MODELING *the* MELT:

what math tells us about the disappearing polar ice caps

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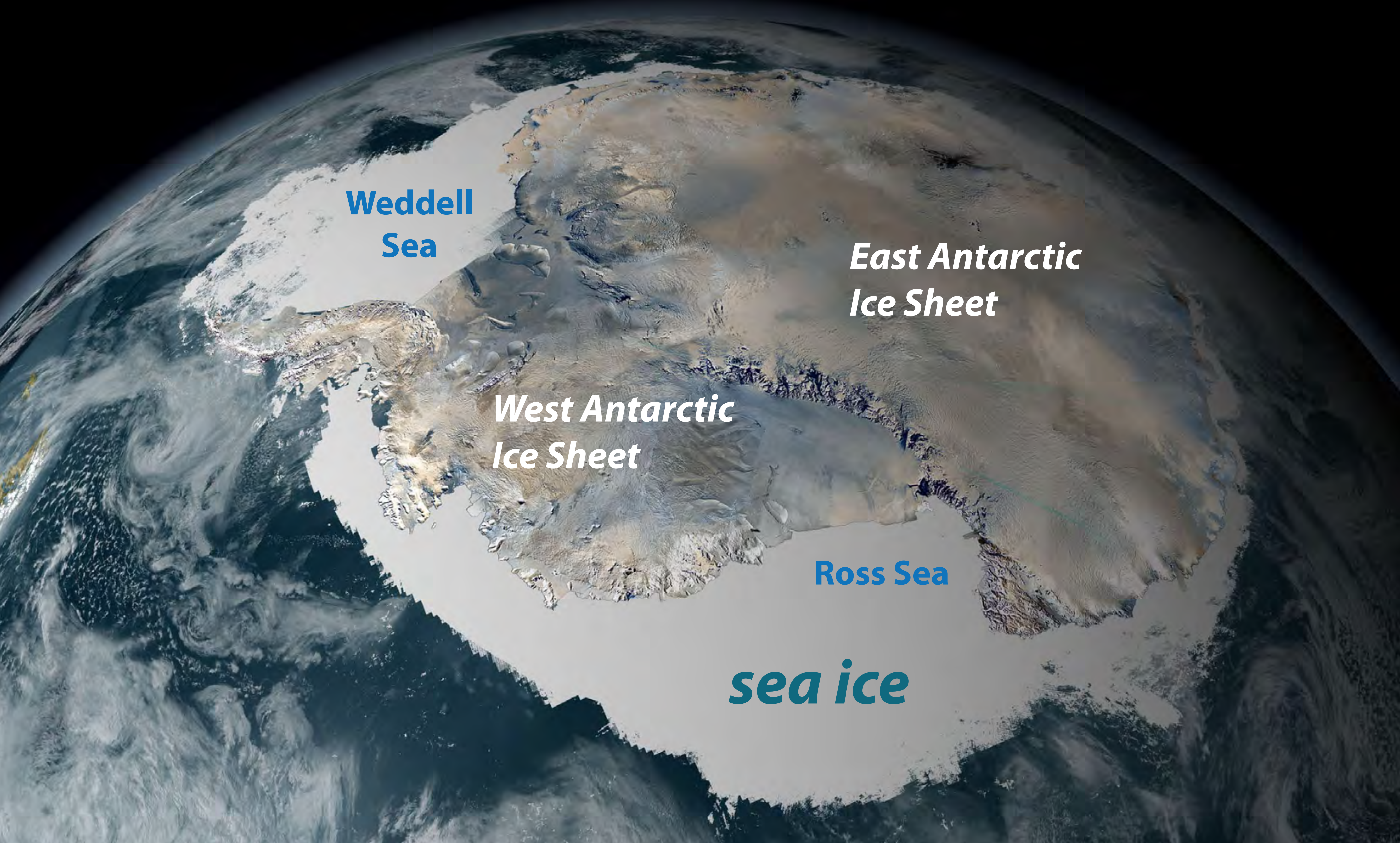
National Math Festival ***April 18, 2015***

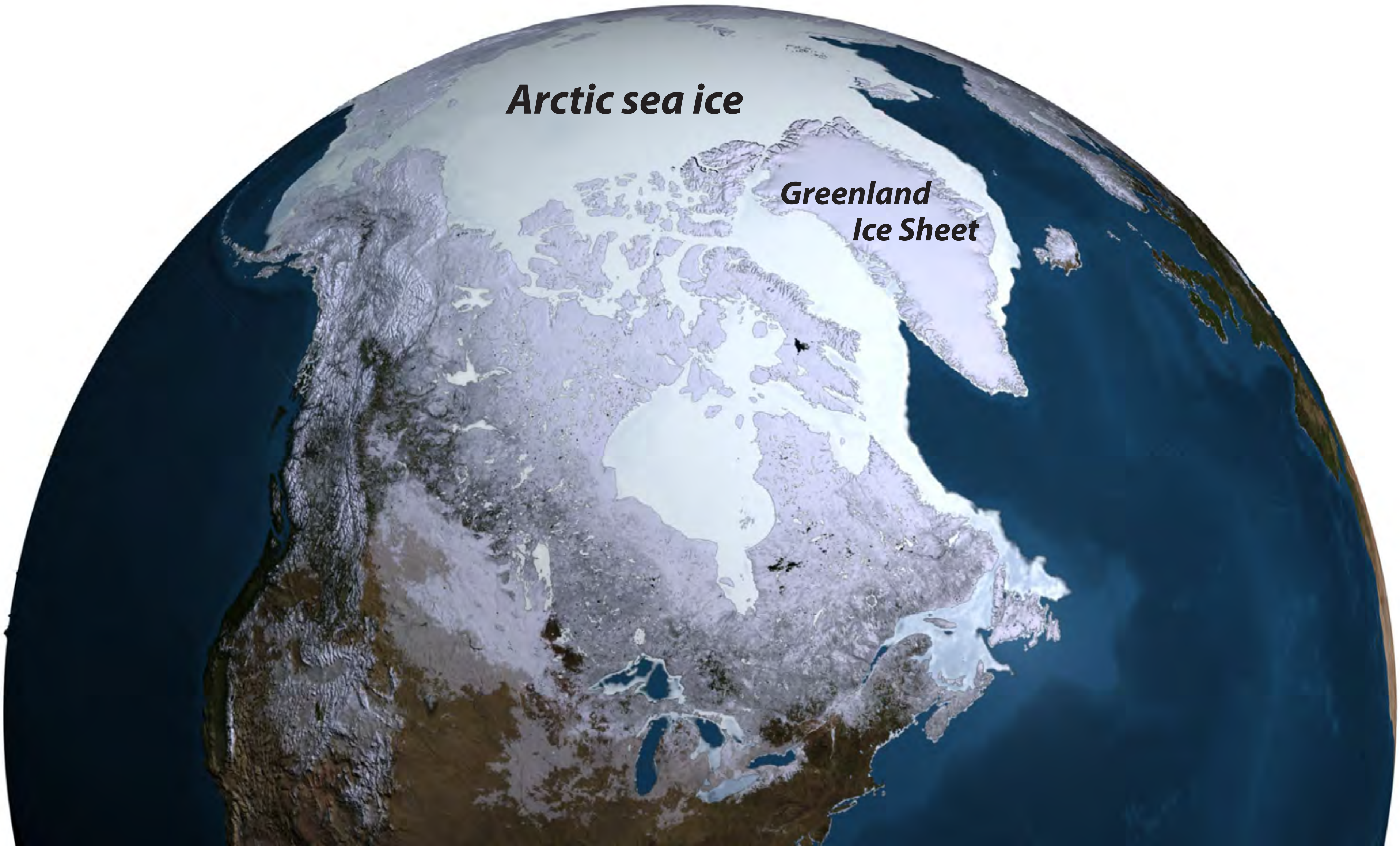
Frey

ANTARCTICA

southern cryosphere



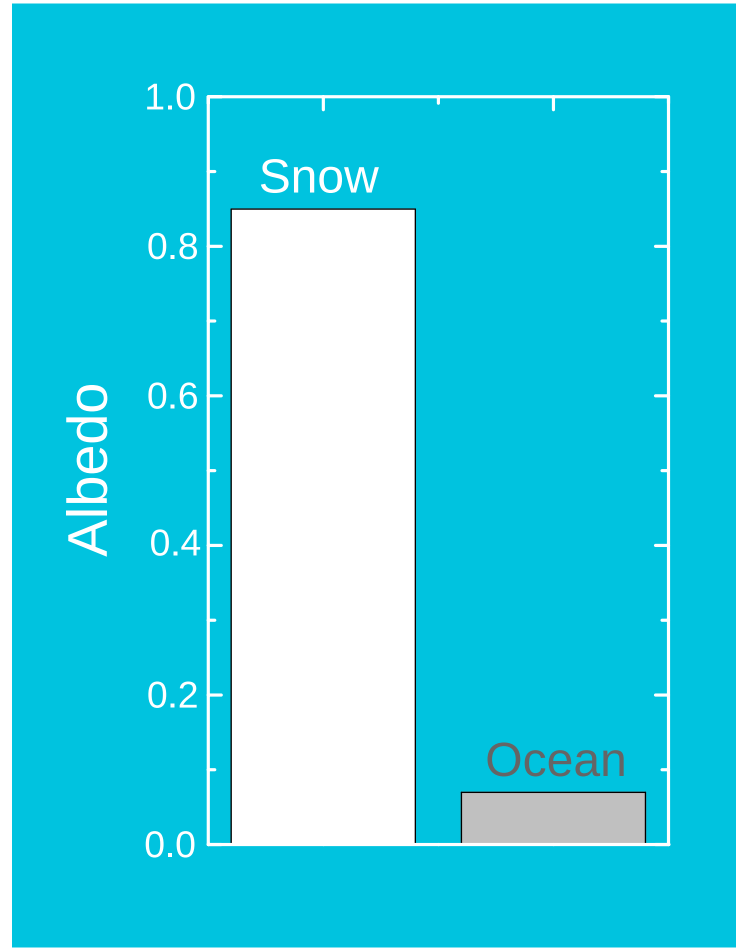
northern cryosphere




SEA ICE covers 7 - 10% of earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, gases, momentum
• global ocean circulation
• indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

