critical exponent $t$ for conductivity and permeability of lattices

numerical estimates in $d = 3$: $t = 2.0$

Derrida, Stauffer, Herrmann, Vannimenus 1983
Gingold, Lobb 1990
Adler, Meir, Aharony, Harris, Klein 1990
Berkowitz, Balberg 1992
⋮

rigorous inequality for $t$ :

$$1 \leq t \leq 2, \qquad d = 2, 3$$

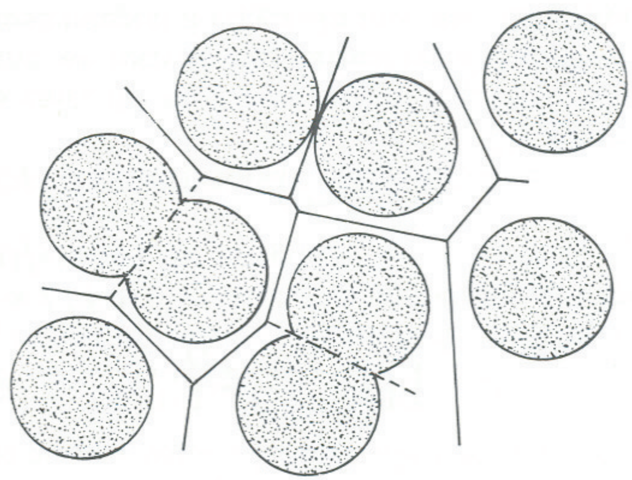
Golden, *Phys. Rev. Lett.* 1990
*Comm. Math. Phys.* 1992

# Non-universal behavior in the continuum:

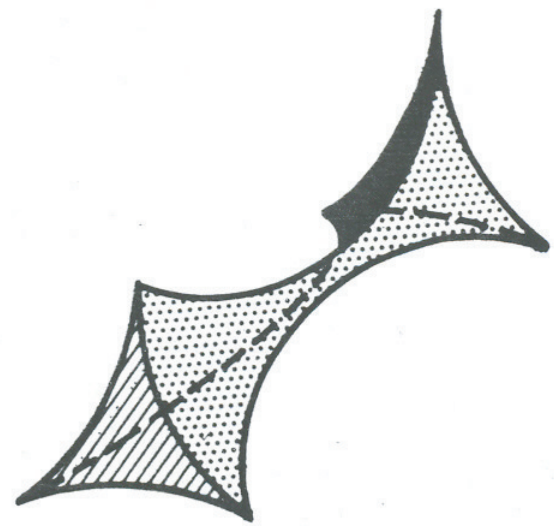
critical exponents for transport in Swiss cheese model take values different than for lattices, e.g. $t > 2$

$e \neq t$

Halperin, Feng, Sen, *Phys. Rev. Lett.* 1985



Swiss cheese model
$d = 2$



conducting neck in $d = 3$
Swiss cheese model

in general, non-universal exponents arise from a singular distribution of local conductances

In sea ice, this distribution is lognormal.

(excluding inclusions below cutoff)

Thus, the permeability exponent for sea ice is **2**, the universal lattice value.

ESTIMATE fluid conductivity scaling factor $k_0 = r^2 / 8$

CRITICAL PATH ANALYSIS ***bottlenecks control flow***

Ambegaokar, Halperin, Langer 1971: CPA in electronic hopping conduction
Friedman, Seaton 1998: CPA in fluid and electrical networks
Golden, Kozlov 1999: rigorous CPA on long-range checkerboard model

$$k_0 \approx r_c^2 / 8$$ critical fluid conductivity

Microstructural analyses yield $r_c \approx$ 0.5 mm

$$k(\phi) \sim 3(\phi - \phi_c)^2 \times 10^{-8}\ \mathrm{m}^2$$