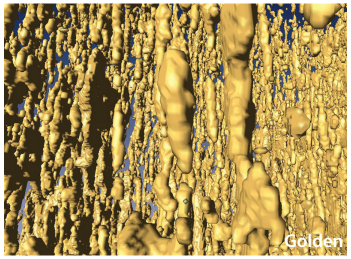
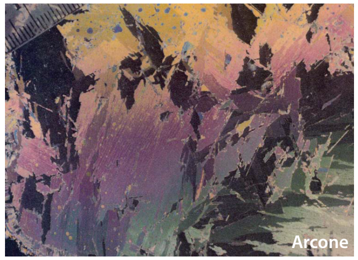
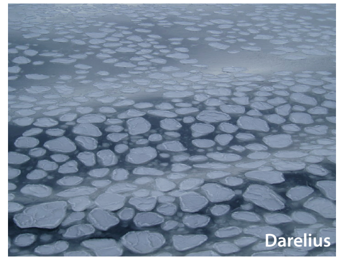
millimeters



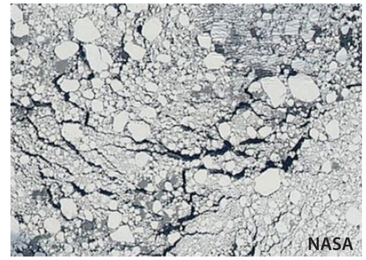
centimeters



meters



kilometers

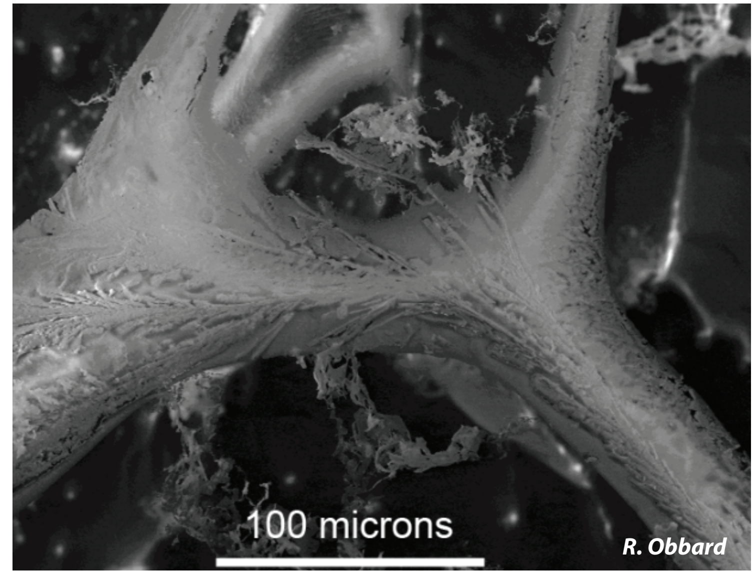


10^3 kilometers

From Micro to Macro in the Physics and Biology of Sea Ice

Kenneth M. Golden
Department of Mathematics
University of Utah

brine inclusions in sea ice (mm)



micro - brine channel (SEM)

brine channels (cm)

sea ice is a porous composite

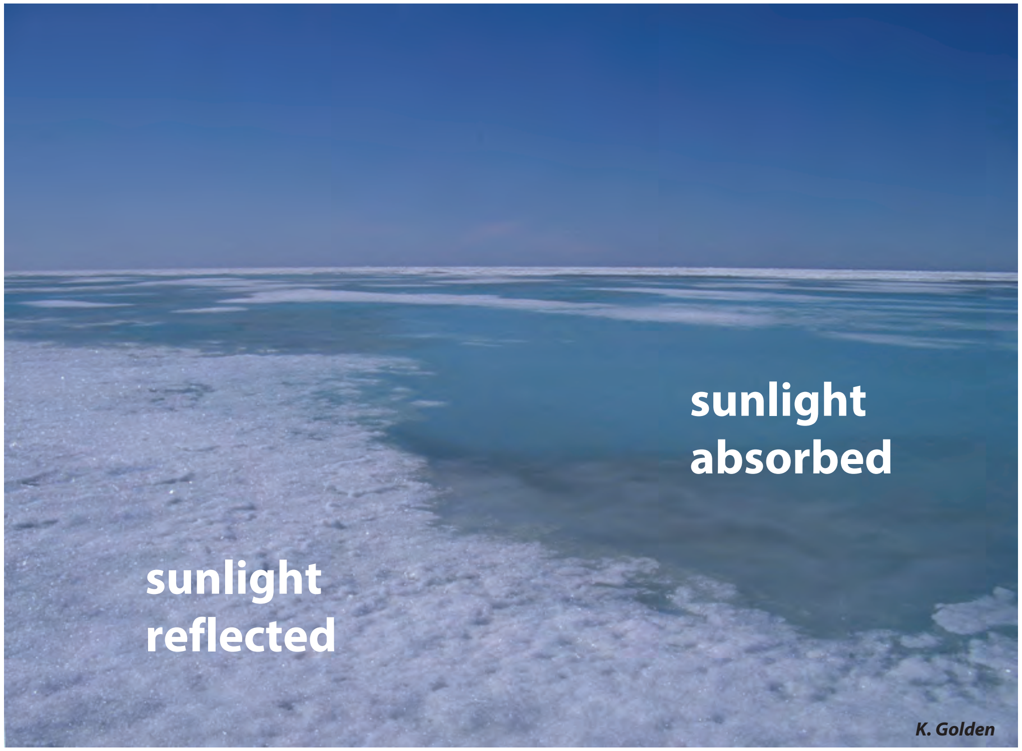
pure ice with brine, air, and salt inclusions

horizontal section

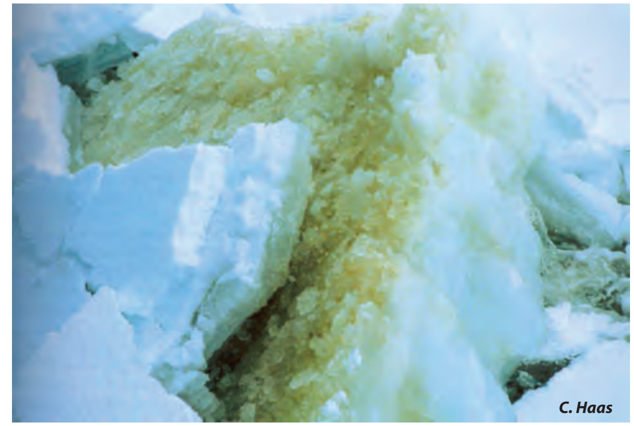
vertical section

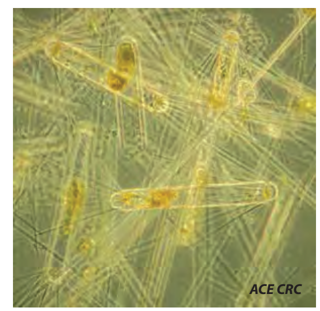
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

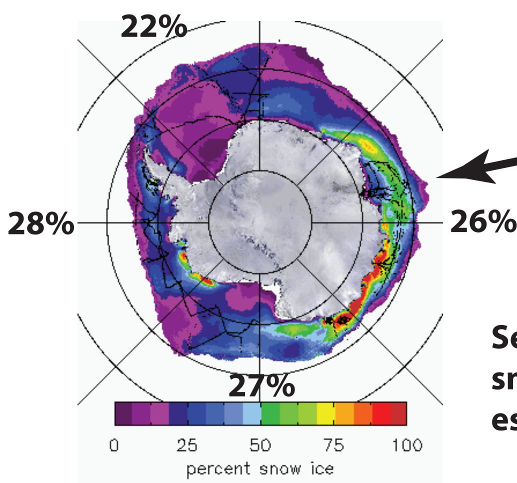
evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







T. Maksym and T. Markus, 2008

September snow-ice estimates

Antarctic surface flooding and snow-ice formation

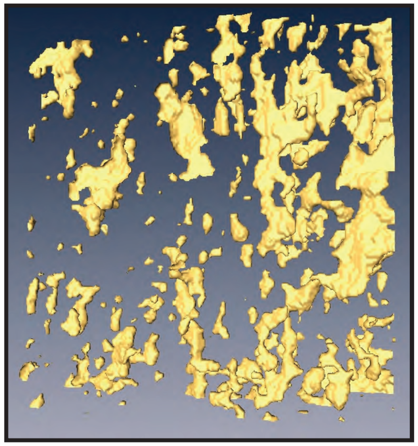
- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO_2*

Sea Ice is a Multiscale Composite Material

microscale

brine inclusions

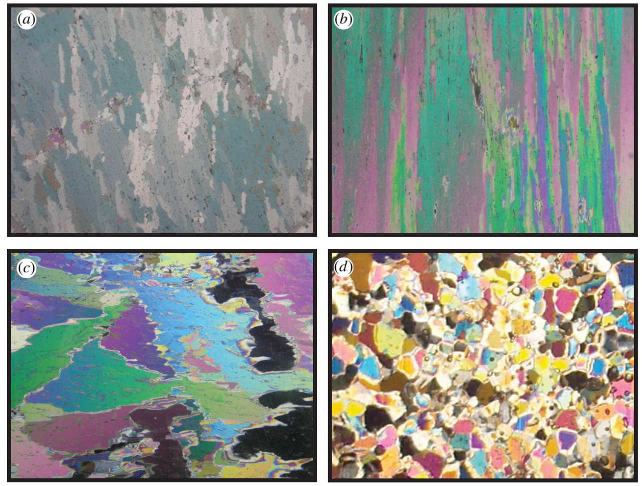
Weeks & Assur 1969



H. Eicken

Golden *et al. GRL* 2007

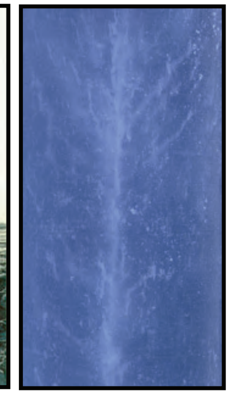
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

millimeters

centimeters

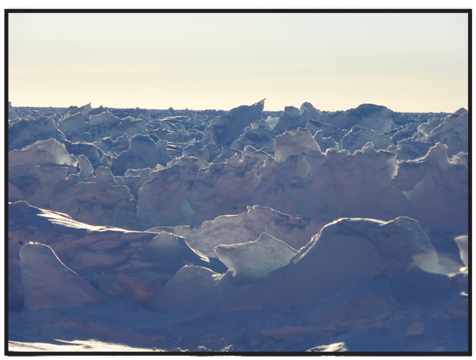
---

mesoscale

Arctic melt ponds

K. Frey

Antarctic pressure ridges



K. Golden

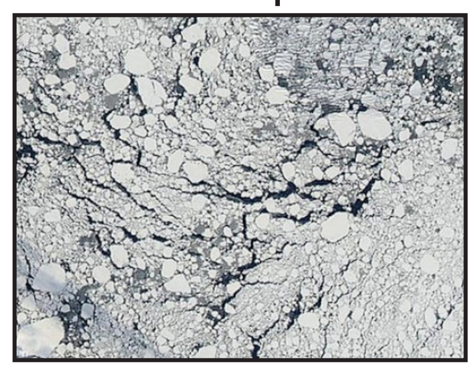
meters

macroscale

sea ice floes

J. Weller

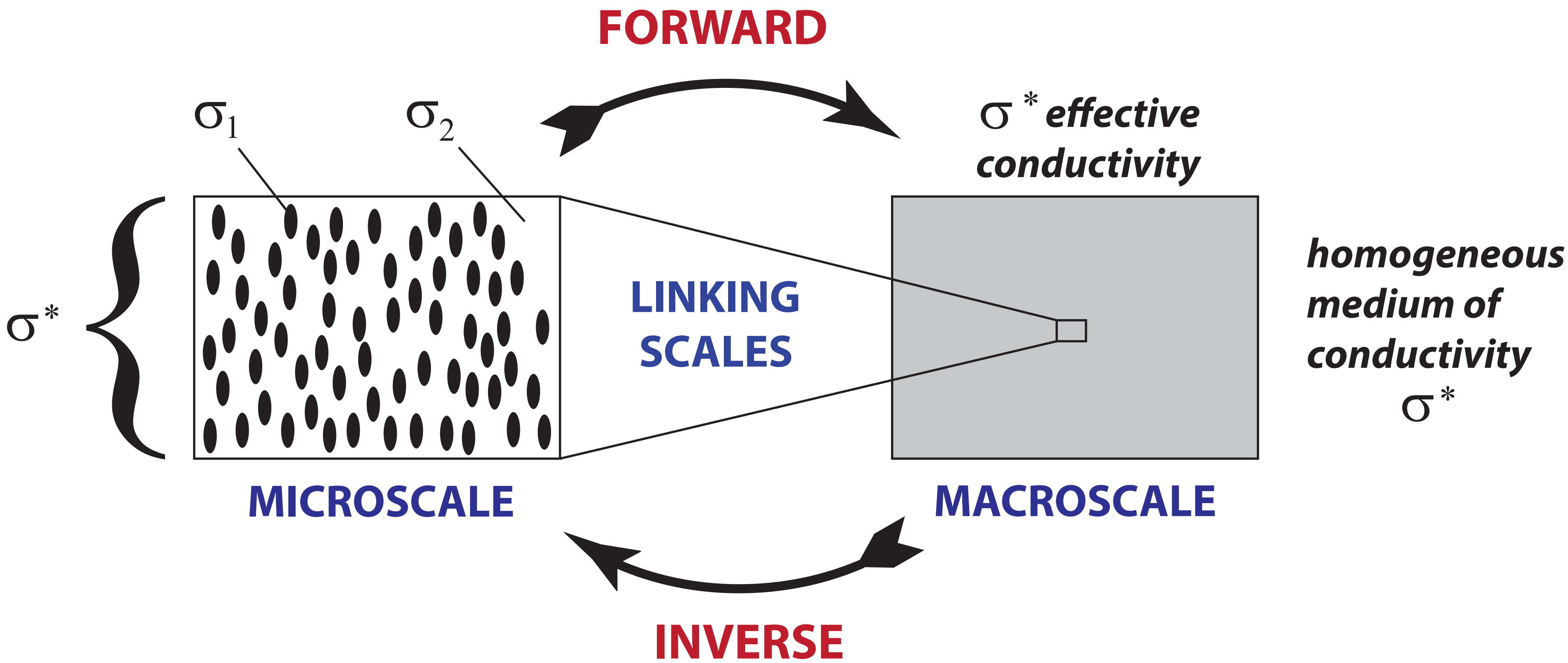
sea ice pack



NASA

kilometers

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

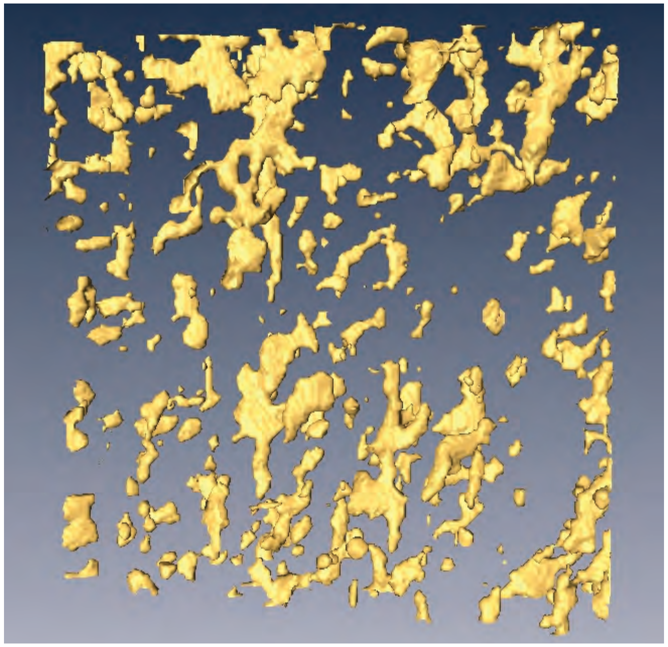
widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is this talk about?

