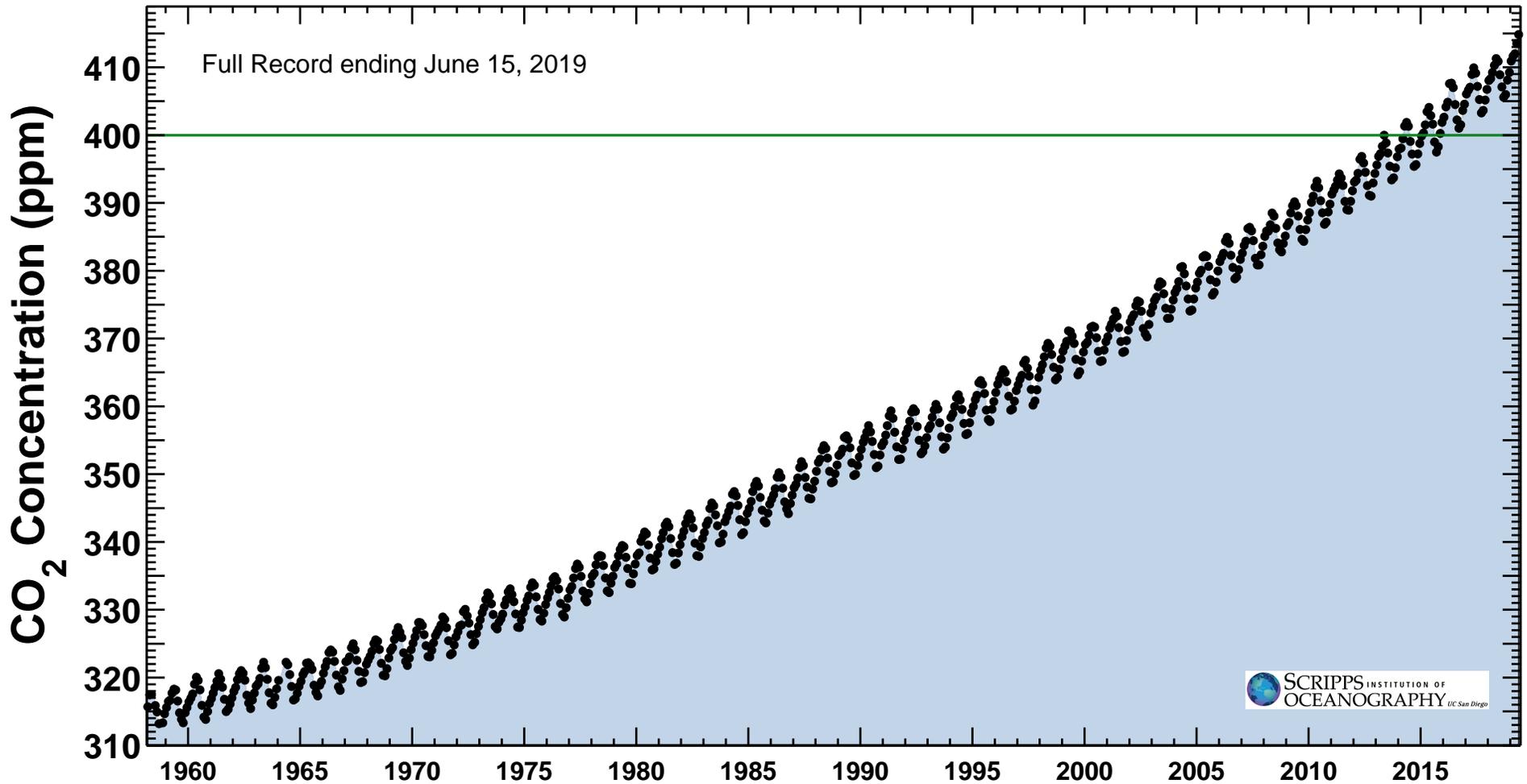
Scientific basis of CO₂ greenhouse effect was given by Tyndall (1861); while the first extensive calculations of its magnitude were made by Arrhenius (1896).

4.

Latest CO₂ reading
June 13, 2019

414.71 ppm

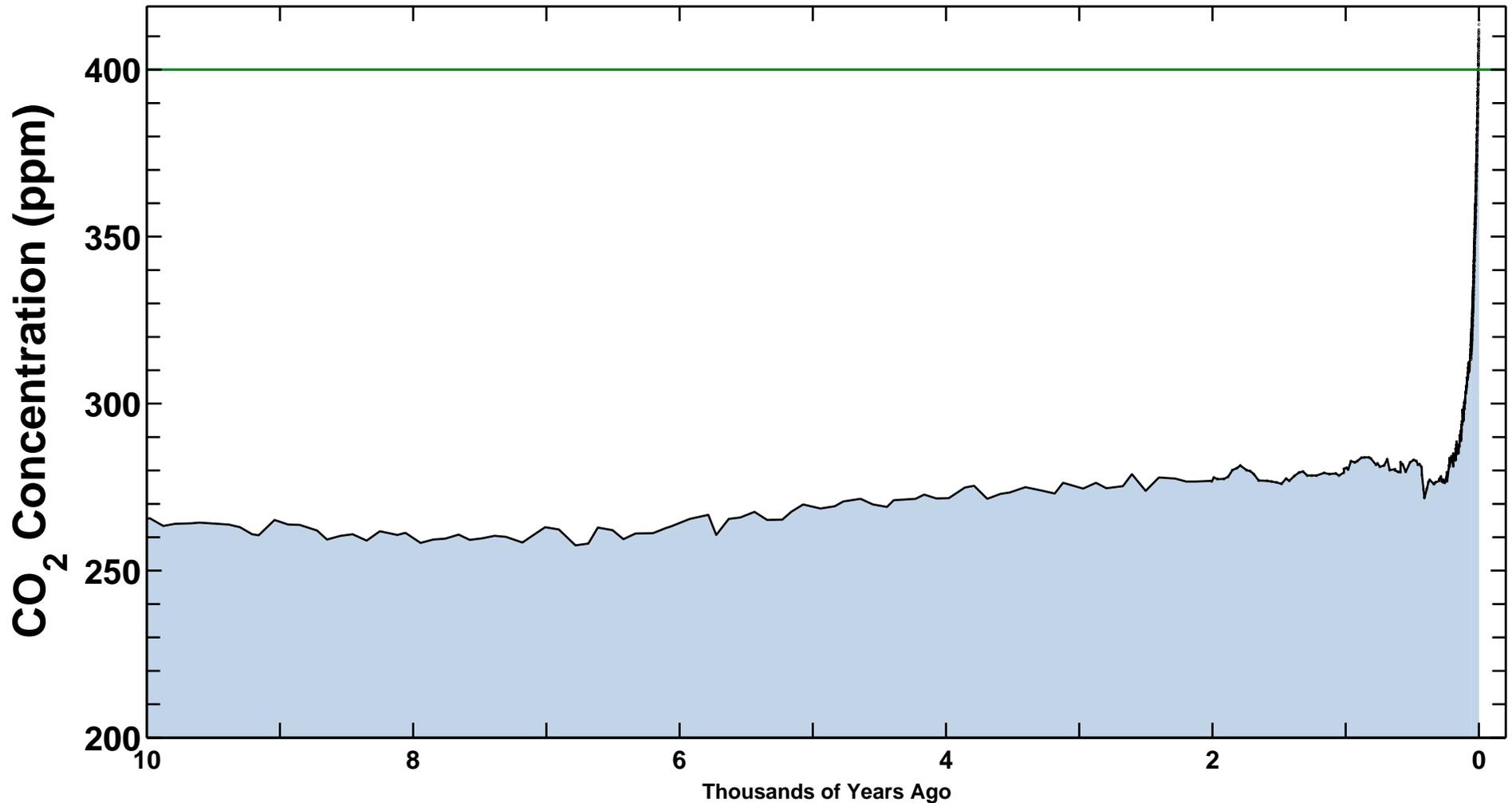
Carbon dioxide concentration at Mauna Loa Observatory



Latest CO₂ reading
June 13, 2019

414.71 ppm

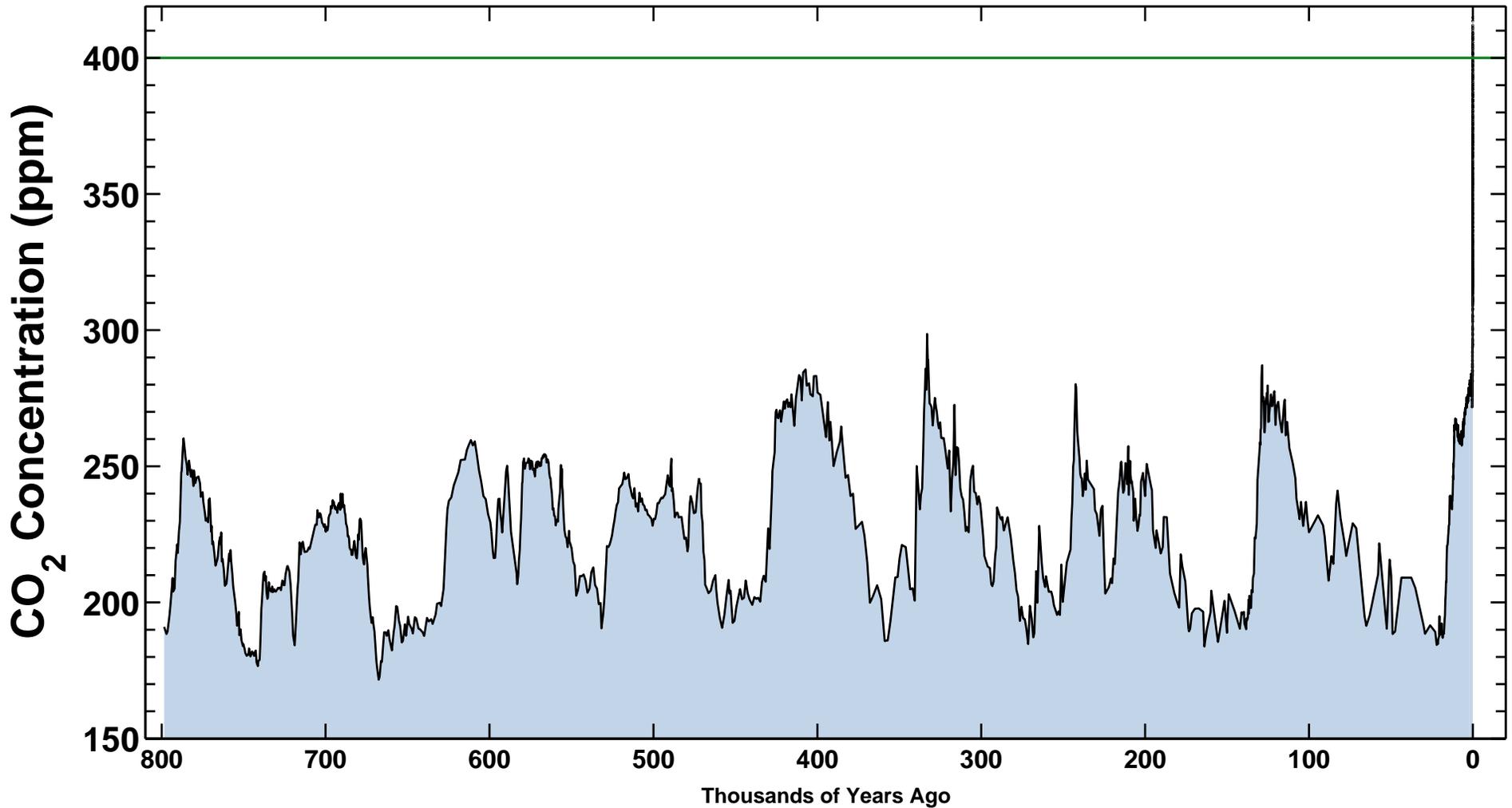
Ice-core data before 1958. Mauna Loa data after 1958.



Latest CO₂ reading
June 13, 2019

414.71 ppm

Ice-core data before 1958. Mauna Loa data after 1958.



Global Climate Models

climate fueled by the nonuniform spatial distribution of incoming solar radiation.

Stute et al., PNAS 2001

Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

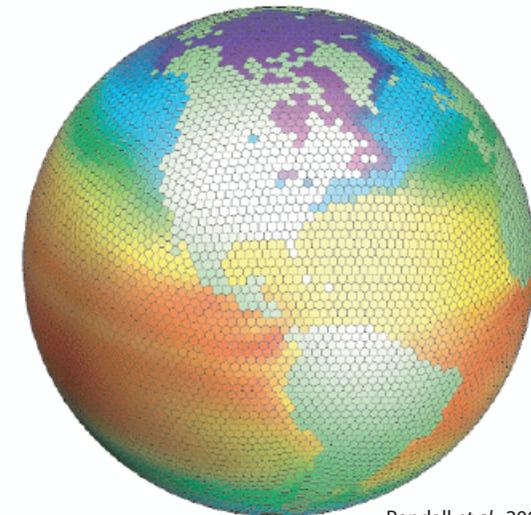
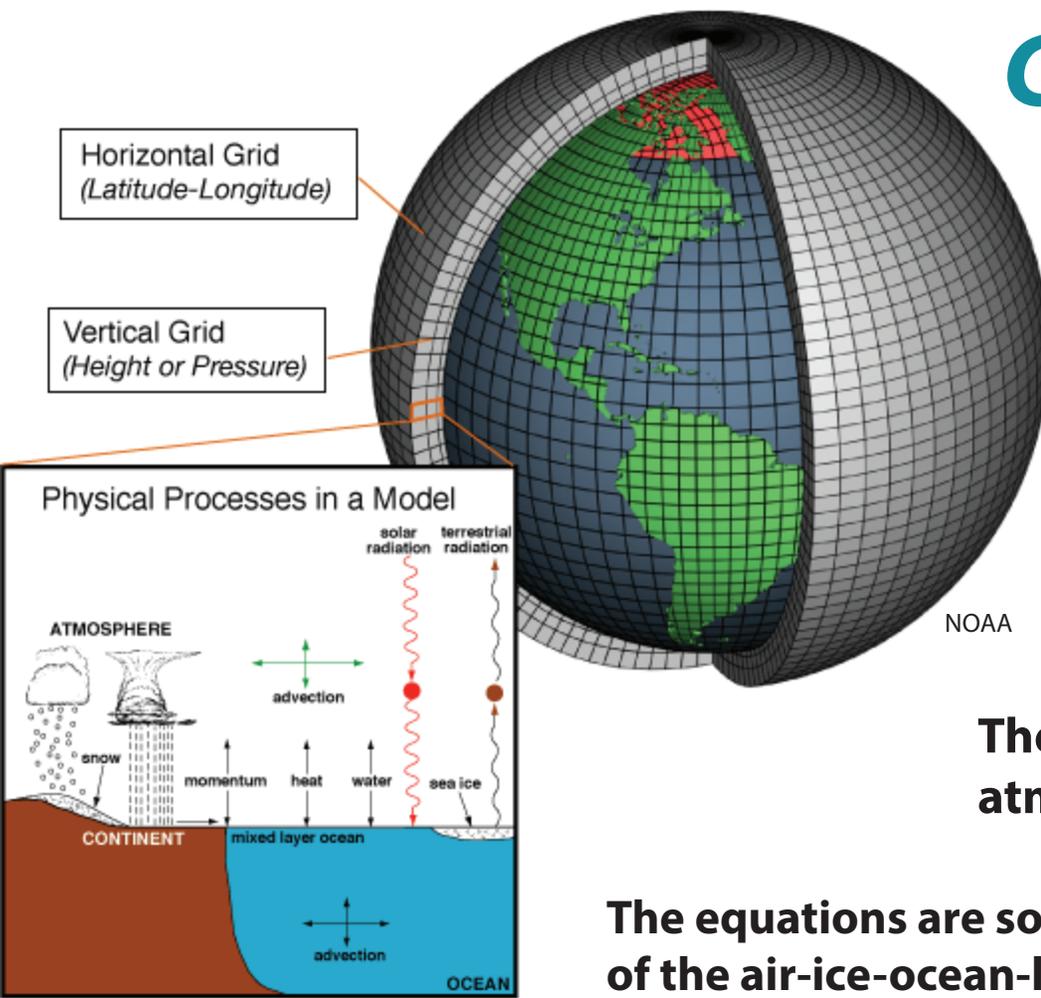
They describe the state of the ocean, land, ice atmosphere, biosphere, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 100 km), using very powerful computers.

key challenge :

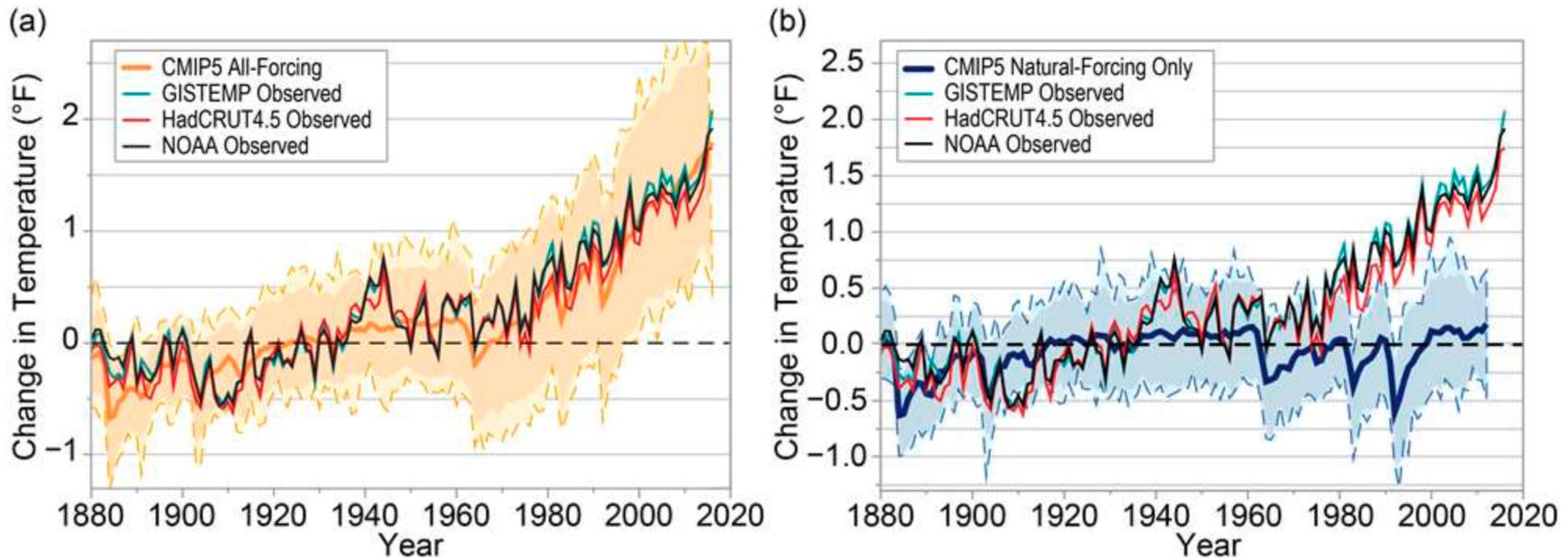
incorporating sub - grid scale processes