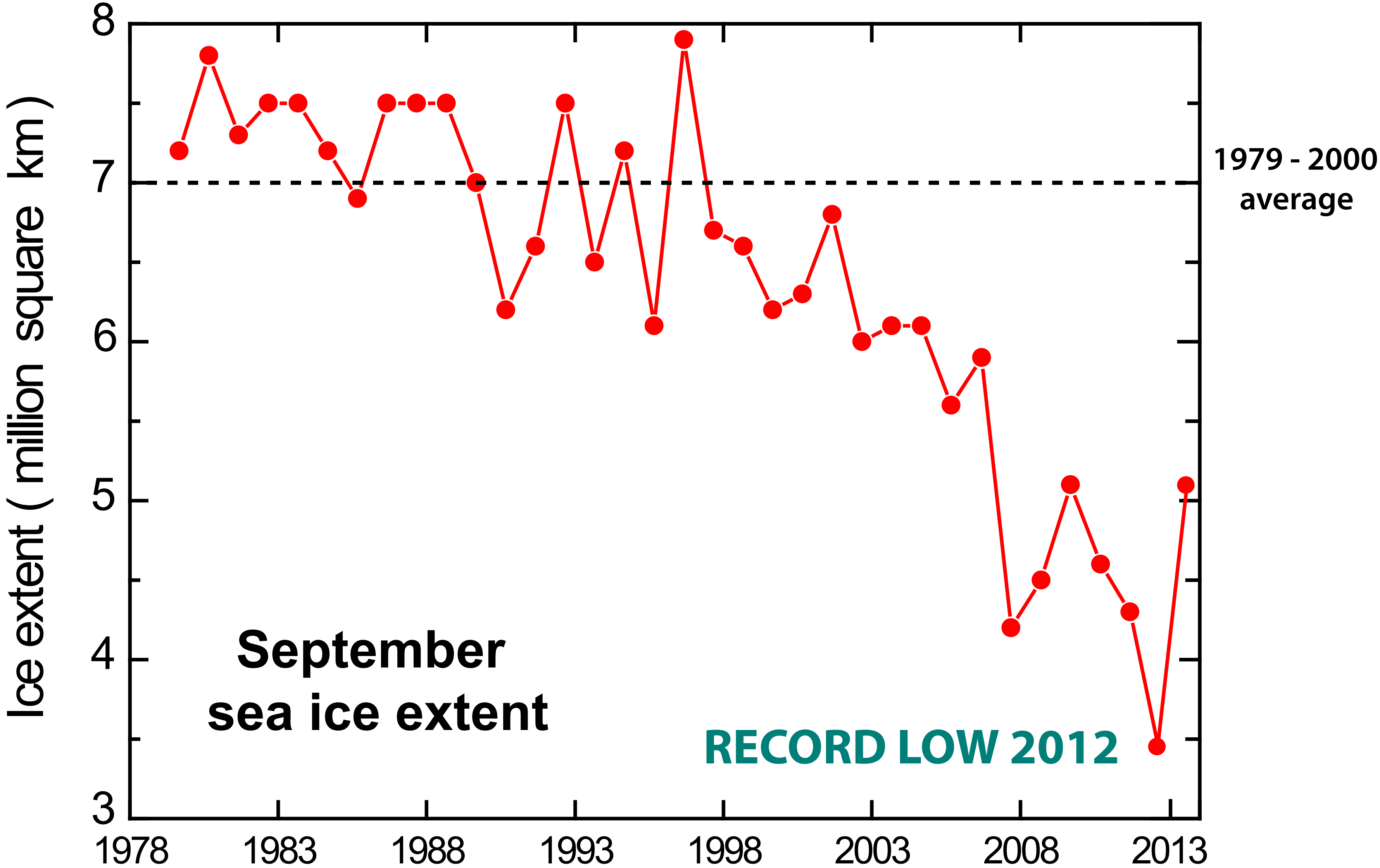
white snow and ice reflect



dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

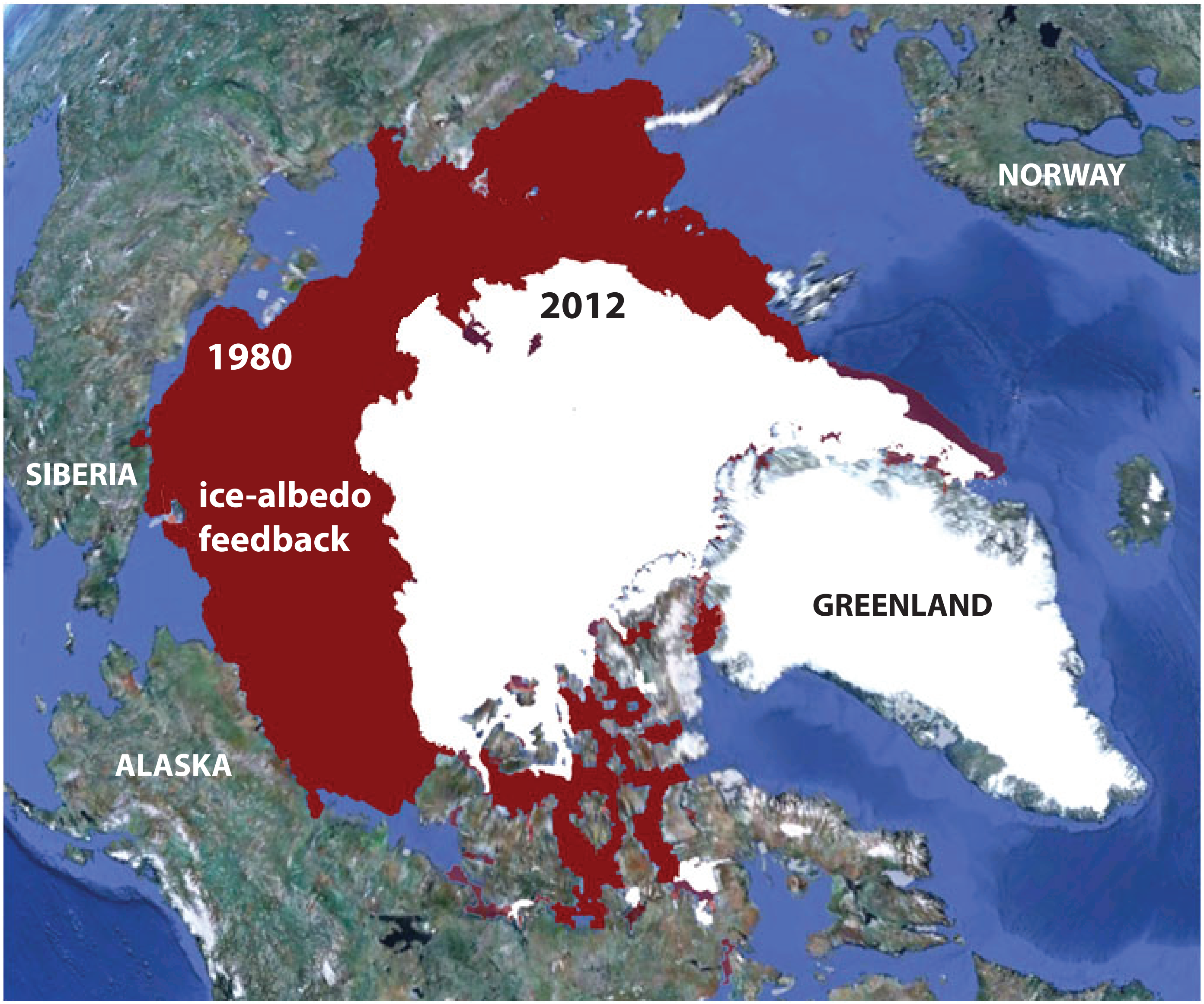
the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

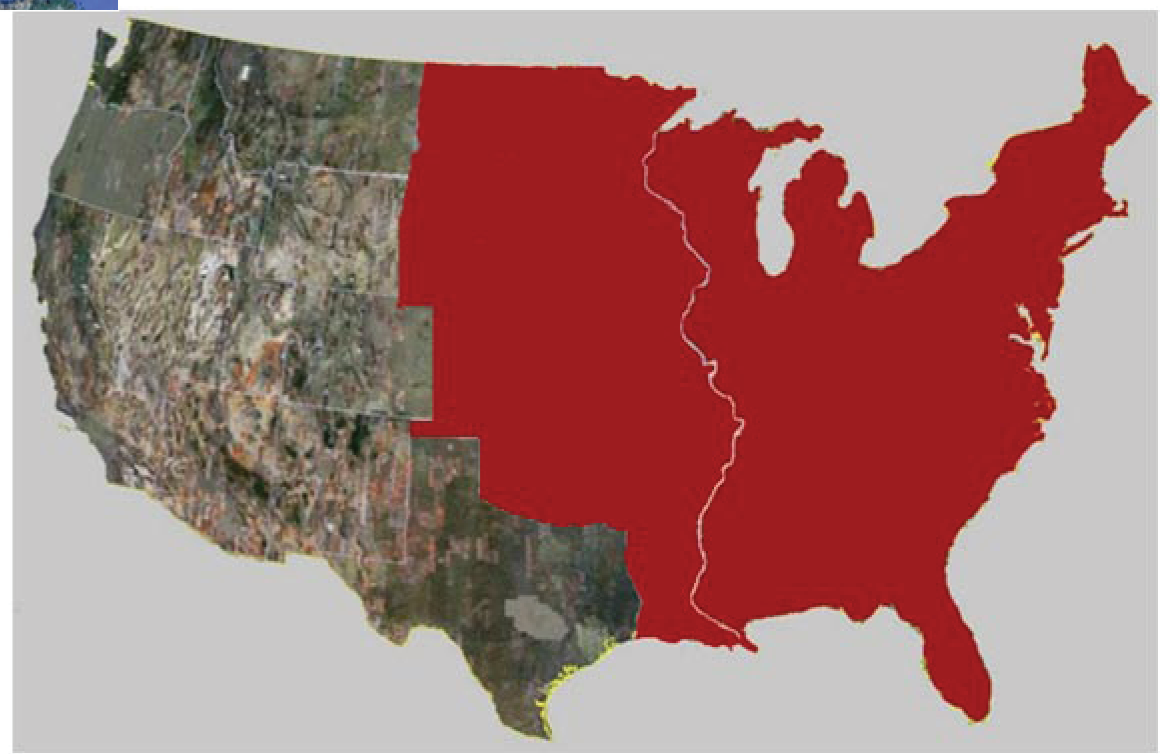
Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers
September 2012 -- 3.4 million square kilometers



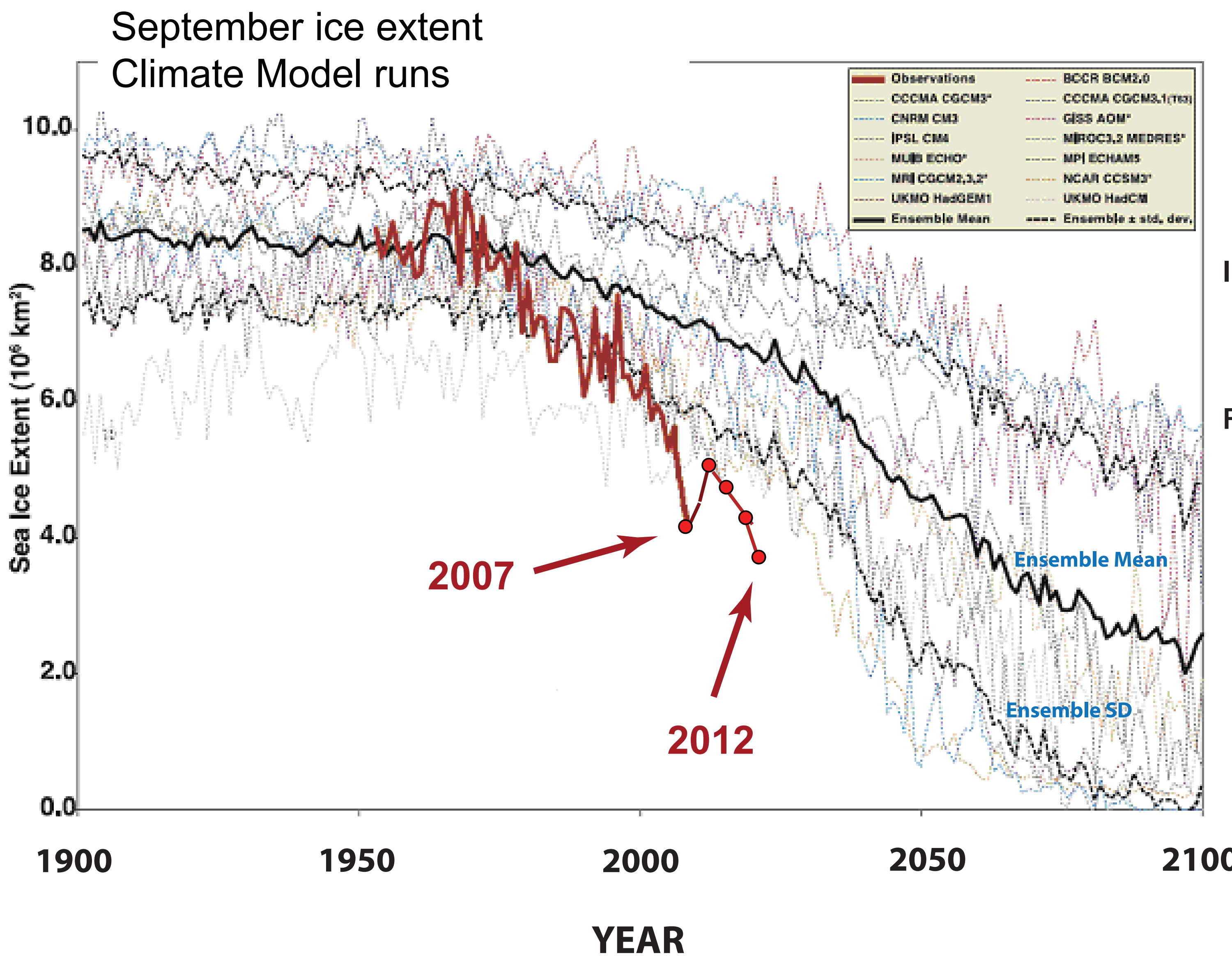
Perovich

recent losses
in comparison to
the United States



Arctic sea ice decline - faster than predicted by climate models

Stroeve et al., GRL, 2007



IPCC AR4 Models

Intergovernmental Panel on Climate Change (IPCC)