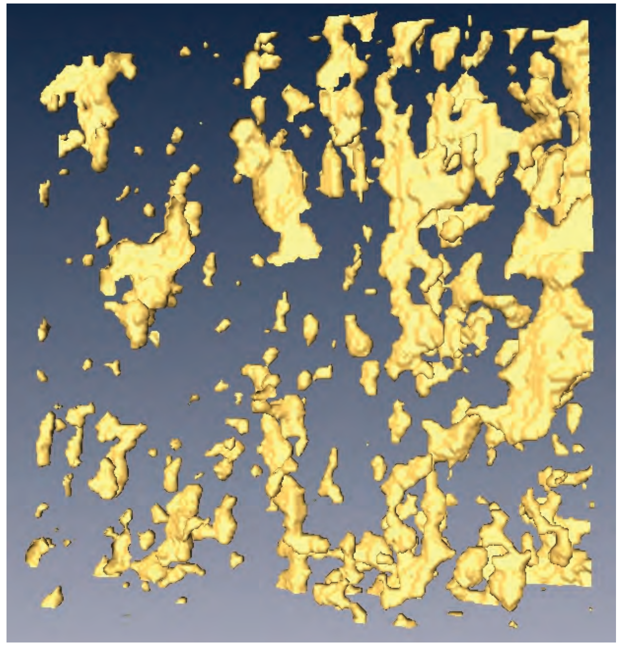
A tour of recent results on modelling macroscopic behaviour in the sea ice system, with a focus on novel mathematics.

microscale

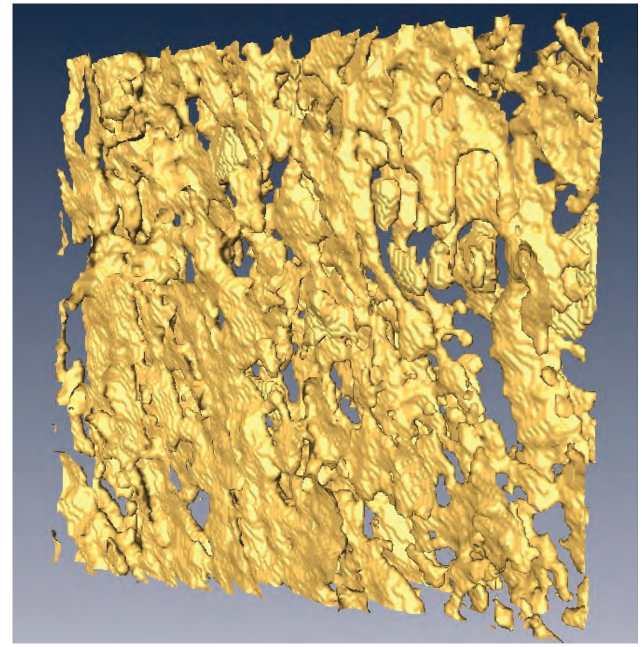
brine volume fraction and ***connectivity*** increase with temperature



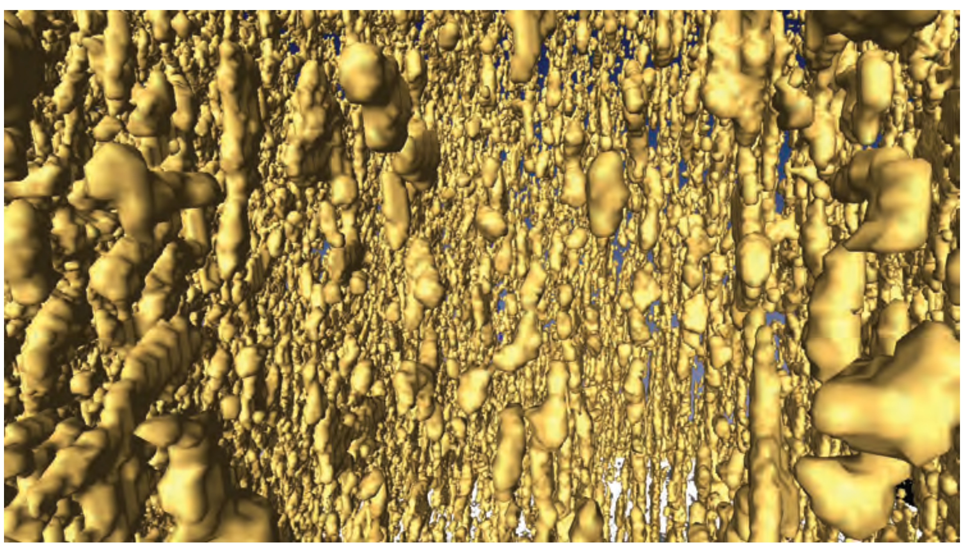
T = -15 °C, $\phi = 0.033$



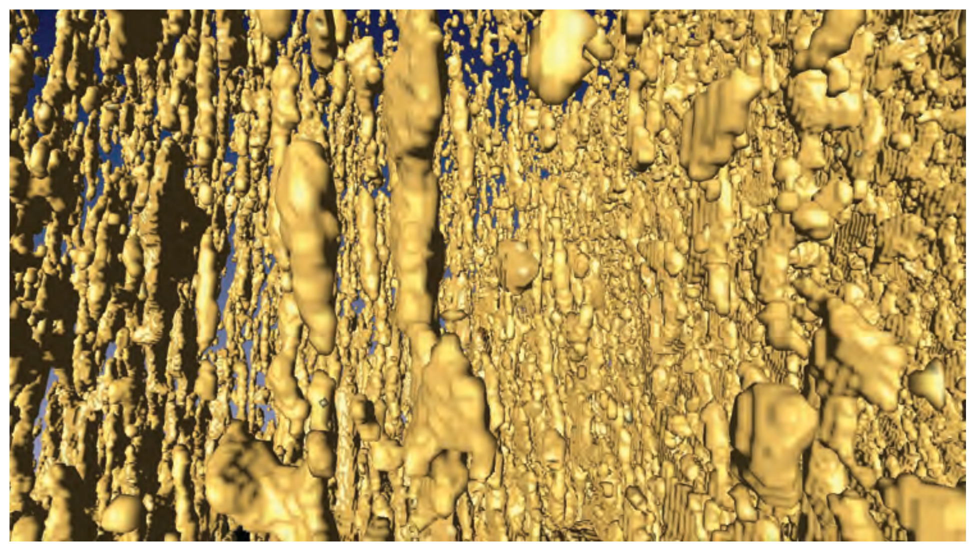
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$



T = -8 °C, $\phi = 0.057$



T = -4 °C, $\phi = 0.113$

X-ray tomography for brine in sea ice

Golden *et al.*, *Geophysical Research Letters*, 2007

Critical behavior of fluid transport in sea ice

impermeable permeable

"on - off" switch
for bulk fluid flow

Arctic field data

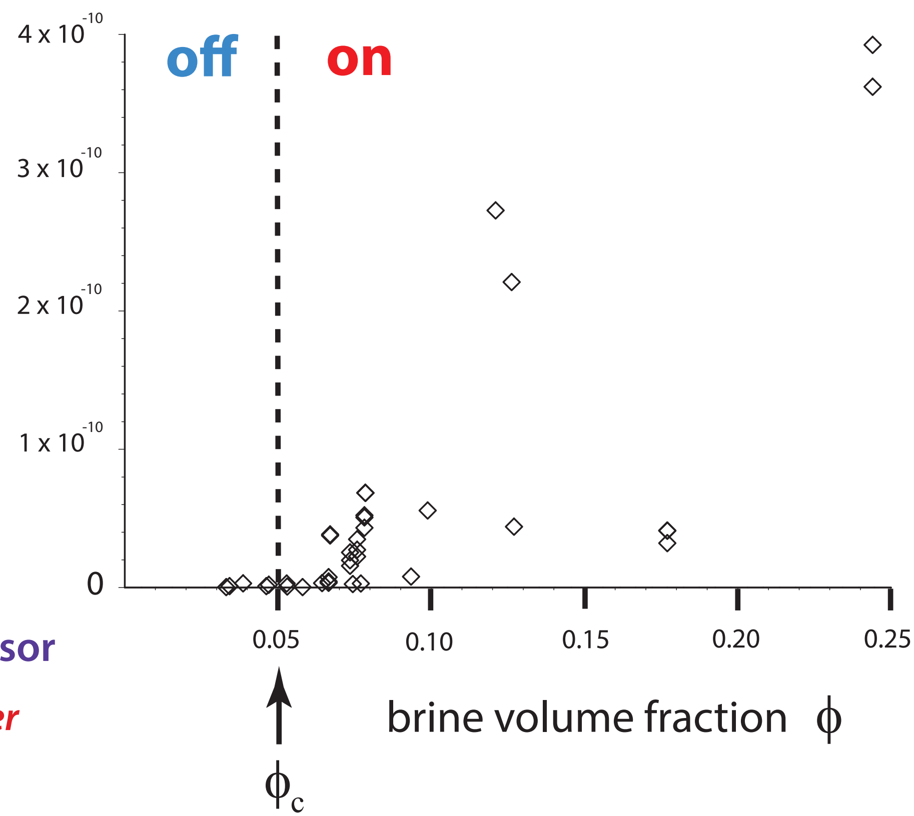
vertical fluid permeability k (m^2)

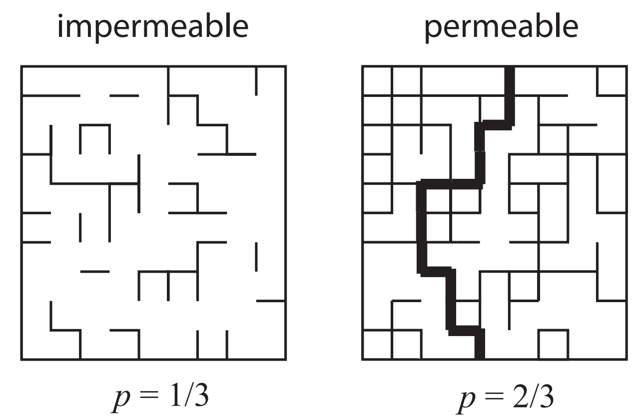
Darcy's Law

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

$\mathbf{k}$ = fluid permeability tensor

homogenized parameter





lattice percolation

PERCOLATION THRESHOLD $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ C$, $S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

sea ice algal communities

D. Thomas 2004

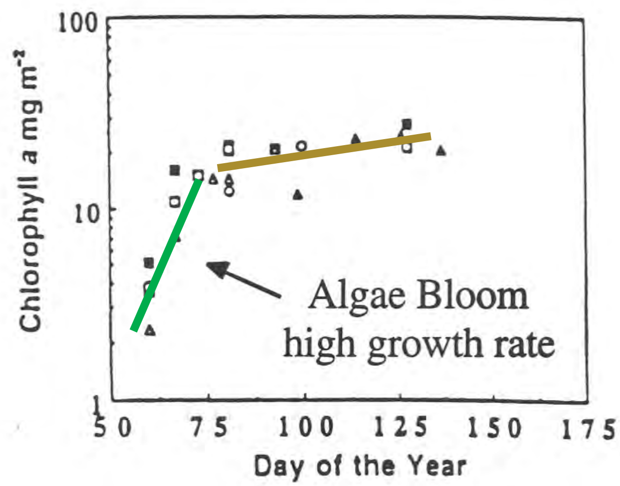
nutrient replenishment
controlled by ice permeability

biological activity turns on or off according to
rule of fives

Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

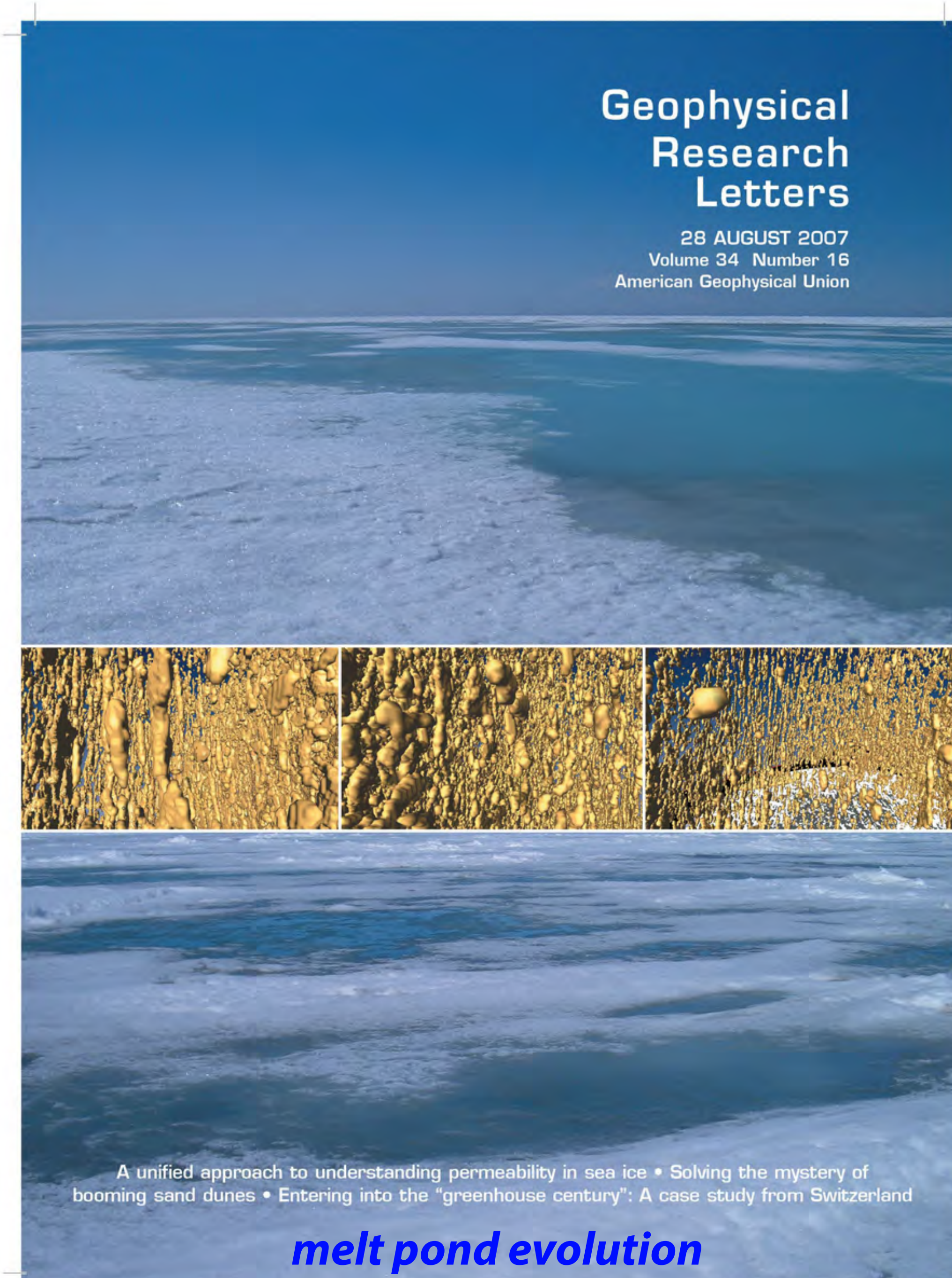
critical behavior of microbial activity



Convection-fueled algae bloom
Ice Station Weddell

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



microscale

governs

mesoscale processes

melt pond evolution

percolation theory for fluid permeability

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8}\ \mathrm{m}^2$$

from critical path analysis in hopping conduction

hierarchical model

rock physics

network model

rigorous bounds

X-ray tomography for brine inclusions

confirms rule of fives

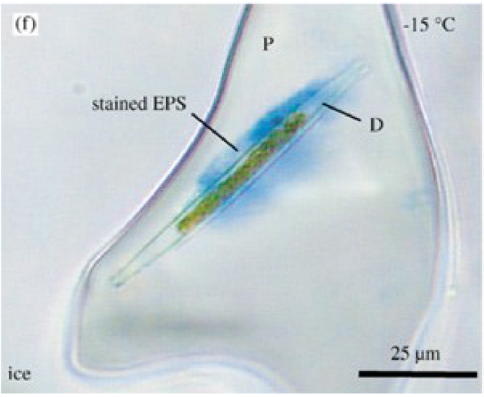
brine percolation threshold of $\phi = 5\%$ for bulk fluid flow

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

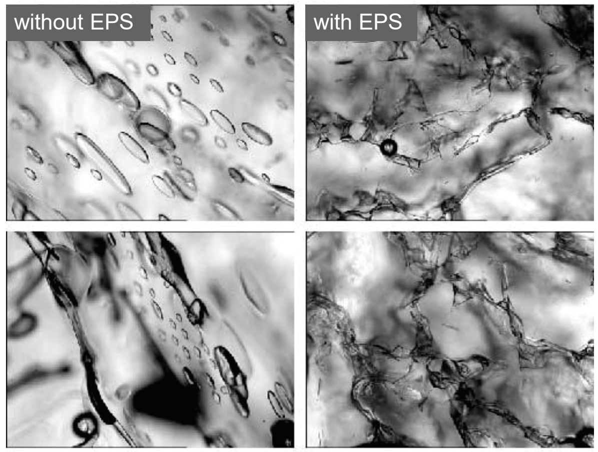
theories agree closely with field data

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

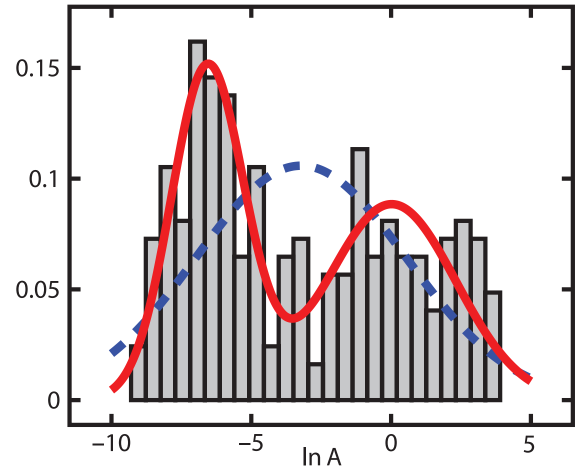
How does EPS affect fluid transport? **How does the biology affect the physics?**



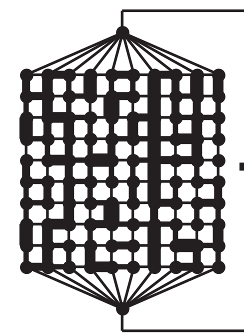
Krembs



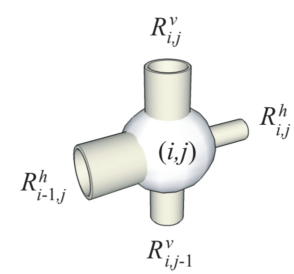
Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability *k;* results predict observed drop in *k*

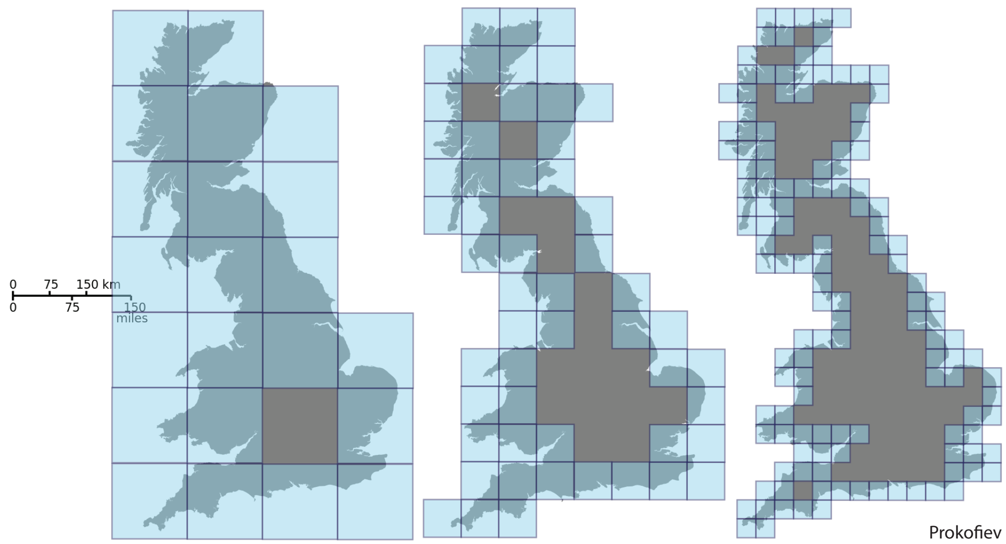


Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden, Eicken, Morris
Ann. Glac. 2006