linking scales



Human influence dominant

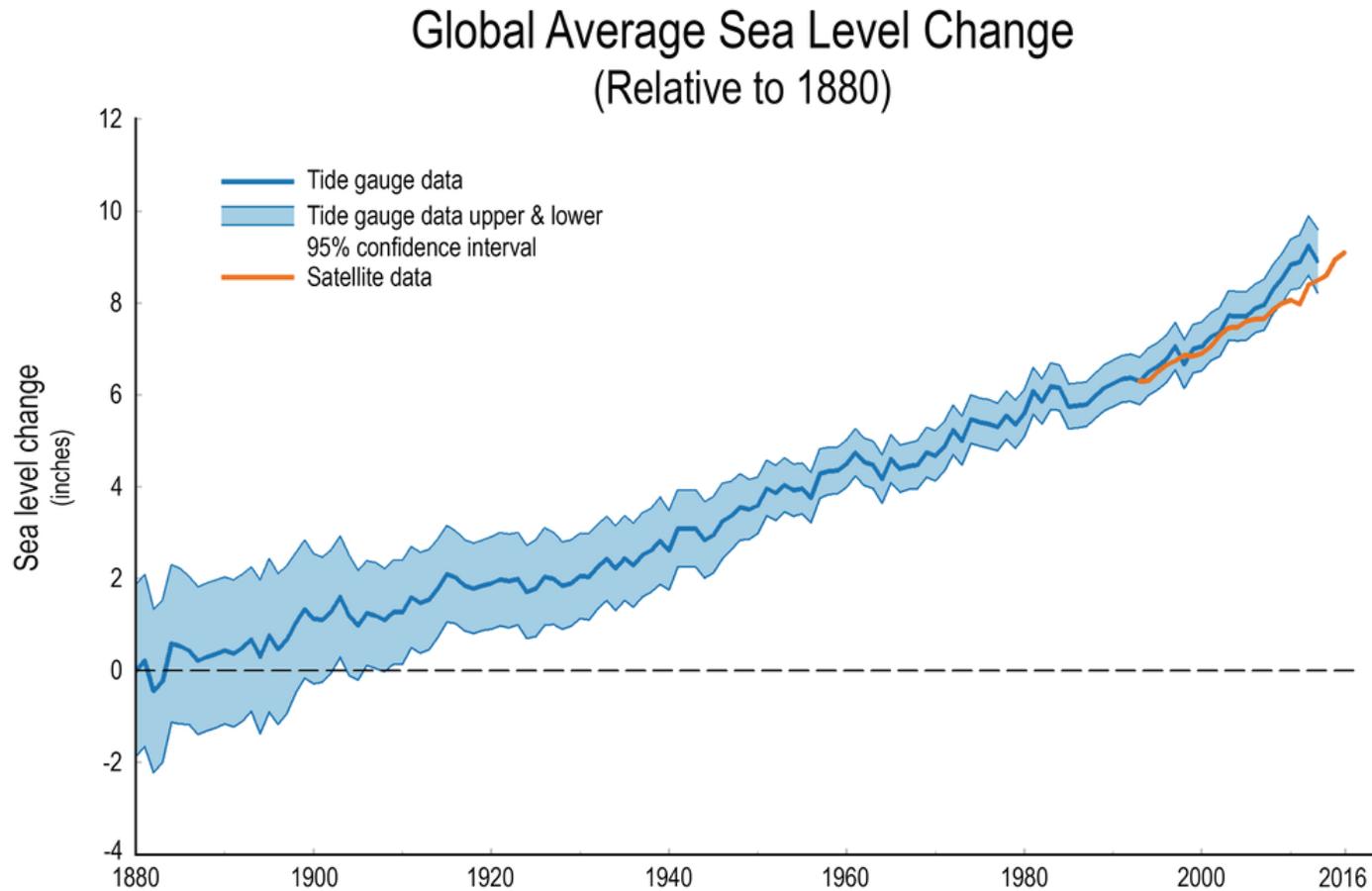
Global Mean Temperature Change



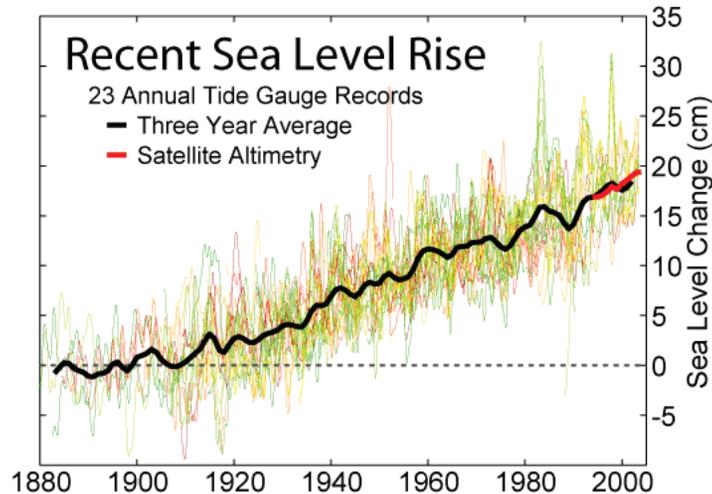
- We cannot explain observed warming without including effects of anthropogenic greenhouse gas emissions
- Observed temperature curve reproduced reasonably well only when taking into consideration all forcing factors

Why is sea level rising?

(melting sea ice does not directly impact rising sea level)



As Earth's climate warms, why does sea level rise?



- **melting land ice: Antarctica, Greenland, mountain glaciers**
(if all melted : 70 m or ~230 ft of sea level rise)

- **thermal expansion of warming ocean**

accounts for about 25% of rise in last half of 20th century, rate ~ tripled in 21st century

- **continental rebound**

rise of land masses that were depressed by the huge weight of ice sheets



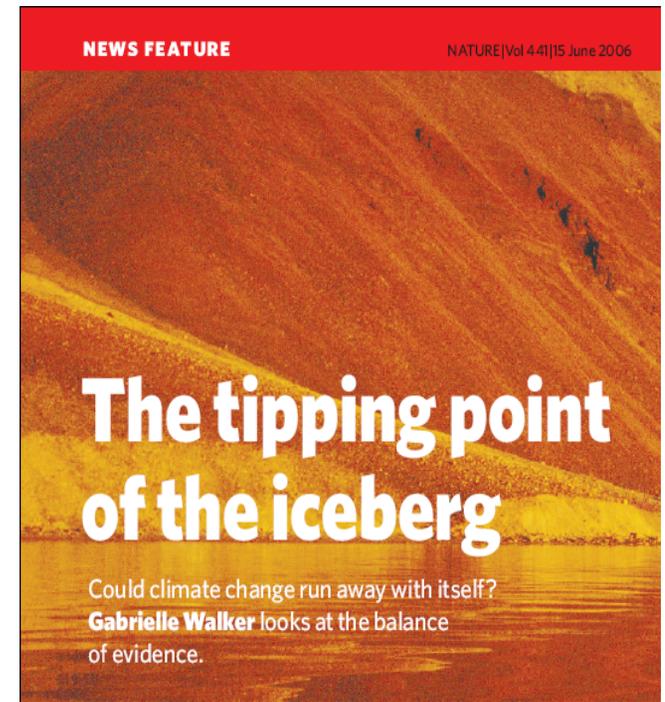
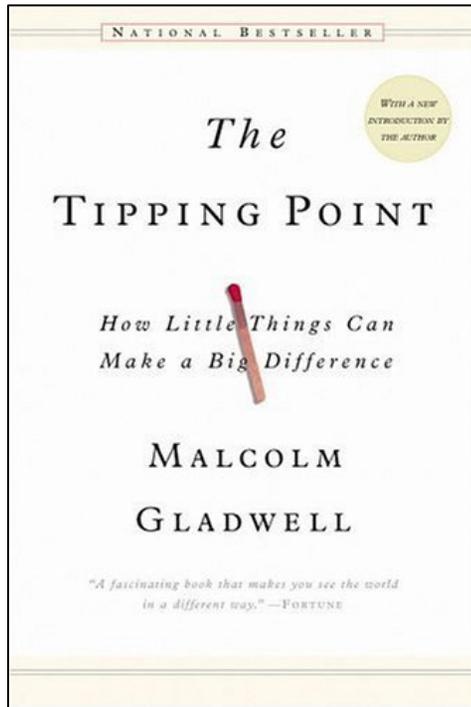
The interaction of warm waters with the periphery of the large ice sheets represents one of the most significant possibilities for abrupt change in the climate system.

no coupling yet of ice sheets and ocean in climate models - no feedback effects



tipping points in the mainstream

Increasing emphasis in recent years on idea of **climate tipping points**, with September Arctic sea ice cover receiving much of the attention.



- Melting of the Greenland ice sheet
- Melting of the West Antarctic ice sheet
- Permafrost and tundra loss, leading to the release of methane
- Formation of Atlantic deep water near the Arctic ocean ●●●

opposite “pole” from GCM’s

active area of mathematical research on sea ice:

Has Arctic sea ice loss passed through a “tipping point”?

an irreversible downward slide to ice-free Arctic summers, driven by ice-albedo feedback

Eisenman, Wettlaufer, PNAS 2009 :

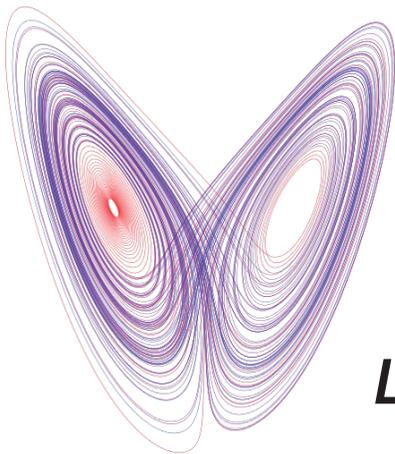
analyze a single nonlinear differential equation for the energy in the upper ocean

look for “bifurcations” in solutions

- unlikely in current loss of summer ice
- more likely in further loss of winter ice

Abbot, Silber, Pierrehumbert, JGR 2011 :

bifurcations when include clouds and ice loss



Lorenz butterfly

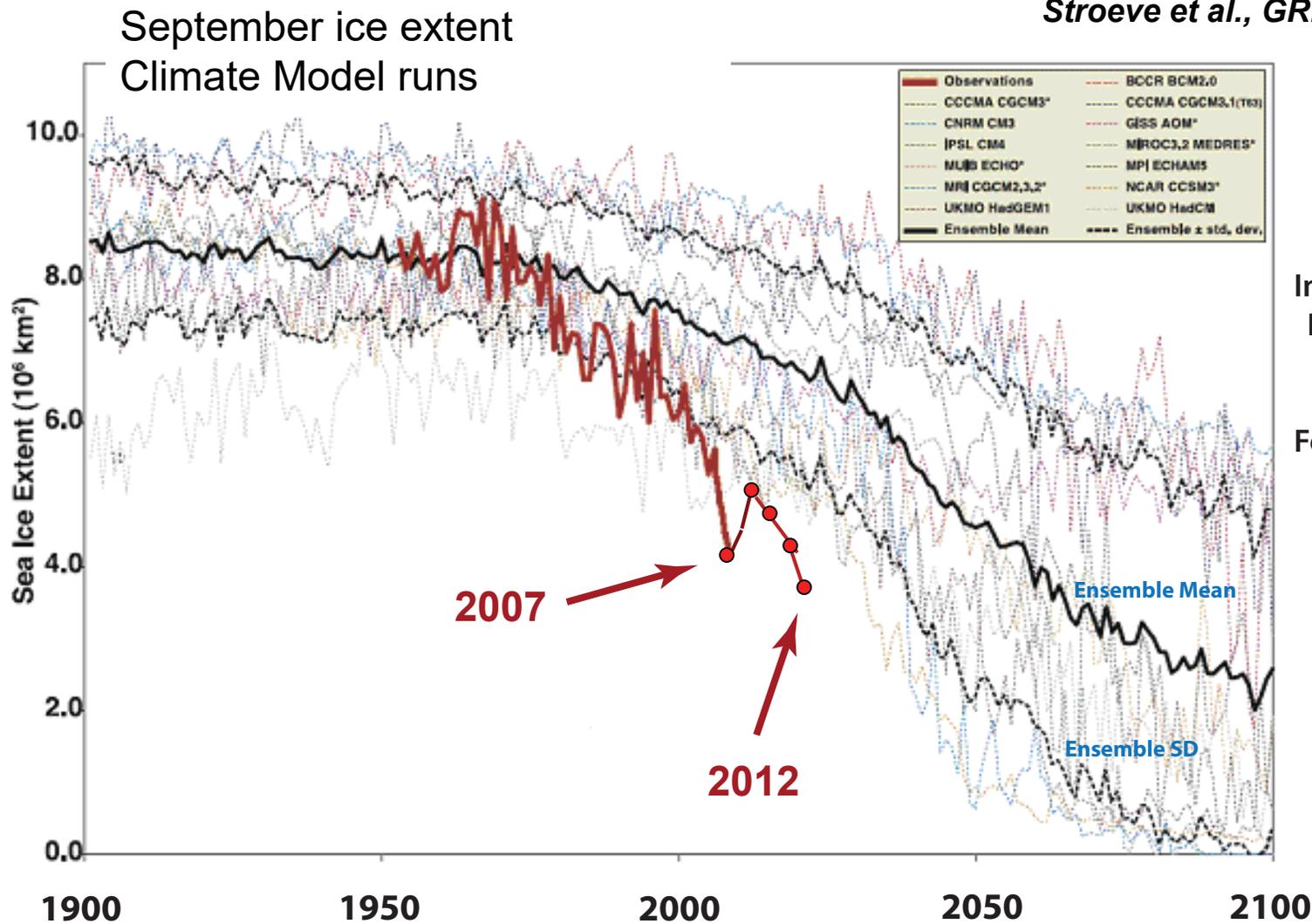
low order (toy) model of Arctic climate change

dynamical systems

Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007

Stroeve et al., GRL, 2012



IPCC AR4
Models

Intergovernmental
Panel on Climate
Change (IPCC)

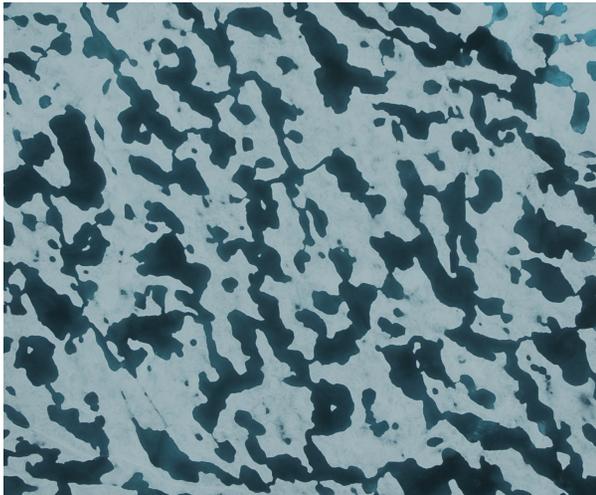
Fourth Assessment
AR4, 2007

challenge:

Represent sea ice more realistically in climate models to improve projections.



How do patterns of dark and light evolve?



Account for key processes

e.g. melt pond evolution

Including PONDS in simulations **LOWERS** predicted sea ice volume over time by 40%.

Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

... and other sub-grid scale structures and processes.

linkage of scales

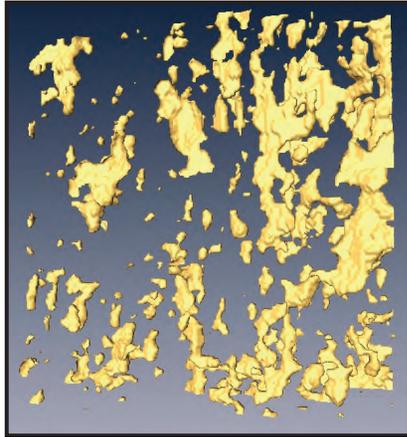
Sea Ice is a Multiscale Composite Material

microscale

brine inclusions

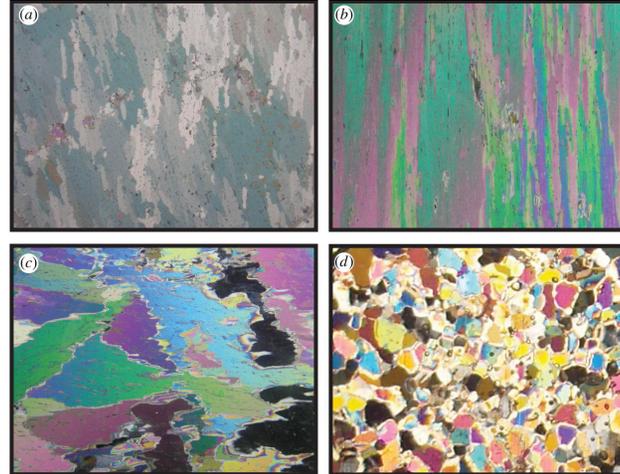


Weeks & Assur 1969



H. Eicken
Golden et al. GRL 2007

polycrystals

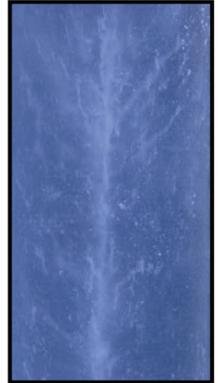


Gully et al. Proc. Roy. Soc. A 2015

brine channels



D. Cole



K. Golden

millimeters

centimeters

mesoscale

Arctic melt ponds



K. Frey

Antarctic pressure ridges



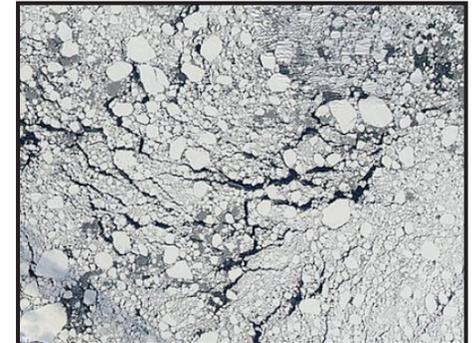
K. Golden

sea ice floes



J. Weller

sea ice pack



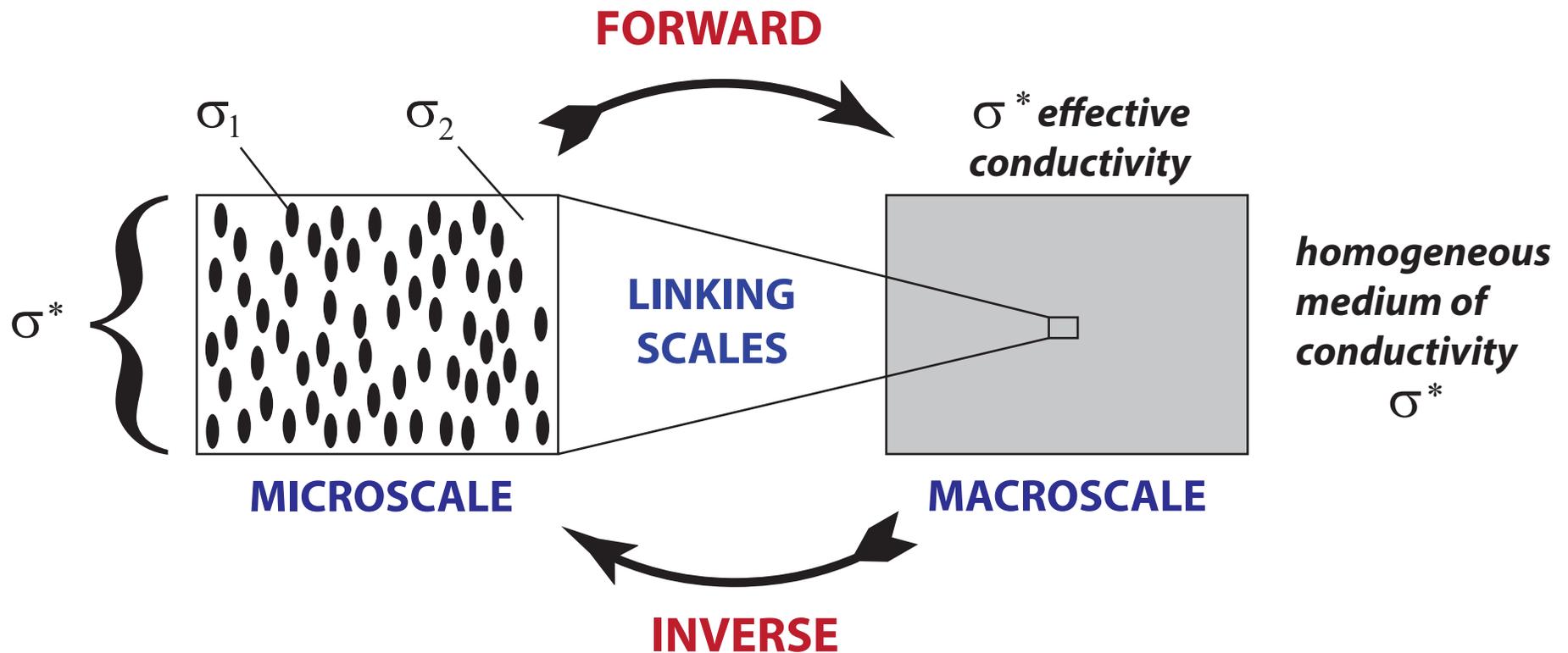
NASA

meters

kilometers

macroscale

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres

Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

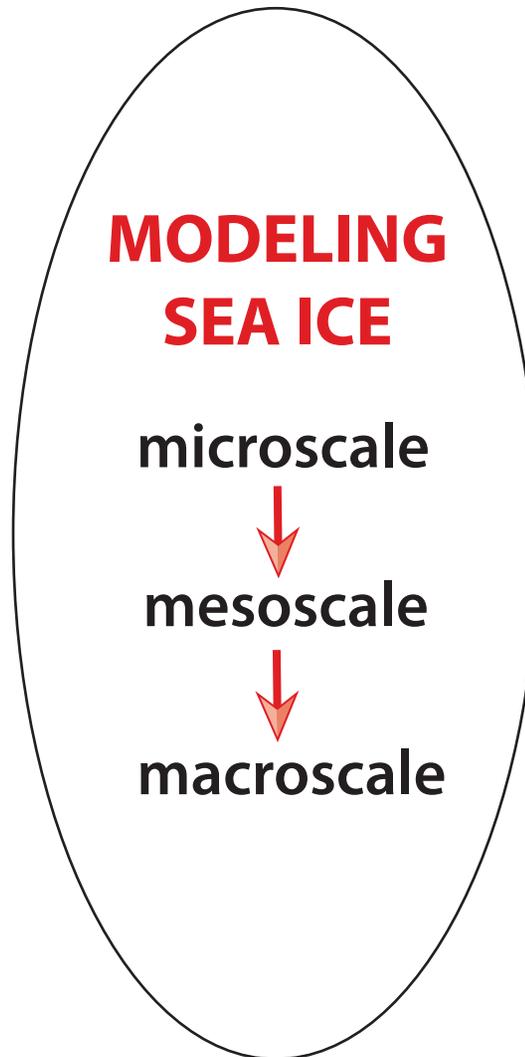
*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*

*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is our research about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...