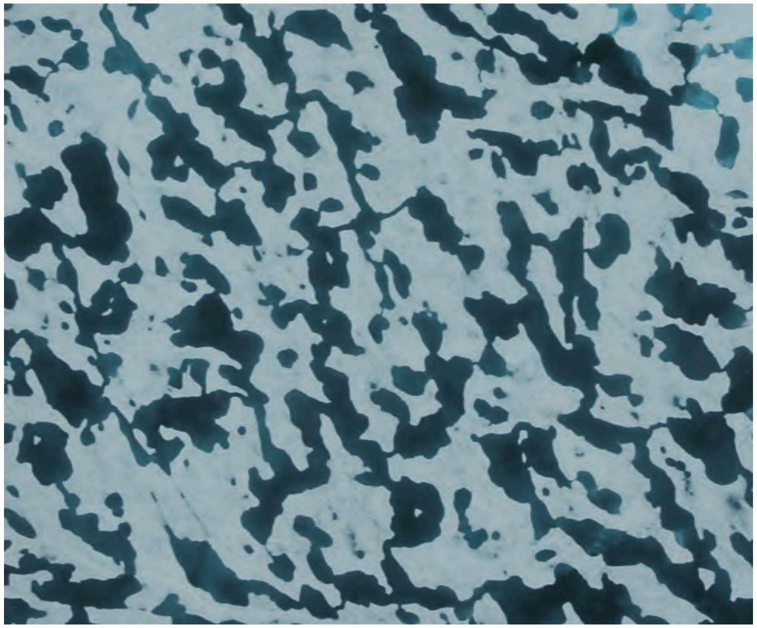
Fourth Assessment AR4, 2007

challenge

represent sea ice more rigorously in climate models

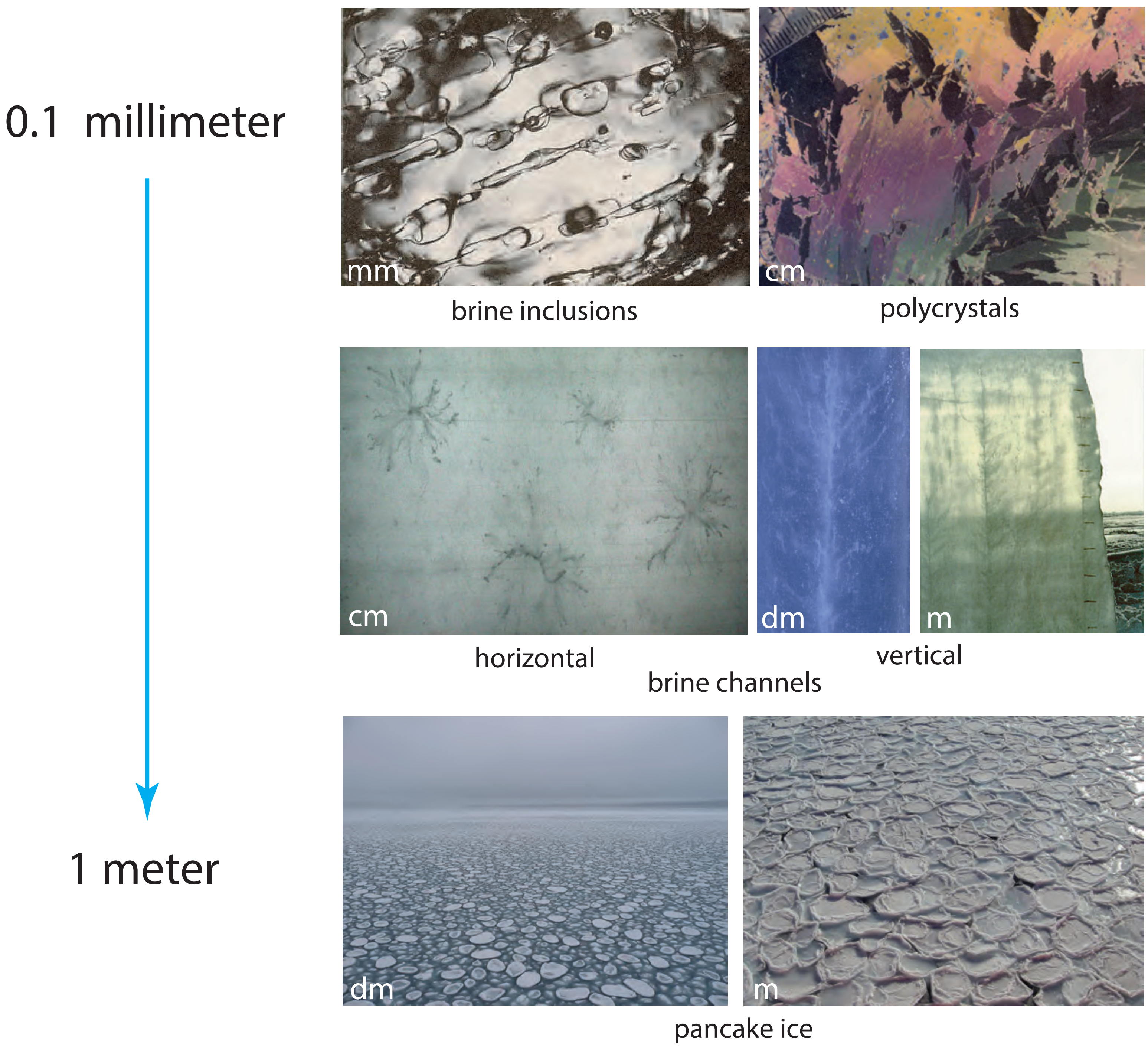
account for key processes

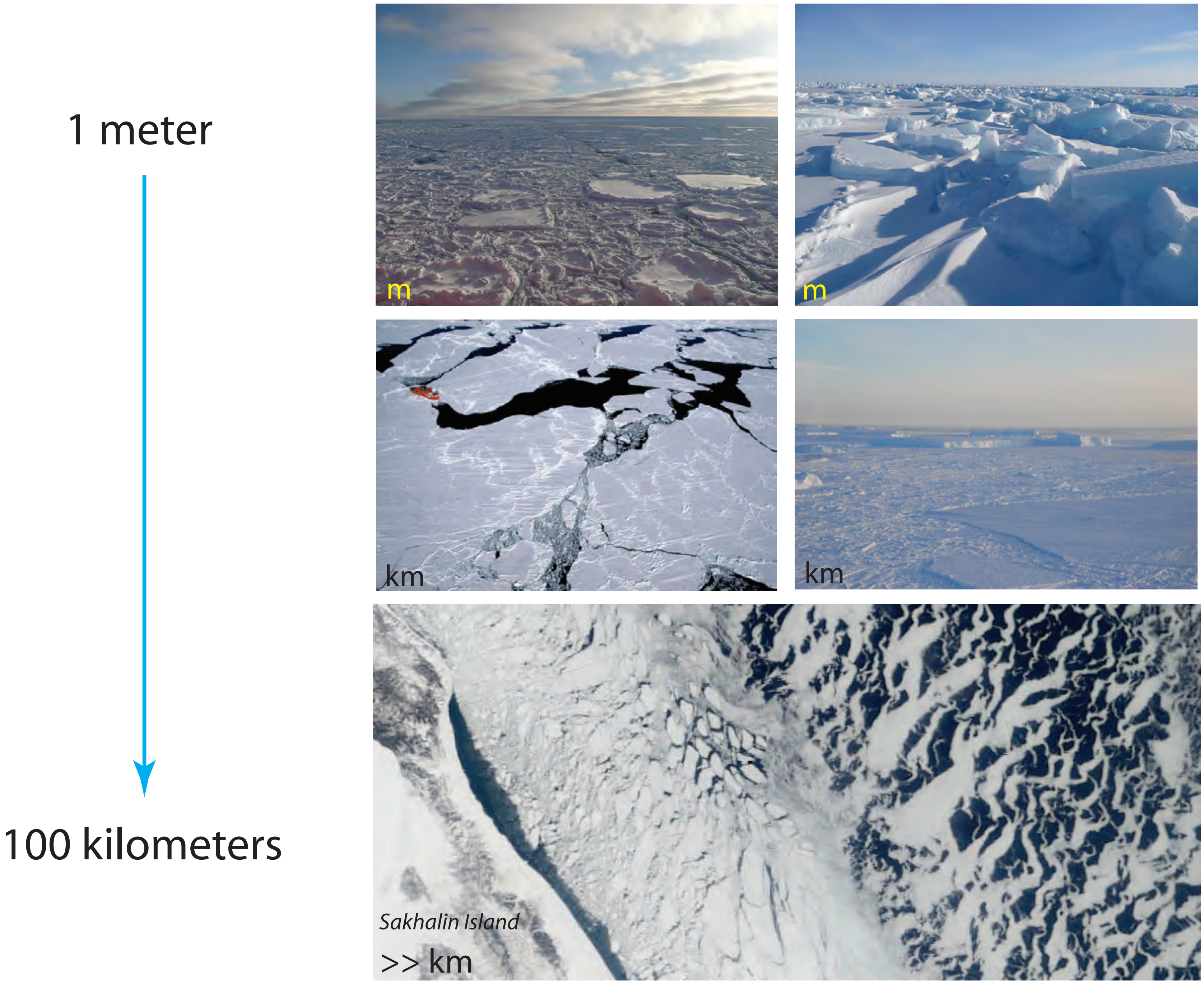
such as melt pond evolution



sea ice is a multiscale composite

displaying structure over 10 orders of magnitude




1 meter
100 kilometers
m
m
km
km
Sakhalin Island
>> km

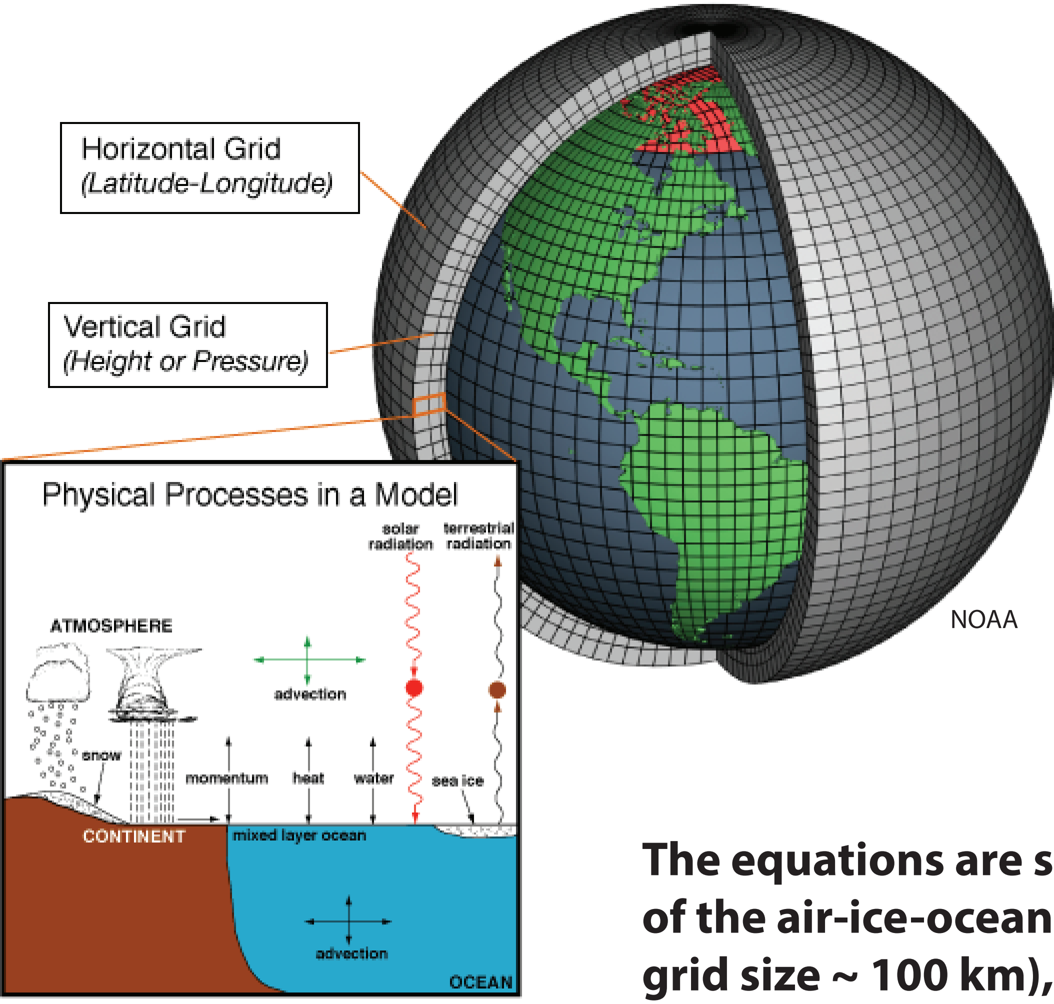
What is this talk about?

Using the mathematics of composite materials and statistical physics to study sea ice structures and processes ... to improve projections of climate change.

1. ***Opposite poles of climate modeling***
2. ***Fluid flow through sea ice - percolation***
3. ***Electromagnetic monitoring of sea ice***
4. ***Fractal geometry of Arctic melt ponds***
5. ***Video of 2012 Antarctic expedition***

cross-pollination

Global Climate Models



Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

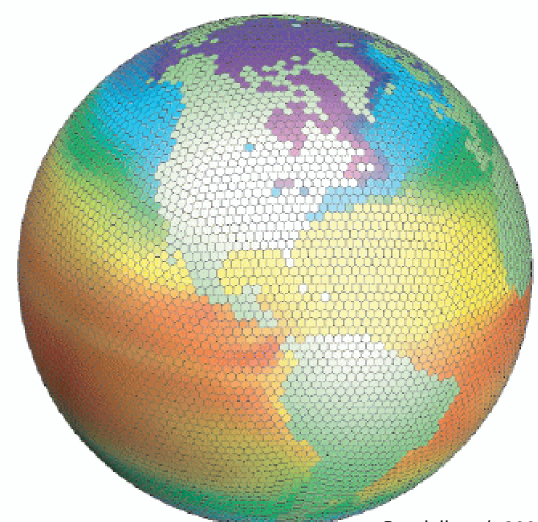
They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 100 km), using very powerful computers.

sub - grid scale processes

sea ice components of GCM:

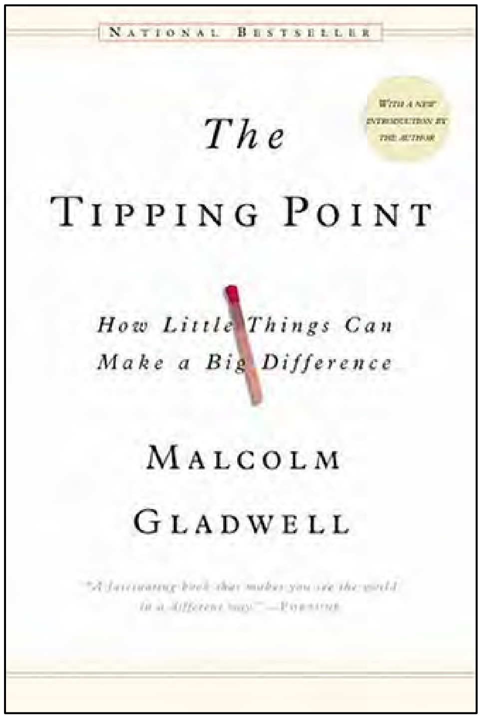
- ***ice thickness distribution***
- ***balance of forces (F=ma)***
- ***heat and radiation fluxes***

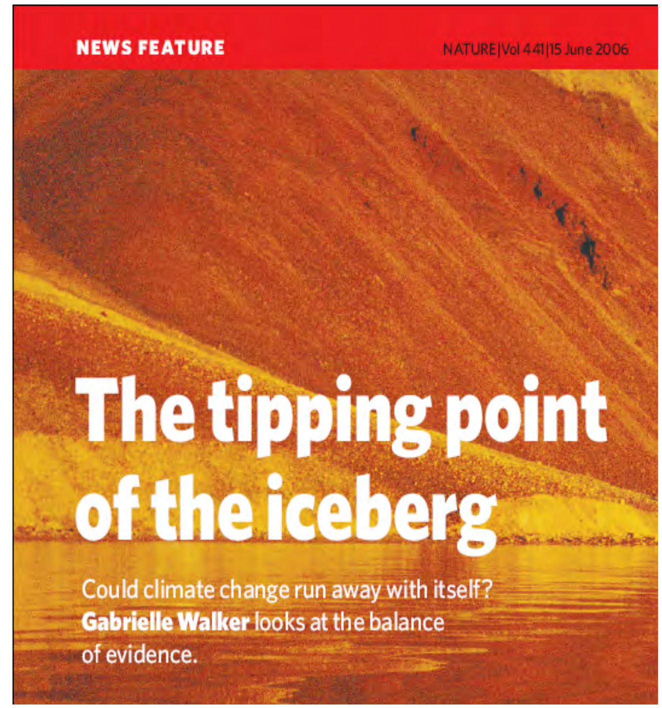


Randall *et al.*, 2002

tipping points in the mainstream

climate tipping points – September Arctic sea ice cover





Melting of the Greenland ice sheet
Melting of the West Antarctic ice sheet
Permafrost and tundra loss, leading to the release of methane
Shutoff of N. Atlantic thermohaline conveyor (Gulf Stream) •••

Lenton, et al., PNAS 2008

opposite "pole" from GCM's

active area of mathematical research on sea ice:

Has Arctic sea ice loss passed through a "tipping point"?

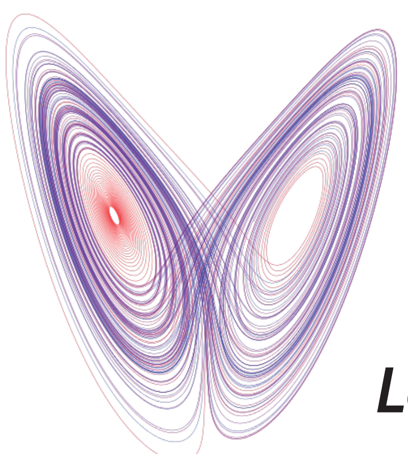
an irreversible downward slide to ice-free Arctic summers, driven by ice-albedo feedback

Eisenman, Wettlaufer, PNAS 2009 : analyze a single nonlinear differential equation for the energy in the upper ocean

look for "bifurcations" in solutions

- unlikely in current loss of summer ice
- more likely in further loss of winter ice

Abbot, Silber, Pierrehumbert, JGR 2011 : *bifurcations when include clouds and ice loss*



low order (toy) model of Arctic climate change

Lorenz butterfly

dynamical systems

Who cares if Arctic sea ice disappears?