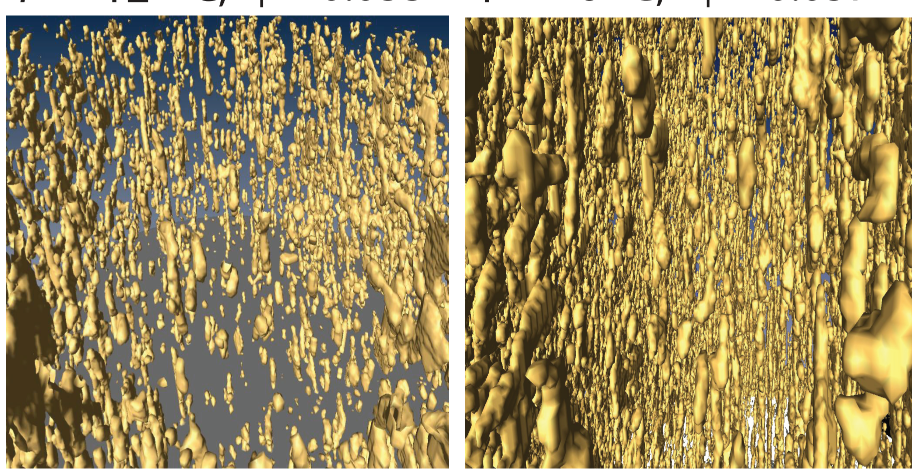
Thermal Evolution of Brine Fractal Geometry in Sea Ice

Nash Ward, Daniel Hallman, Benjamin Murphy, Jody Reimer,
Marc Oggier, Megan O'Sadnick, Elena Cherkaev and Kenneth Golden, 2022



fractal dimension of the British coastline by box counting

$T = -12°$ C, $\phi = 0.033$ $T = -8°$ C, $\phi = 0.057$



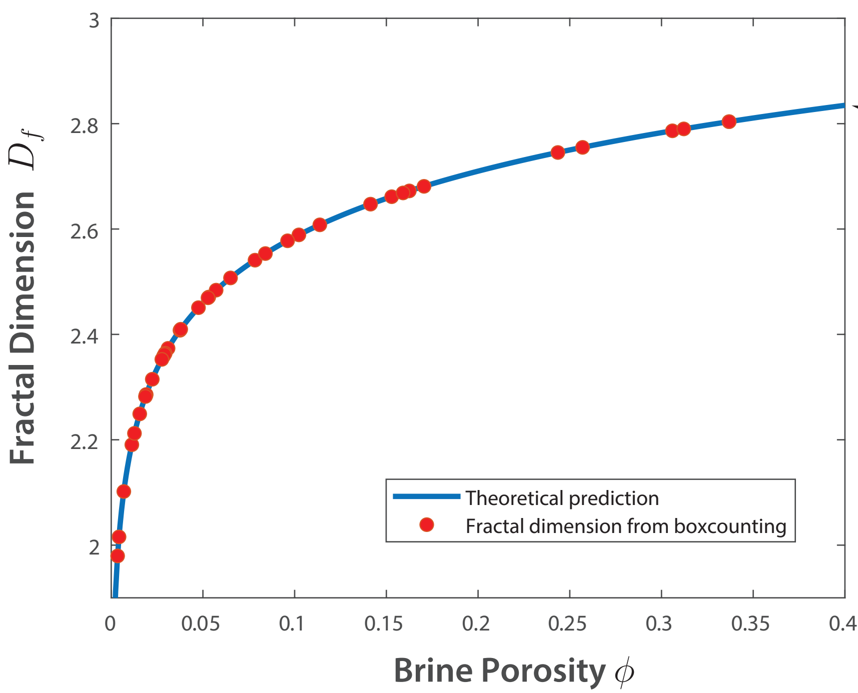
Golden, Eicken, et al. *GRL*, 2007

brine channels and inclusions "look" like fractals (from 30 yrs ago)

X-ray computed tomography of brine in sea ice

columnar and granular

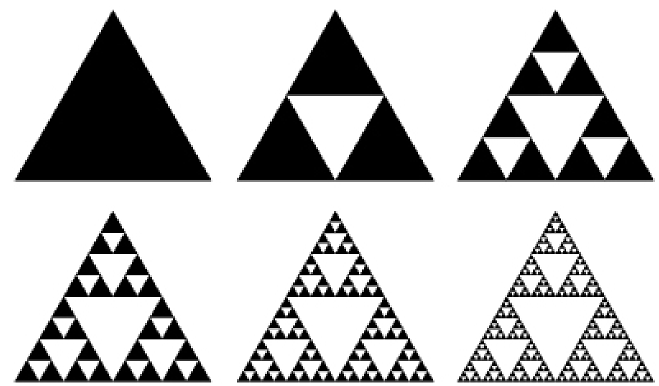
The first comprehensive, quantitative study of the fractal dimension of brine in sea ice and its strong dependence on temperature and porosity.

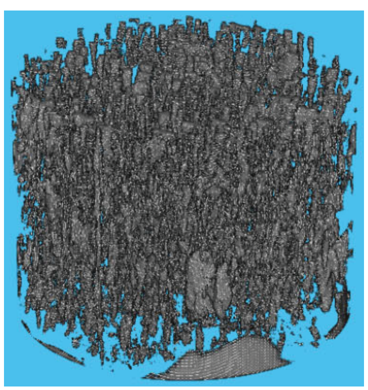


$$D_f = 3 - \frac{\ln \phi}{\ln(\lambda_{min}/\lambda_{max})}$$

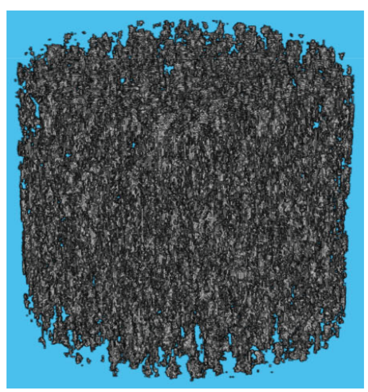
The blue curve is exact for the Sierpinski gasket (an exactly self-similar geometry); discovered for sandstones - statistically self-similar porous media.

Katz and Thompson, 1985
Yu and Li, 2001

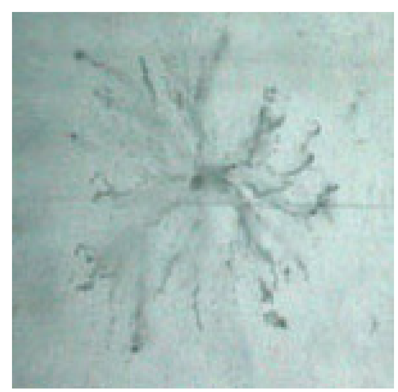




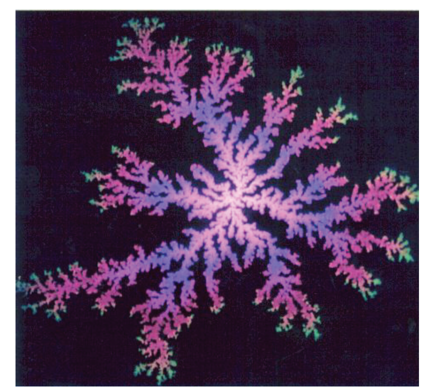
X-ray tomography



DLA model

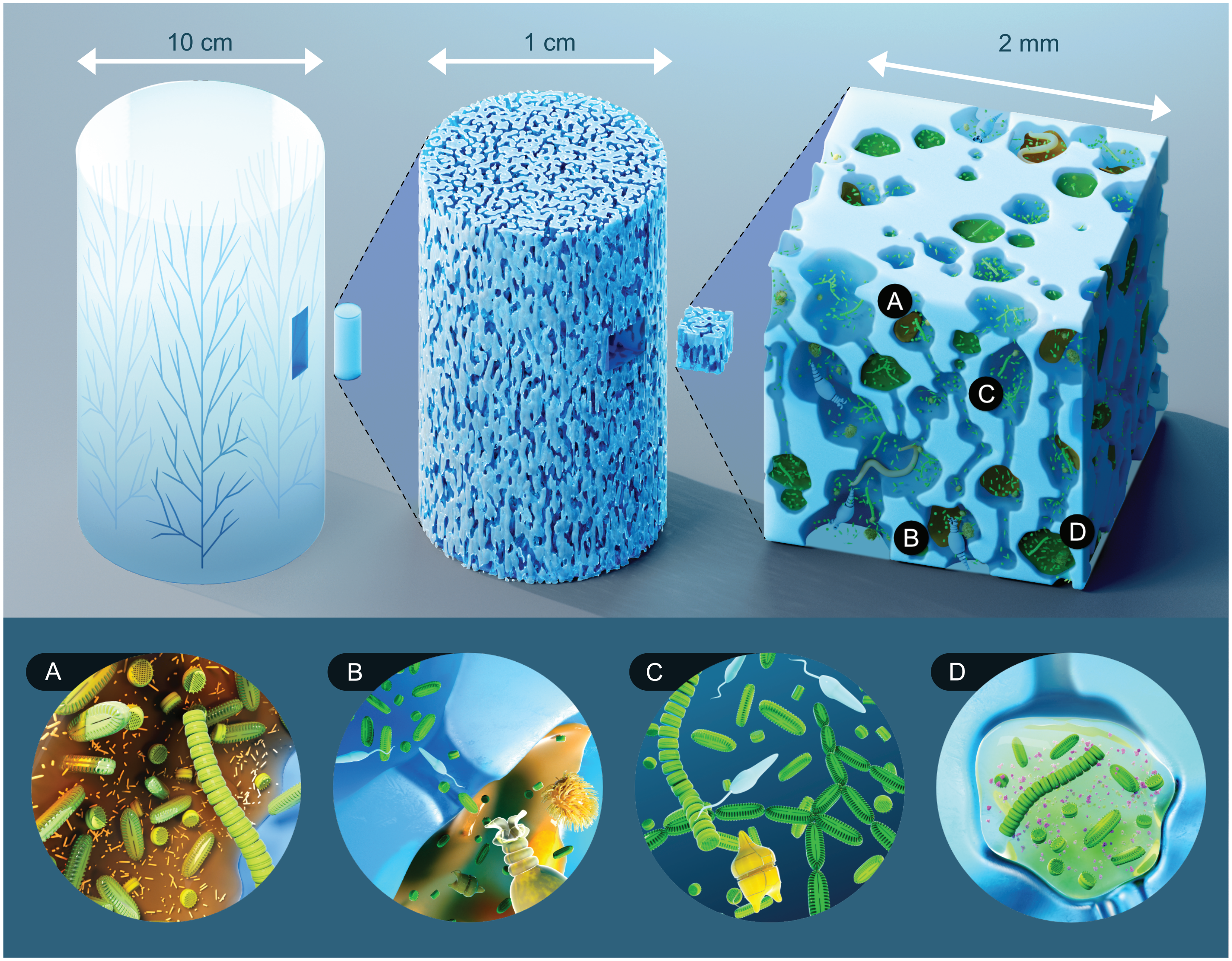


brine channel in sea ice



diffusion limited aggregation

Implications of brine fractal geometry on sea ice ecology and biogeochemistry



Brine inclusions are home to ice endemic organisms, e.g., bacteria, diatoms, flagellates, rotifers, nematodes.

The habitability of sea ice for these organisms is inextricably linked to its complex brine geometry.

(A) Many sea ice organisms attach themselves to inclusion walls; inclusions with a higher fractal dimension have greater surface area for colonization.

(B) Narrow channels prevent the passage of larger organisms, leading to refuges where smaller organisms can multiply without being grazed, as in (C).

(D) Ice algae secrete extracellular polymeric substances (EPS) which alter incusion geometry and may further increase the fractal dimension.


Golden

Remote sensing of sea ice

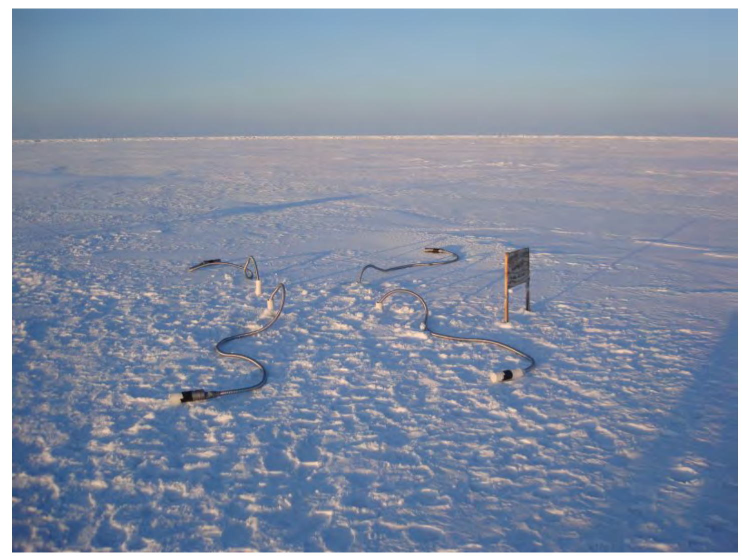
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

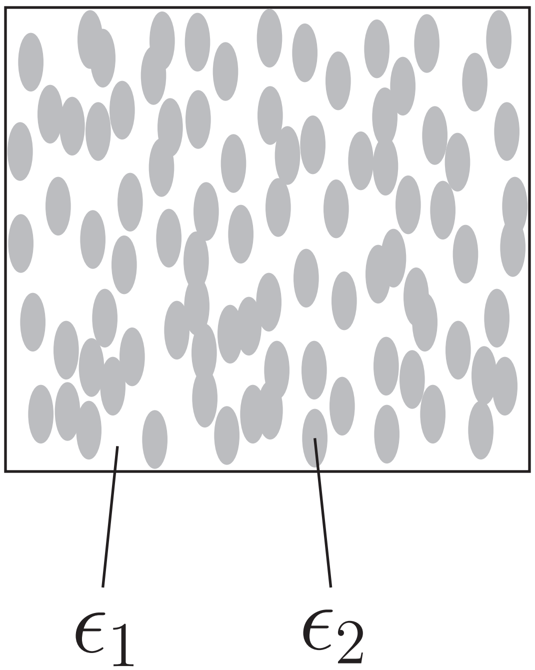
$$\varepsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



$$\Bigg\} \ \epsilon^*$$

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \ , \text{composite geometry} \right)$$

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

Analytic Continuation Method for Homogenization

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)

Stieltjes integral representation for homogenized parameter

separates geometry from parameters

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s-z}$$

geometry → $d\mu(z)$

material parameters → $s - z$

$$s = \frac{1}{1 - \epsilon_1/\epsilon_2}$$

μ:
- spectral measure of self adjoint operator $\Gamma\chi$
- mass = p_1
- higher moments depend on n-point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$$

$\chi =$ characteristic function of the brine phase

$$E = s\,(s + \Gamma\chi)^{-1} e_k$$

$\Gamma\chi$: microscale → macroscale

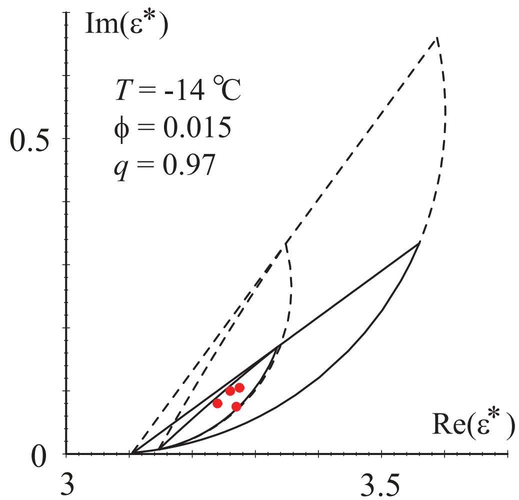
$\Gamma\chi$ *links scales*

Golden and Papanicolaou, Comm. Math. Phys. 1983

This representation distills the complexities of mixture geometry into the spectral properties of an operator like the Hamiltonian in physics.