What is our research about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...

Inputs, Ingredients

COMPOSITE MATERIALS

electrical engineering,
stealth technology

porous media,
oil extraction

statistical mechanics
of ferromagnets

Anderson localization,
semiconductor physics

random matrix theory

differential equations



**MODELING
SEA ICE**

microscale

mesoscale

macroscale

What is our research about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...

Inputs, Ingredients

COMPOSITE MATERIALS

electrical engineering,
stealth technology

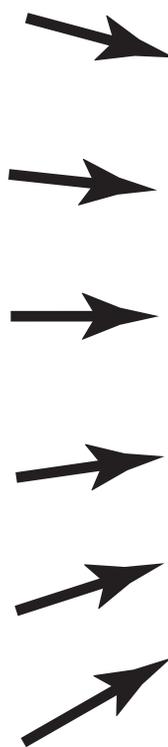
porous media,
oil extraction

statistical mechanics
of ferromagnets

Anderson localization,
semiconductor physics

random matrix theory

differential equations

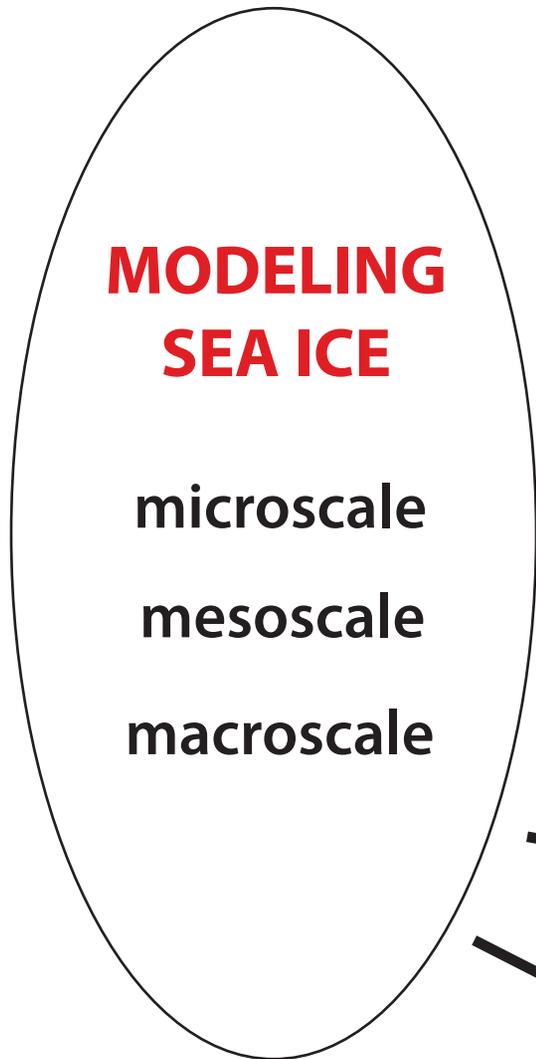


MODELING SEA ICE

microscale

mesoscale

macroscale



Outputs, Impacts

CLIMATE MODELING

sea ice physics
& biology

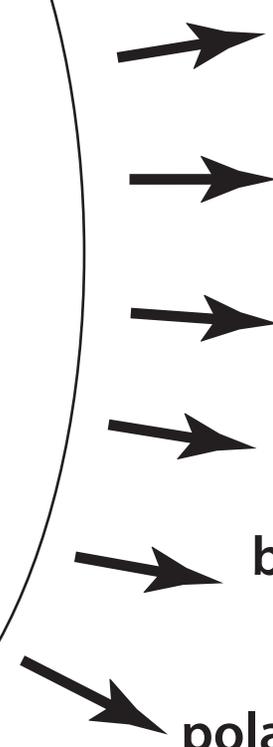
composites,
polycrystals

remote sensing

advection diffusion

biomedical imaging,
biomaterials, EPS

polar microbial ecology



What is our research about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...

Inputs, Ingredients

COMPOSITE MATERIALS

electrical engineering,
stealth technology

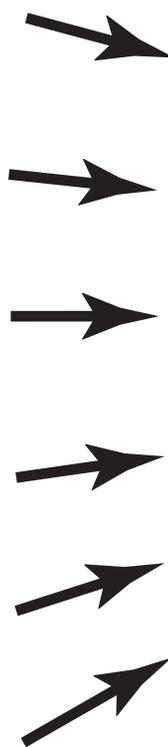
porous media,
oil extraction

statistical mechanics
of ferromagnets

Anderson localization,
semiconductor physics

random matrix theory

differential equations

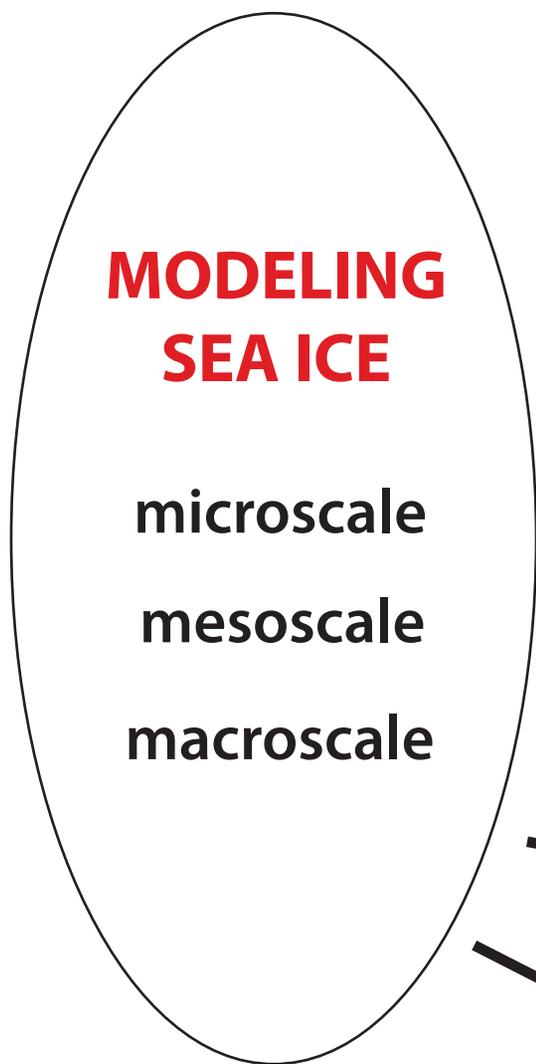


MODELING SEA ICE

microscale

mesoscale

macroscale



Outputs, Impacts

CLIMATE MODELING

sea ice physics
& biology

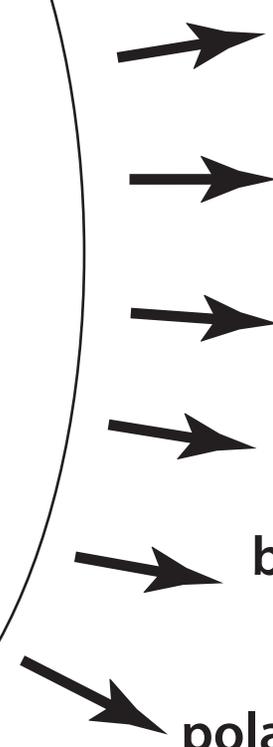
composites,
polycrystals

remote sensing

advection diffusion

biomedical imaging,
biomaterials, EPS

polar microbial ecology



Physics of sea ice drives advances in many areas of science and engineering.

What is our research about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...

Inputs, Ingredients

COMPOSITE MATERIALS

electrical engineering,
stealth technology

porous media,
oil extraction

statistical mechanics
of ferromagnets

Anderson localization,
semiconductor physics

random matrix theory

differential equations

Cross pollination

magnets
radar absorbers
human bone
rat brains

**MODELING
SEA ICE**

Outputs, Impacts

CLIMATE MODELING

sea ice physics
& biology

composites,
polycrystals

remote sensing

advection diffusion

biomedical imaging,
biomaterials, EPS

polar microbial ecology

Physics of sea ice drives advances in many areas of science and engineering.

How do scales interact in the sea ice system?



basin scale -
grid scale
albedo

NASA

Linking Scales

km
scale
melt
ponds



Perovich

Linking Scales

mm
scale
brine
inclusions



meter
scale
snow
topography

sea ice formation



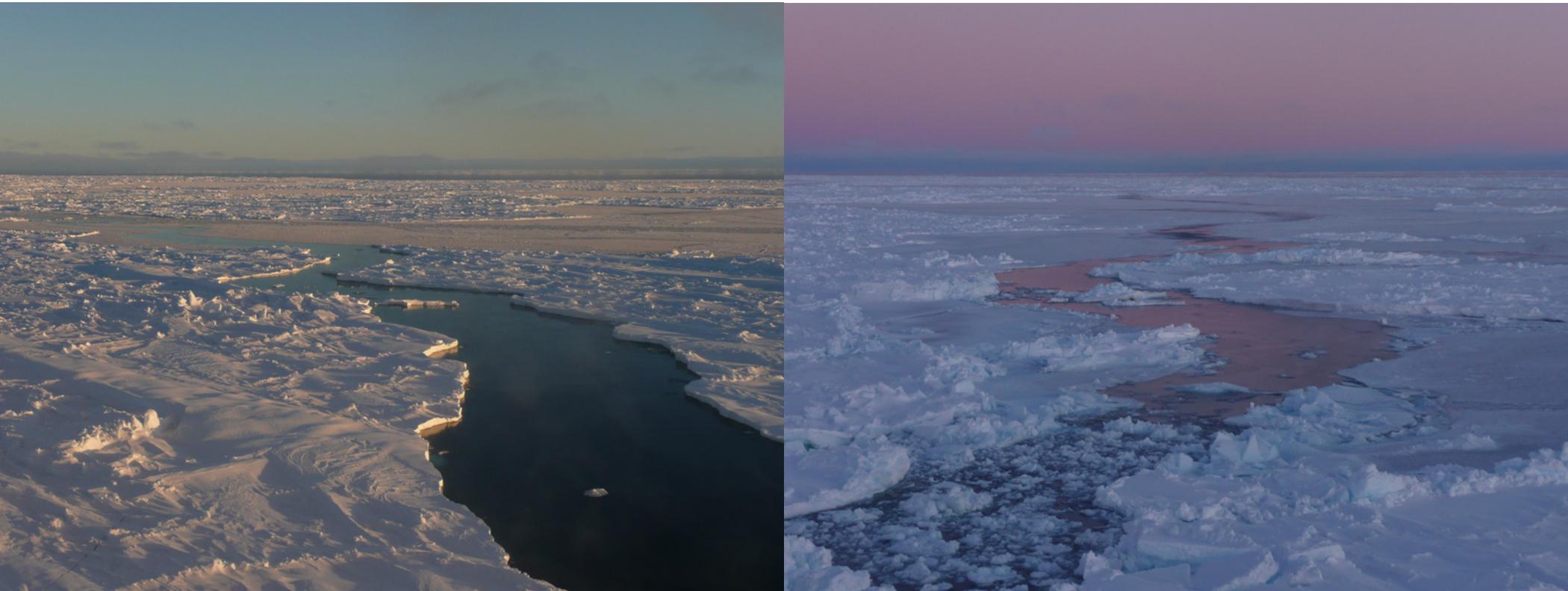




sea ice dynamics
plate tectonics on a fast time scale



leads



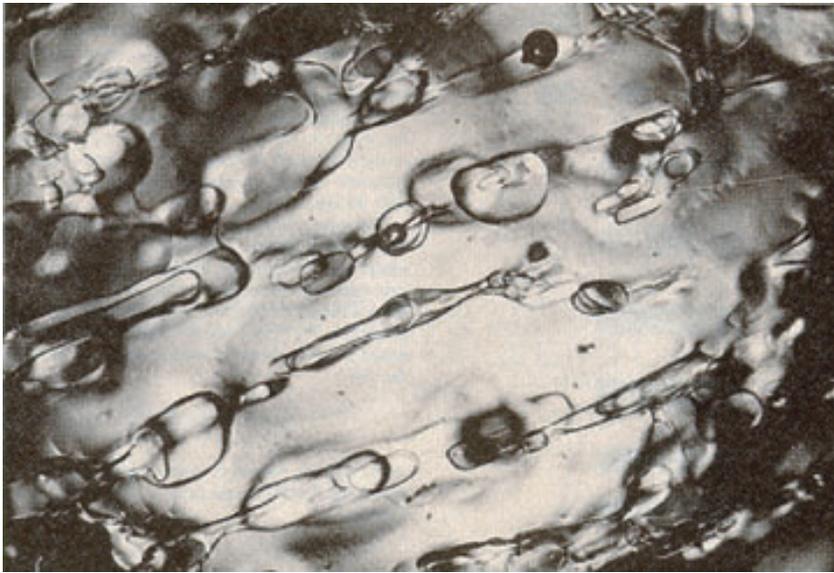
heat flows directly from ocean to atmosphere

sea ice microphysics

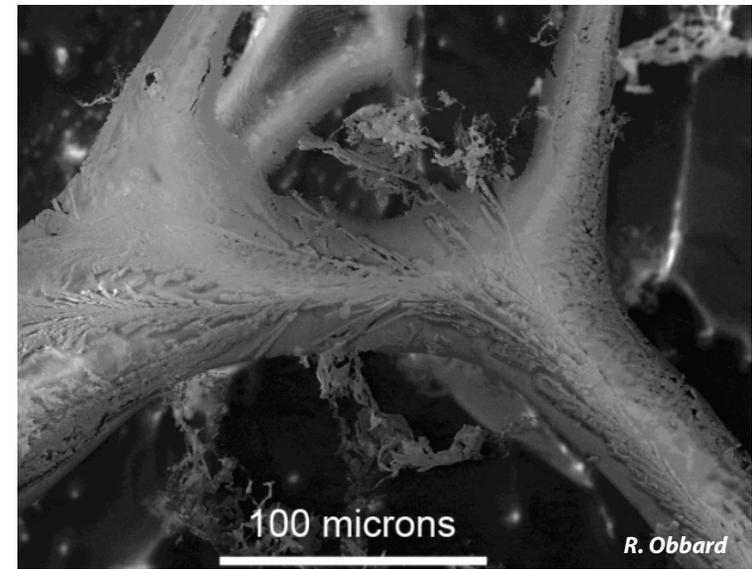
fluid transport



sea ice may appear to be a barren, impermeable cap ...



brine inclusions in sea ice (mm)



micro - brine channel (SEM)

sea ice is a porous composite

pure ice with brine, air, and salt inclusions

brine channels (cm)



horizontal section



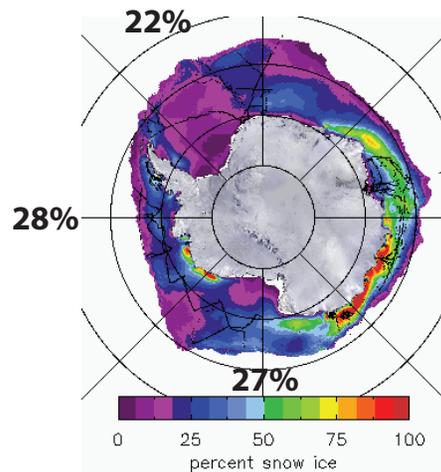
vertical section

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities



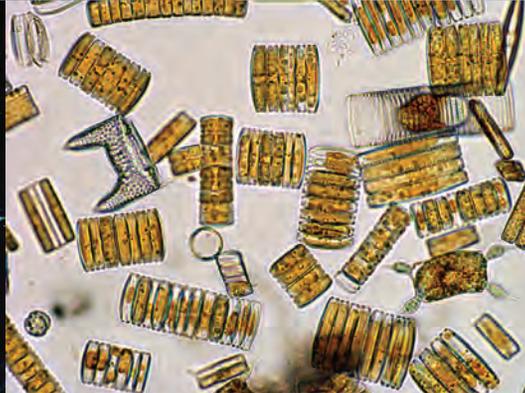
September
snow-ice
estimates

T. Maksym and T. Markus, 2008

*Antarctic surface flooding
and snow-ice formation*

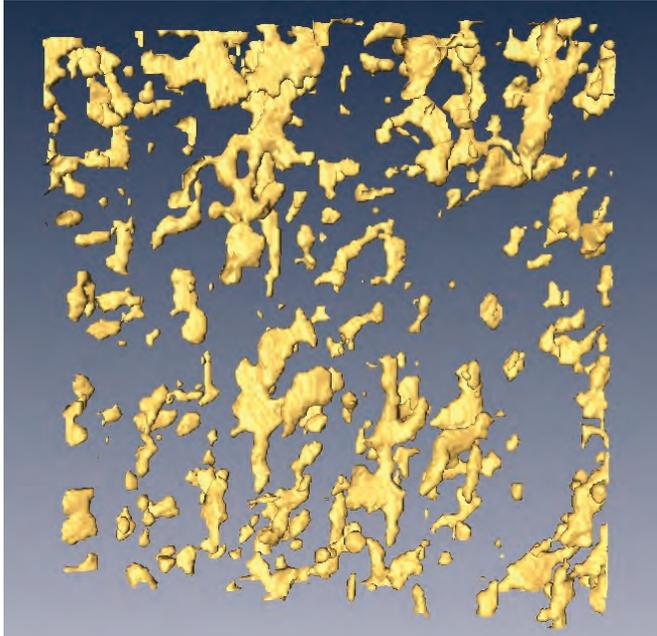
- evolution of salinity profiles
- ocean-ice-air exchanges of heat, CO₂

sea ice ecosystem

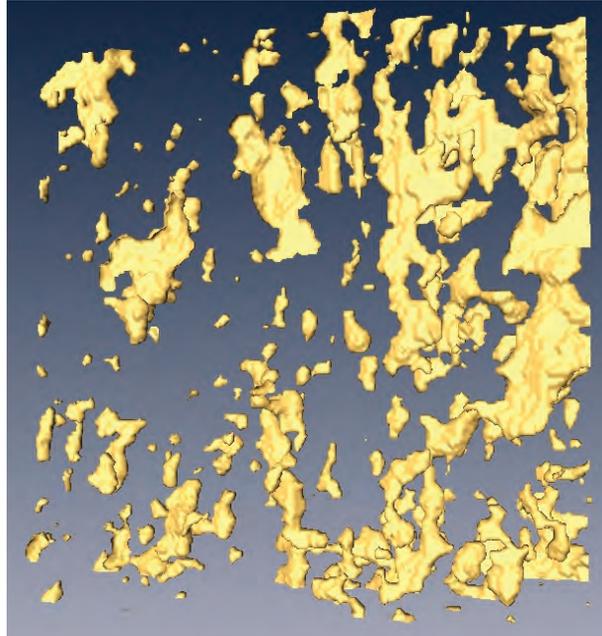


sea ice algae
support life in the polar oceans

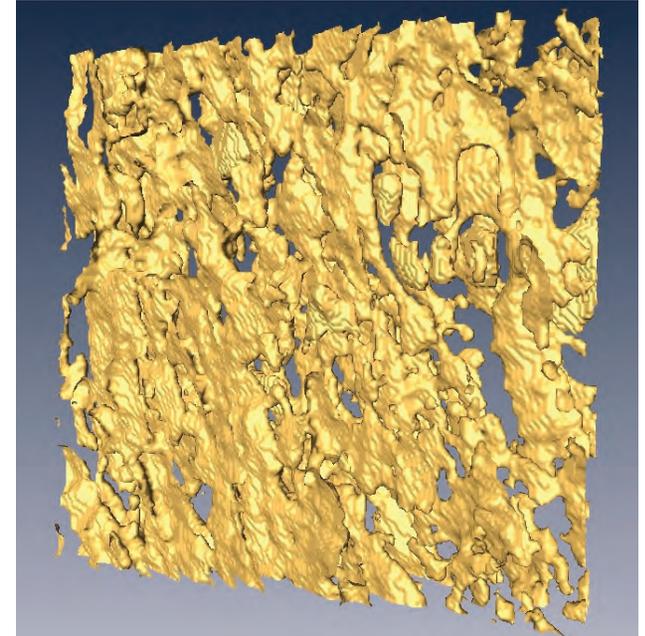
brine volume fraction and **connectivity** increase with temperature