Ralph (Malik) Ahkivgak, c. 20 Oct 1988

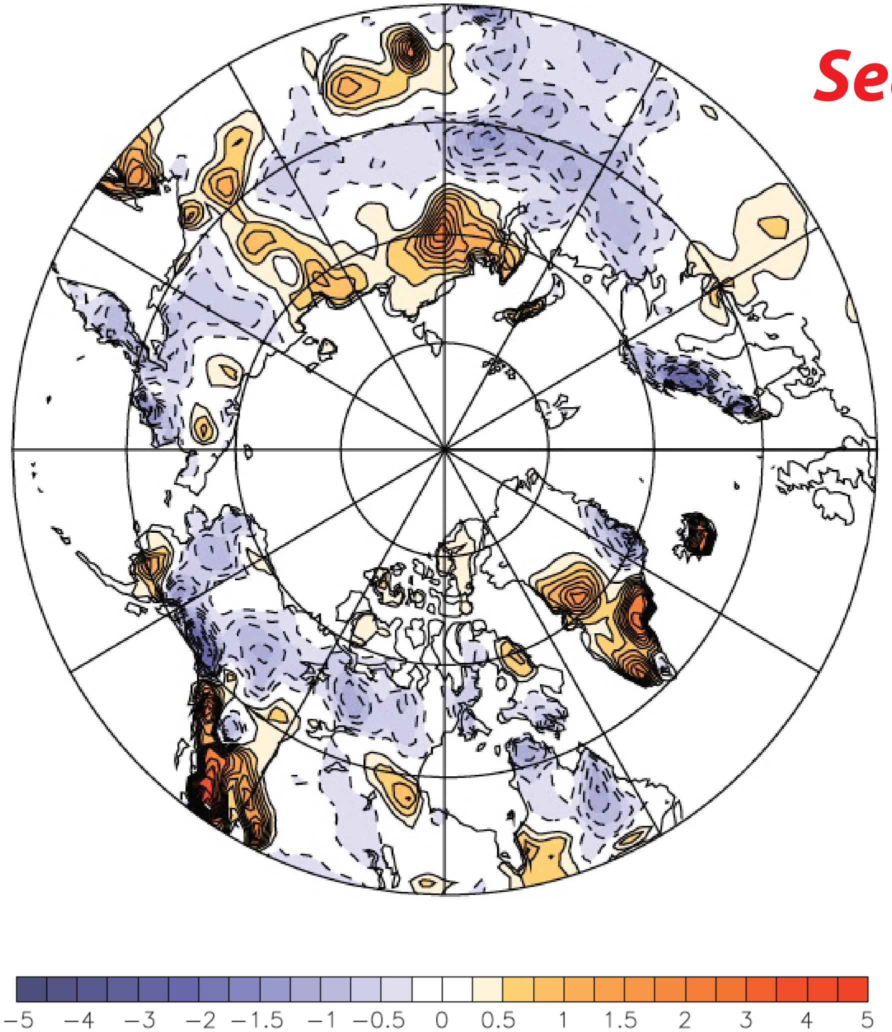
• The Arctic holds 25% of the world's undiscovered oil & gas reserves

• Sea ice is both a hazard and a supporting feature for hydrocarbon exploration & production

oil companies care about Arctic sea ice loss

Source: UNEP/GRID-Arendal

Sea-ice loss: impacts beyond the Arctic



Orsolini et al., 2011

changes in precipitation and temperature patterns, storm tracks, ...

- One climate model projects reduced precipitation in American West (Sewall & Sloan, 2005)

  Utah - greatest snow on Earth?

- Analysis of 2007 ice minimum suggests above normal snow deposition in NW North America (Orsolini et al., 2011)

- Colder weather in SE Asia, possibly in Eastern US (Hondo et al., 2009)

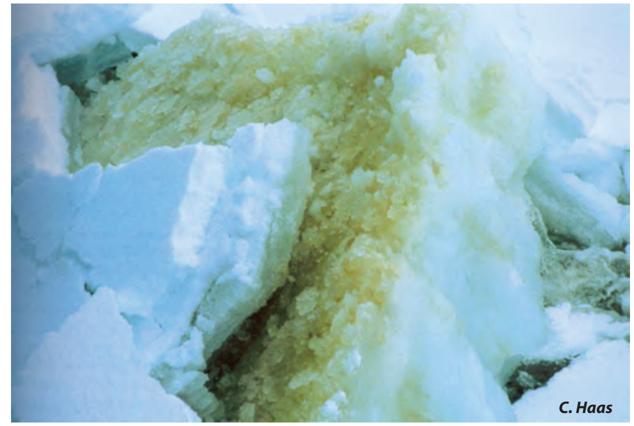
sea ice microphysics

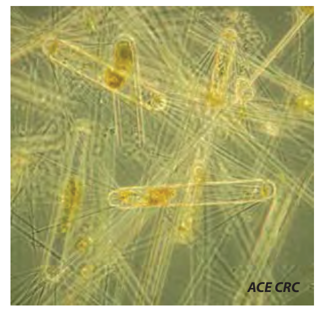
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

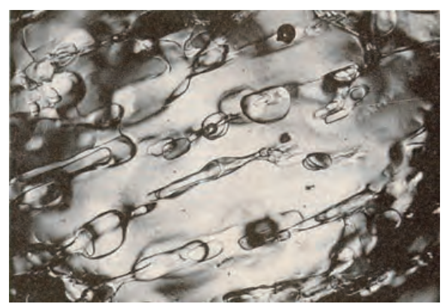
nutrient flux for algal communities





- *drainage of brine and melt water*
- *ocean-ice-air exchanges of heat, CO_2*
- *Antarctic surface flooding and snow-ice formation*
- *evolution of salinity profiles*

↕ **linkage of scales**



fluid permeability *k* of a porous medium

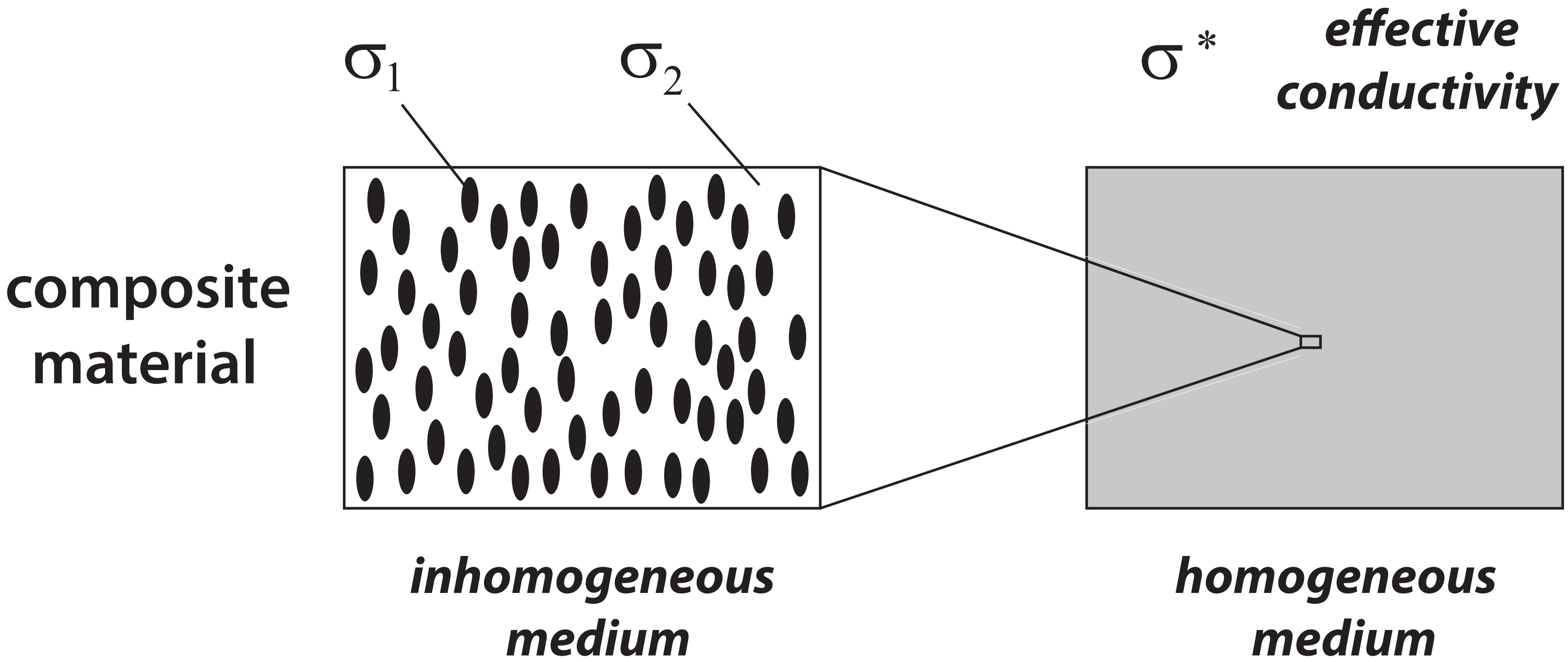
porous concrete

how much water gets through the sample per unit time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

HOMOGENIZATION

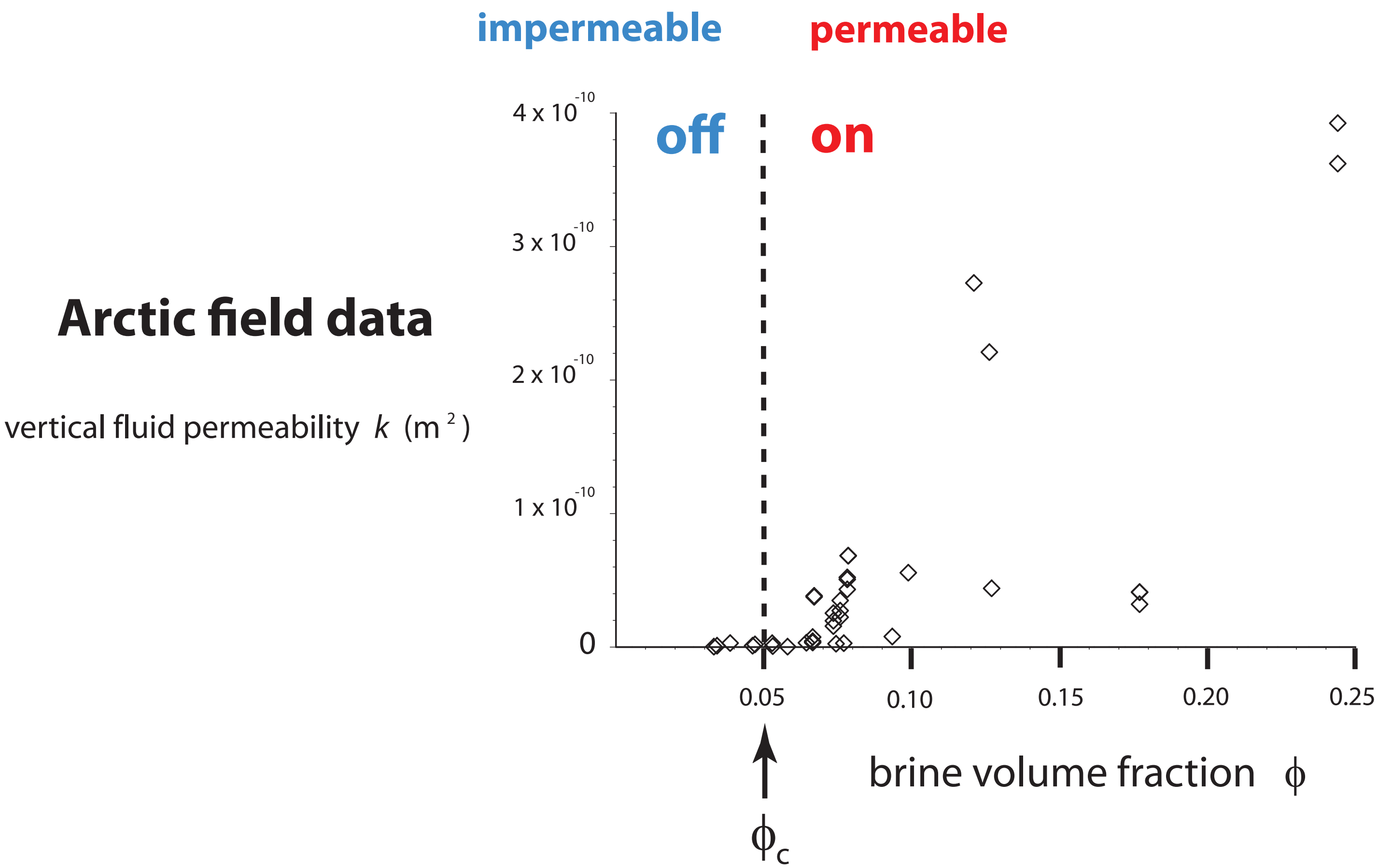


find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

Critical behavior of fluid transport in sea ice



on - off switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ C$, $S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007
Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

sea ice algal communities

D. Thomas 2004

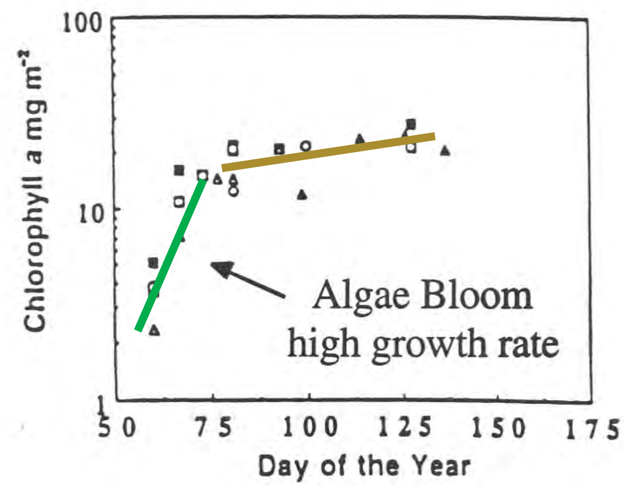
nutrient replenishment controlled by ice permeability

biological activity turns on or off according to ***rule of fives***

Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

critical behavior of microbial activity



Convection-fueled algae bloom
Ice Station Weddell

sea ice ecosystem

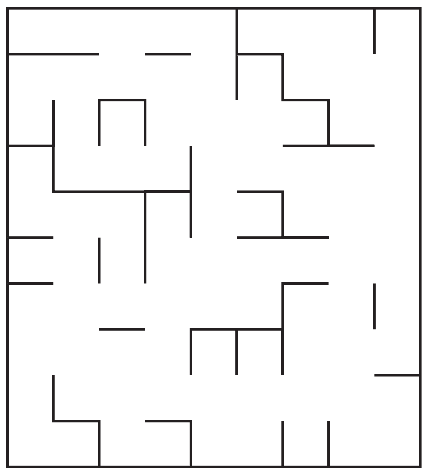
sea ice algae
support life in the polar oceans

Why is the rule of fives true?