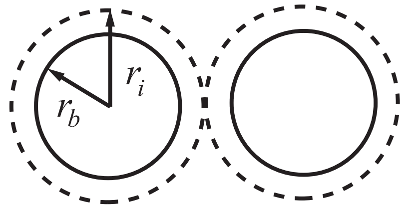
forward and inverse bounds on the complex permittivity of sea ice

forward bounds



4.75 GHz data

matrix particle

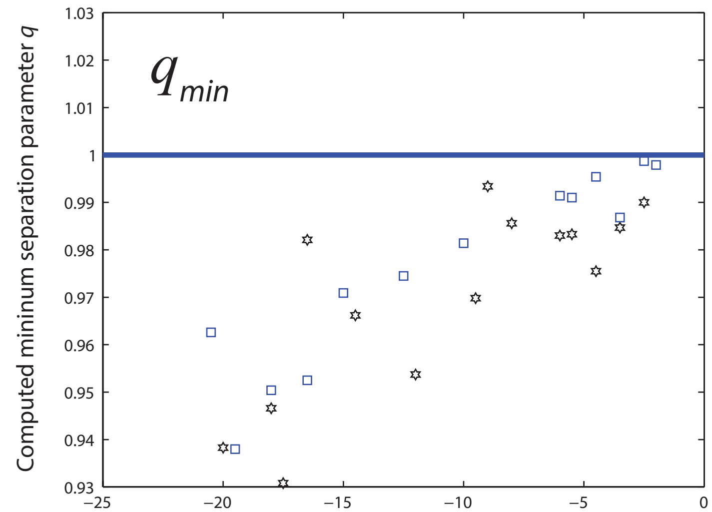


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



Inverse Homogenization

Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001), McPhedran, McKenzie, Milton (1982), *Theory of Composites*, Milton (2002)

ε^* ⟶ ***composite geometry*** **(spectral measure μ)**

inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

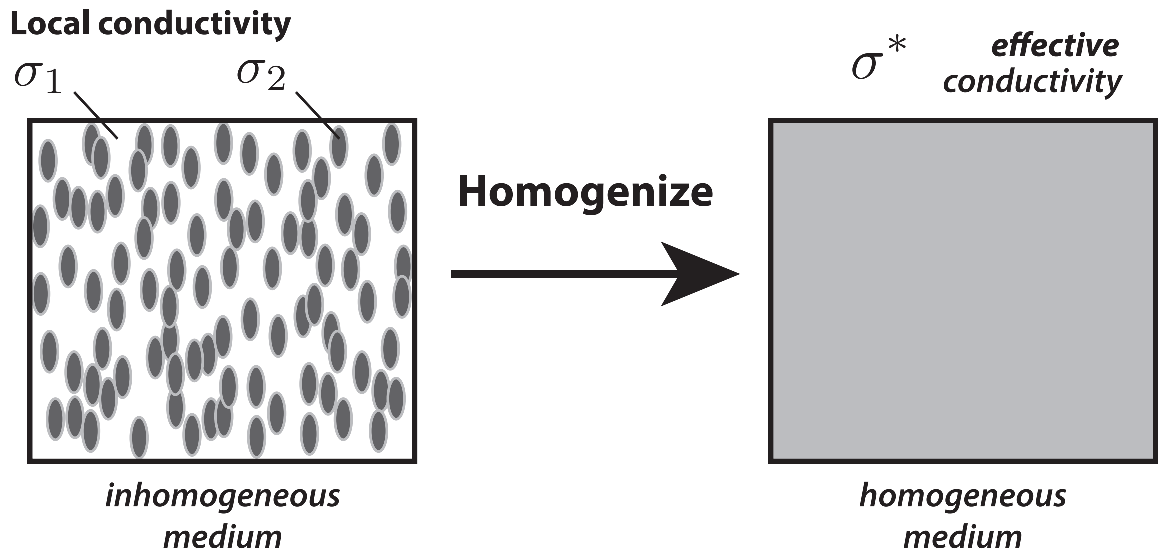
rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

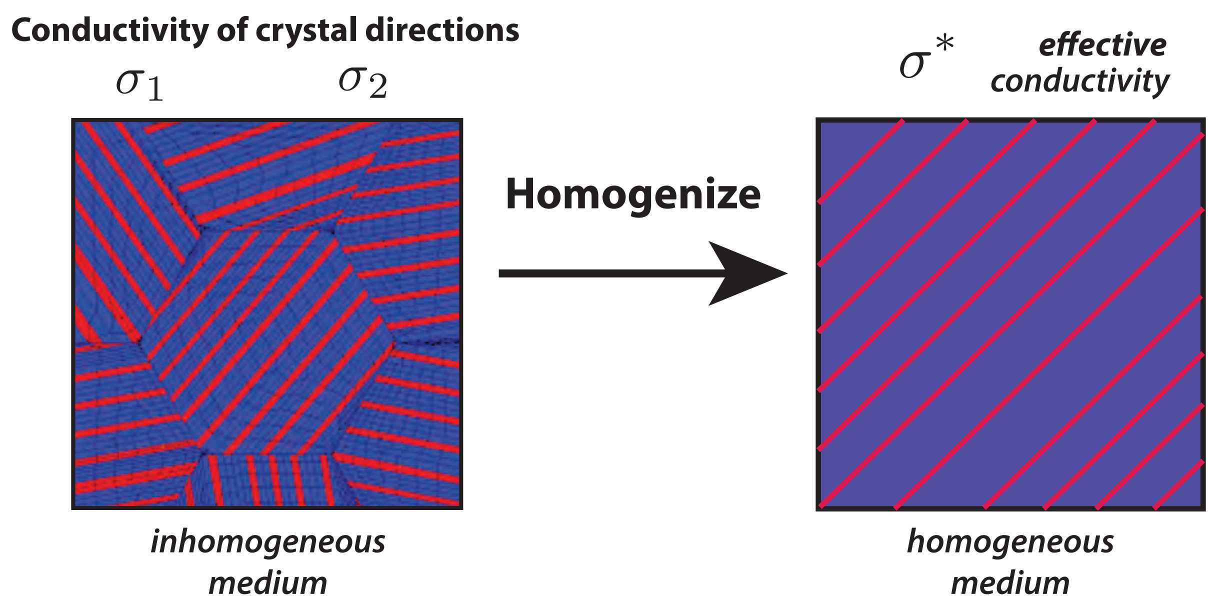
Homogenization for polycrystalline materials

Two-component composites



Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Polycrystalline media



Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**

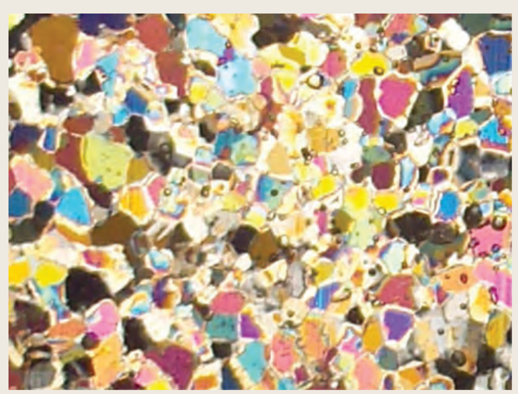
  Milton (1981, 2002), Barabash and Stroud (1999), ...

- **Forward and inverse bounds**

  orientation statistics

- **Applied to sea ice using two-scale homogenization**

- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



Proc. Roy. Soc. A 8 Feb 2015

higher threshold for fluid flow in granular sea ice

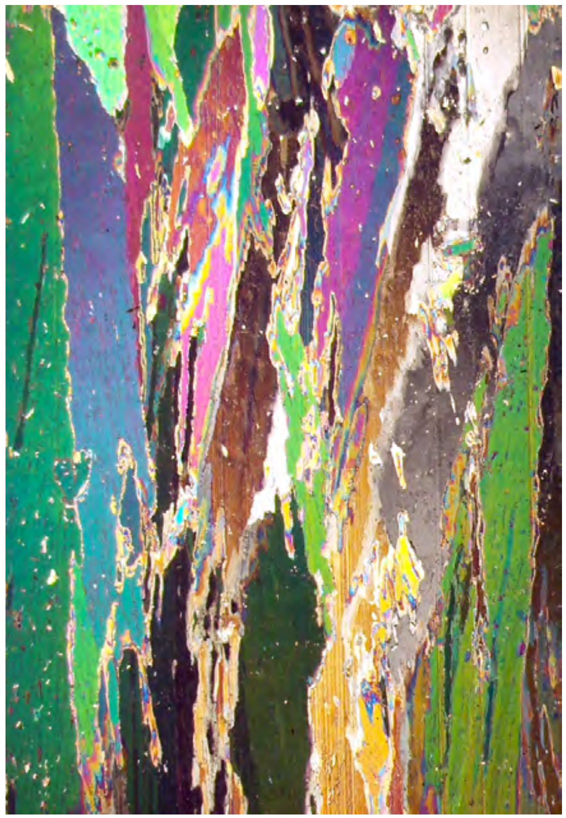
microscale details impact "mesoscale" processes

nutrient fluxes for microbes
melt pond drainage
snow-ice formation

columnar

granular

5%



10%

Golden, Sampson, Gully, Lubbers, Tison 2022

electromagnetically distinguishing ice types
Kitsel Lusted, Elena Cherkaev, Ken Golden

Rigorous bounds on the complex permittivity tensor of sea ice with polycrystalline anisotropy in the horizontal plane

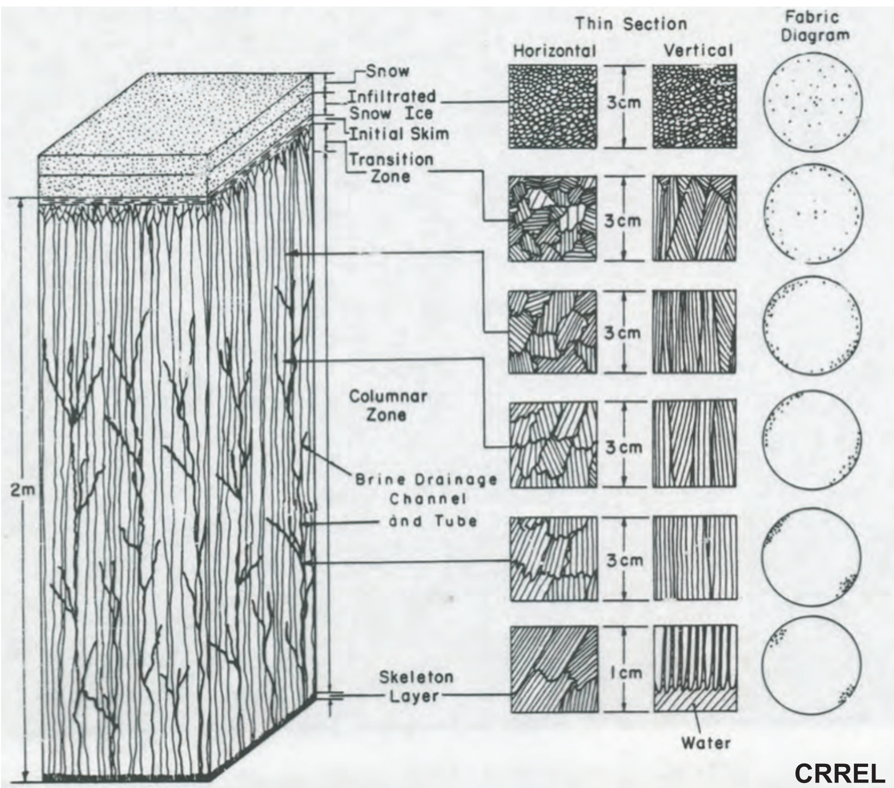
Kenzie McLean, Elena Cherkaev, Ken Golden 2022

motivated by

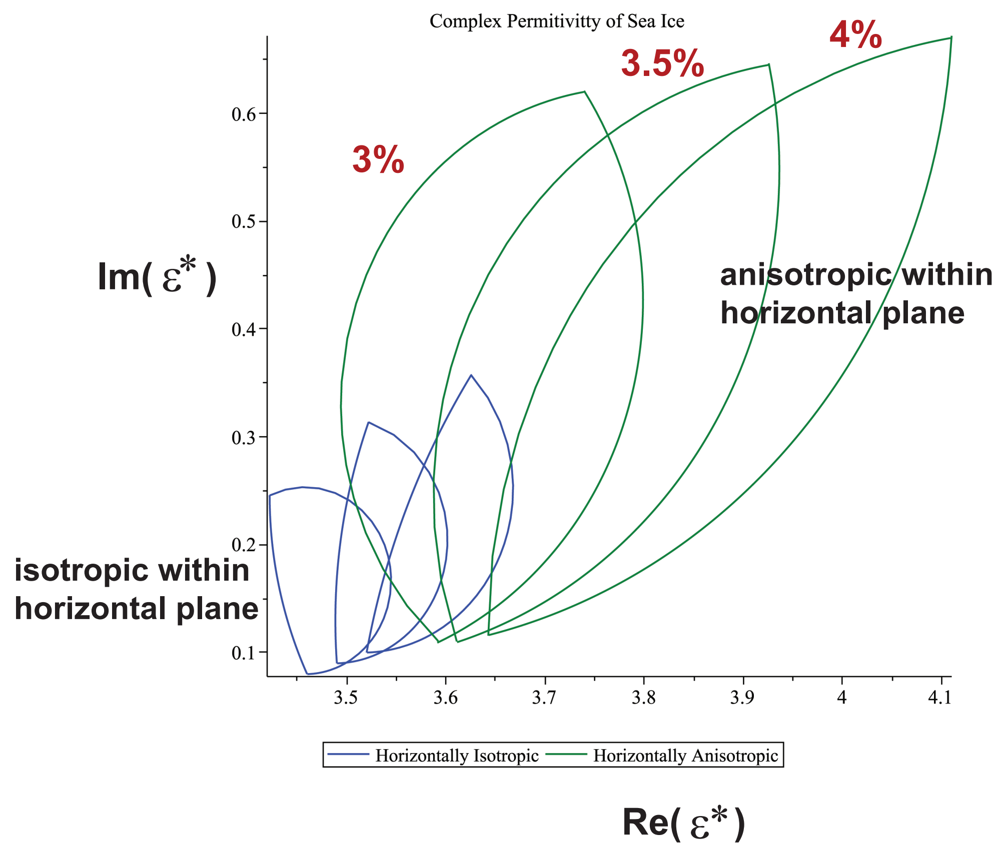
Weeks and Gow, *JGR* 1979: c-axis alignment in Arctic fast ice off Barrow

Golden and Ackley, *JGR* 1981: radar propagation model in aligned sea ice

input: orientation statistics



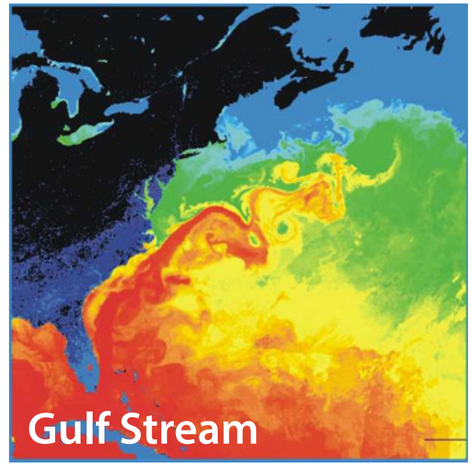
output: bounds

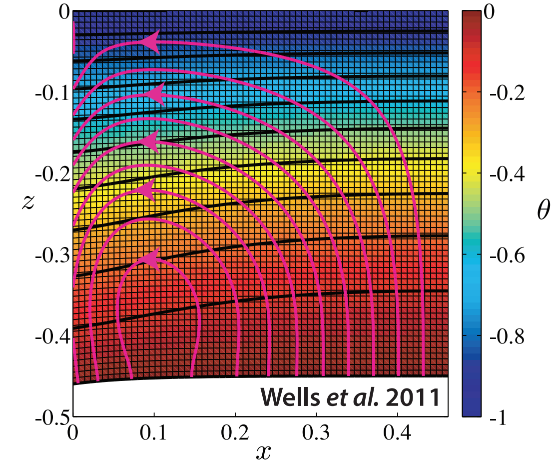


advection enhanced diffusion

effective diffusivity

nutrient and salt transport in sea ice
heat transport in sea ice with convection
sea ice floes in winds and ocean currents
tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere





advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

homogenize

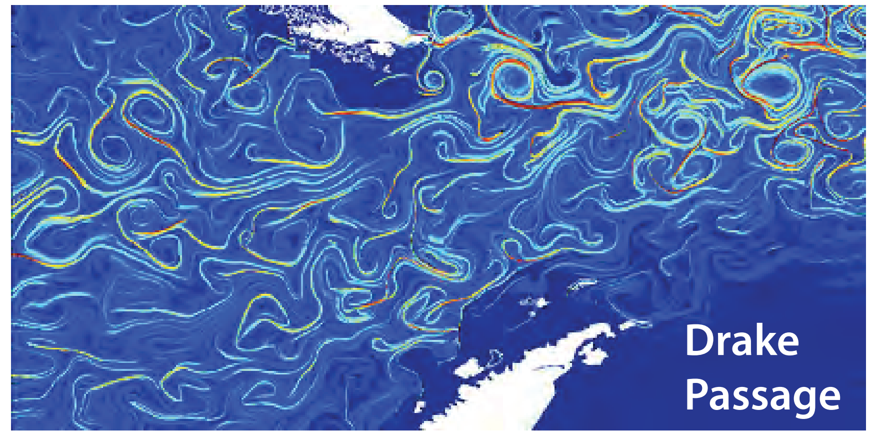
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* ***effective diffusivity***

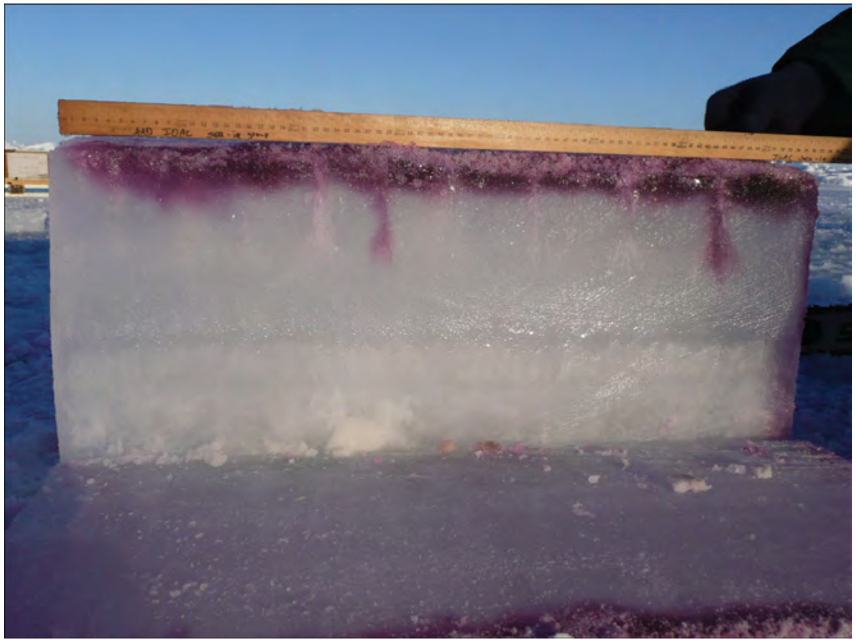
Stieltjes integral for κ^* with spectral measure