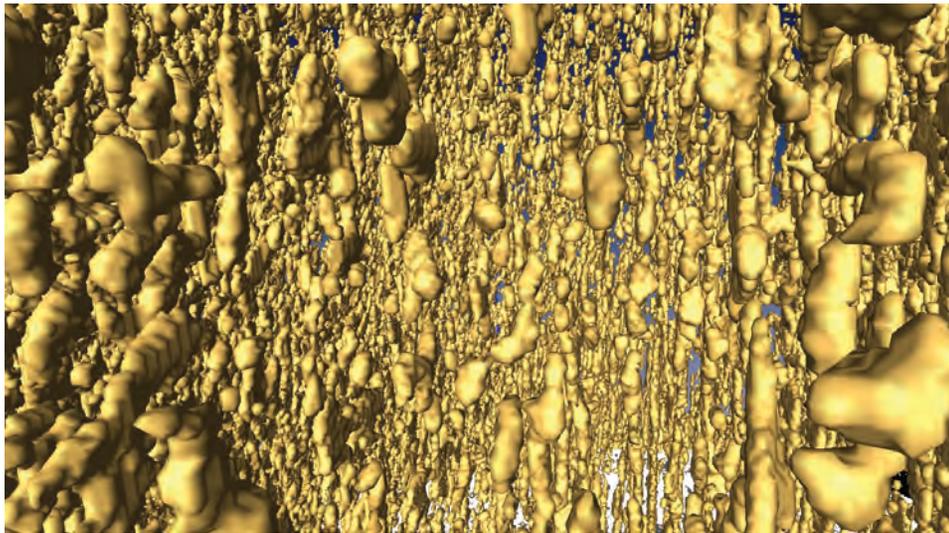
$T = -15\text{ }^{\circ}\text{C}, \phi = 0.033$



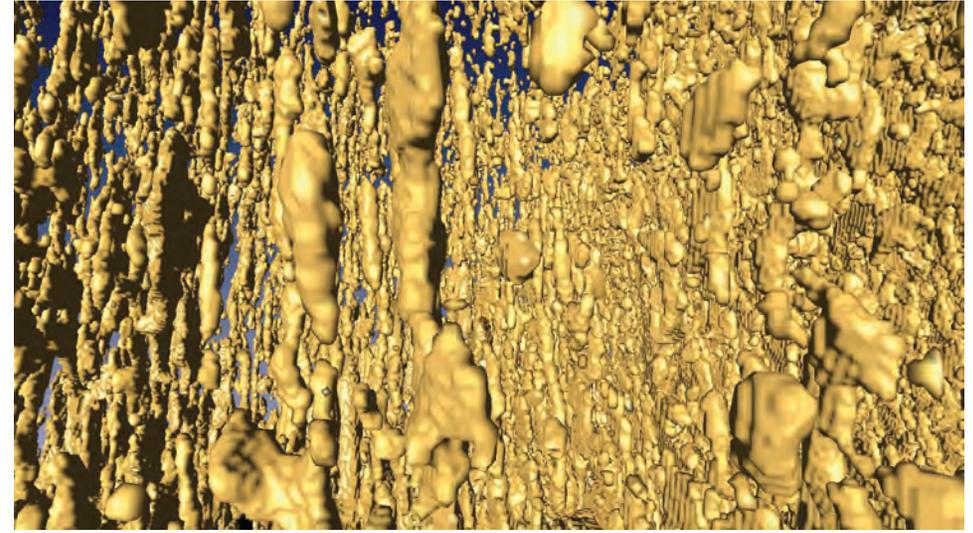
$T = -6\text{ }^{\circ}\text{C}, \phi = 0.075$



$T = -3\text{ }^{\circ}\text{C}, \phi = 0.143$

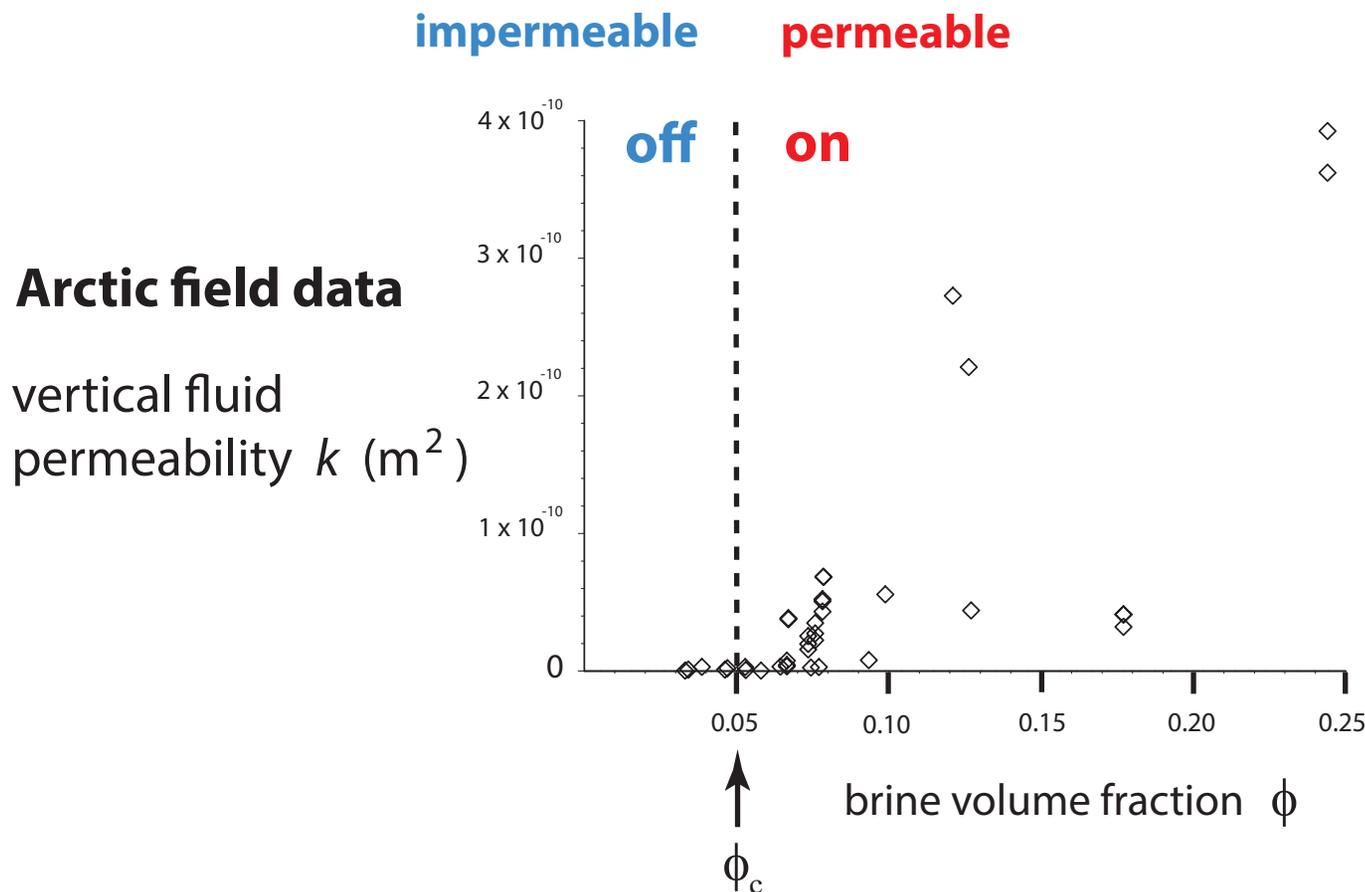


$T = -8\text{ }^{\circ}\text{C}, \phi = 0.057$



$T = -4\text{ }^{\circ}\text{C}, \phi = 0.113$

Critical behavior of fluid transport in sea ice



**“on - off” switch
for fluid flow**

critical brine volume fraction $\phi_c \approx 5\%$ \longleftrightarrow $T_c \approx -5^\circ \text{C}$, $S \approx 5 \text{ ppt}$

RULE OF FIVES

Golden, Ackley, Lytle Science 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009



sea ice algal communities

D. Thomas 2004

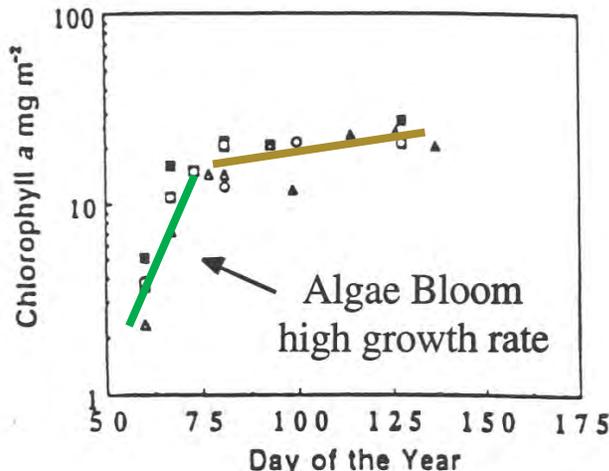
nutrient replenishment controlled by ice permeability

biological activity turns on or off according to **rule of fives**

Golden, Ackley, Lytle Science 1998

Fritsen, Lytle, Ackley, Sullivan Science 1994

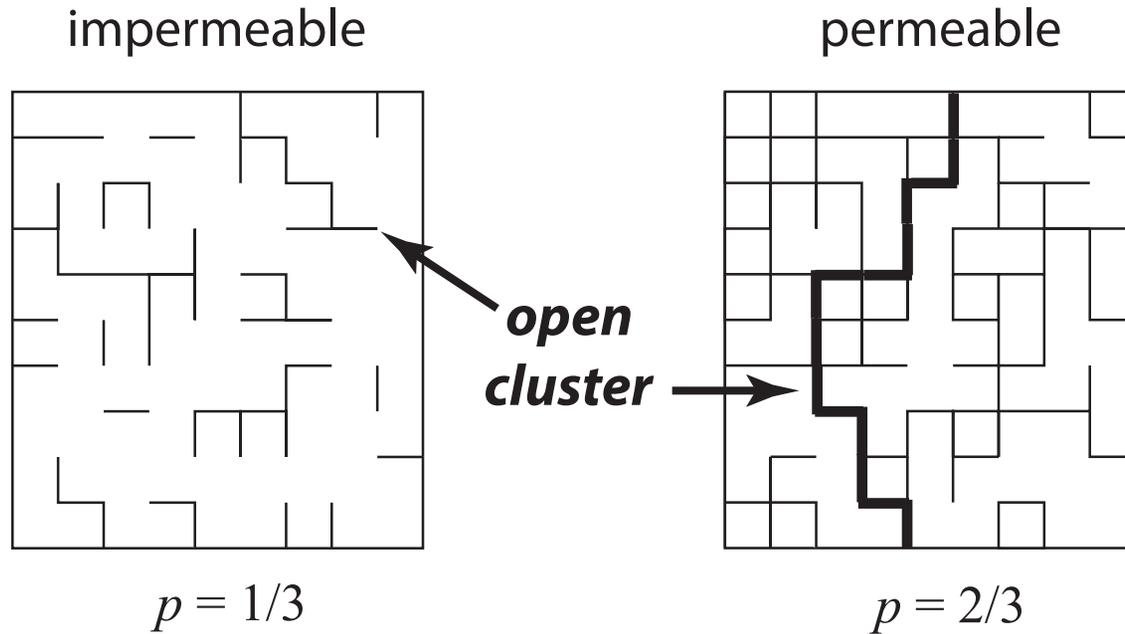
critical behavior of microbial activity



Convection-fueled algae bloom
Ice Station Weddell

percolation theory

probabilistic theory of connectedness



bond \longrightarrow *open* with probability p
closed with probability $1-p$

percolation threshold

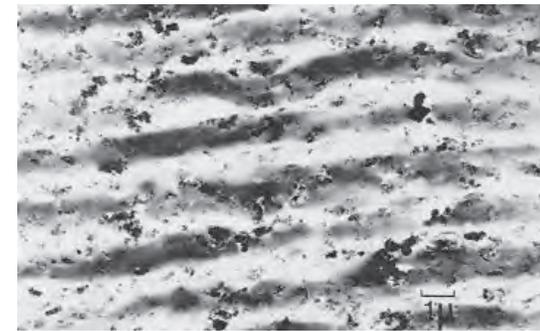
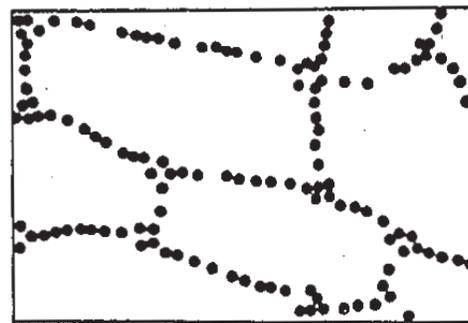
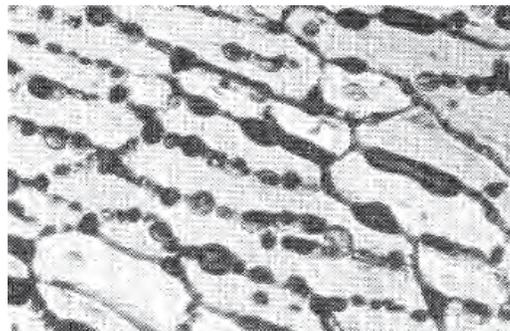
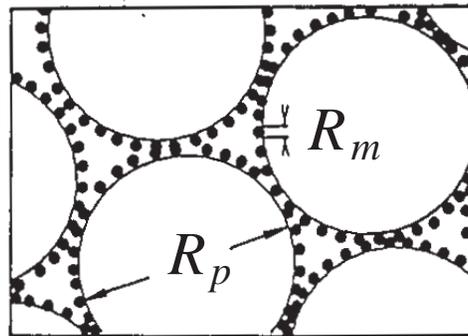
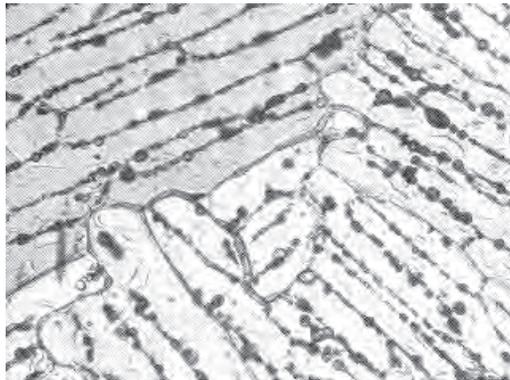
$$p_c = 1/2 \quad \text{for } d = 2$$

smallest p for which there is an infinite open cluster

Continuum percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5\%$$

Golden, Ackley, Lytle, *Science*, 1998



sea ice

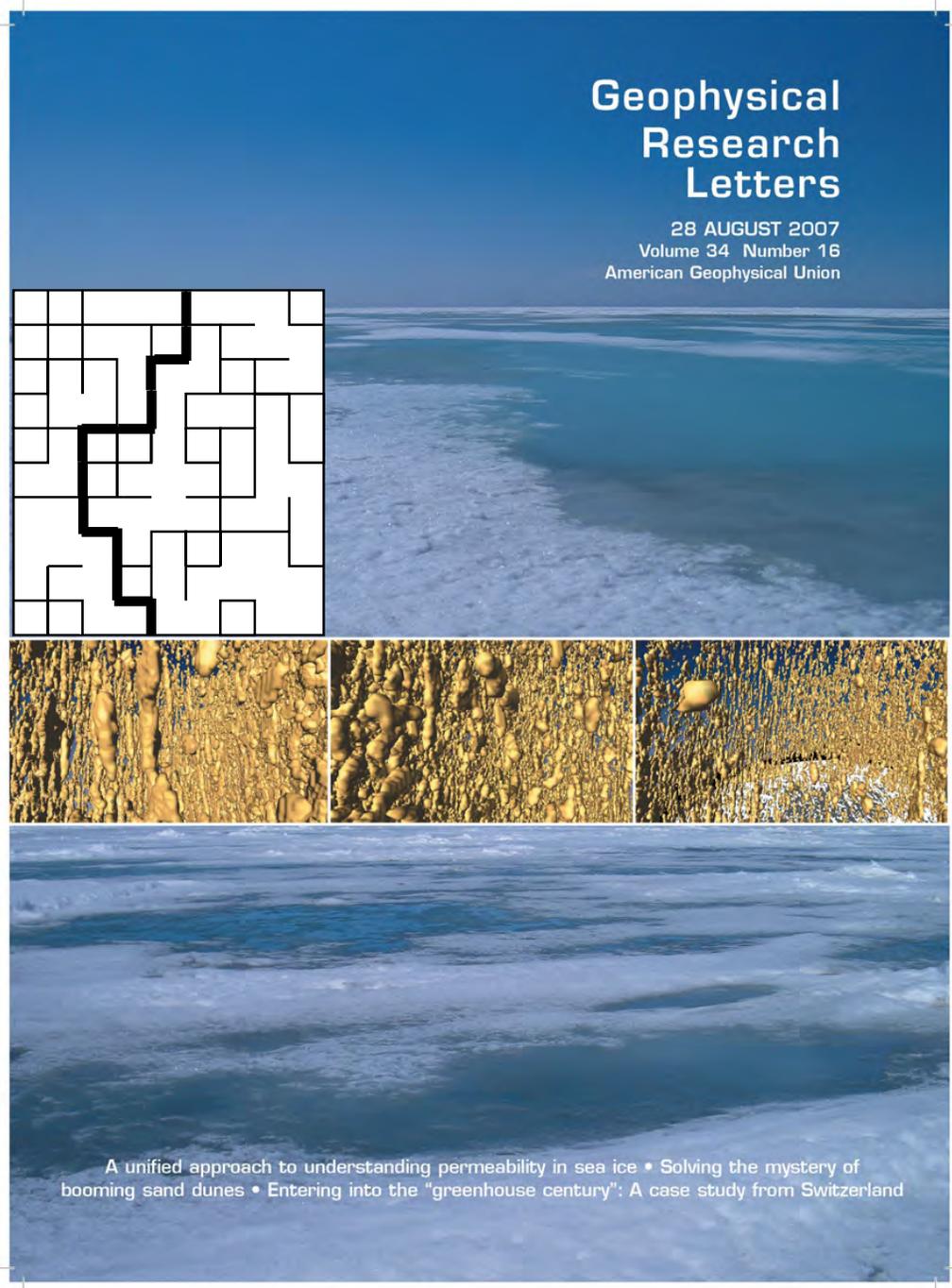
compressed powder

radar absorbing composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton*, Miner, Pringle, Zhu, *Geophysical Research Letters* 2007



percolation theory for fluid permeability

$$k(\phi) = k_0 (\phi - 0.05)^2$$

← critical exponent t

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

from critical path analysis
in hopping conduction

hierarchical model
rock physics
network model
rigorous bounds

X-ray tomography for
brine inclusions

confirms rule of fives

Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009

theories agree closely
with field data

microscale
governs

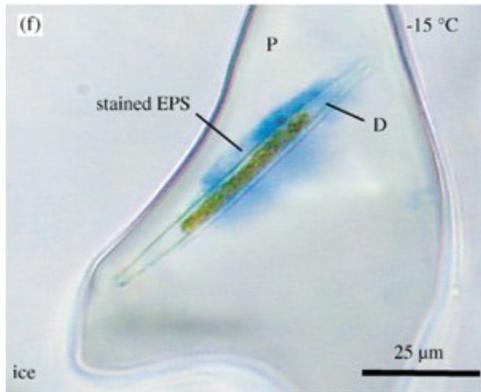
mesoscale
processes

**melt pond
evolution**

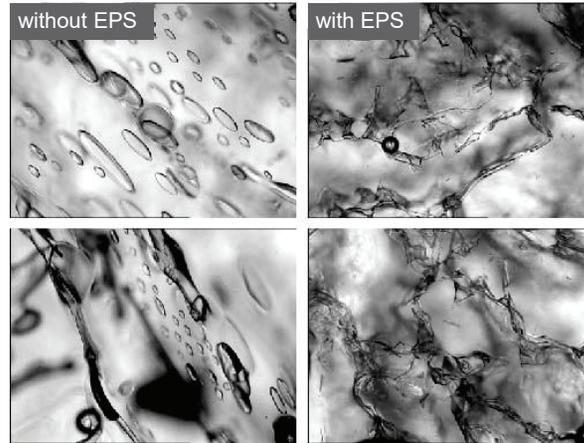
A unified approach to understanding permeability in sea ice • Solving the mystery of booming sand dunes • Entering into the "greenhouse century": A case study from Switzerland