percolation theory

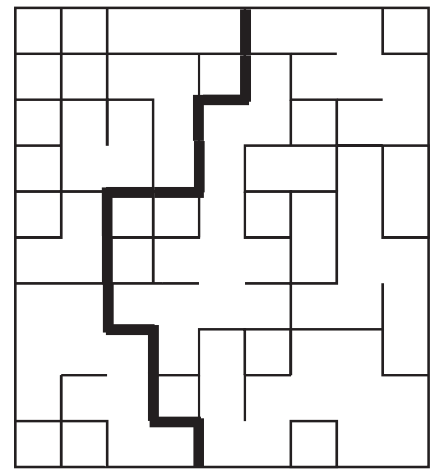
mathematical theory of connectedness

impermeable



$p = 1/3$

permeable



$p = 2/3$

bond → *open* with probability p
closed with probability $1\text{-}p$

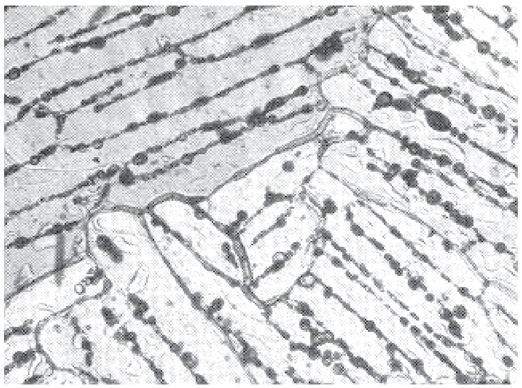
percolation threshold

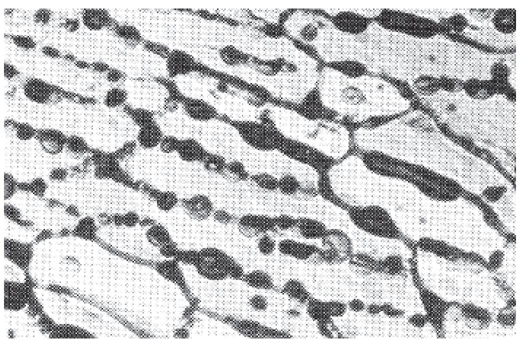
$$p_c = 1/2 \quad \text{for} \quad d = 2$$

first appearance of infinite cluster ***"tipping point" for connectivity***

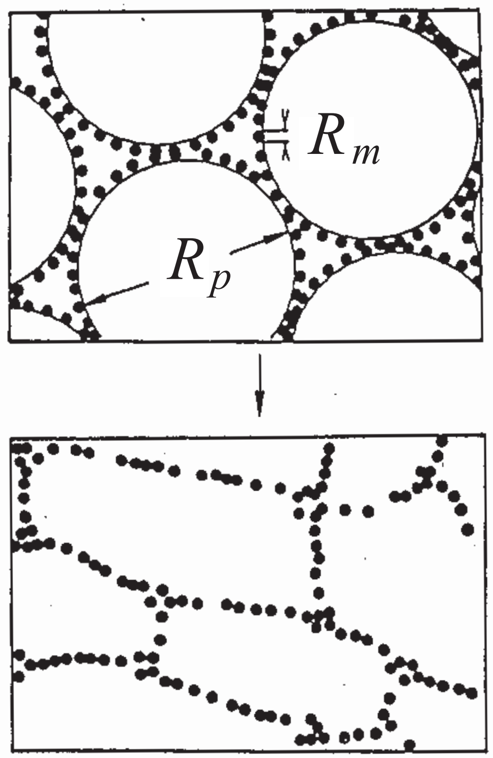
Continuum percolation model for ***stealthy*** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

$\phi_c \approx 5\,\%$ Golden, Ackley, Lytle, *Science,* 1998

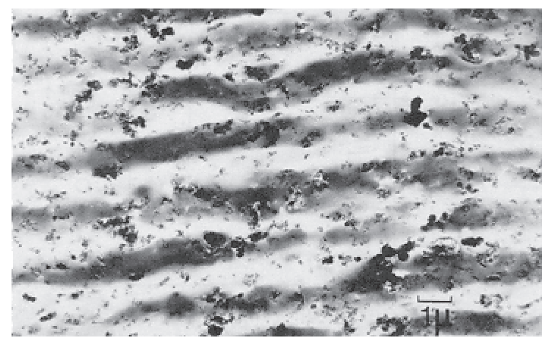




sea ice



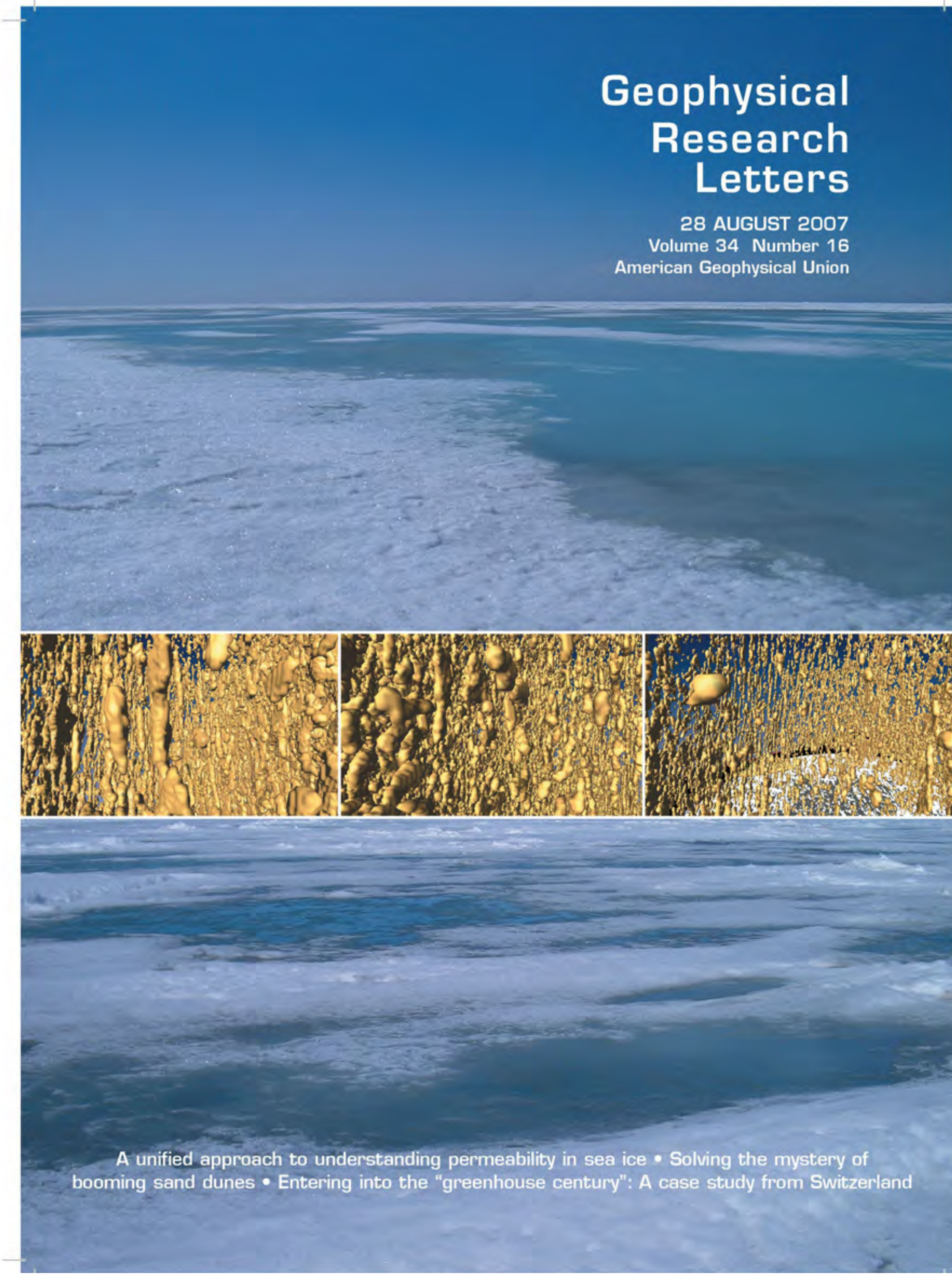
compressed powder



radar absorbing composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice **Golden, Eicken, Heaton, Miner, Pringle, Zhu**



rigorous bounds
percolation theory
hierarchical model
network model

field data

micro-scale

controls

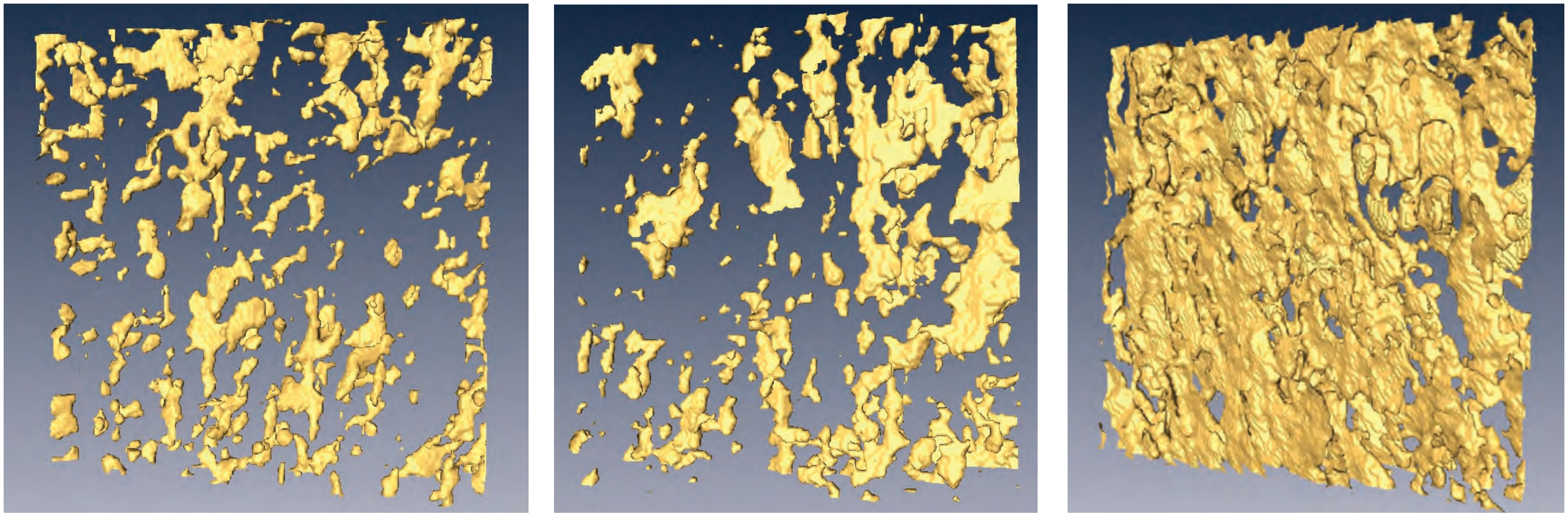
macro-scale processes

X-ray tomography for brine inclusions

unprecedented look at thermal evolution of brine phase and its connectivity

brine connectivity (over cm scale)

8 x 8 x 2 mm



-15 °C, $\phi = 0.033$ -6 °C, $\phi = 0.075$ -3 °C, $\phi = 0.143$

X-ray tomography confirms percolation threshold

3-D images
pores and throats → 3-D graph
nodes and edges

analyze graph connectivity as function of temperature and sample size

- ***use finite size scaling techniques to confirm rule of fives***
- ***order parameter data from a natural material***

Pringle, Miner, Eicken, Golden, J. Geophys. Res. 2009

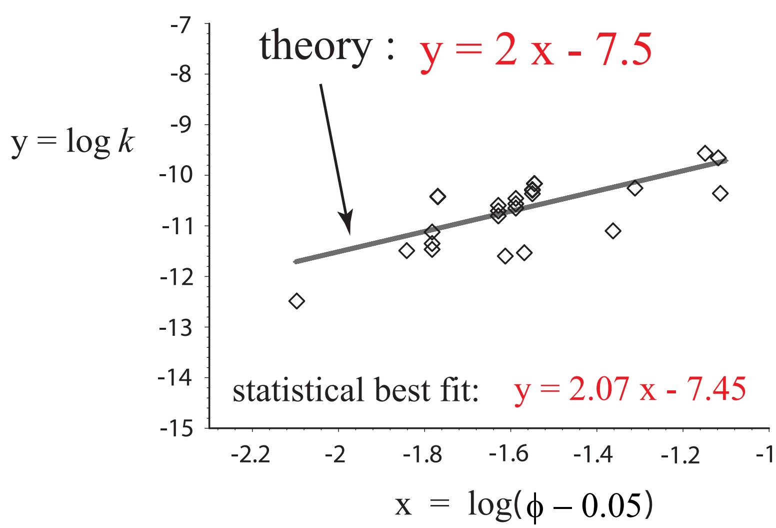
lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8}\ \mathrm{m}^2$$

- exponent is UNIVERSAL lattice value $t \approx 2.0$
- sedimentary rocks like sandstones also exhibit universality
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0



Remote sensing of sea ice

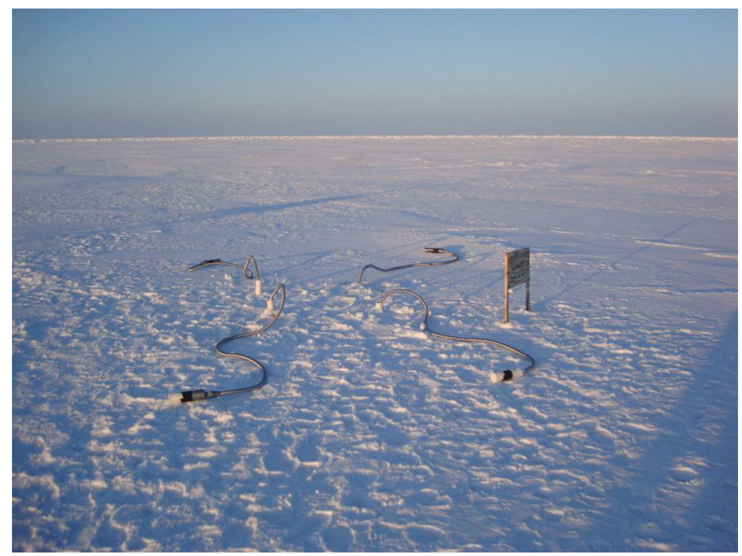
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

ε^*

effective complex permittivity
(dielectric constant, conductivity)