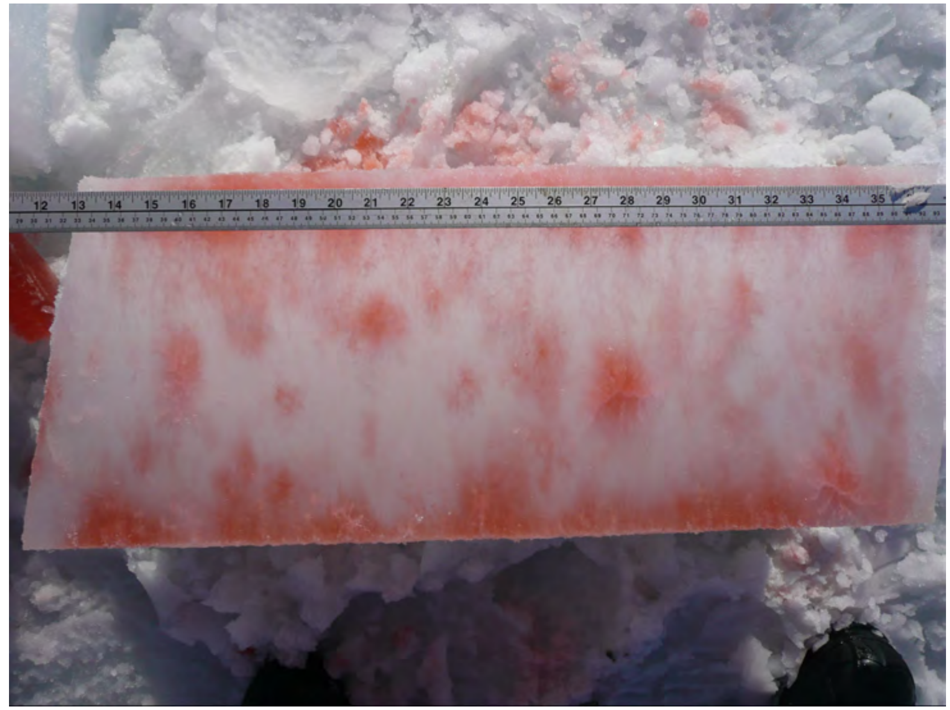
Avellaneda and Majda, PRL 89, CMP 91

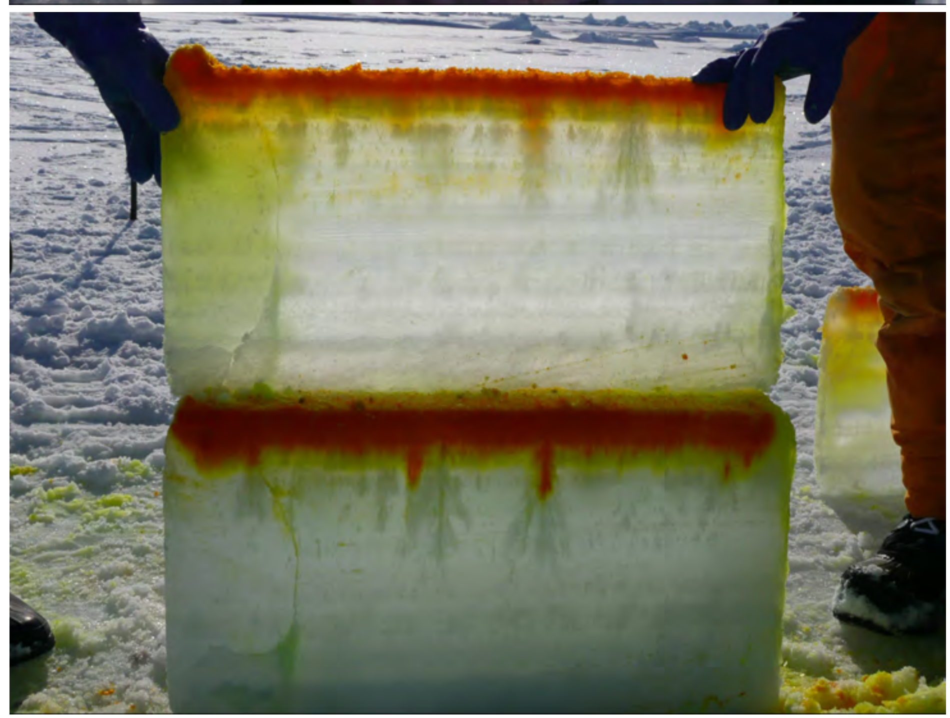
Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017
Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020



tracers flowing through inverted sea ice blocks







wave propagation in the marginal ice zone (MIZ)

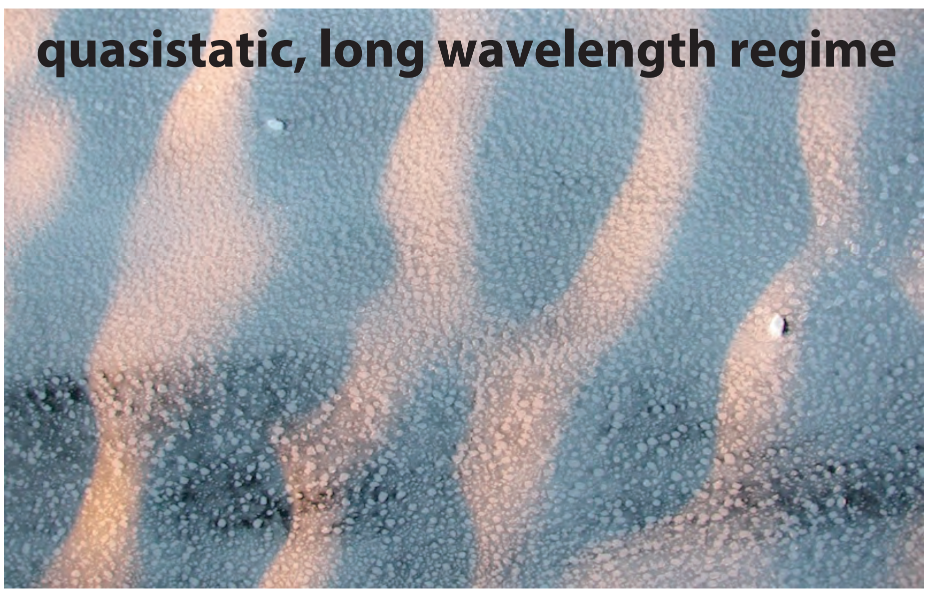
first theory of key parameter in wave-ice interactions only fitted to wave data before

Keller, 1998
Mosig, Montiel, Squire, 2015
Wang, Shen, 2012

Analytic Continuation Method

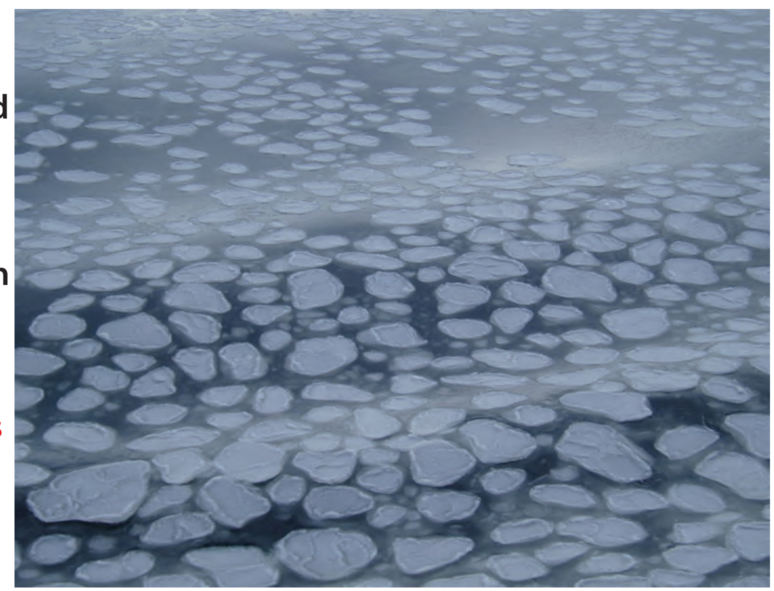
Bergman (78) - Milton (79)
integral representation for ε^*
Golden and Papanicolaou (83)

Milton, *Theory of Composites* (02)



homogenized parameter depends on sea ice concentration and ice floe geometry

like EM waves



direct calculation of spectral measures

Murphy, Hohenegger, Cherkaev, Golden, *Comm. Math. Sci.* 2015

- depends only on the composite geometry
- discretization of microstructural image gives binary network
- fundamental operator becomes a random matrix
- spectral measure computed from eigenvalues and eigenvectors

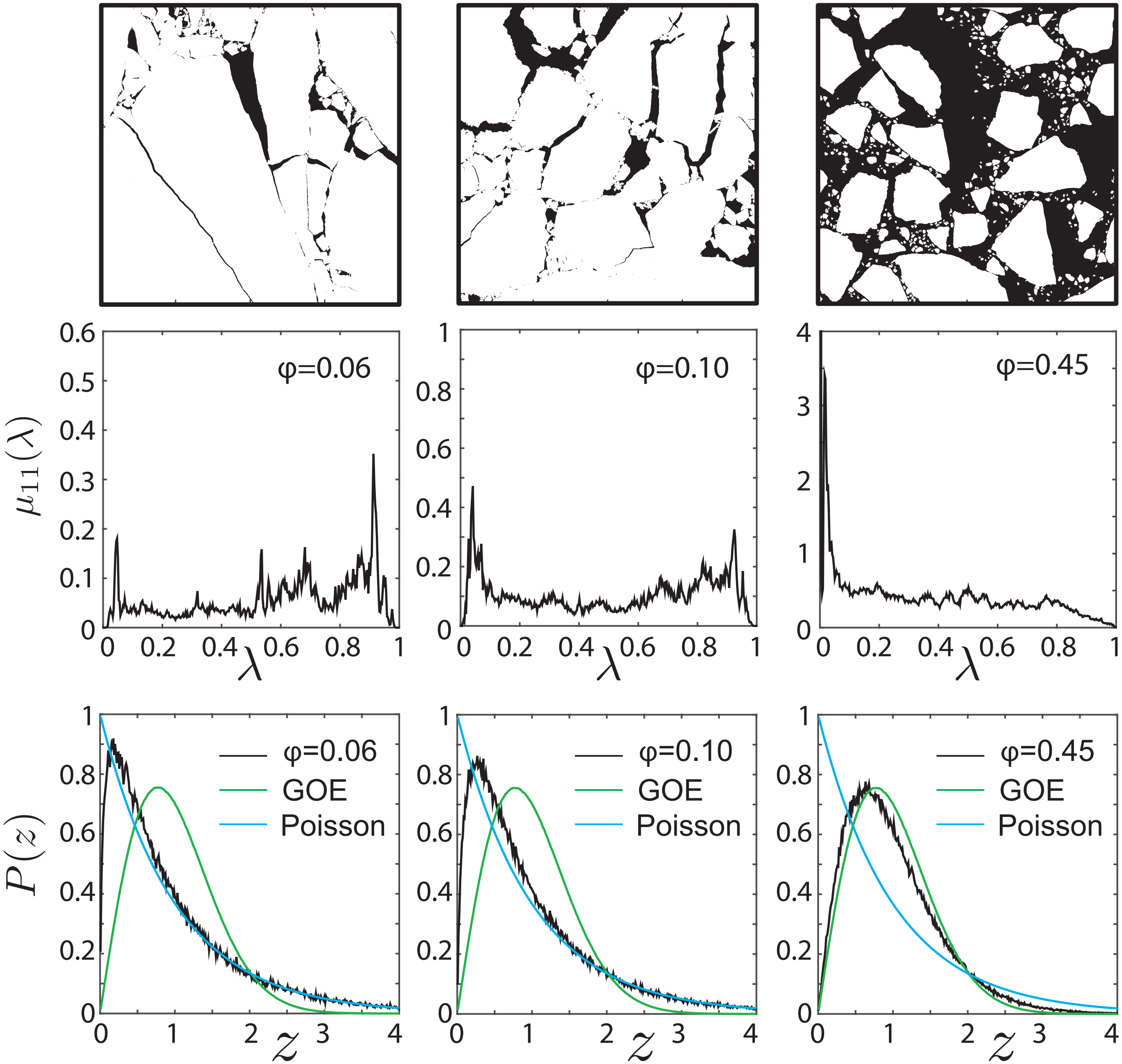
once we have the spectral measure μ it can be used in Stieltjes integrals for other transport coefficients:

electrical and thermal conductivity, complex permittivity, magnetic permeability, diffusion, fluid flow properties

earlier studies of spectral measures

Day and Thorpe 1996
Helsing, McPhedran, Milton 2011

Spectral computations for sea ice floe configurations

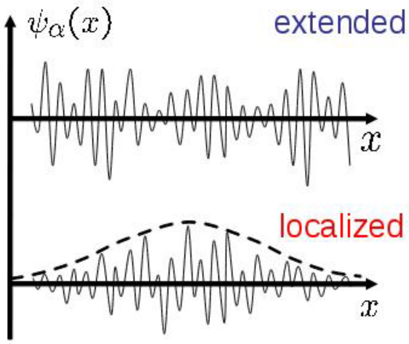


spectral measures

eigenvalue spacing distributions

uncorrelated → **level repulsion**

UNIVERSAL Wigner-Dyson distribution



electronic transport in semiconductors

metal / insulator transition

localization

Anderson 1958
Mott 1949
Shklovshii et al 1993
Evangelou 1992

Anderson transition in wave physics:
quantum, optics, acoustics, water waves, ...

---

from analysis of spectral measures for brine, melt ponds, ice floes

we find percolation-driven

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden   Phys. Rev. Lett.   2017

PERCOLATION TRANSITION → universal eigenvalue statistics (GOE)
extended states, mobility edges

-- but with NO wave interference or scattering effects ! --

Order to disorder in quasiperiodic composites

Morison, Murphy, Cherkaev, Golden, Comm. Phys. 2022

sea ice inspired - high tech spin off

tunable quasiperiodic composites with exotic properties

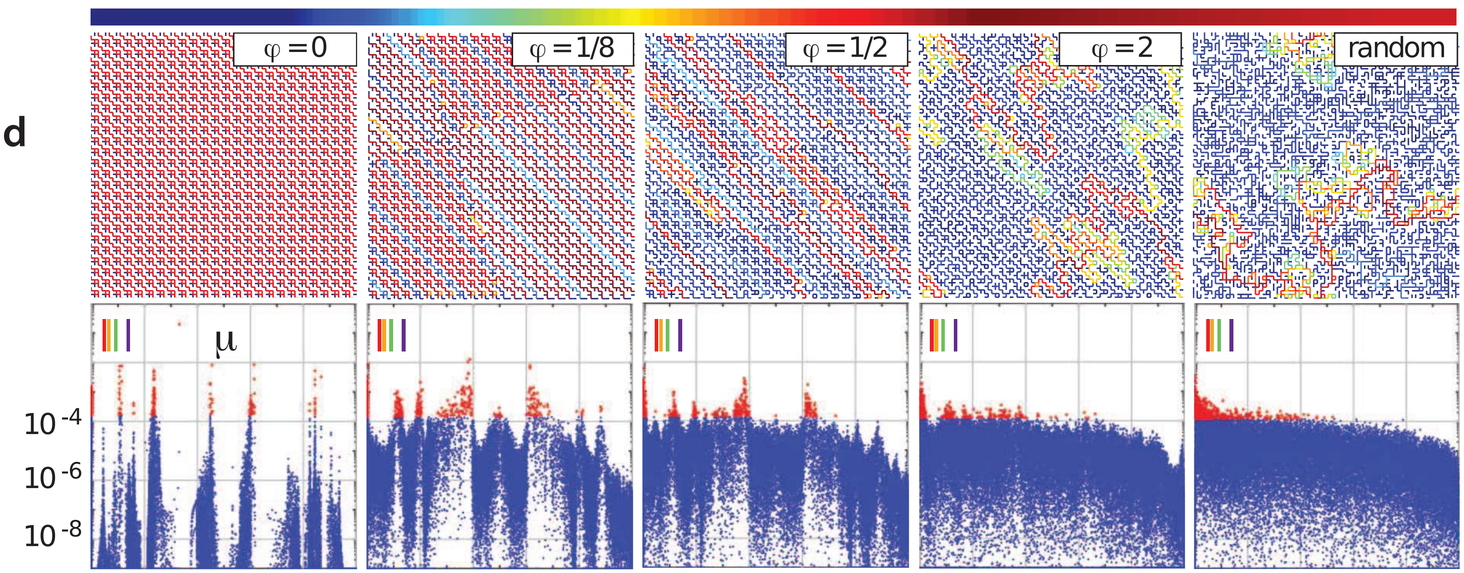
(optical, electrical, thermal, ...), Anderson localization; our Moiré patterned geometries are similar to twisted bilayer graphene