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

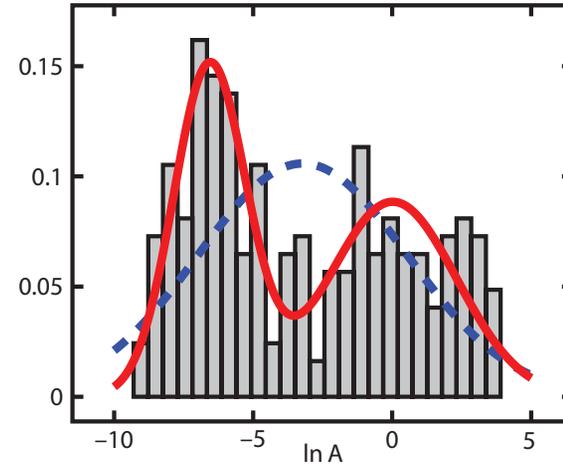
How does EPS affect fluid transport? **How does the biology affect the physics?**



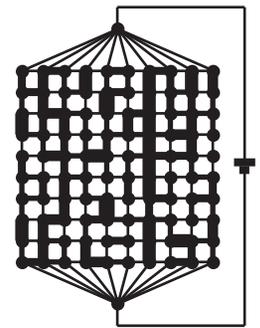
Krembs



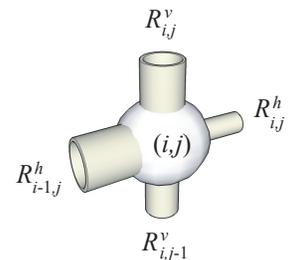
Krembs, Eicken, Deming, PNAS 2011



**RANDOM
PIPE
MODEL**



- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability k ; results predict observed drop in k



Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden, Eicken, Morris
Ann. Glac. 2006

Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and
the Mathematics of
Transport in Sea Ice

page 562

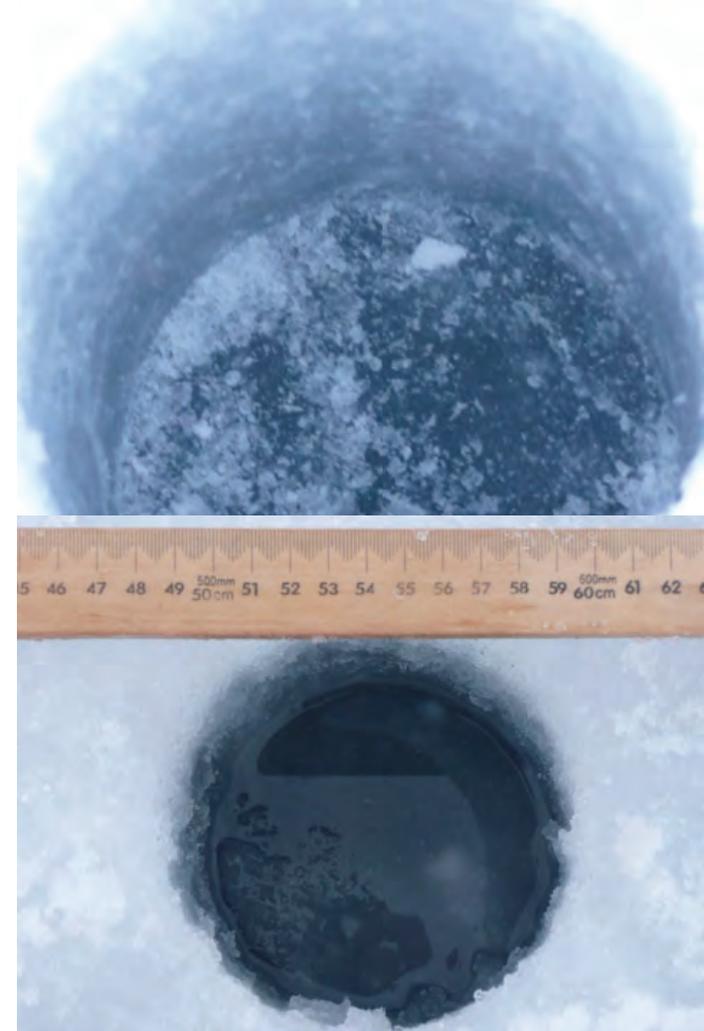
Mathematics and the
Internet: A Source of
Enormous Confusion
and Great Potential

page 586



photo by Jan Lieser

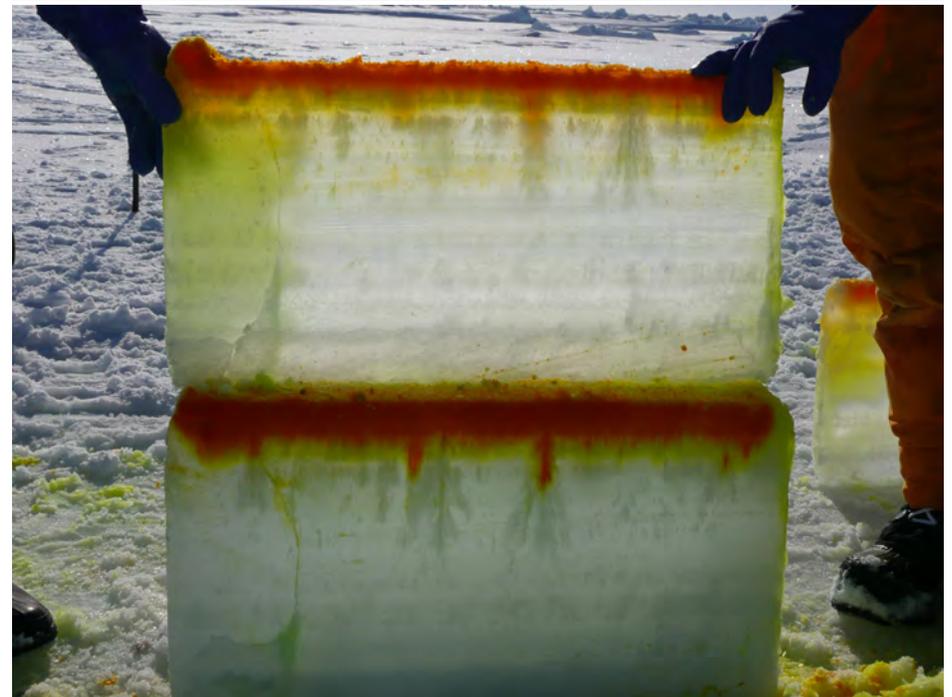
Real analysis in polar coordinates (see page 613)



**measuring
fluid permeability
of Antarctic sea ice**

SIPEX 2007

tracers flowing through inverted sea ice blocks



Arctic and Antarctic field experiments

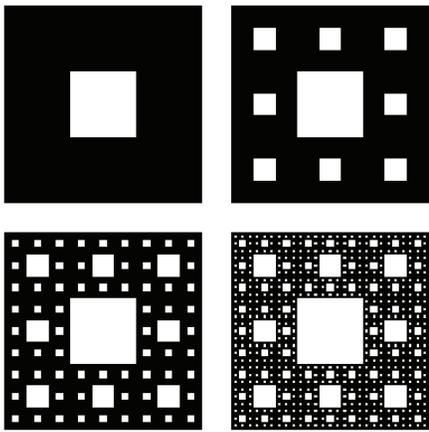
*develop electromagnetic methods
of monitoring fluid transport and
microstructural transitions*

extensive measurements of fluid and
electrical transport properties of sea ice:

2007	Antarctic	SIPEX
2010	Antarctic	McMurdo Sound
2011	Arctic	Barrow AK
2012	Arctic	Barrow AK
2012	Antarctic	SIPEX II
2013	Arctic	Barrow AK
2014	Arctic	Chukchi Sea

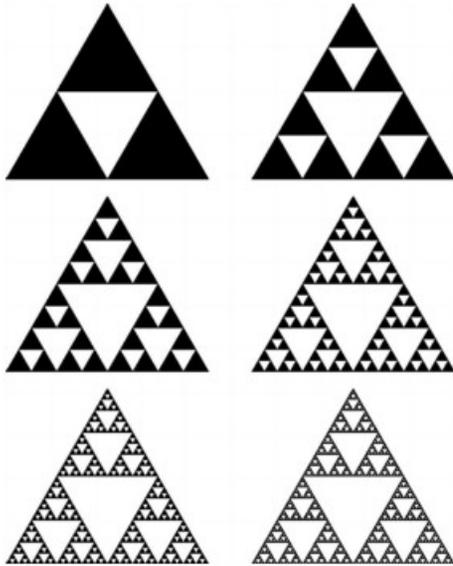


mesoscale



fractals

self-similar structure
non-integer dimension



$$D = \frac{\log 3}{\log 2} = 1.585\dots$$

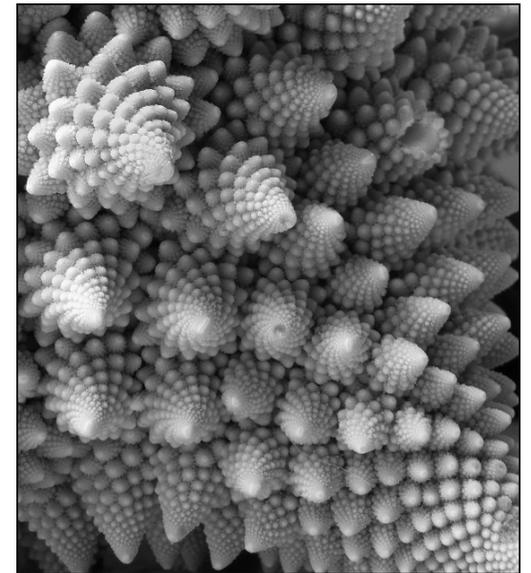
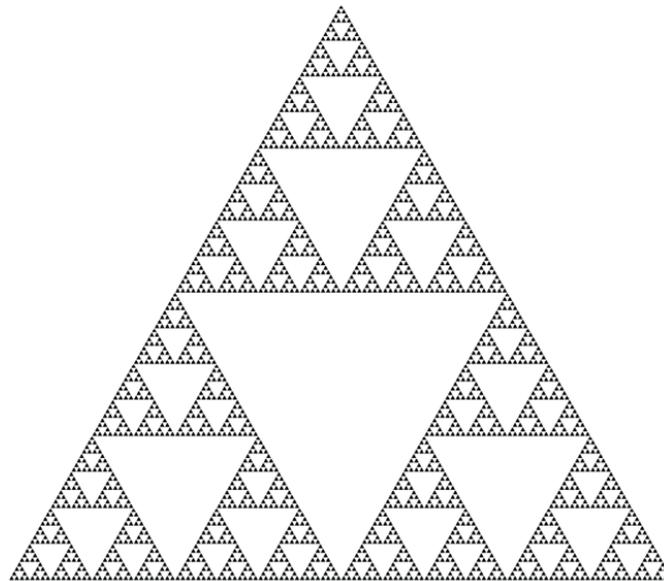
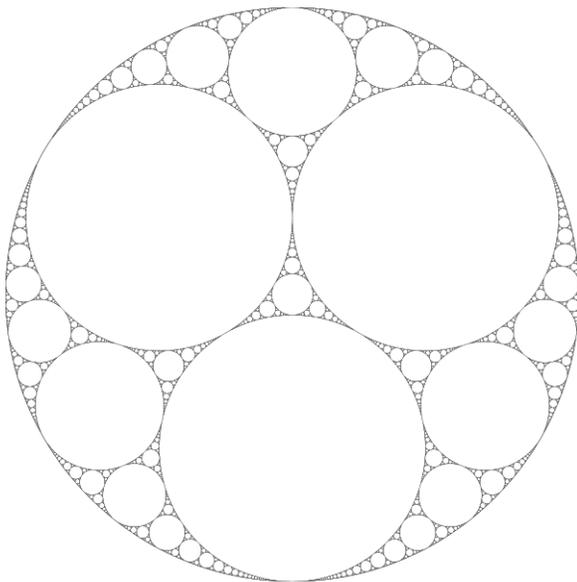


the sea ice pack is a *fractal*

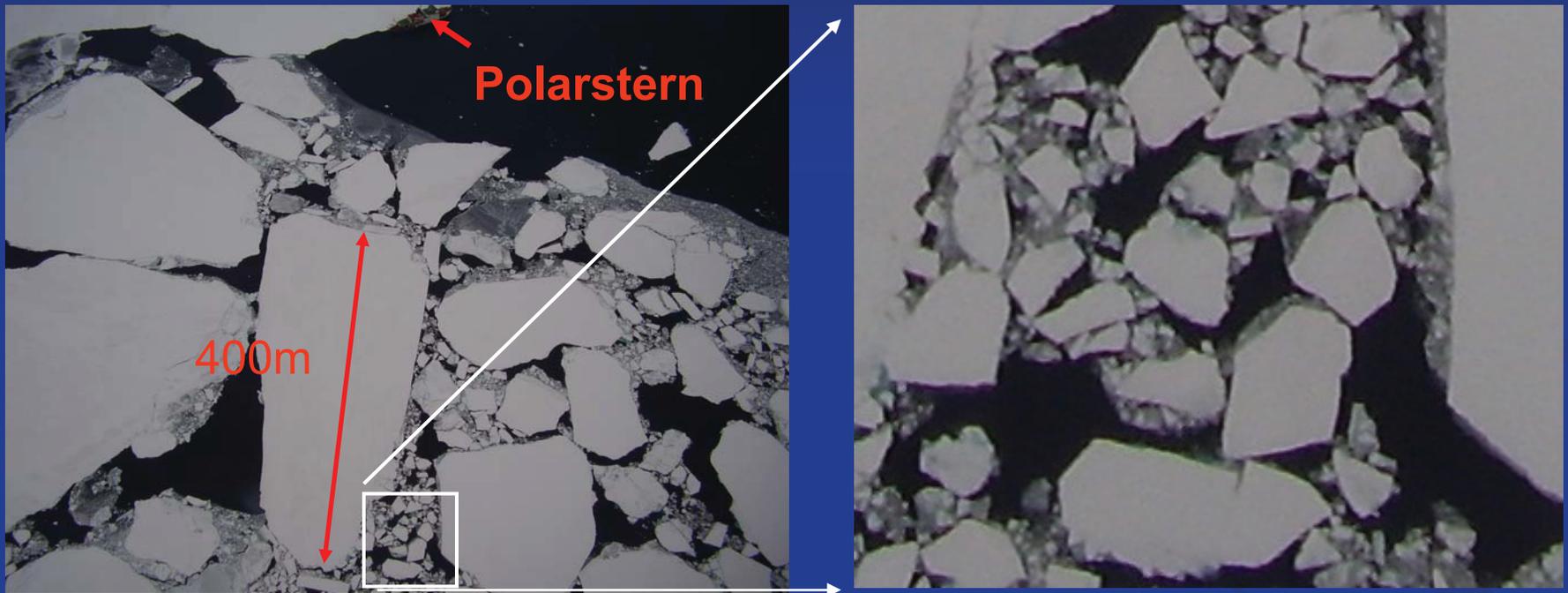
displaying self-similar structure on many scales

floe size distribution important in
dynamics (fracture), thermodynamics (melting)

bigger floes easier to break, smaller floes easier to melt



Self-similarity of sea ice floes



Weddell Sea, Antarctica
2006

Takenobu Toyota

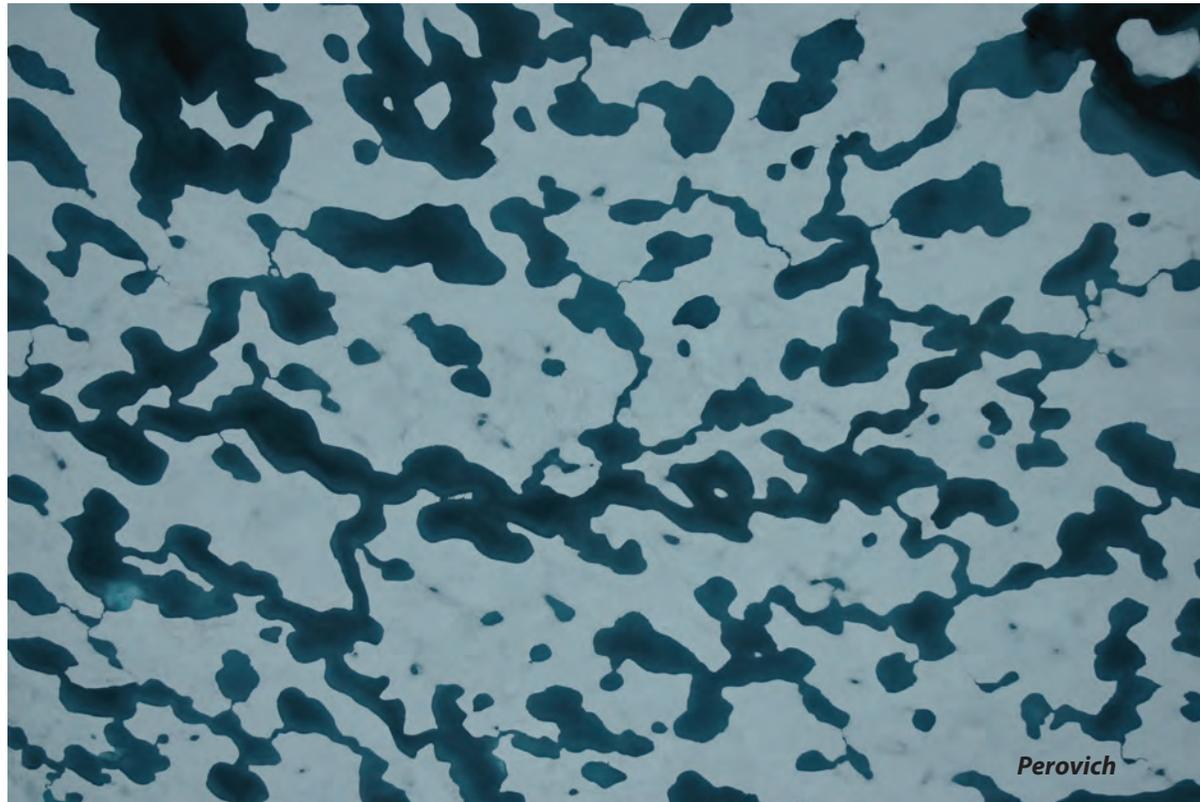
melt pond formation and albedo evolution:

- *major drivers in polar climate*
- *key challenge for global climate models*

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham,
Taylor, Worster 2006
Flocco, Feltham 2007

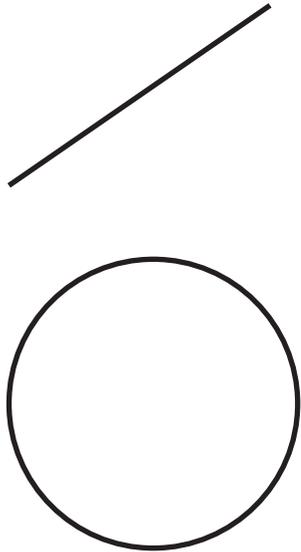
Skyllingstad, Paulson,
Perovich 2009
Flocco, Feltham,
Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

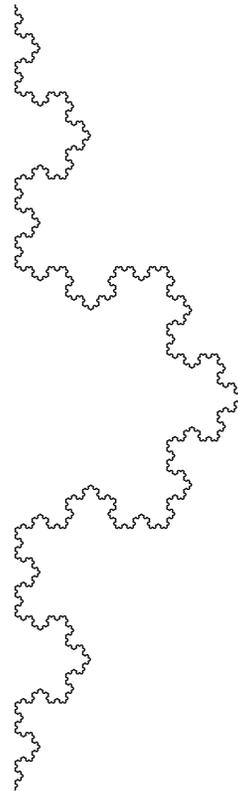
fractal curves in the plane

they wiggle so much that their dimension is >1



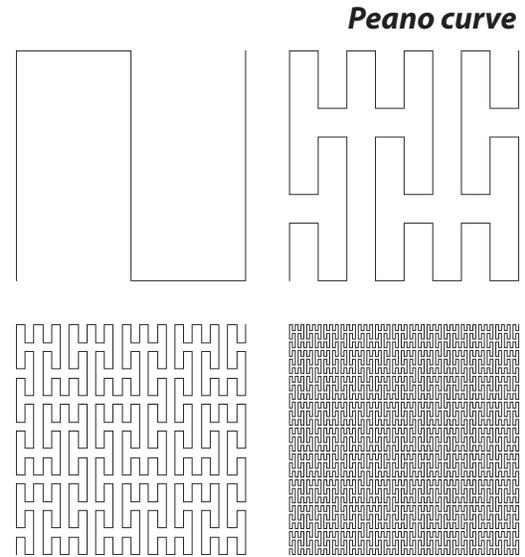
simple curves

$D = 1$



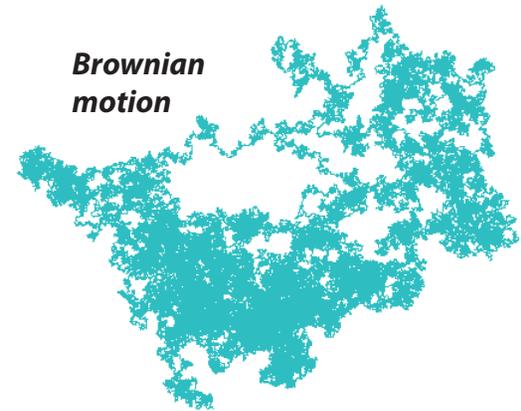
Koch snowflake

$D = 1.26$



Peano curve

Brownian motion



space filling curves

$D = 2$



30th Congressional District, Texas, 1991-1996



clouds exhibit fractal behavior from 1 to 1000 km



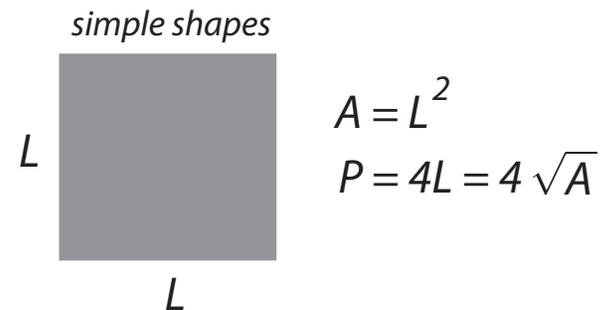
use **perimeter-area** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

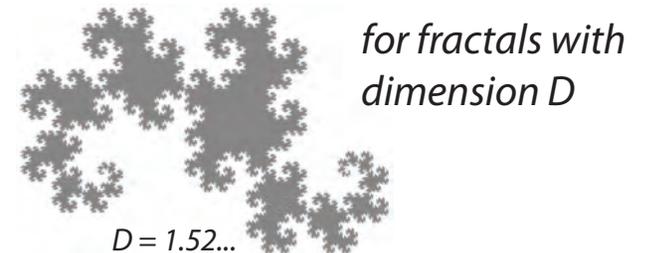
S. Lovejoy, Science, 1982



$$P \sim \sqrt{A}$$



$$P \sim \sqrt{A}^D$$

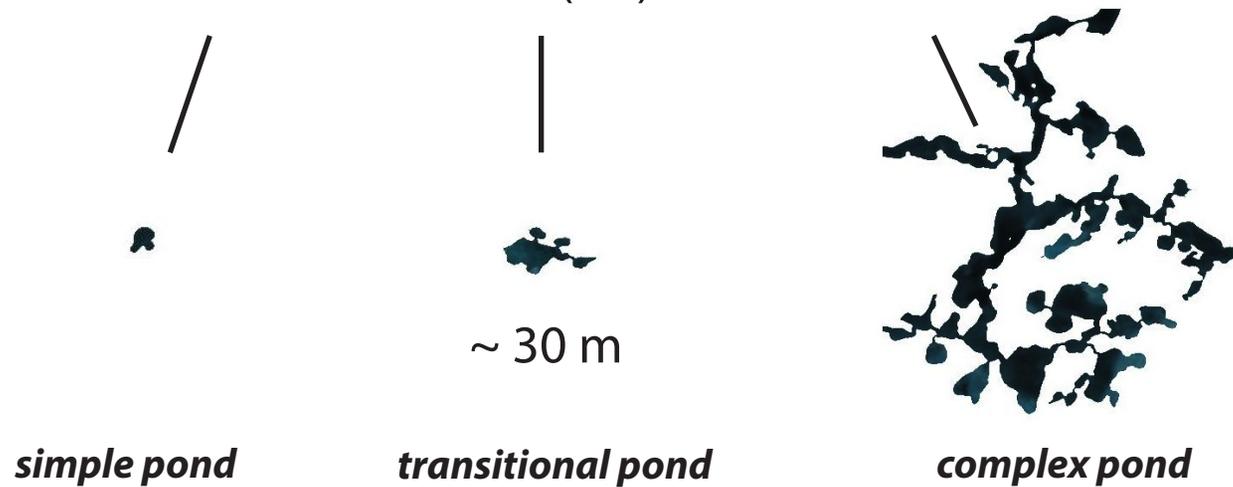
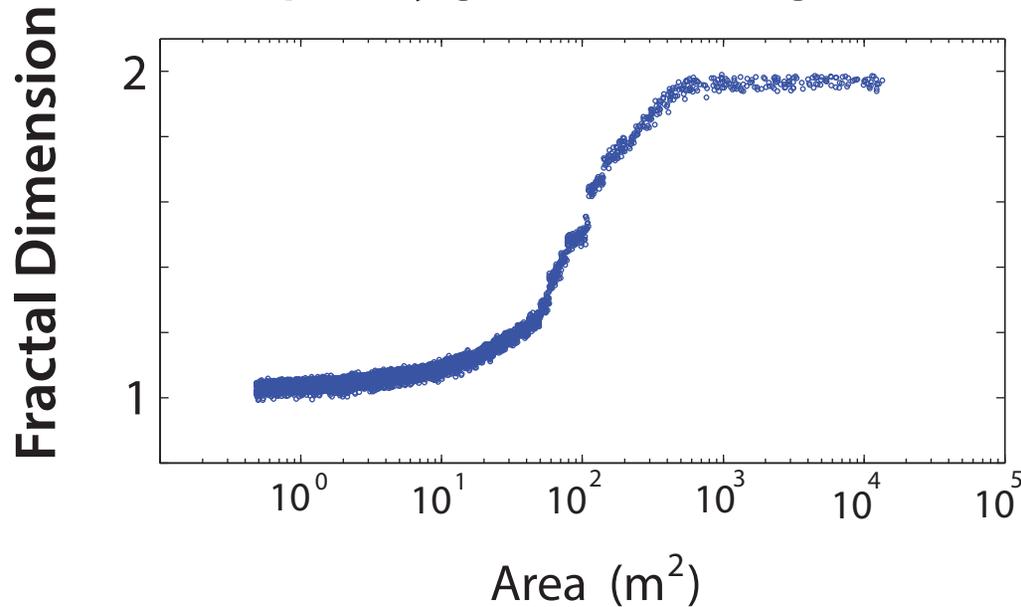


Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

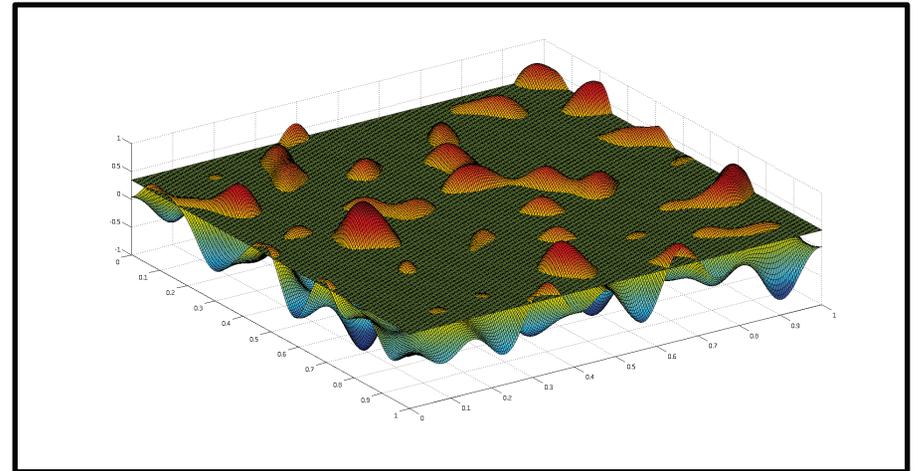
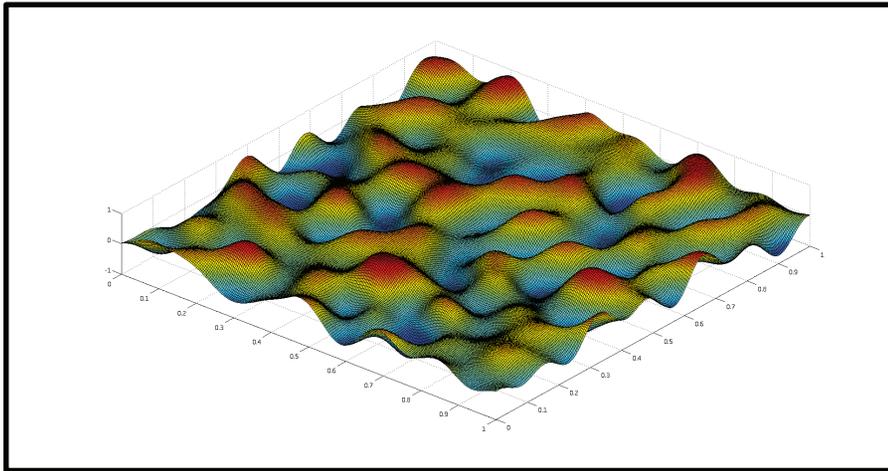
complexity grows with length scale



Continuum percolation model for melt pond evolution

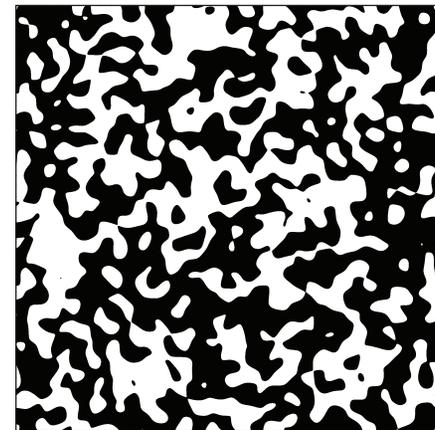
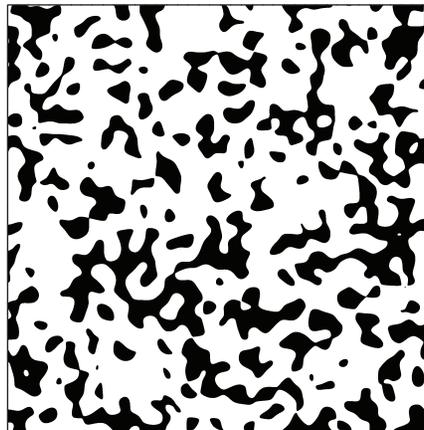
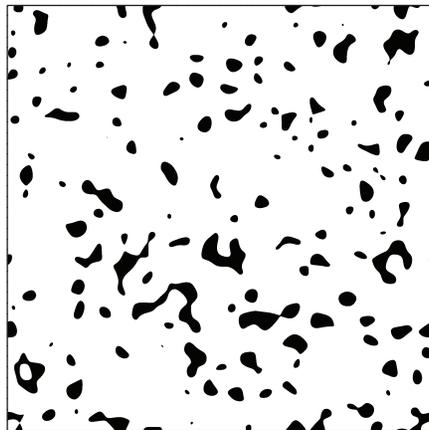
level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds



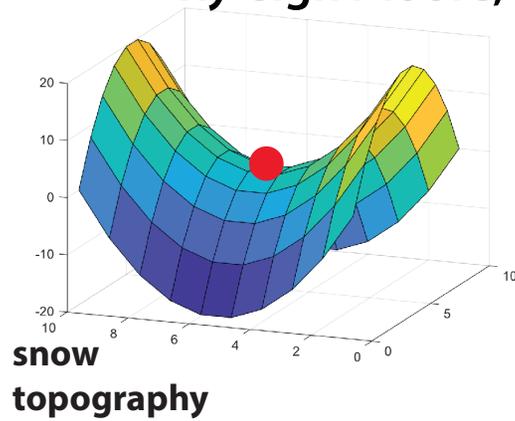
electronic transport in disordered media

diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

Saddle Points, Morse Theory and the Fractal Geometry of Melt Ponds

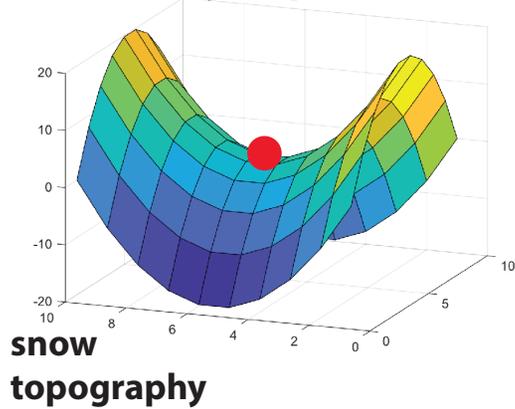
Ryleigh Moore, Jacob Jones, Dane Gollero, Court Strong, Ken Golden 2021



As ponds coalesce at saddle points, fractal dimension proxy isoperimetric quotient $P^2/4\pi A$ jumps, driving the transition.

Saddle Points, Morse Theory and the Fractal Geometry of Melt Ponds

Ryleigh Moore, Jacob Jones, Dane Gollero, Court Strong, Ken Golden 2021



As ponds coalesce at saddle points, fractal dimension proxy isoperimetric quotient $P^2/4\pi A$ jumps, driving the transition.



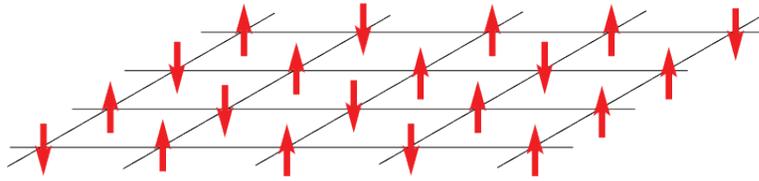
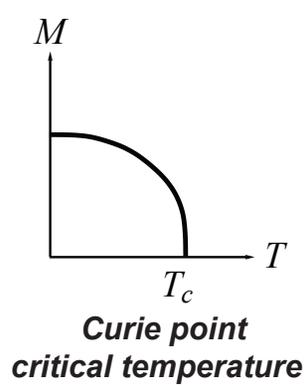
Ryleigh Moore
Department of Mathematics
University of Utah

Multidisciplinary drifting Observatory
for the Study of Arctic Climate (**MOSAiC**)

MOSAiC School aboard the icebreaker *RV Akademik Federov*

20 grad students from around the world
(**3 from U.S., 1 mathematician**)

Ising Model for a Ferromagnet



$$s_i = \begin{cases} +1 & \text{spin up} & \text{blue} \\ -1 & \text{spin down} & \text{white} \end{cases}$$

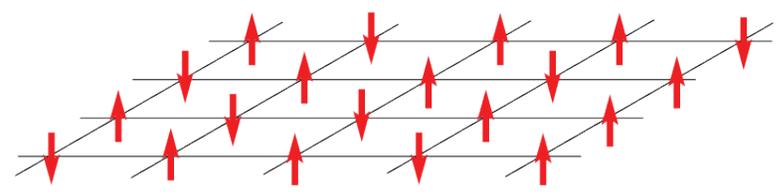
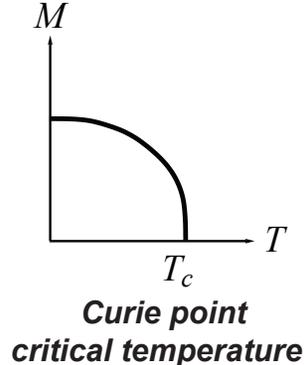
$$\mathcal{H} = -H \sum_i s_i - J \sum_{\langle i,j \rangle} s_i s_j$$

nearest neighbor Ising Hamiltonian

$$M(T, H) = \lim_{N \rightarrow \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

effective magnetization

Ising Model for a Ferromagnet

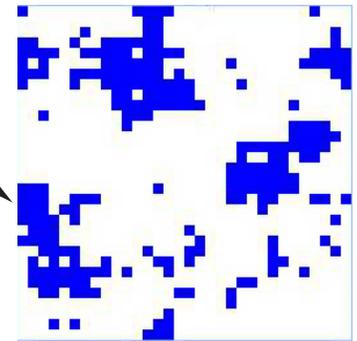


$$s_i = \begin{cases} +1 & \text{spin up} & \text{blue} \\ -1 & \text{spin down} & \text{white} \end{cases}$$

$$\mathcal{H} = -H \sum_i s_i - J \sum_{\langle i,j \rangle} s_i s_j$$

nearest neighbor Ising Hamiltonian

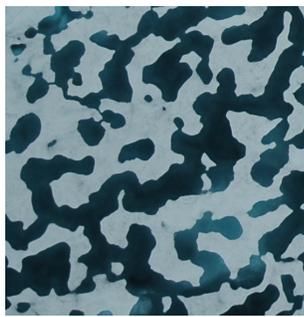
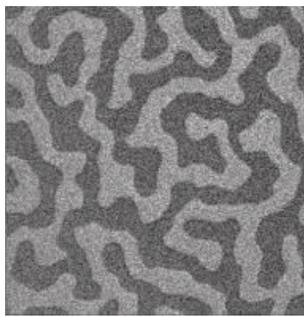
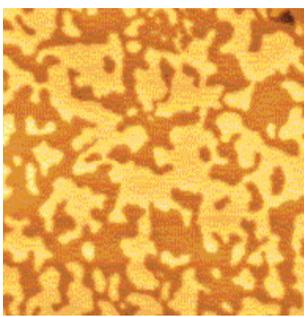
islands of like spins



$$M(T, H) = \lim_{N \rightarrow \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

effective magnetization

energy is lowered when nearby spins align with each other, forming **magnetic domains**



magnetic domains in cobalt

melt ponds (Perovch)

magnetic domains in cobalt-iron-boron

melt ponds (Perovch)

Ising model for ferromagnets \longrightarrow Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

$$\mathcal{H} = - \sum_i^N H_i s_i - J \sum_{\langle i,j \rangle}^N s_i s_j \quad s_i = \begin{cases} \uparrow & +1 \text{ water (spin up)} \\ \downarrow & -1 \text{ ice (spin down)} \end{cases}$$

random magnetic field
represents snow topography

magnetization M pond area fraction $F = \frac{(M+1)}{2}$ only nearest neighbor patches interact
~ albedo

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system “flows” toward metastable equilibria.