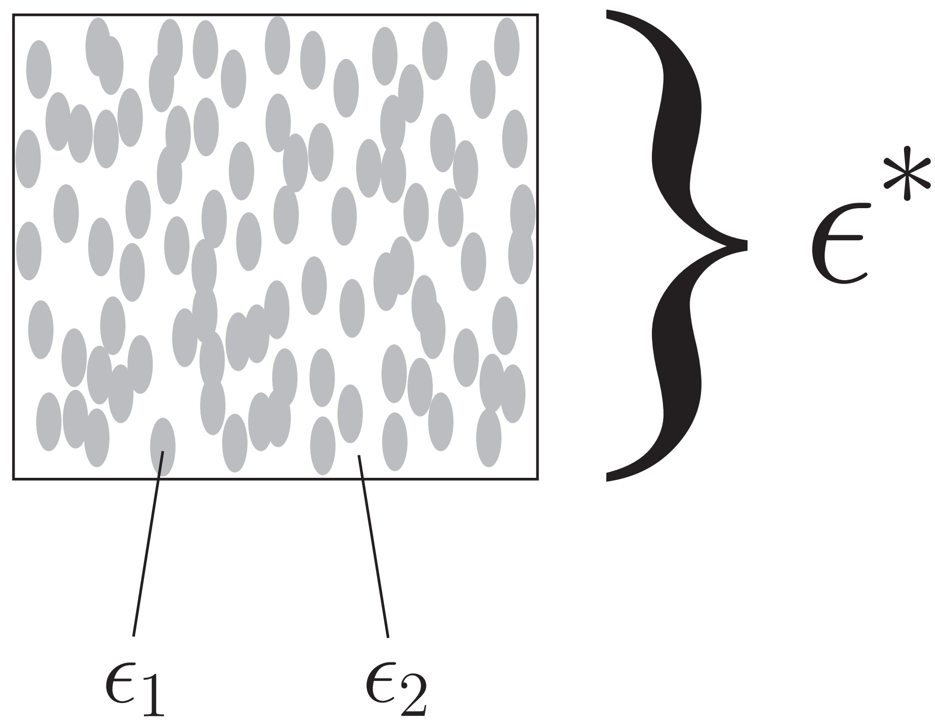


brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

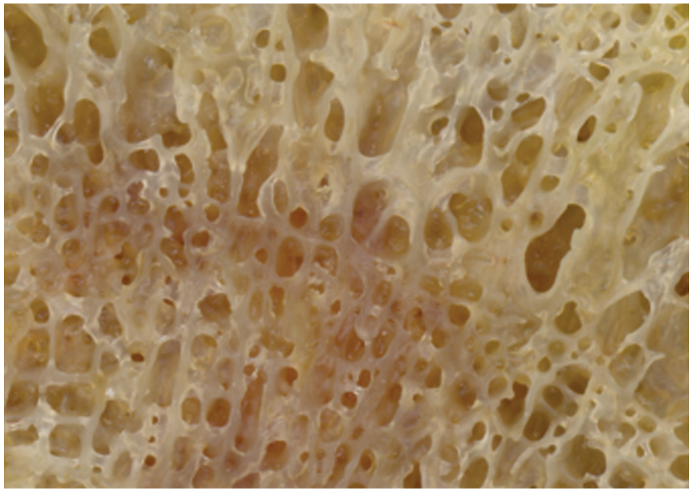
HOMOGENIZATION



p_1 , p_2 = volume fractions of brine and ice

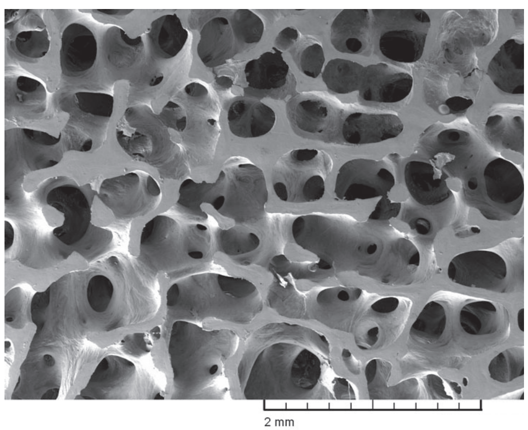
the math doesn't care if it's sea ice or bone!

HUMAN BONE

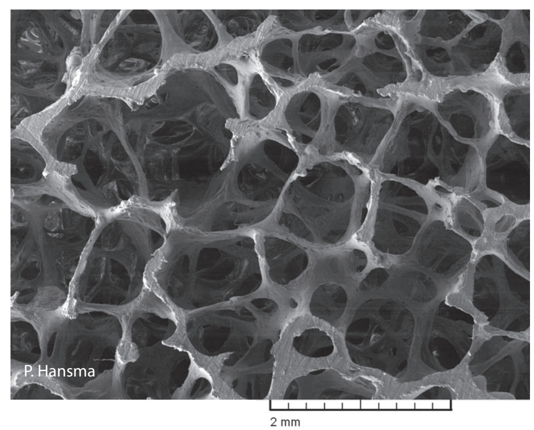


SEA ICE

young healthy trabecular bone



old osteoporotic trabecular bone



loss of bone connectivity

apply analysis of brine connectivity to EM monitoring osteoporosis

Golden, Murphy, Cherkaev, J. Biomechanics 2011

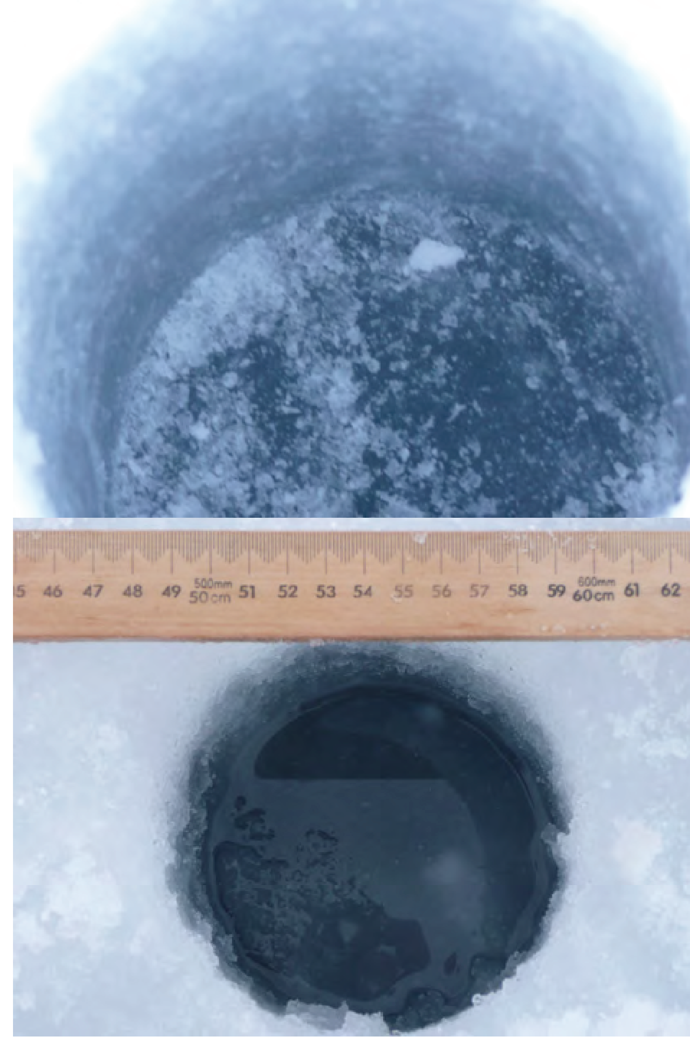
Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	*Antarctic*	*SIPEX*
2010	*Antarctic*	*McMurdo Sound*
2011	*Arctic*	*Barrow AK*
2012	*Arctic*	*Barrow AK*
2012	*Antarctic*	*SIPEX II*
2013	*Arctic*	*Barrow AK*
2014	*Arctic*	*Chukchi Sea*

photo by Jan Lieser

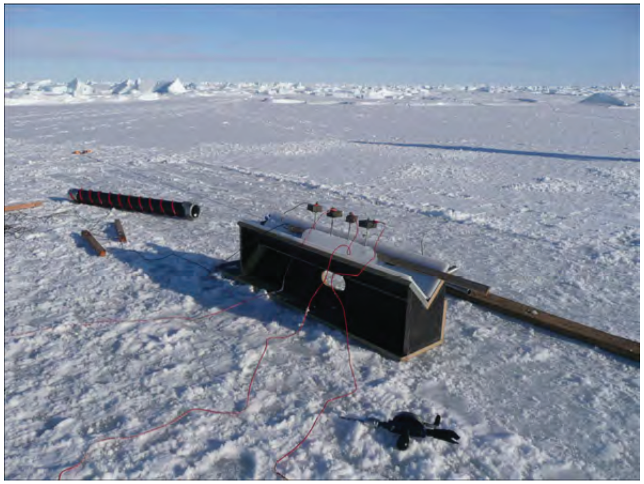


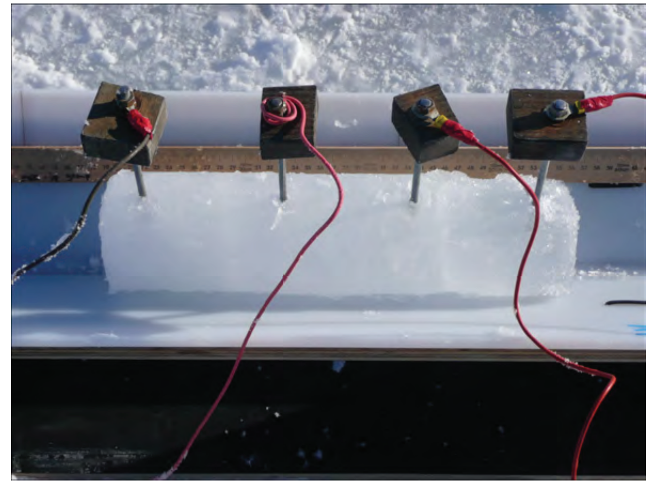
measuring fluid permeability of Antarctic sea ice

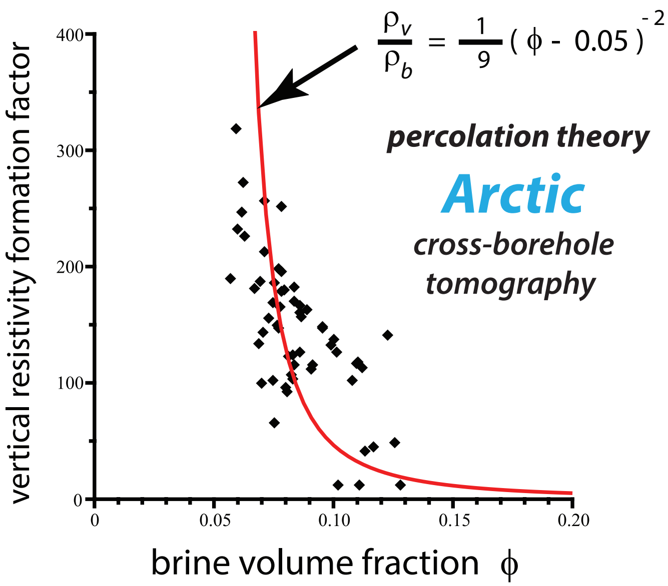
SIPEX 2007

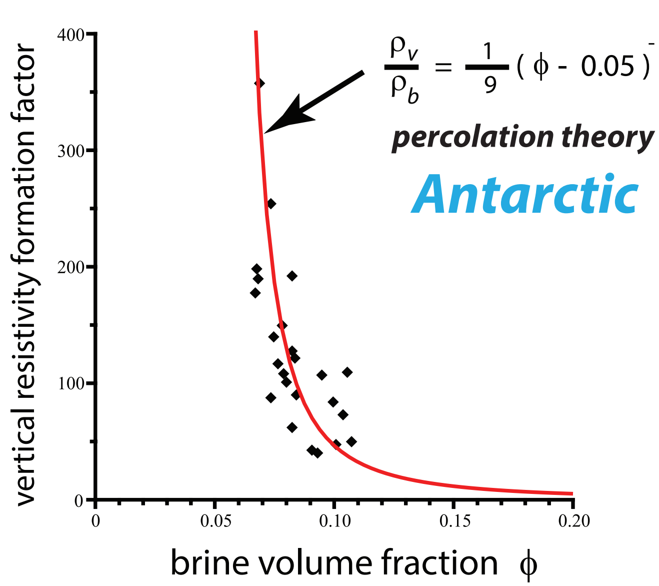
critical behavior of electrical transport in sea ice

electrical signature of the on-off switch for fluid flow





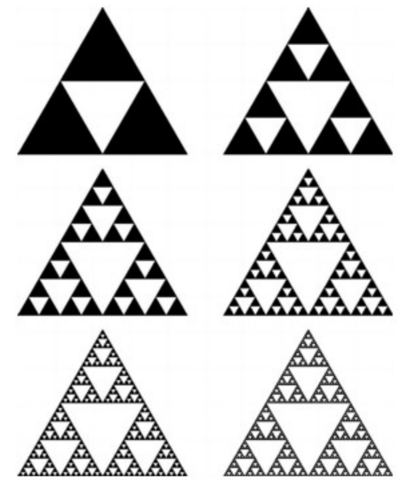




cross-borehole tomography - electrical classification of sea ice layers

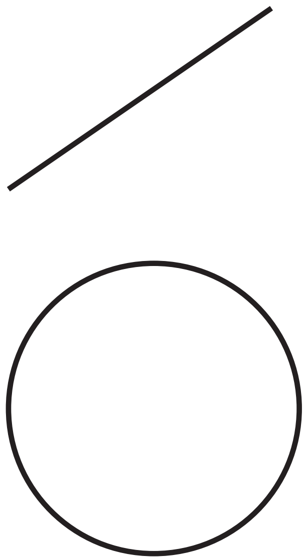
Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2014