increasing twist angle between two lattices

periodic → **quasiperiodic**

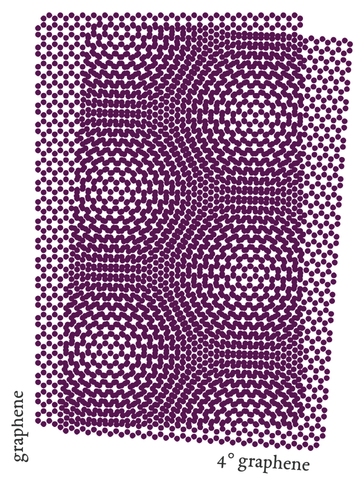
electric field strength

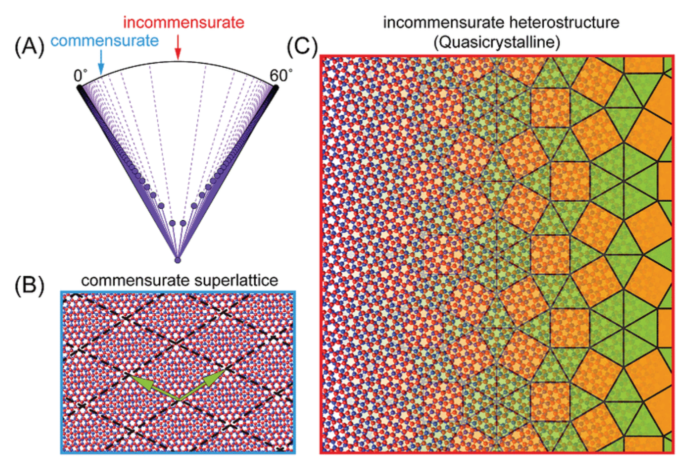
spectral measure



RRN at percolation threshold

twisted bilayer graphene





Yao et al., 2018

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

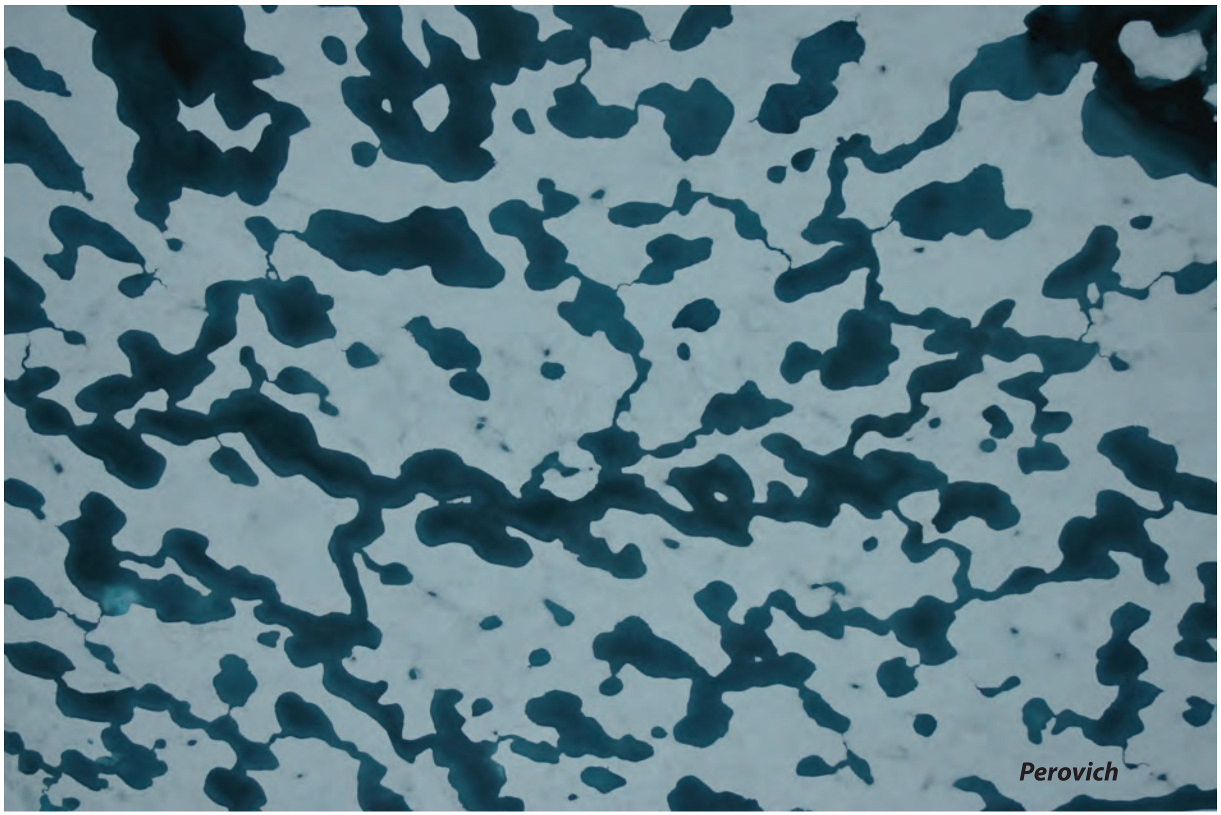
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

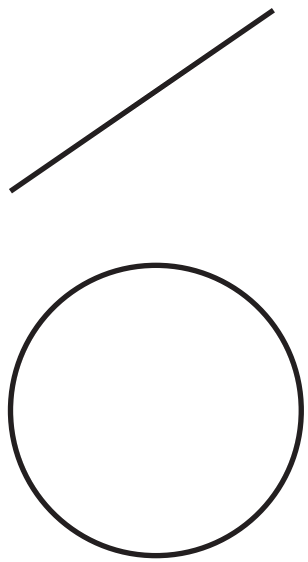
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

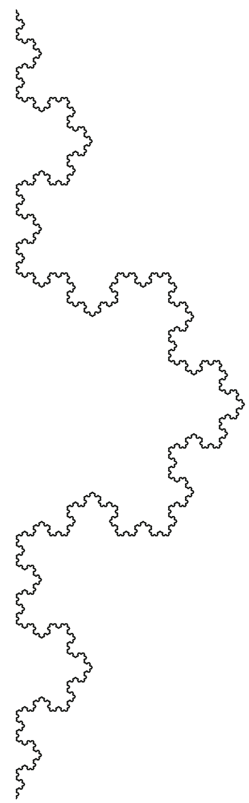
fractal curves in the plane

they wiggle so much that their dimension is >1



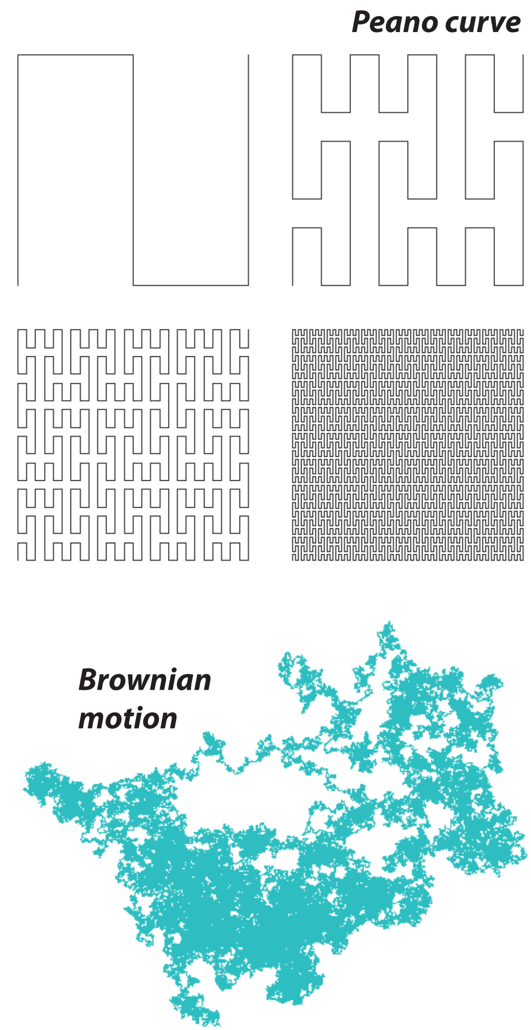
simple curves

D = 1



Koch snowflake

D = 1.26



space filling curves

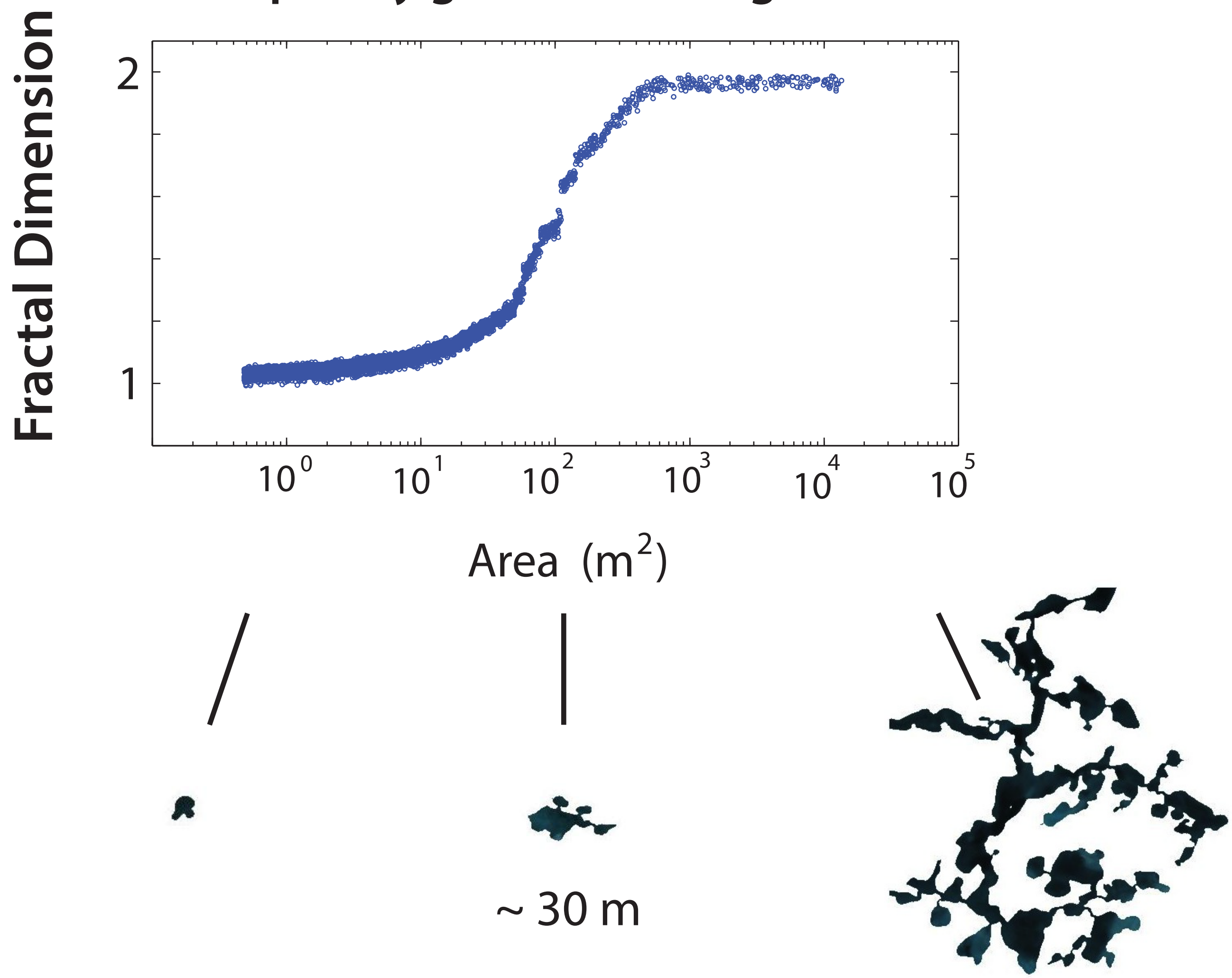
D = 2

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

complexity grows with length scale

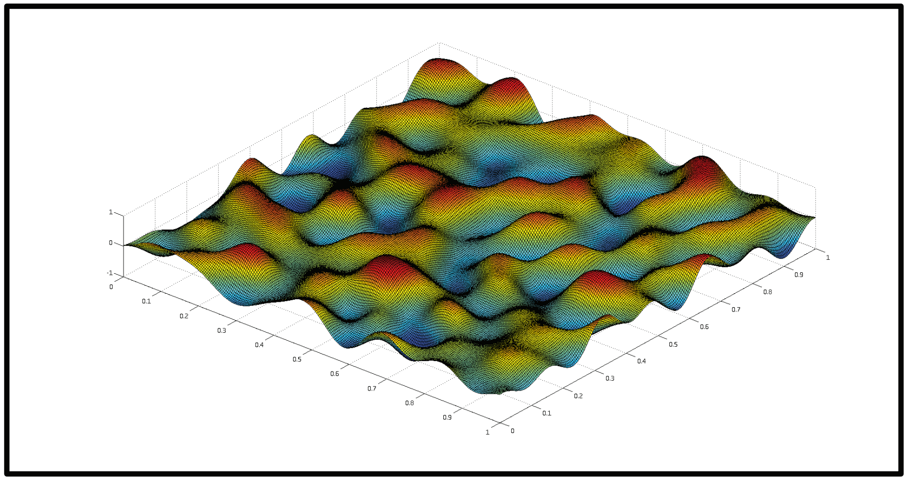


simple pond ***transitional pond*** ***complex pond***

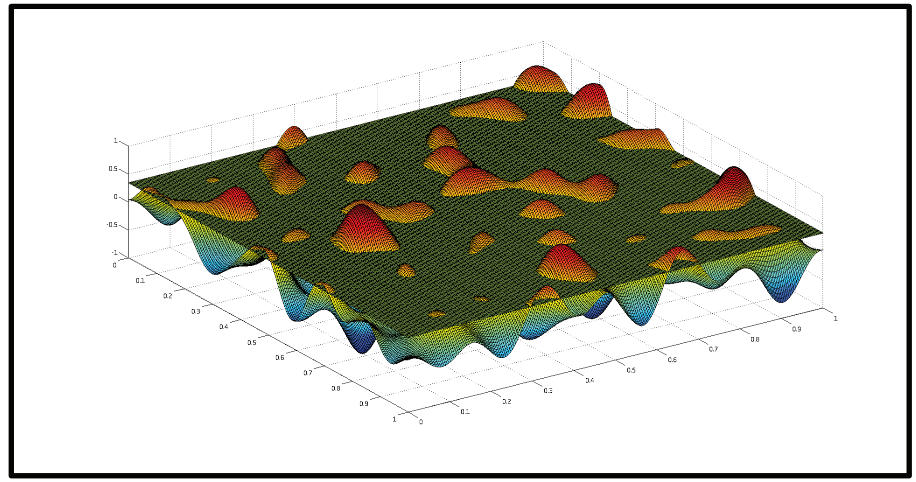
Continuum percolation model for melt pond evolution

level sets of random surfaces

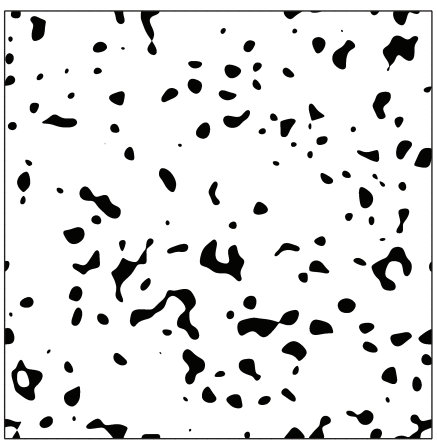
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018

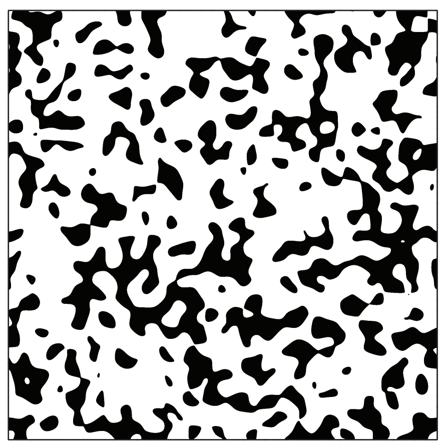


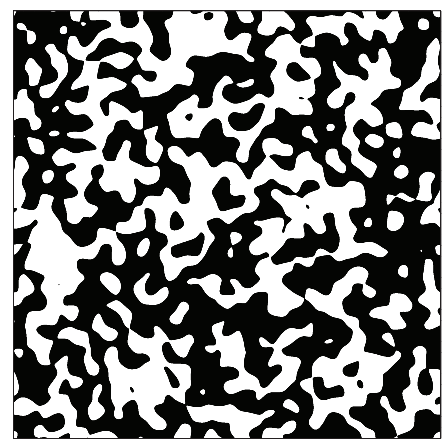
random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds

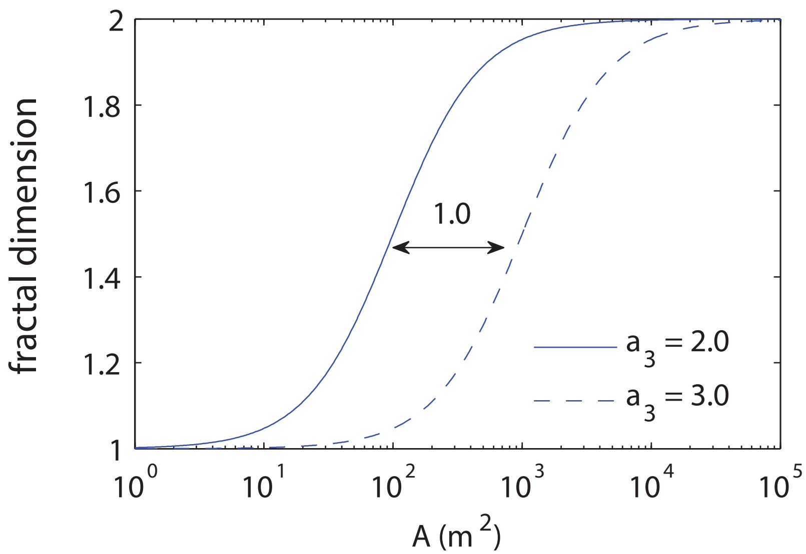


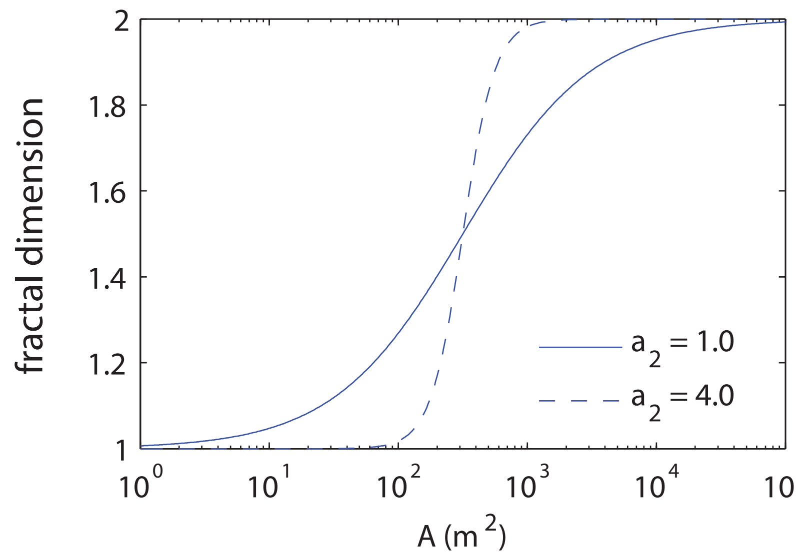




electronic transport in disordered media *diffusion in turbulent plasmas* *Isichenko, Rev. Mod. Phys., 1992*

fractal dimension curves depend on statistical parameters defining random surface





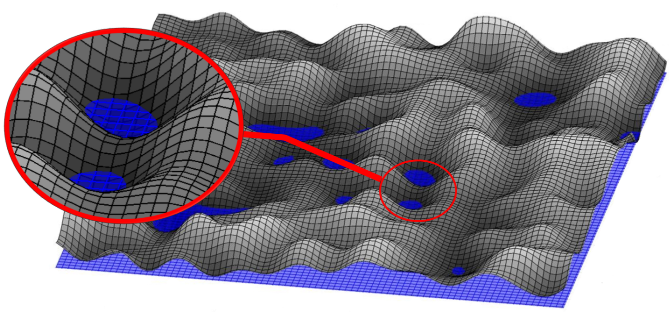
Topology of the sea ice surface and the fractal geometry of Arctic melt ponds

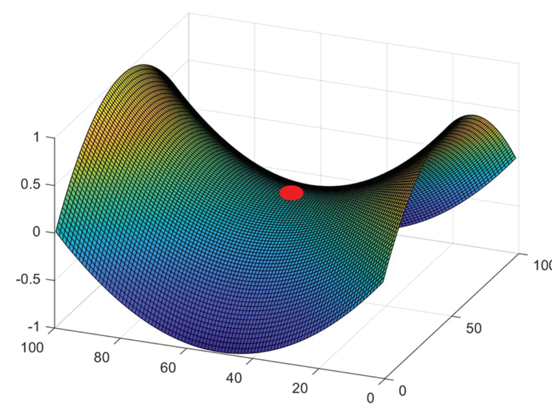
Physical Review Research (invited, under revision)

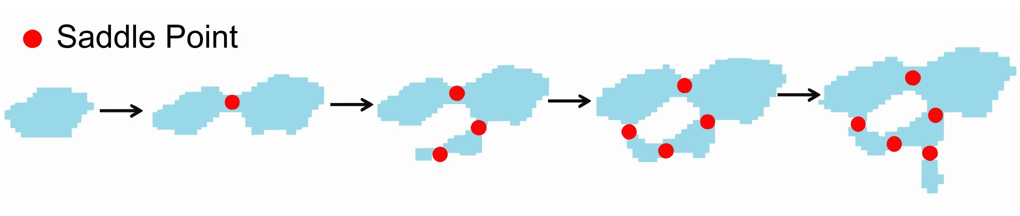
Ryleigh Moore, Jacob Jones, Dane Gollero,
Court Strong, Ken Golden

Several models replicate the transition in fractal dimension, but none explain how it arises.