Order from Disorder

Ising model for ferromagnets \longrightarrow Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

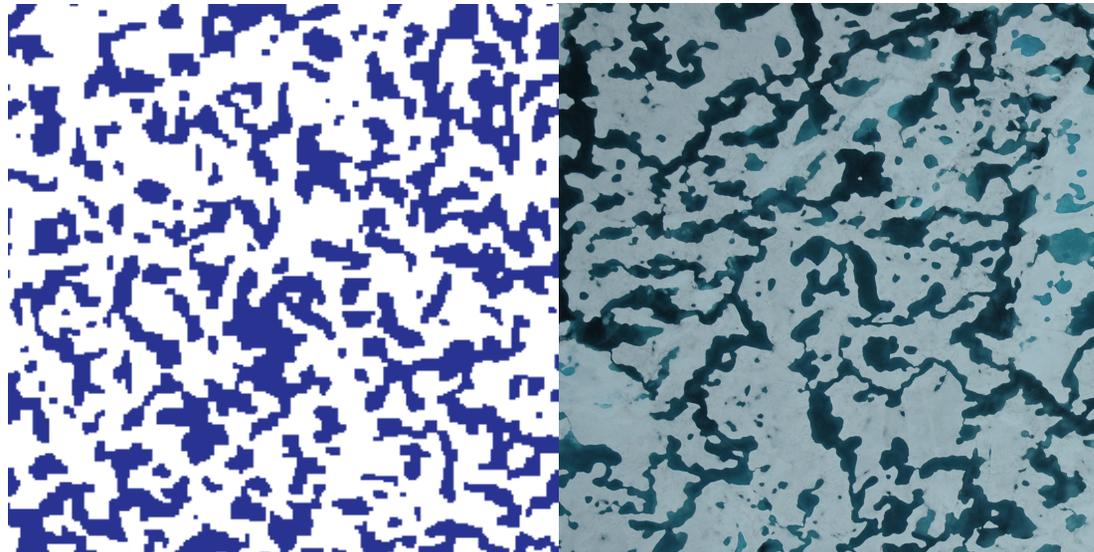
$$\mathcal{H} = - \sum_i^N H_i s_i - J \sum_{\langle i,j \rangle}^N s_i s_j \quad s_i = \begin{cases} \uparrow & +1 \text{ water (spin up)} \\ \downarrow & -1 \text{ ice (spin down)} \end{cases}$$

random magnetic field
represents snow topography

magnetization M pond area fraction $F = \frac{(M+1)}{2}$ only nearest neighbor patches interact
~ albedo

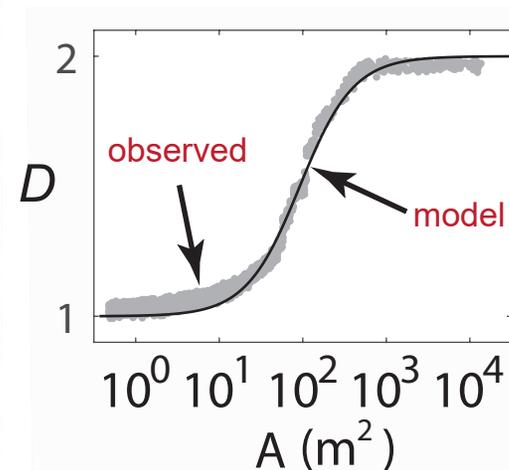
Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system “flows” toward metastable equilibria.

Order from Disorder



Ising
model

melt pond
photo (Perovich)



pond size
distribution exponent

observed -1.5

(Perovich, et al. 2002)

model -1.58

*Scientific American
EOS, PhysicsWorld, ...*

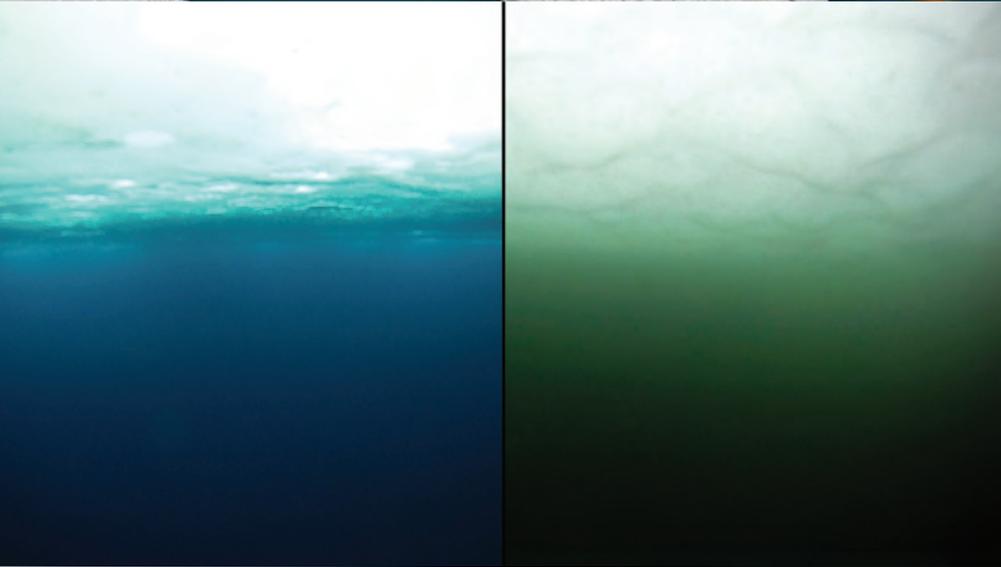
ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data



Perovich

Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

WINDOWS



no bloom

bloom

massive under-ice **algal bloom**

Arrigo et al., *Science* 2012

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances* 2017

The effect of melt pond geometry on the distribution of solar energy under first year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden
Geophys. Res. Lett. 2019

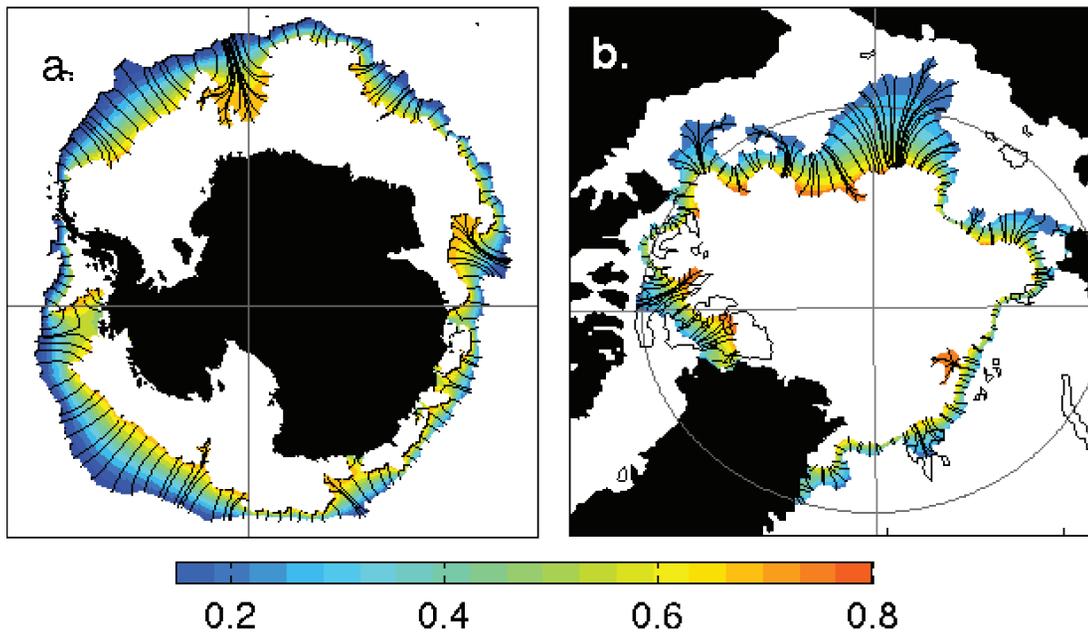
(2015 AMS MRC)

macroscale

Marginal Ice Zone

MIZ

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



MIZ WIDTH
fundamental length scale of
ecological and climate dynamics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

transitional region between
dense interior pack ($c > 80\%$)
sparse outer fringes ($c < 15\%$)

**How to objectively
measure the “width”
of this complex,
non-convex region?**

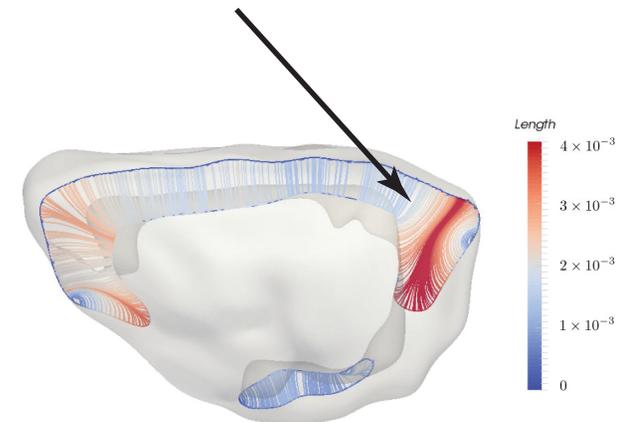
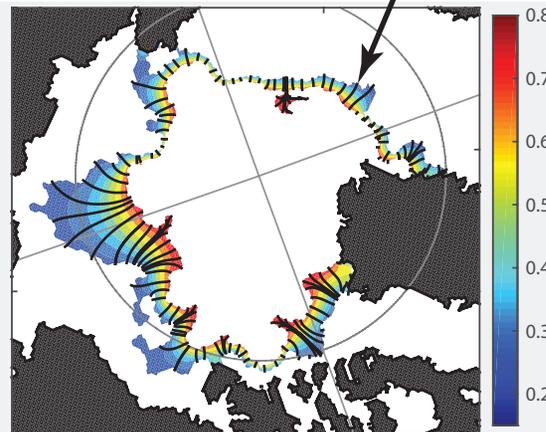
Objective method for measuring MIZ width motivated by medical imaging and diagnostics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

39% widening
1979 - 2012

“average” lengths of streamlines

streamlines of a solution
to Laplace’s equation



Arctic Marginal Ice Zone

**cross-section of the
cerebral cortex of a rodent brain**

analysis of different MIZ WIDTH definitions

Strong, Foster, Cherkaev, Eisenman, Golden
J. Atmos. Oceanic Tech. 2017

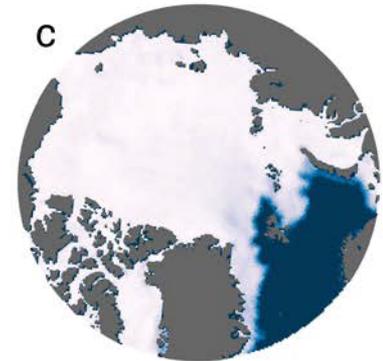
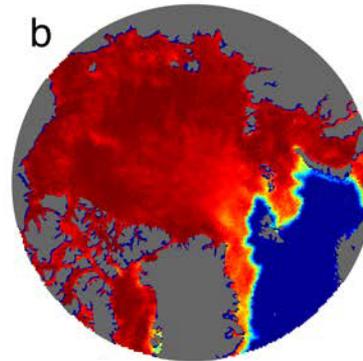
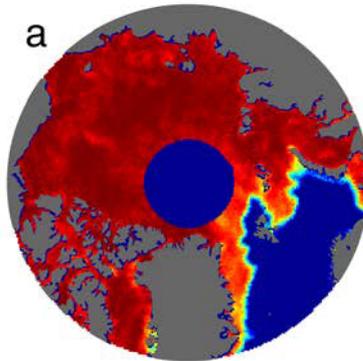
Strong and Golden
Society for Industrial and Applied Mathematics News, April 2017

Filling the polar data gap with partial differential equations

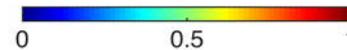
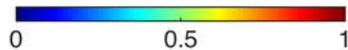
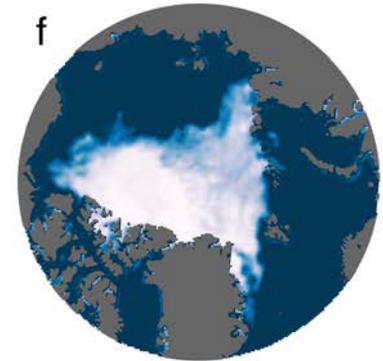
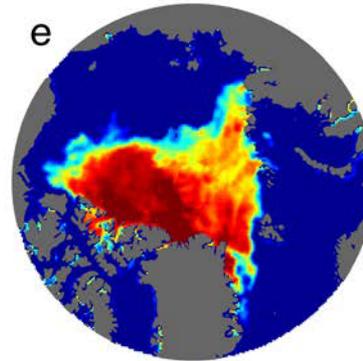
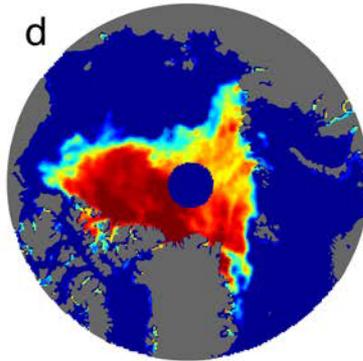
hole in satellite coverage
of sea ice concentration field

previously assumed
ice covered

Gap radius: 611 km
06 January 1985



Gap radius: 311 km
30 August 2007



$$\Delta\psi=0$$

fill with harmonic function satisfying
satellite BC's plus stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

**NOAA/NSIDC Sea Ice Concentration CDR
product update will use our PDE method.**



**Who cares if
Arctic sea ice
disappears?**



Ralph (Malik) Ahkivgak, c. 20 Oct 1988

© Bill Hess – Running Dog Publications; <http://wasillaalaskaby300.squarespace.com/>

Drew Barrymore

John Krasinski



INSPIRED BY THE INCREDIBLE TRUE STORY
that united the world

BIG MIRACLE

seaice.alaska.edu/gi

Use of sea ice as a platform

- Walrus life cycle tied to sea-ice cycle
- Ice floes as diving platforms for feeding over shallow shelf

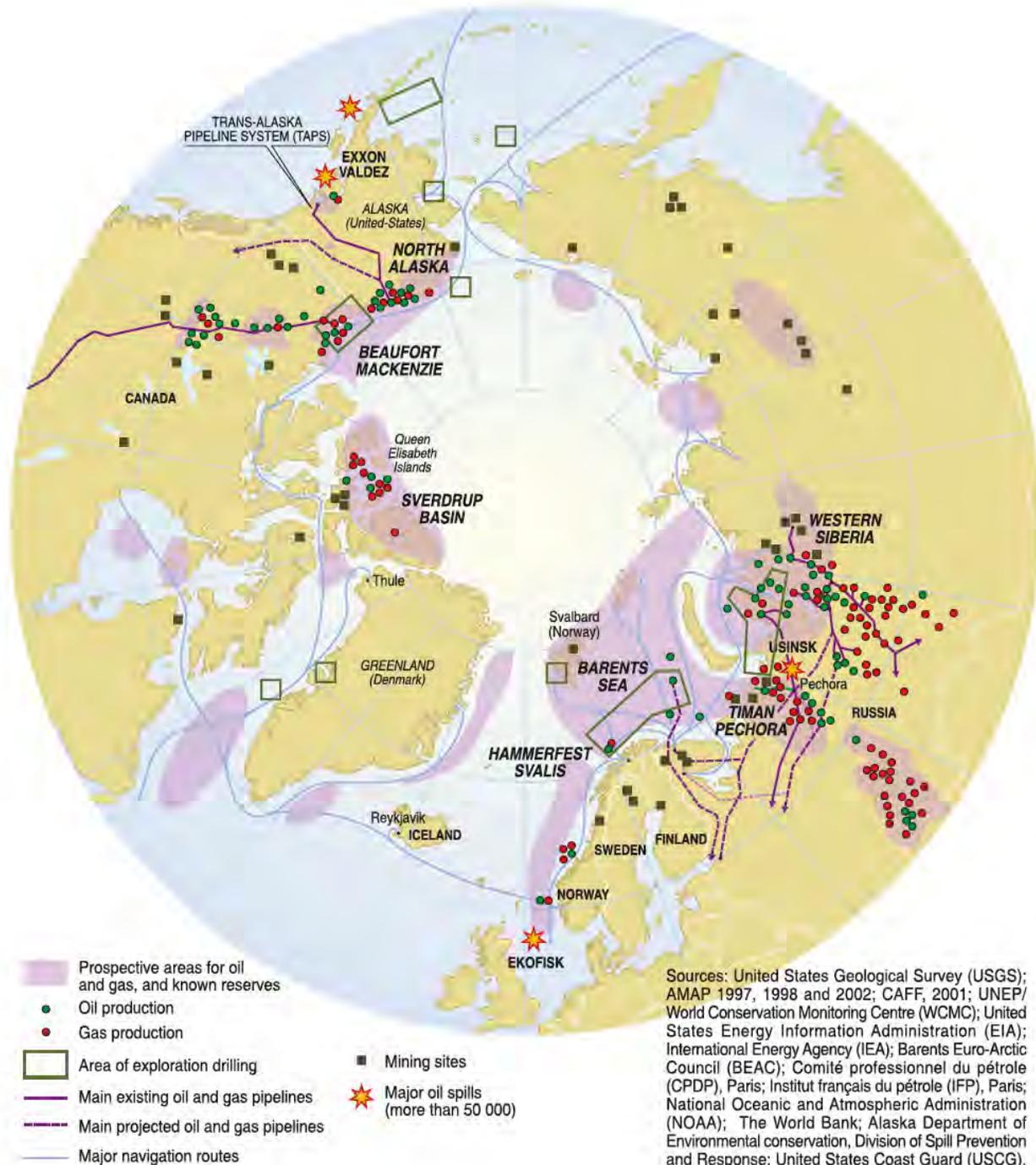


Photo: Marc Webber, US Fish & Wildlife Service

BMCM Tim Sullivan

oil companies care about Arctic sea ice loss

- The Arctic holds 25% of the world's undiscovered oil & gas reserves
- Sea ice is both a hazard and a supporting feature for hydrocarbon exploration & production



Source: UNEP/GRID-Arendal

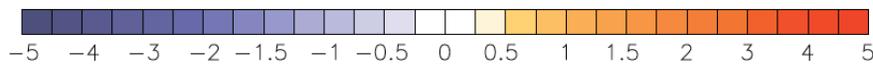
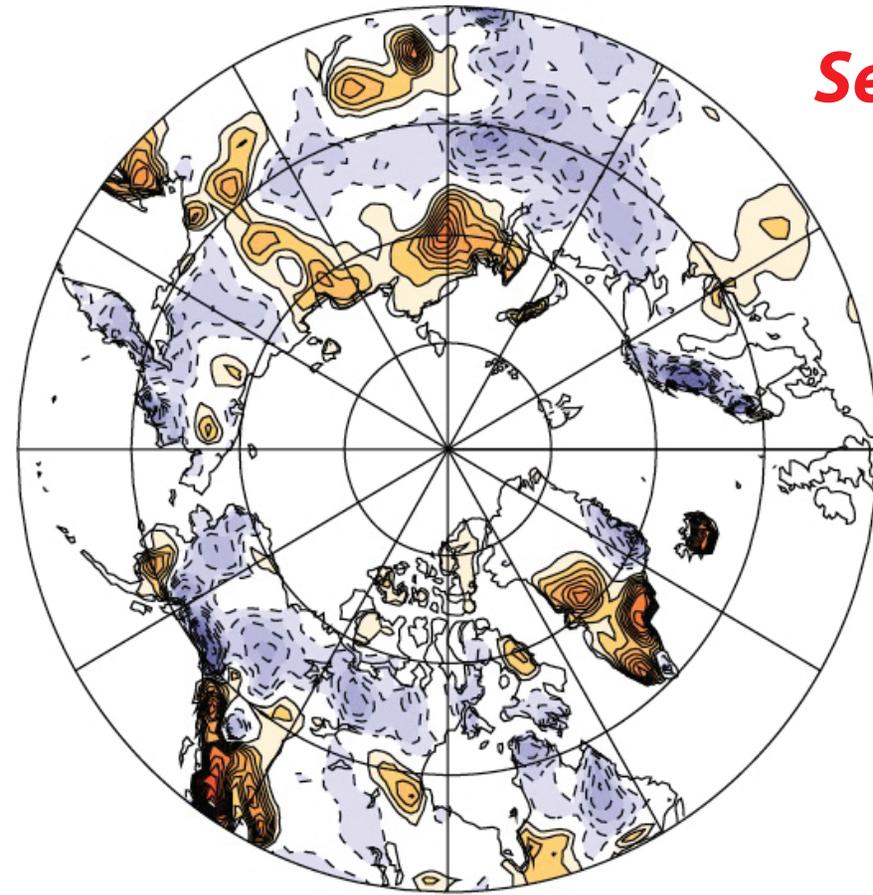
Sea-ice loss: impacts beyond the Arctic

changes in precipitation and temperature patterns, storm tracks, ...