fractals and multiscale structure



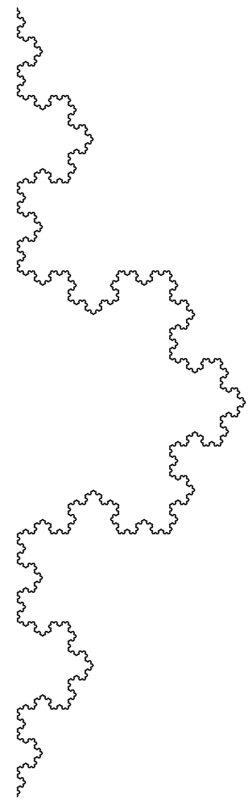
fractal curves in the plane

they wiggle so much that their dimension is >1



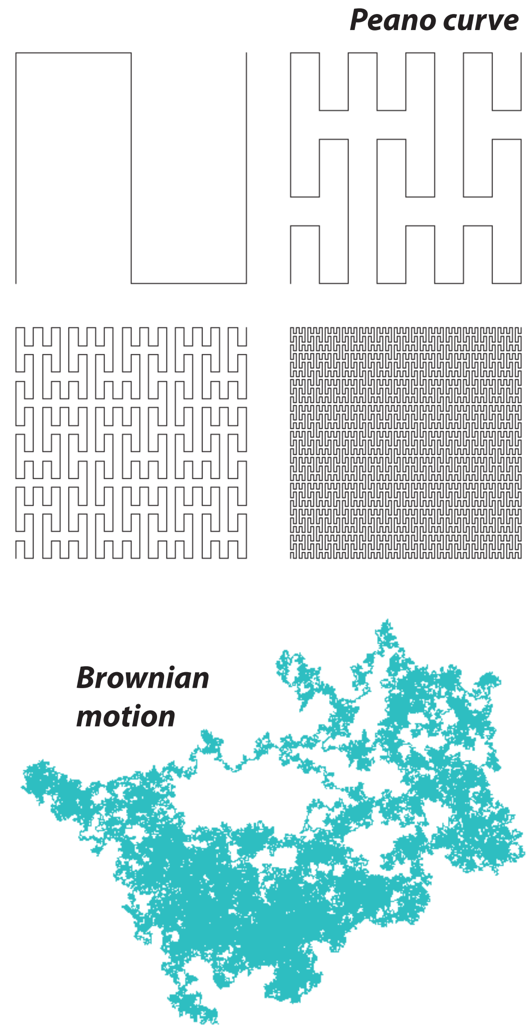
simple curves

D = 1



Koch snowflake

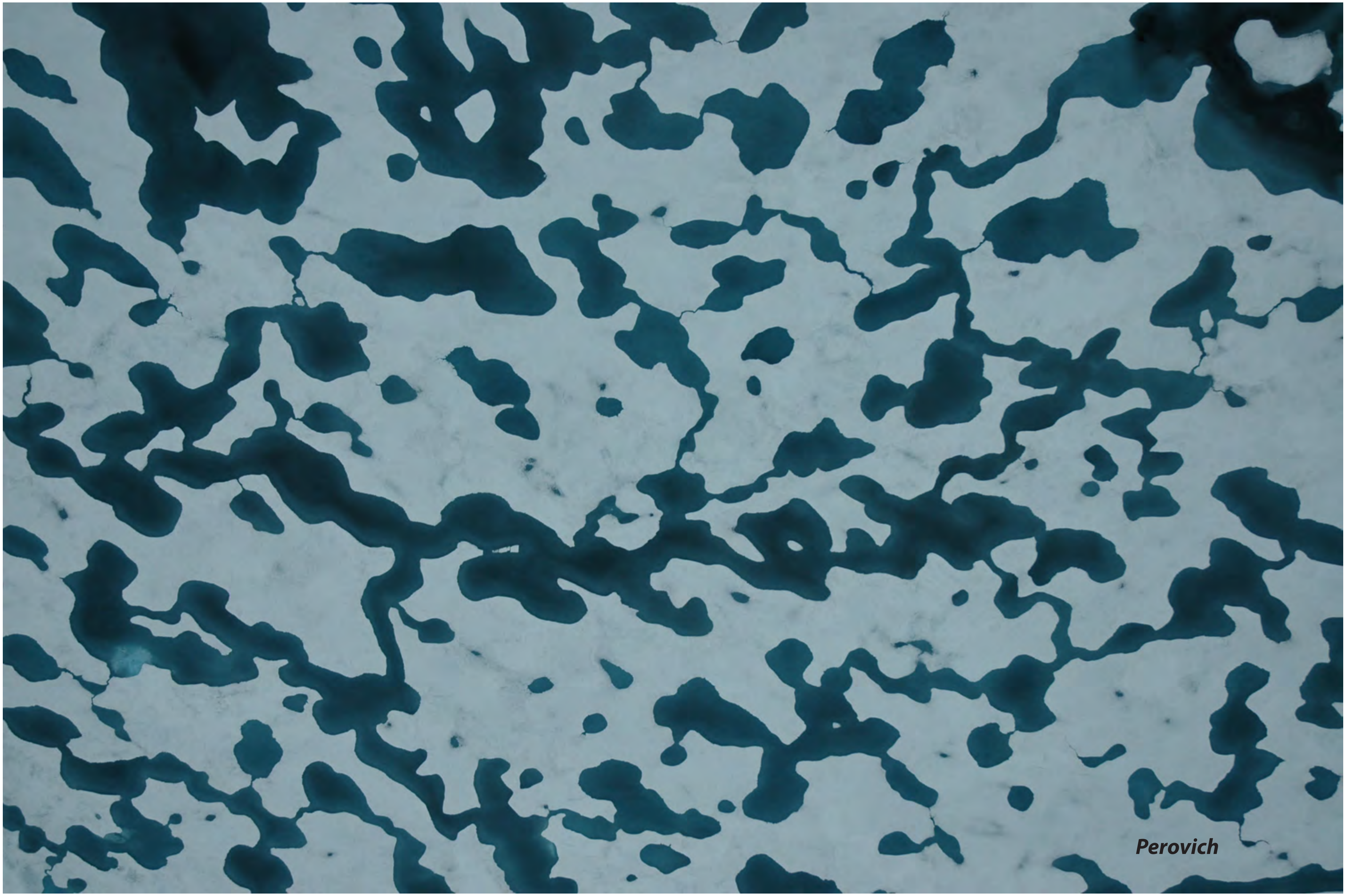
D = 1.26



space filling curves

D = 2

Arctic melt ponds



melt pond formation and albedo evolution -- major drivers in polar climate

key challenge for global climate models

clouds exhibit fractal behavior from 1 to 1000 km

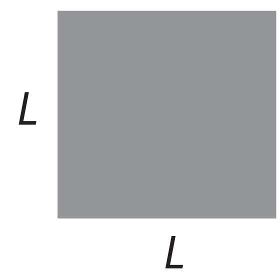
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

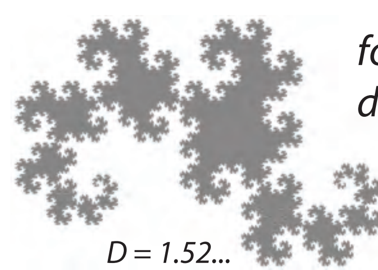
simple shapes



$A = L^2$

$P = 4L = 4\sqrt{A}$

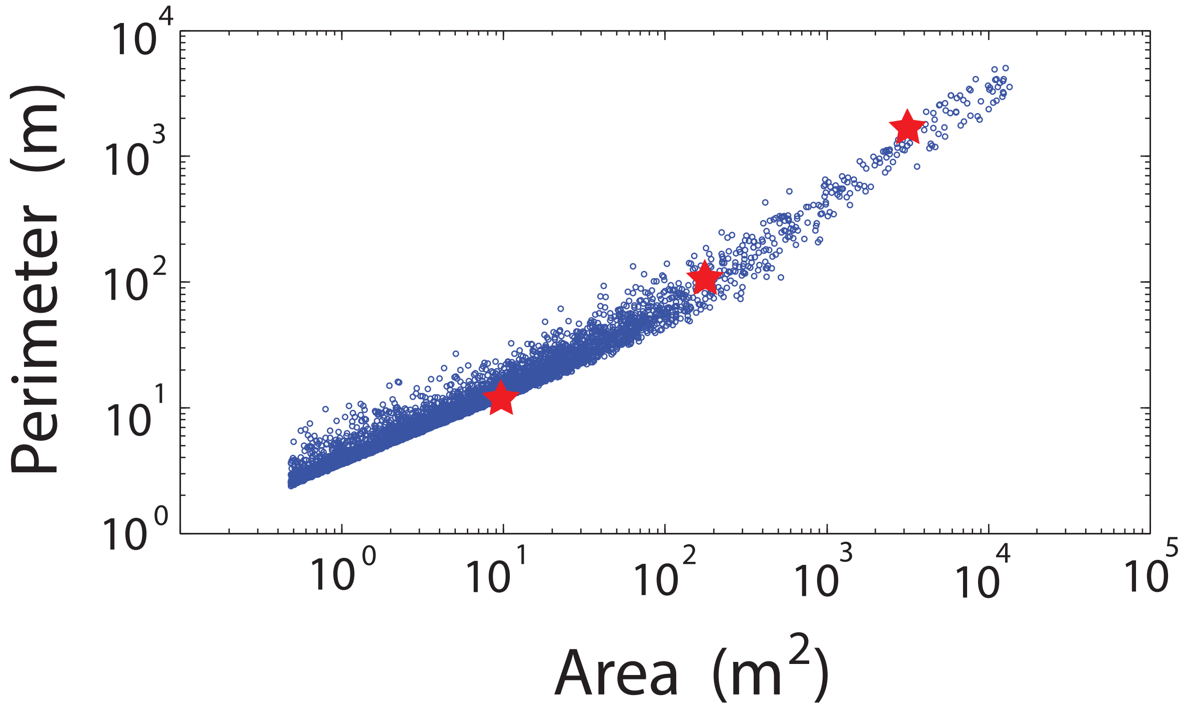
$$P \sim \sqrt{A}^{\,D}$$



for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds *The Cryosphere, 2012*

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

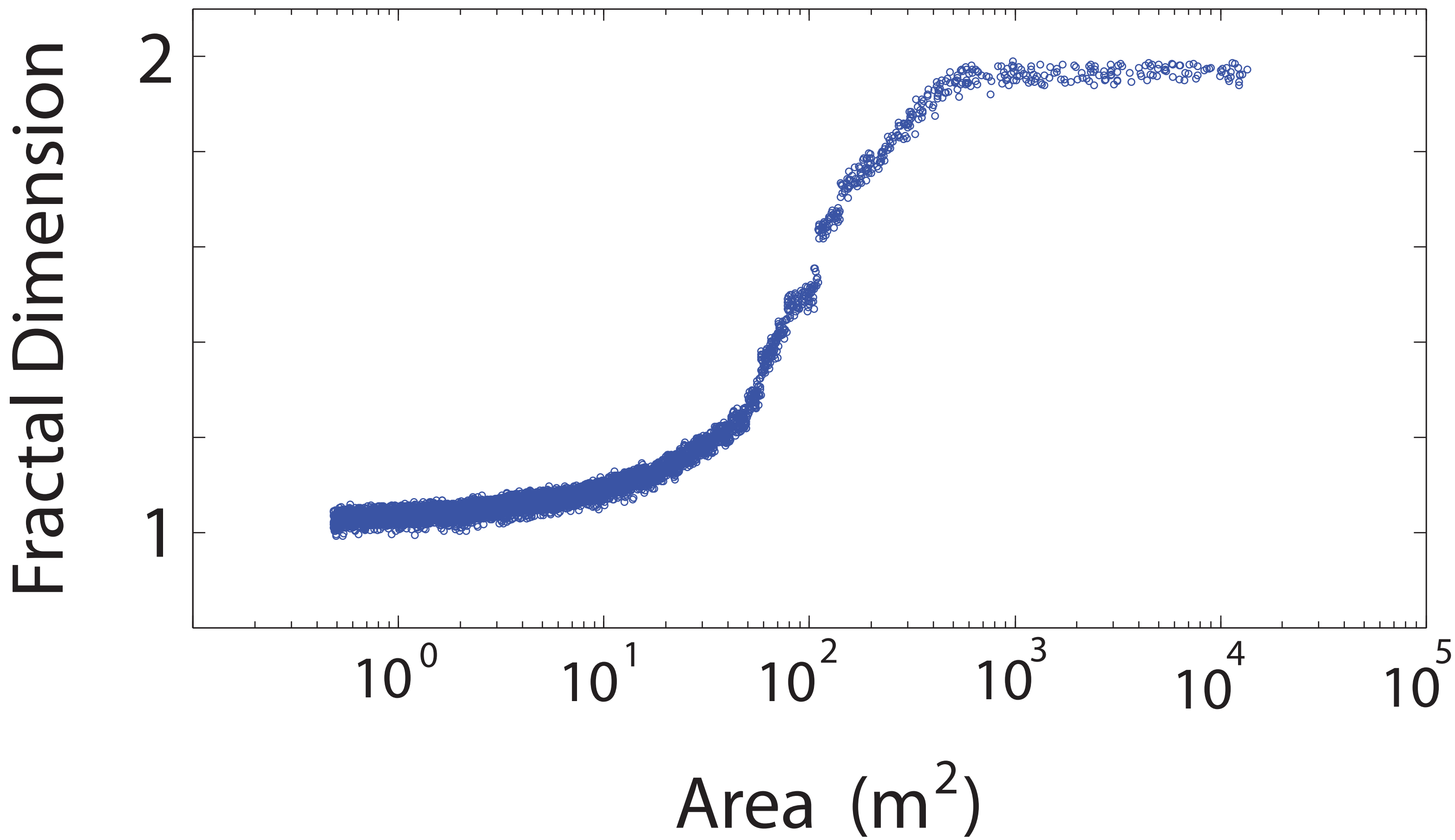


data from
5269 Arctic
melt ponds

simple pond ***transitional pond*** ***complex pond***

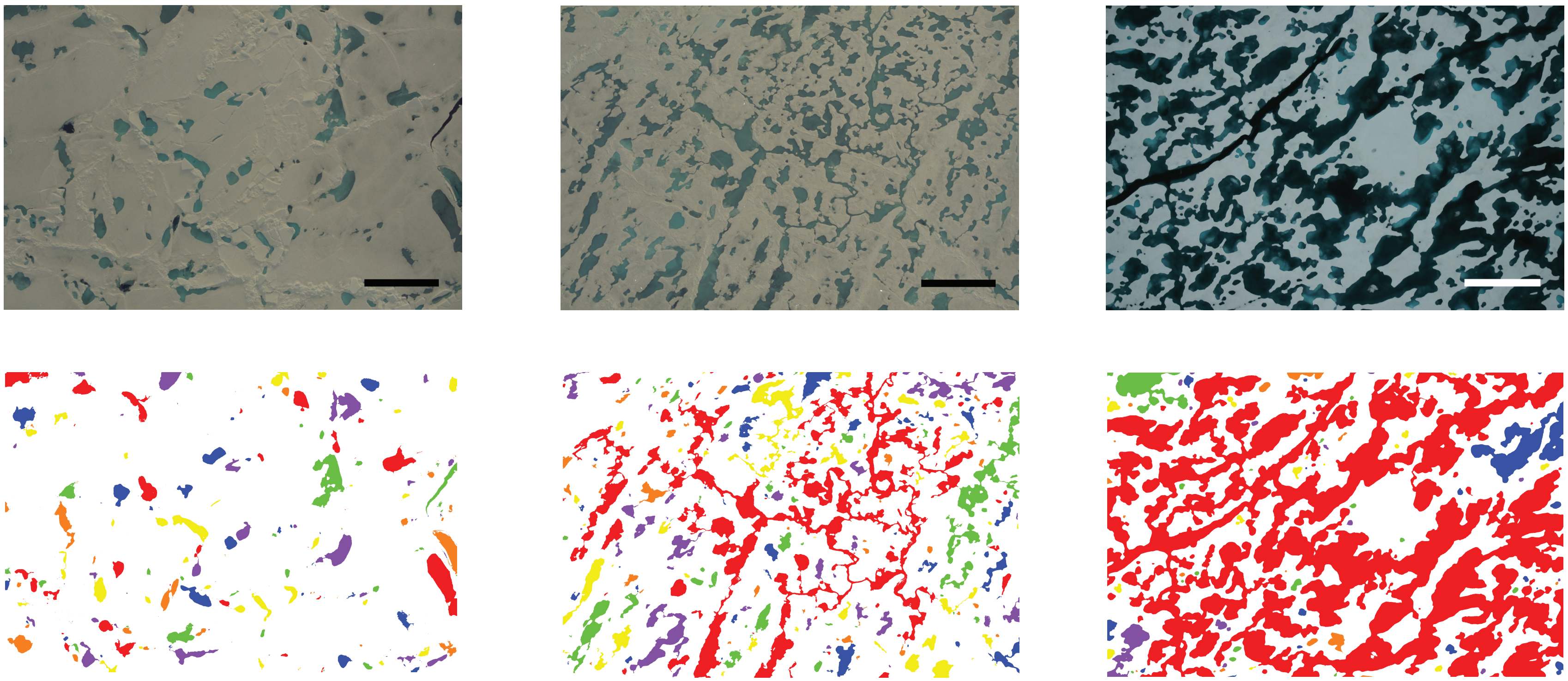
transition in the fractal dimension

complexity grows with length scale



compute "derivative" of area - perimeter data

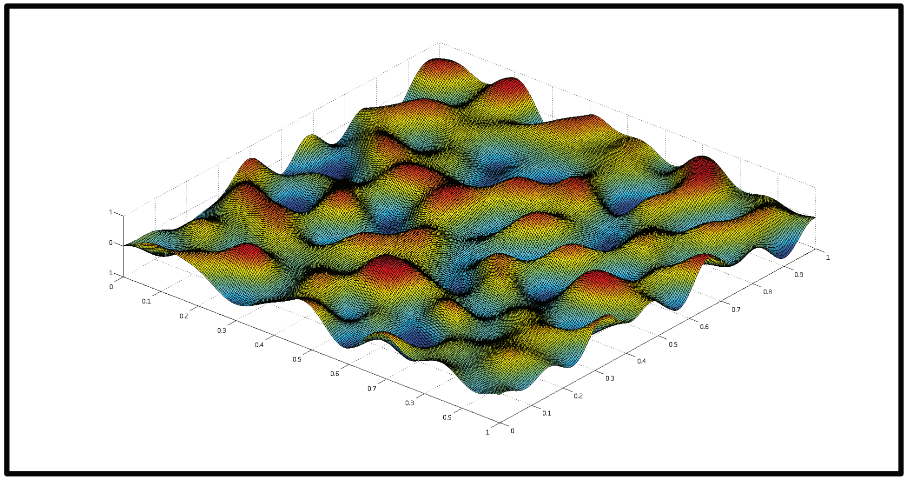
small simple ponds coalesce to form large connected structures with complex boundaries