We use Morse theory applied to the random surface model to show that saddle points play the critical role in the fractal transition.

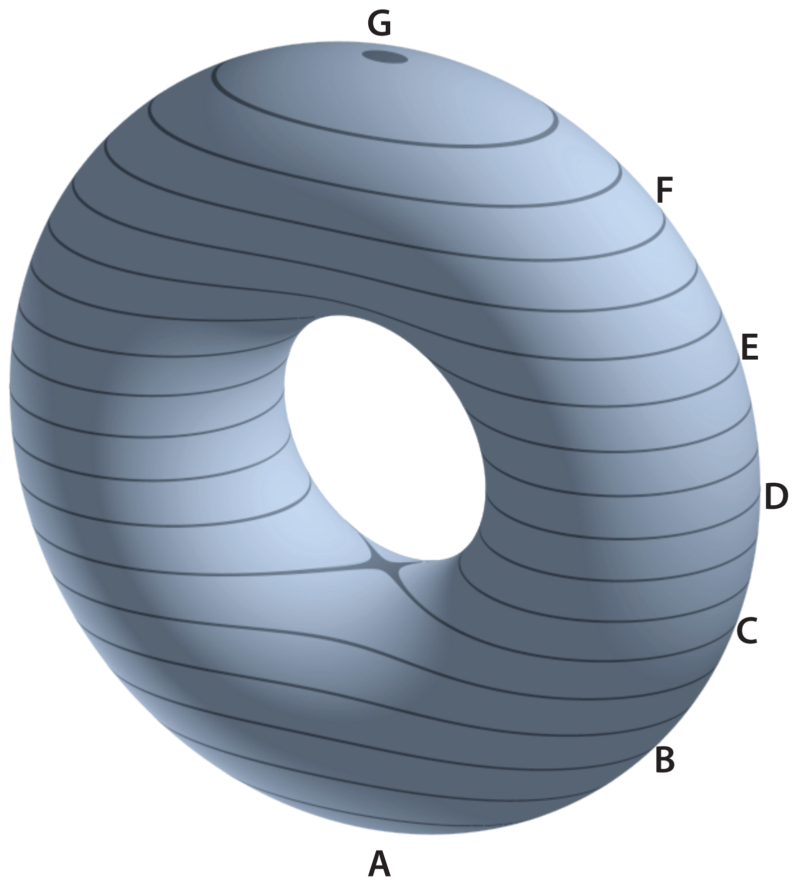






ponds coalesce
(change topology) and
complexify at saddle points

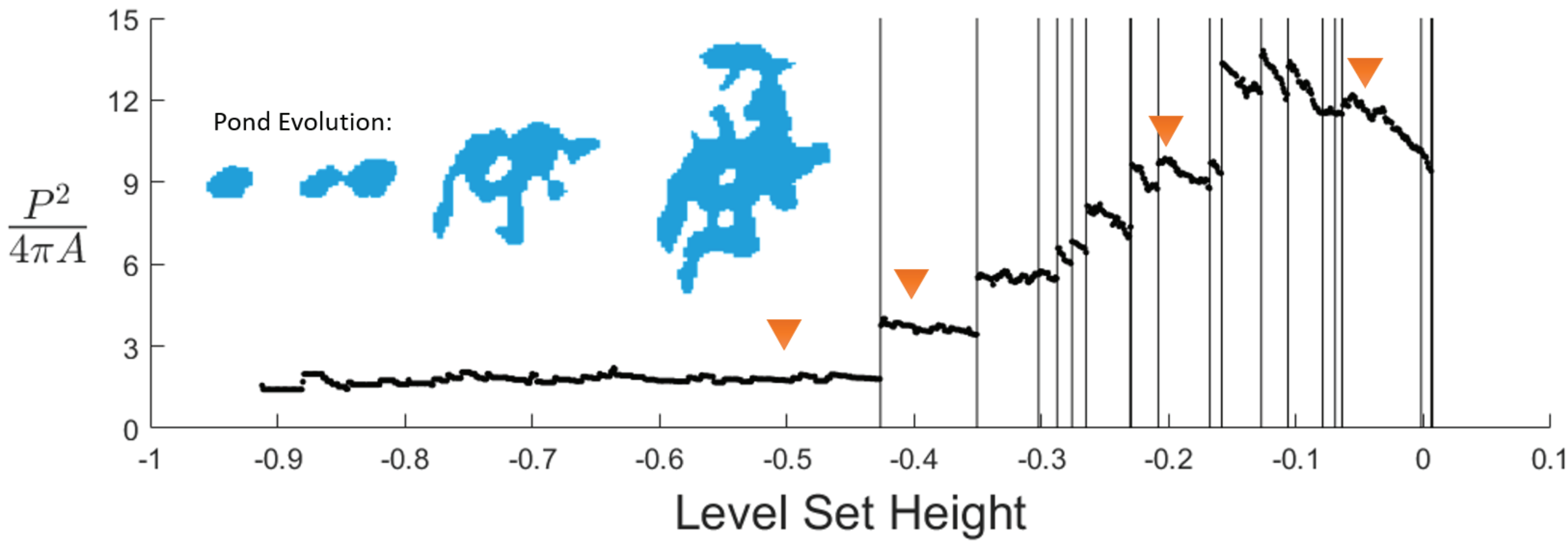
Morse theory



Morse theory tells us that changes in the topology of a surface occur at critical points of smooth functions on the surface: maxima, minima, and saddles.

Main results

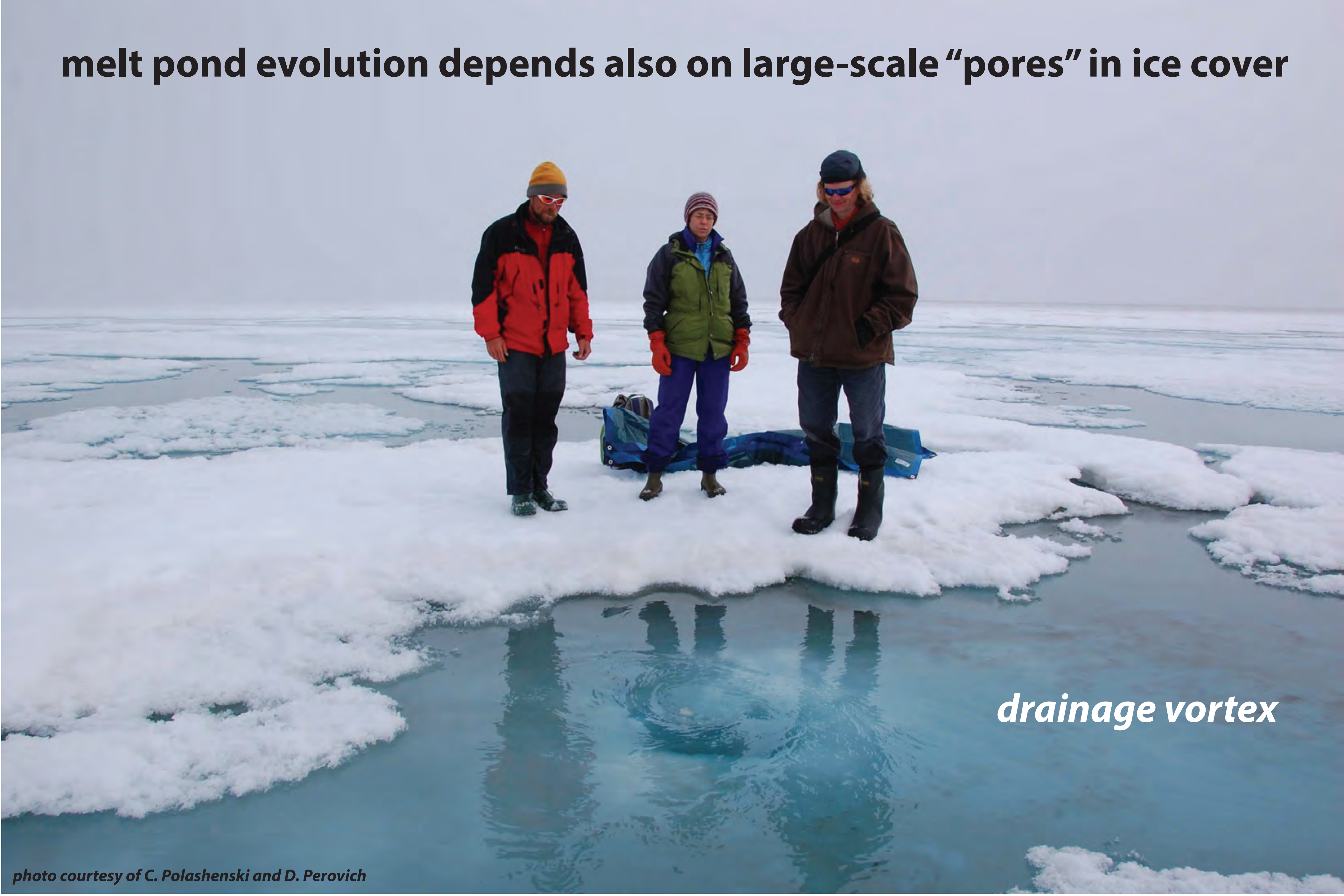
Isoperimetric quotient - as a proxy for fractal dimension - increases in discrete jumps when ponds coalesce at saddle points.



Horizontal fluid permeability "controlled" by saddles ~ electronic transport in 2D random potential.

drainage processes, seal holes

melt pond evolution depends also on large-scale "pores" in ice cover



Melt pond connectivity enables vast expanses of melt water to drain down seal holes, thaw holes, and leads in the ice.

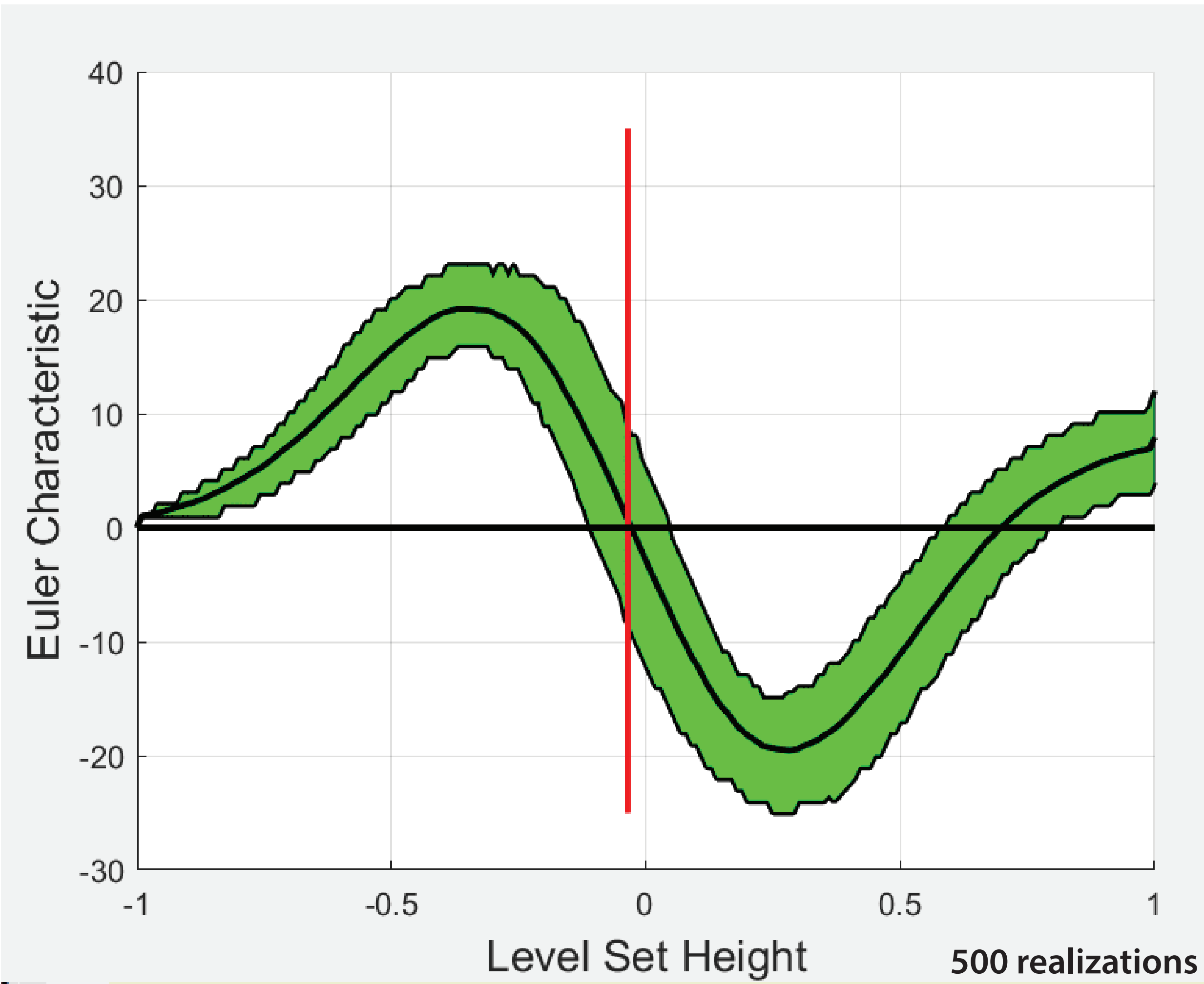
Topological Data Analysis

Euler characteristic = # maxima + # minima - # saddles

topological invariant

persistent homology

filtration - sequence of nested topological spaces, indexed by water level



Expected
Euler Characteristic Curve (ECC)

tracks the evolution of the EC of the flooded surface as water rises

zero of ECC ~ percolation

percolation on a torus
creates a giant cycle

Bobrowski & Skraba, 2020

Carlsson, 2009

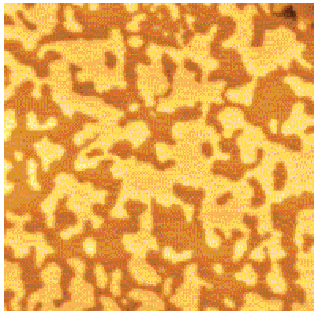
Vogel, 2002 GRF

porous media
cosmology
brain activity

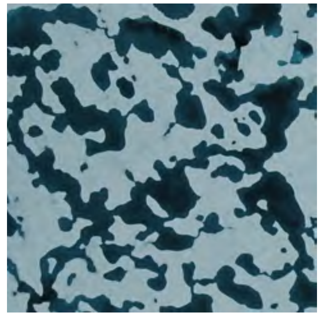
melt pond donuts

From magnets to melt ponds

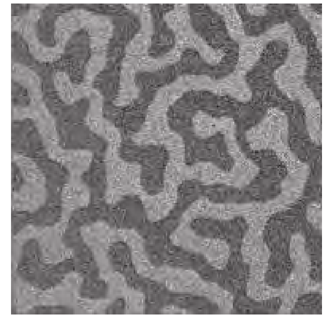
100 year old model for magnetic materials used to explain melt pond geometry