- One climate model projects reduced precipitation in American West (Sewall & Sloan, 2005)

Utah - greatest snow on Earth?

- Analysis of 2007 ice minimum suggests above normal snow deposition in NW North America (Orsolini et al., 2011)
- Colder weather in SE Asia, possibly in Eastern US (Hondo et al., 2009)



Orsolini et al., 2011

Conclusions

1. Sea ice is a fascinating multiscale composite with structure similar to many other natural and man-made materials.
2. Mathematical methods developed for sea ice advance the theory of composites and other areas of science and engineering.
3. **Homogenization and statistical physics help *link scales in sea ice and composites***; provide rigorous methods for finding effective behavior; advance sea ice representations in climate models.
4. **Fluid flow** through sea ice mediates **melt pond evolution** and many processes important to climate change and polar ecosystems.
5. Field experiments are essential to developing relevant mathematics.
6. Our research is helping to **improve projections of climate change**, the fate of Earth's sea ice packs, and the ecosystems they support.

University of Utah Sea Ice Modeling Group (2017-2021)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics
Elena Cherkaev, Professor of Mathematics
Court Strong, Associate Professor of Atmospheric Sciences
Ben Murphy, Adjunct Assistant Professor of Mathematics

Postdoctoral Researchers: Noa Kraitzman (now at ANU), Jody Reimer

Graduate Students: Kyle Steffen (now at UT Austin with Clint Dawson)
Christian Sampson (now at UNC Chapel Hill with Chris Jones)
Huy Dinh (now a sea ice MURI Postdoc at NYU/Courant)
Rebecca Hardenbrook
David Morison (Physics Department)
Ryleigh Moore
Delaney Mosier
Daniel Hallman

Undergraduate Students: Kenzie McLean, Jacqueline Cinella Rich,
Dane Gollero, Samir Suthar, Anna Hyde,
Kitsel Lusted, Ruby Bowers, Kimball Johnston,
Jerry Zhang, Nash Ward, David Gluckman

High School Students: Jeremiah Chapman, Titus Quah, Dylan Webb

Sea Ice Ecology Group Postdoc Jody Reimer, Grad Student Julie Sherman,
Undergraduates Kayla Stewart, Nicole Forrester

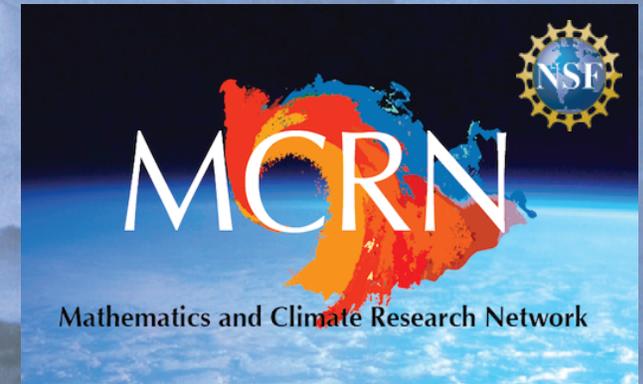
THANK YOU

Office of Naval Research

Applied and Computational Analysis Program
Arctic and Global Prediction Program

National Science Foundation

Division of Mathematical Sciences
Division of Polar Programs



Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship Aurora Australis without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 186km off the coast.

The extent of the damage is not known.

Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an inert gas.

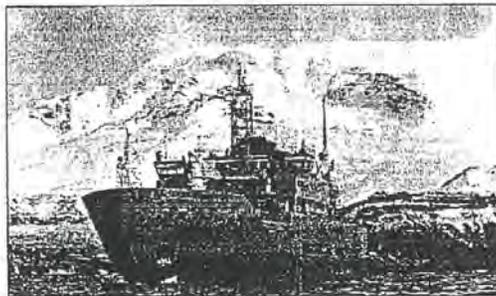
The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known.

The 25 crew and 54 expeditioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year but at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Div-



A file photo of the Aurora Australis in Antarctica.

ision's Hobart office.

He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available," Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The Aurora Australis is owned by P&O Australia and chartered by the Antarctic Div-

ision for about \$11 million a year.

P&O Australia managing director Richard Hein said yesterday the company was assessing the situation and a number of rescue options were being considered.

It was too early to say whether P&O would be liable for the cost of the aborted mission.

The vessel left Hobart last Wednesday for a seven-week voyage mainly to study a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.



Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

Fire strands Antarctic ship in sea ice

AN engine room fire has disabled the icebreaker Aurora Australis in sea ice, deep in Antarctic waters.

There were no injuries and the ship was not in danger after Tuesday night's fire,

Australian Antarctic Division director Mr Rex Moncur said. But Mr Moncur said he expected it would have to abandon its pioneering mid-winter voyage to the edge of the Ant-

arctic continent and return to Hobart for repairs.

The cause of the fire was not known but the engines have been turned off, with the ship 100 nautical miles from the Antarctic coast.

THE CANBERRA TIMES

Thursday 23 July 1998

Page 4

Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Australian icebreaker Aurora Australis in sea ice, deep in Antarctic waters.

Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected Aurora Australis would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs.

The fire had been extinguished and the engines were turned off, leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good.

Crew had to wear breathing apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully assessed.

The Aurora, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

Mr Moncur said the cause of the fire was not yet known.

2:45 am July 22, 1998

"Please don't be alarmed but we have an uncontrolled fire in the engine room ..."

about 10 minutes later ...

"Please don't be alarmed but we're lowering the lifeboats ..."

Sydney Morning Herald
23 July, 1998

ICEBREAKER BURNS

A pioneering \$2-million Australian scientific voyage to the mid-winter Antarctic polynya is expected to be scrapped following an engine room fire on the Aurora Australis yesterday. The 54 people on board were forced on deck in the



ISSN 0002-9920 (print)
ISSN 1088-9477 (online)



Notices

of the American Mathematical Society

November 2020

Volume 67, Number 10



AMS AMERICAN
MATHEMATICAL
SOCIETY
Advancing research. Creating connections.

*The cover is based on "Modeling Sea Ice,"
page 1535.*

Modeling Sea Ice



*Kenneth M. Golden, Luke G. Bennetts,
Elena Cherkaev, Ian Eisenman, Daniel Feltham,
Christopher Horvat, Elizabeth Hunke,
Christopher Jones, Donald K. Perovich,
Pedro Ponte-Castañeda, Courtenay Strong,
Deborah Sulsky, and Andrew J. Wells*

Kenneth M. Golden is a Distinguished Professor of Mathematics at the University of Utah. His email address is golden@math.utah.edu.

Luke G. Bennetts is an associate professor of applied mathematics at the University of Adelaide. His email address is luke.bennetts@adelaide.edu.au.

Elena Cherkaev is a professor of mathematics at the University of Utah. Her email address is elena@math.utah.edu.

Ian Eisenman is an associate professor of climate, atmospheric science, and physical oceanography at the Scripps Institution of Oceanography at the University of California San Diego. His email address is eisenman@ucsd.edu.

Daniel Feltham is a professor of climate physics at the University of Reading. His email address is d.f.feltham@reading.ac.uk.

Christopher Horvat is a NOAA Climate and Global Change Postdoctoral Fellow at the Institute at Brown for Environment and Society at Brown University. His email address is christopher_horvat@brown.edu.

Elizabeth Hunke is a deputy group leader, T-3 fluid dynamics and solid mechanics group at the Los Alamos National Laboratory. Her email address is ec1are@lanl.gov.

Christopher Jones is a Bill Guthridge Distinguished Professor of Mathematics

at the University of North Carolina, Chapel Hill. His email address is ckrtj@unc.edu.

Donald K. Perovich is a professor of engineering at the Thayer School of Engineering at Dartmouth College. His email address is donald.k.perovich@dartmouth.edu.

Pedro Ponte-Castañeda is a Raymond S. Markowitz Faculty Fellow and professor of mechanical engineering and applied mechanics and of mathematics at the University of Pennsylvania. His email address is ponte@seas.upenn.edu.

Courtenay Strong is an associate professor of atmospheric sciences at the University of Utah. His email address is court_strong@utah.edu.

Deborah Sulsky is a professor of mathematics and statistics and of mechanical engineering at the University of New Mexico. Her email address is sulsky@math.unm.edu.

Andrew J. Wells is an associate professor of physical climate science at the University of Oxford. His email address is Andrew.Wells@physics.ox.ac.uk.

Communicated by Notices Associate Editor Reza Malek-Madani.

*For permission to reprint this article, please contact:
reprint-permission@ams.org.*

Special Issue on the Mathematics of Planet Earth

Read about the application of mathematics and computational science to issues concerning invasive populations, Arctic sea ice, insect flight, and more in this Planet Earth **special issue!**

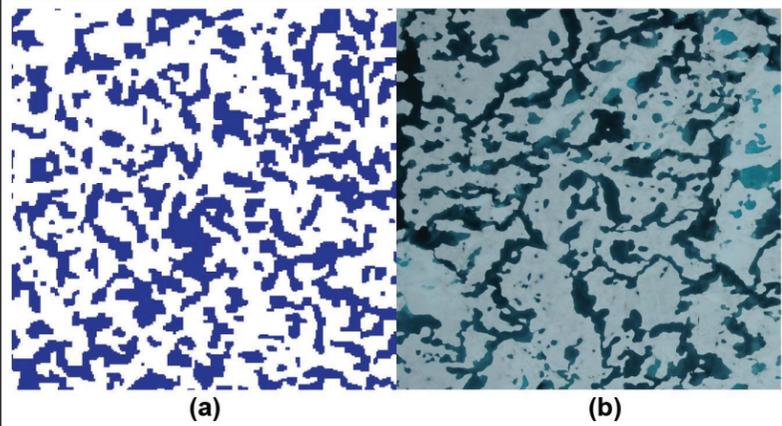


Figure 3. Comparison of real Arctic melt ponds with metastable equilibria in our melt pond Ising model. **3a.** Ising model simulation. **3b.** Real melt pond photo. Figure 3a courtesy of Yiping Ma, 3b courtesy of Donald Perovich.

Vast labyrinthine ponds on the surface of melting Arctic sea ice are key players in the polar climate system and upper ocean ecology. Researchers have adapted the Ising model, which was originally developed to understand magnetic materials, to study the geometry of meltwater's distribution over the sea ice surface. In an article on page 5, Kenneth Golden, Yiping Ma, Courtenay Strong, and Ivan Sudakov explore model predictions.

Controlling Invasive Populations in Rivers

By Yu Jin and Suzanne Lenhart

Flow regimes can change significantly over time and space and strongly impact all levels of river biodiversity, from the individual to the ecosystem. Invasive species in rivers—such as bighead and silver carp, as well as quagga and zebra mussels—continue to cause damage. Management of these species may include targeted adjustment of flow rates in rivers, based on recent research that examines the effects of river morphology and water flow on rivers' ecological statuses. While many previous methodologies rely on habitat suitability models or oversimplification of the hydrodynamics, few studies have focused on the integration of ecological dynamics into water flow assessments.

Earlier work yielded a hybrid modeling approach that directly links river hydrology with stream population models [3]. The hybrid model's hydrodynamic component is based on the water depth in a gradually varying river structure. The model derives the steady advective flow from this structure and relates it to flow features like water discharge, depth, velocity, cross-

sectional area, bottom roughness, bottom slope, and gravitational acceleration. This approach facilitates both theoretical understanding and the generation of quantitative predictions, thus providing a way for scientists to analyze the effects of river fluctuations on population processes.

When a population spreads longitudinally in a one-dimensional (1D) river with spatial heterogeneities in habitat and temporal fluctuations in discharge, the resulting hydrodynamic population model is

$$N_t = -A_t(x,t) \frac{N}{A(x,t)} + \frac{1}{A(x,t)} (D(x,t)A(x,t)N_x)_x - \frac{Q(t)}{A(x,t)} N_x + rN \left(1 - \frac{N}{K}\right)$$

$$\begin{aligned} N(0,t) &= 0 && \text{on } (0,T), x=0, \\ N_x(L,t) &= 0 && \text{on } (0,T), x=L, \\ N(x,0) &= N_0(x) && \text{on } (0,L), t=0 \end{aligned} \quad (1)$$

See **Invasive Populations** on page 4

Modeling Resource Demands and Constraints for COVID-19 Intervention Strategies

By Erin C.S. Acquesta, Walt Beyeler, Pat Finley, Katherine Klise, Monear Makvandi, and Emma Stanislawski

As the world desperately attempts to control the spread of COVID-19, the need for a model that accounts for realistic trade-offs between time, resources, and corresponding epidemiological implications is apparent. Some early mathematical models of the outbreak compared trade-offs for non-pharmaceutical interventions [3], while others derived the necessary level of test coverage for case-based interventions [4] and demonstrated the value of prioritized testing for close contacts [7].

Isolated analyses provide valuable insights, but real-world intervention strategies are interconnected. Contact tracing is the lynchpin of infection control [6] and forms the basis of prioritized testing. Therefore, quantifying the effectiveness of contact tracing is crucial to understanding the real-life implications of disease control strategies.

Contact Tracing Demands

Contact tracers are skilled, culturally competent interviewers who apply their knowledge of disease and risk factors when notifying people who have come into contact with COVID-19-infected individuals. They also continue to monitor the situation after case investigations [1].

Case investigation consists of four steps:

1. Identify and notify cases
2. Interview cases
3. Locate and notify contacts
4. Monitor contacts.

Most health departments are implementing case investigation, contact identification, and quarantine to disrupt COVID-19 transmission. The timeliness of contact tracing is constrained by the length of the infectious period, the turn-around time for testing and result reporting, and the ability to successfully reach and interview patients and their contacts. The European Centre for Disease Prevention and Control approximates that contact tracers spend one to two hours conducting an interview [2]. Estimates regarding the timelines of other steps are limited to subject matter expert elicitation and can vary based on cases' access to phone service or willingness to participate in interviews.

Bounded Exponential

The fundamental structure of our model follows traditional susceptible-exposed-infected-recovered (SEIR) compartmental modeling [5]. We add an asymptomatic population A , a hospitalized population H , and disease-related deaths D , as well as corresponding quarantine states. We define the states $\{S_i, E_i, A_i, I_i, H, R, D\}_{i=0,1}$ for our compartments, such that $i=0$ and $i=1$

correspond to unquarantined and quarantined respectively. Rather than focus on the dynamics that are associated with the state transition diagram in Figure 1, we introduce a formulation for the real-time demands on contact tracers' time as a function of infection prevalence, while also respecting constraints on resources.

When the work that is required to investigate new cases and monitor existing contacts exceeds available resources, a backlog develops. To simulate this backlog, we introduce a new compartment C for tracking the dynamic states of cases:

$$\frac{dC}{dt} = [flow_{in}] - [flow_{out}].$$

Flow into the backlog compartment, represented by $[flow_{in}]$, reflects case identification that is associated with the following transitions in the model:

- The rate of random testing: $q_{rA}(t)A_0(t) \rightarrow A_1(t)$ and $q_{rI}(t)I_0(t) \rightarrow I_1(t)$
- Testing triggered by contact tracing: $q_{tA}(t)A_0(t) \rightarrow A_1(t)$, $q_{tI}(t)I_0(t) \rightarrow I_1(t)$, and $q_{tE}(t)E_1(t) \rightarrow \{A_1(t), I_1(t)\}$
- The population that was missed by the non-pharmaceutical interventions that require hospitalization: $\tau_{IH}(t)I_0(t) \rightarrow H(t)$.

Here, $q_{rA}(t)$ defines the time-dependent rate of random testing, $q_{tA}(t)$ signifies the time-dependent rate of testing that is triggered by contact tracing, and τ_{IH} is the inverse of the expected amount of time for which an infected individual is symptomatic before hospitalization. These terms collectively provide the simulated number of newly-identified positive COVID-19 cases. However, we also need the average number of contacts per case. We thus define function $\mathcal{K}(\kappa, T_S, \phi_\kappa)$ that depends on the average number of contacts a day (κ), the average number of days for which an individual is infectious before going into isolation (T_S), and the likelihood that the individual

See **COVID-19 Intervention** on page 3

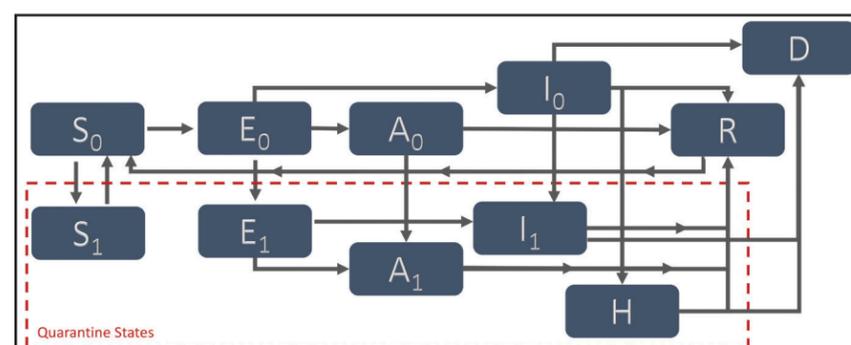


Figure 1. Disease state diagram for the compartmental infectious disease model. Figure courtesy of the authors.

Nonprofit Org
U.S. Postage
PAID
Permit No 360
Bellmawr, NJ

siam
SOCIETY for INDUSTRIAL and APPLIED MATHEMATICS
3600 Market Street, 6th Floor
Philadelphia, PA 19104-2688 USA