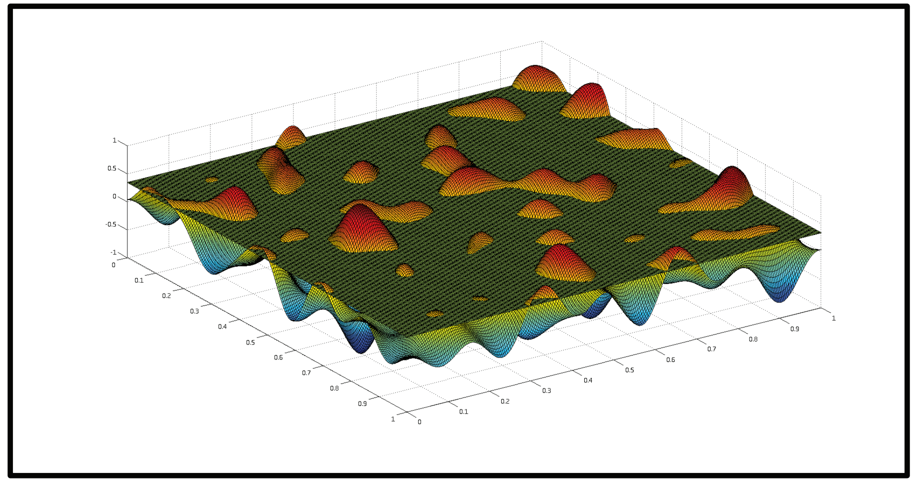
melt pond percolation

Continuum percolation model for melt pond evolution

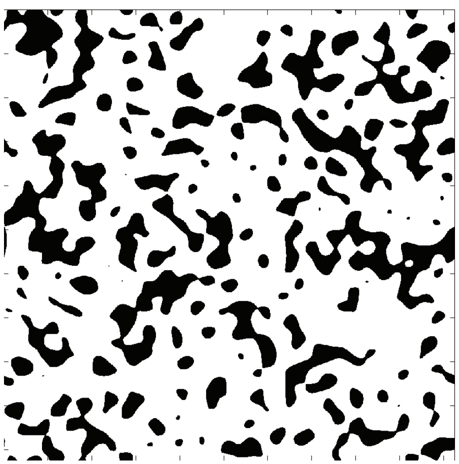
Brady Bowen, Court Strong, Ken Golden, 2015

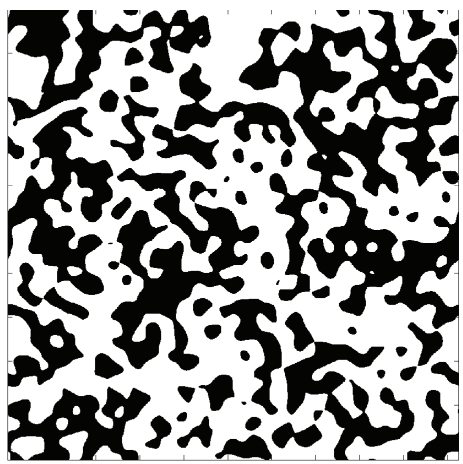


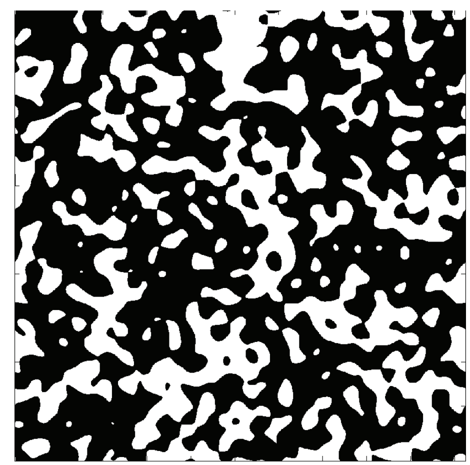
random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds





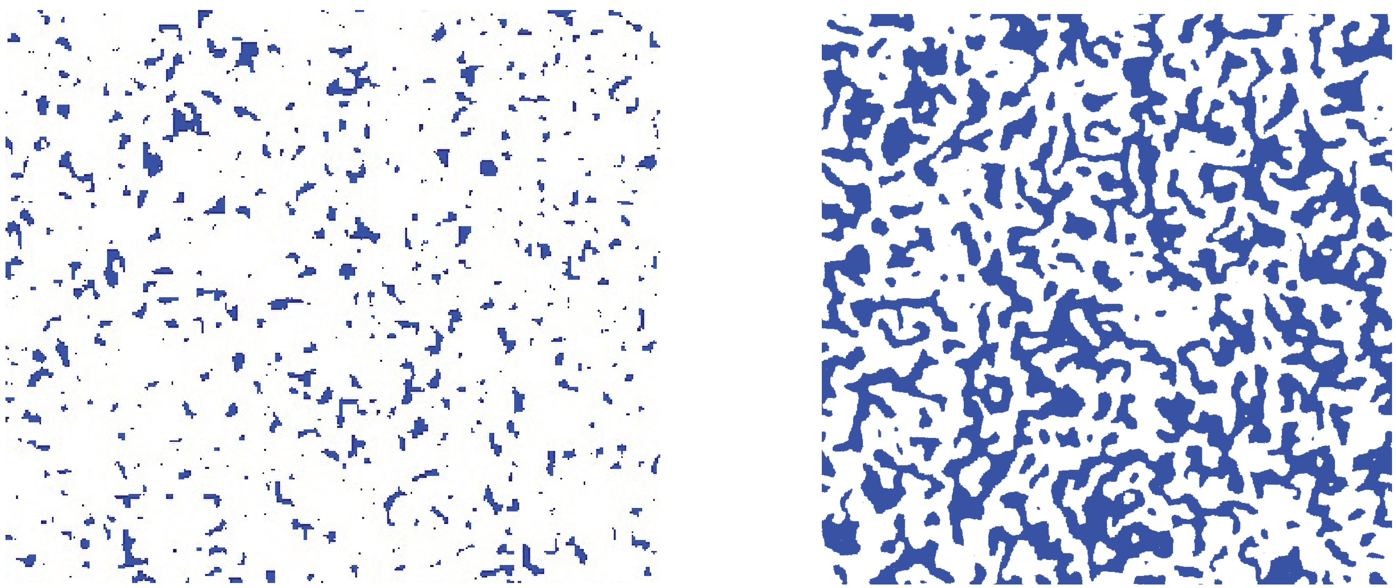


electronic transport in disordered media *diffusion in turbulent plasmas* *(Isichenko, Rev. Mod. Phys., 1992)*

Ising model for ferromagnets → Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow \ +1 & \text{water (spin up)} \\ \downarrow \ -1 & \text{ice (spin down)} \end{cases}$$

magnetization $M = \lim_{N\to\infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$ **pond coverage** $\frac{(M+1)}{2}$

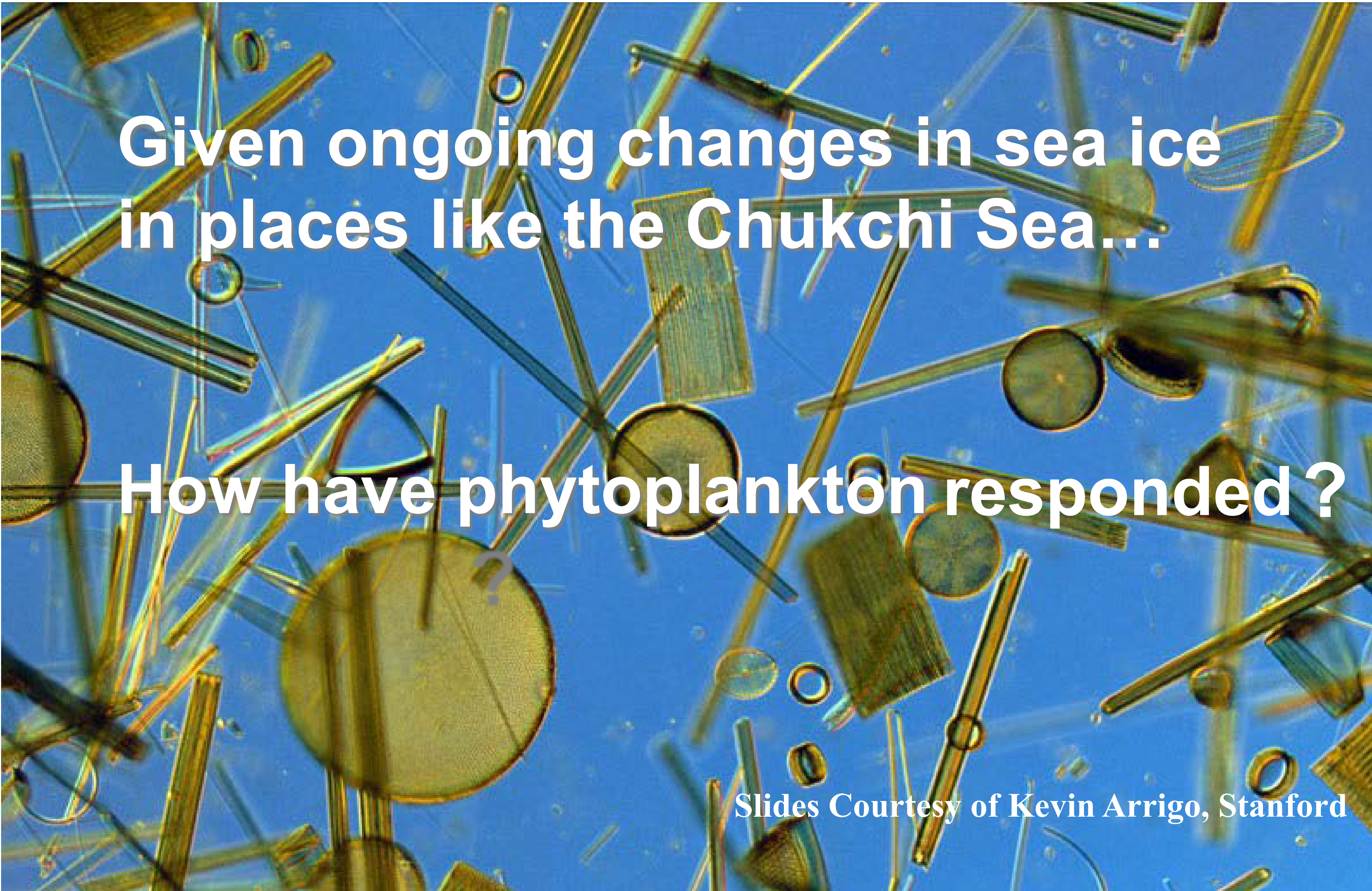


"melt ponds" are clusters of magnetic spins that align with the applied field

accurately predicts fractal transition

Ma, Sudakov, Golden 2015
(Thekkedath, Alali, Strong, Golden)

Question:



Given ongoing changes in sea ice in places like the Chukchi Sea…

How have phytoplankton responded ?

Slides Courtesy of Kevin Arrigo, Stanford

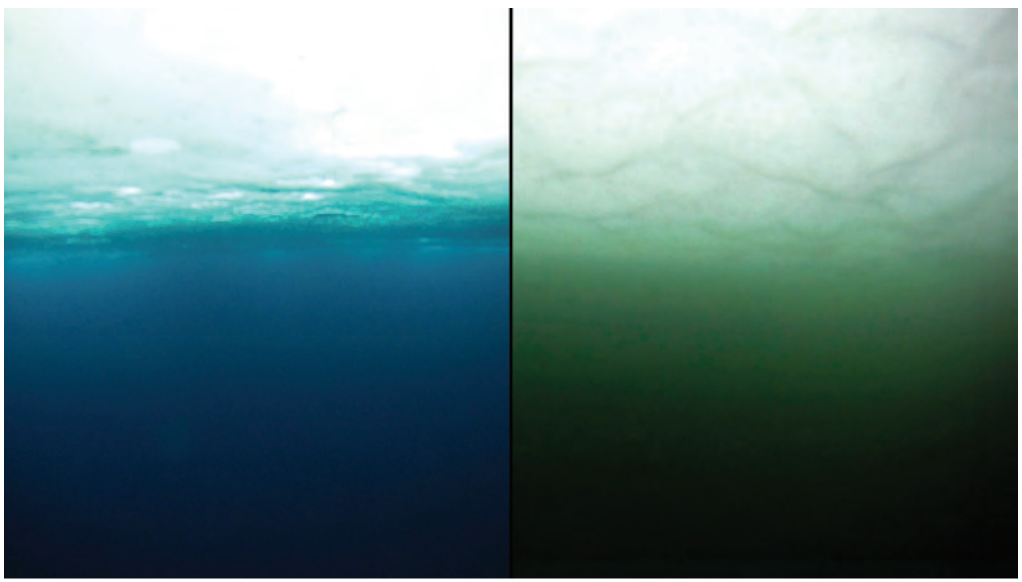
2011 massive under-ice algal bloom

Arrigo et al., *Science* 2012

melt ponds act as

WINDOWS

allowing light through sea ice



no bloom

bloom

Have we crossed into a new ecological regime?

Mathematics

provides
the platform,
the language,
the “operating system”
we use to quantitatively
formulate and answer
such questions!

THANK YOU

National Science Foundation

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Division of Polar Programs

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Arctic and Global Prediction Program

Applied and Computational Analysis Program

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Australian Government · Department of the Environment and Water Resources · Australian Antarctic Division

ace CRC · Antarctic Climate & Ecosystems · Cooperative Research Centre

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Antarctica New Zealand

UAF · University of Alaska Fairbanks

The University of Utah

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999