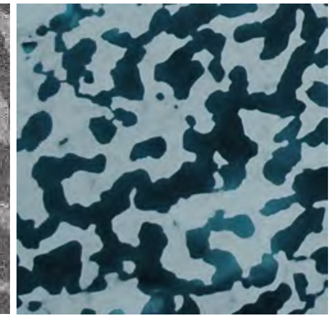
magnetic domains in cobalt



Arctic melt ponds

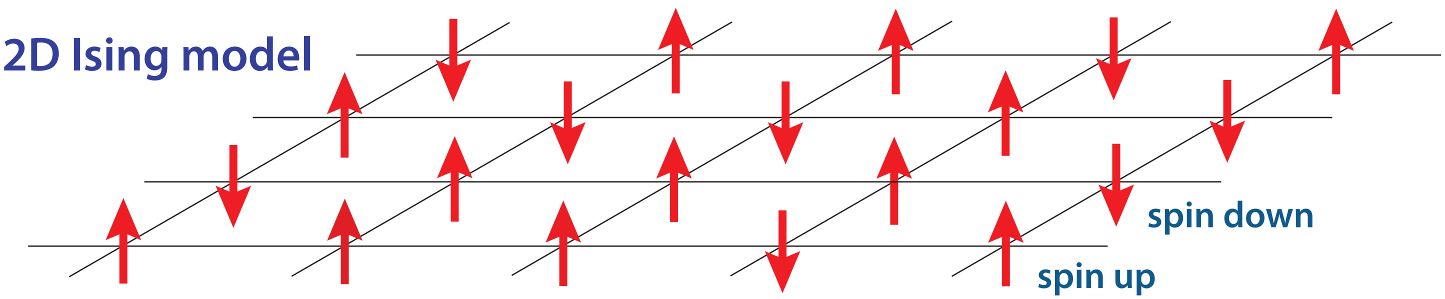


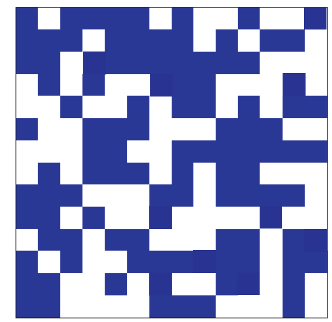
magnetic domains in cobalt-iron-boron



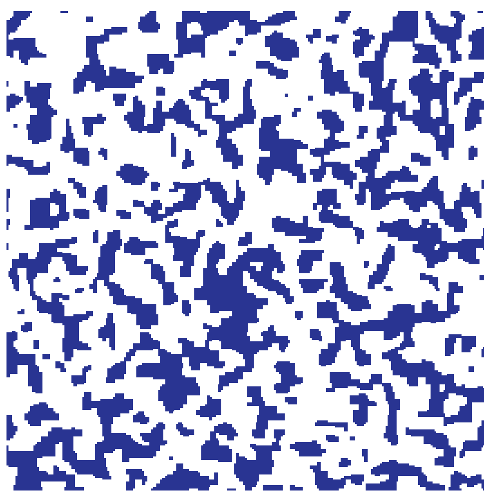
Arctic melt ponds

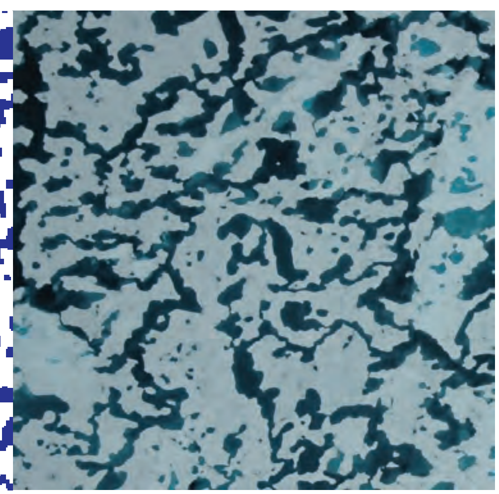
2D Ising model





model





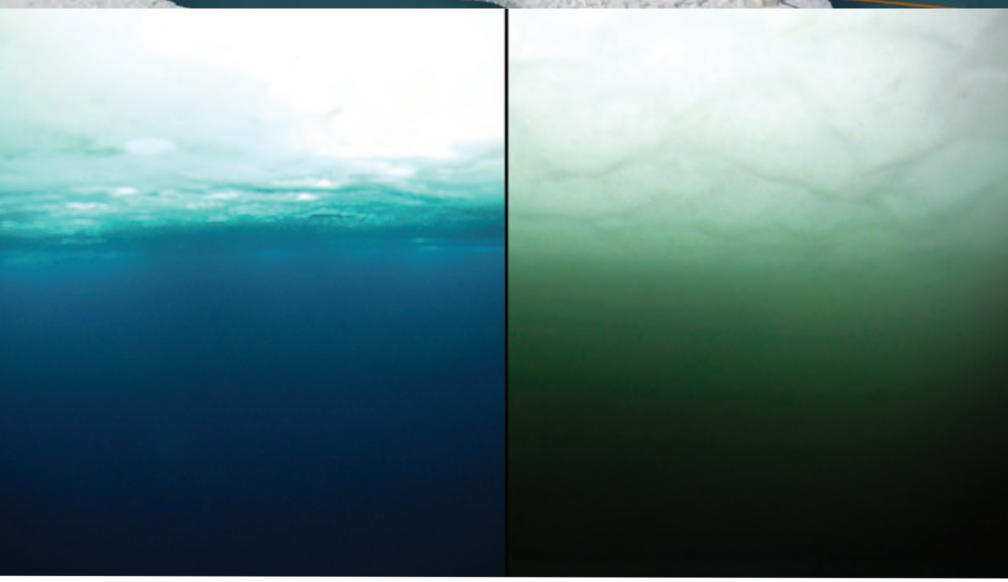
real ponds (Perovich)

Ma, Sudakov, Strong, Golden, *New J. Phys.* 2019

Perovich

Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

WINDOWS



no bloom bloom

massive under-ice algal bloom

Arrigo et al., *Science* 2012

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

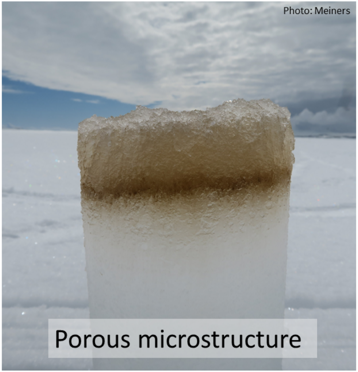
Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, ***Science Advances*** 2017

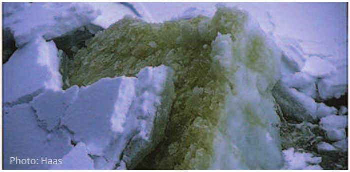
The effect of melt pond geometry on the distribution of solar energy under first year sea ice

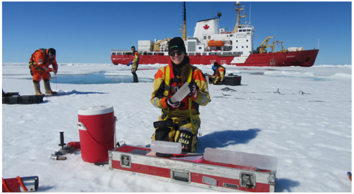
Horvat, Flocco, Rees Jones, Roach, Golden
Geophys. Res. Lett. 2019

(2015 AMS MRC)

SEA ICE ALGAE

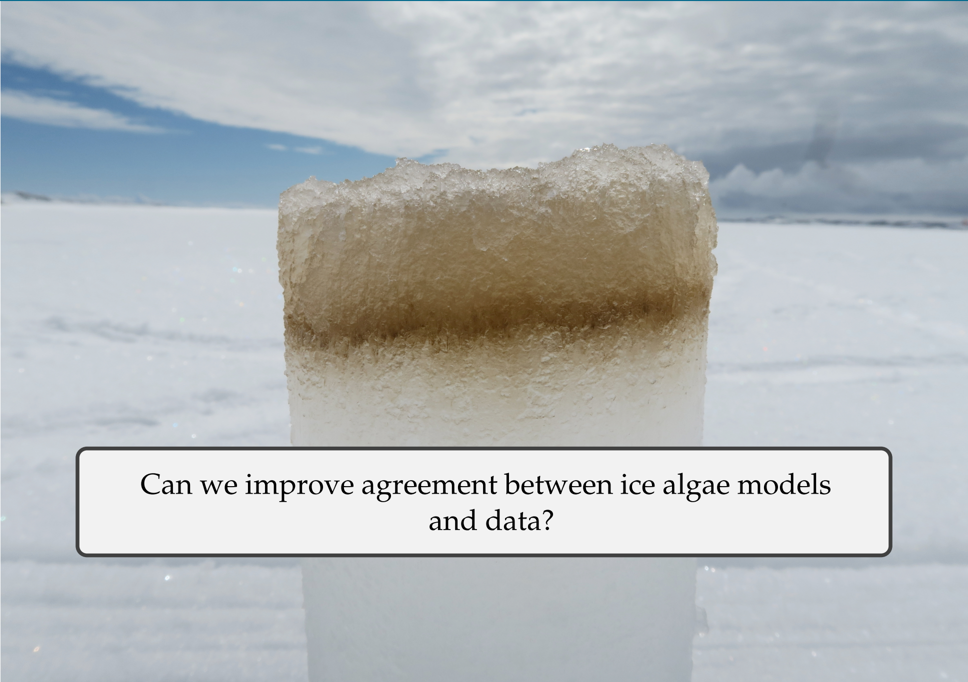






80% of polar bear diet can be traced to ice algae*.

* Brown TA, et al. (2018). *PloS one*, 13(1), e0191631



Can we improve agreement between ice algae models and data?

Algal bloom model*

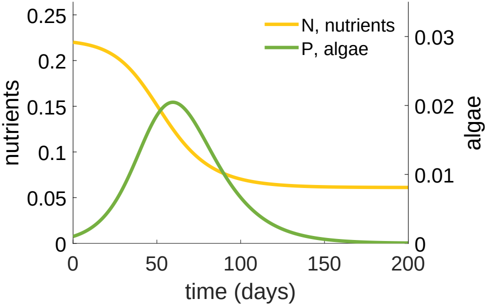
$$\text{nutrients:} \qquad \frac{dN}{dt} = \underbrace{\alpha}_{\text{input}} - \underbrace{\beta NP}_{\text{uptake}} - \underbrace{\eta N}_{\text{loss}}$$

$$\text{algae:} \qquad \frac{dP}{dt} = \underbrace{\gamma\beta NP}_{\text{growth}} - \underbrace{\delta P}_{\text{death}},$$

$$N(0) = n_0, \qquad P(0) = p_0$$

* Huppert, A., et al. (2002). *American Naturalist*, 159(2), 156-171

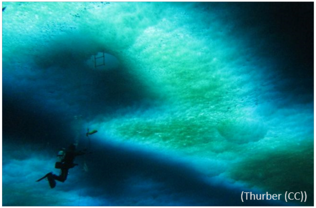
Algal bloom model

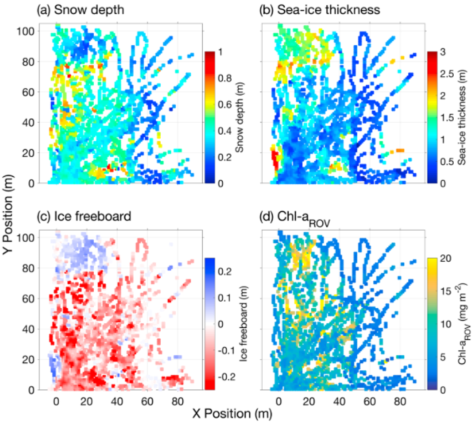


- poor agreement with data
- poor agreement between models

Steinacher, M., et al. (2010). *Biogeosciences*, 7(3), 979-1005

HETEROGENEITY





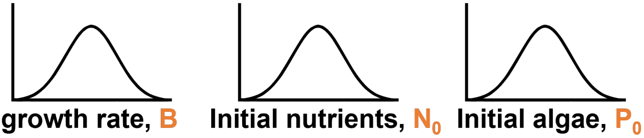
Meiners, K.M., et al. (2017). *Geophysical Research Letters*, 44(14), 7382-7390

Heterogeneity in initial conditions

At each location within a larger region, we could consider

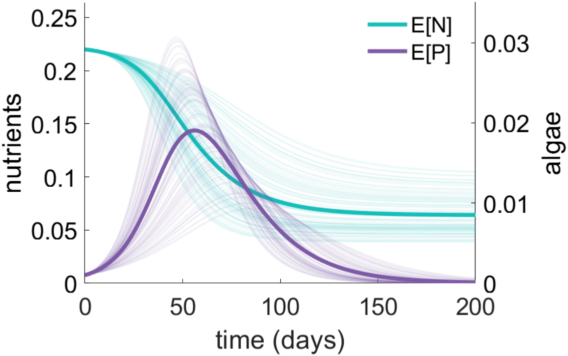
$$\frac{dN}{dt} = \alpha - BNP - \eta N$$
$$\frac{dP}{dt} = \gamma BNP - \delta P$$

$$N(0) = N_0, \qquad P(0) = P_0$$



HOW DO WE ANALYZE THIS MODEL?

Monte Carlo simulations?



Too slow! Full algae model takes **8 hours** (cloud computing).

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DOI: 10.1111/ele.14095

ECOLOGY LETTERS WILEY

METHOD

Uncertainty quantification for ecological models with random parameters

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Email: reimer@math.utah.edu

Abstract

There is often considerable uncertainty in parameters in ecological models. This uncertainty can be incorporated into models by treating parameters as random variables with distributions, rather than fixed quantities. Recent advances in uncertainty quantification methods, such as polynomial chaos approaches, allow for the analysis of models with random parameters. We introduce these methods with a motivating case study of sea ice algal blooms in heterogeneous environments. We compare Monte Carlo methods with polynomial chaos techniques to help understand the dynamics of an algal bloom model with random parameters.

Polynomial chaos expansions

$$N(t;B,P_0,N_0) \approx N_V(t;B,P_0,N_0) := \sum_{j=1}^{n} \widetilde{N}_j(t)\phi_j(B,P_0,N_0),$$

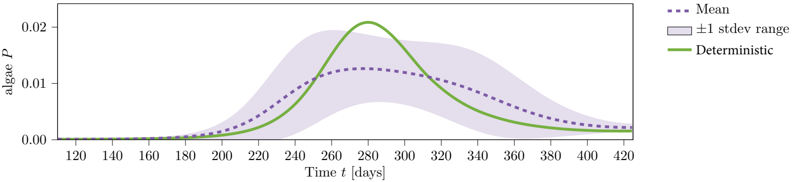
$$P(t;B,P_0,N_0) \approx P_V(t;B,P_0,N_0) := \sum_{j=1}^{n} \widetilde{P}_j(t)\phi_j(B,P_0,N_0),$$

where

- $V := \text{span}\{\phi_j\}_{j=1}^{n}$
- ϕ_j are orthogonal polynomials that form a basis for V
- $(\widetilde{N}_j, \widetilde{P}_j)$ need to be computed

Xiu, D. (2010). Numerical methods for stochastic computations. Princeton university press.

Ecological insights



- lower peak bloom intensity
- longer bloom duration
- able to compare variance to data

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Notices

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AMS AMERICAN MATHEMATICAL SOCIETY

Advancing research. Creating connections.

The cover is based on "Modeling Sea Ice," page 1535.

THANK YOU

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Eigenvalue Statistics of Random Matrix Theory

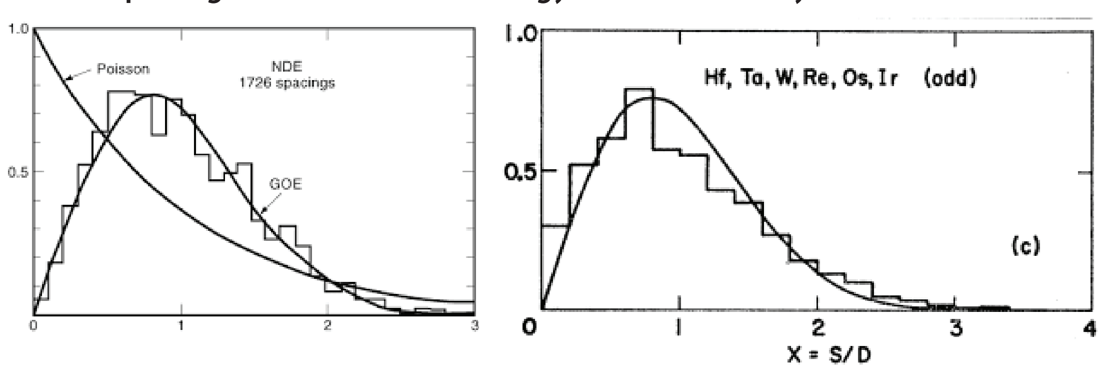
Wigner (1951) and Dyson (1953) first used random matrix theory (RMT) to describe quantized energy levels of heavy atomic nuclei.

$[\mathbf{N}]_{ij} \sim N(0,1)$, $\mathbf{A} = (\mathbf{N}+\mathbf{N}^T)/2$ ***Gaussian orthogonal ensemble (GOE)***

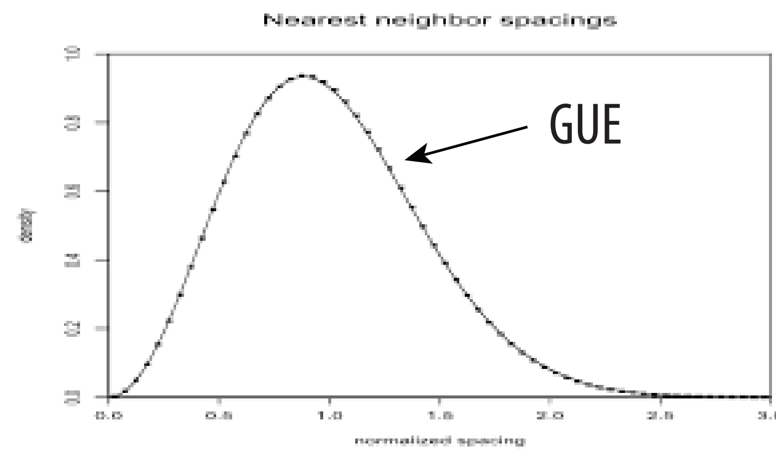
$[\mathbf{N}]_{ij} \sim N(0,1)+iN(0,1)$, $\mathbf{A} = (\mathbf{N}+\mathbf{N}^\dagger)/2$ ***Gaussian unitary ensemble (GUE)***

Short range and long range correlations of eigenvalues are measured by various eigenvalue statistics.

Spacing distributions of energy levels for heavy atomic nuclei



Spacing distributions of the first billion zeros of the Riemann zeta function



Universal eigenvalue statistics arise in a broad range of "unrelated" problems!