

local conductivity in 1D inhomogeneous material

$$\sigma(x) = 3 + \cos x + \cos kx$$

effective conductivity

$$\sigma^*(k) = \begin{cases} \text{constant} & k \text{ irrational} & \text{quasiperiodic} \\ f(k) & k \text{ rational} & \text{periodic} \end{cases}$$

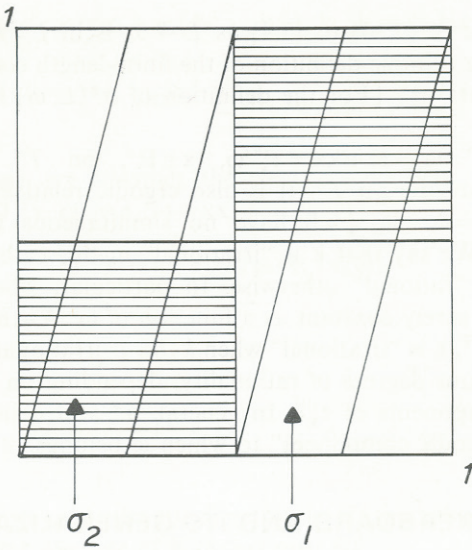
Golden, Goldstein, Lebowitz, Phys. Rev. Lett. 1985

Classical transport in quasiperiodic media

Golden, Goldstein, and Lebowitz

Phys. Rev. Lett. 1985

J. Stat. Phys. 1990



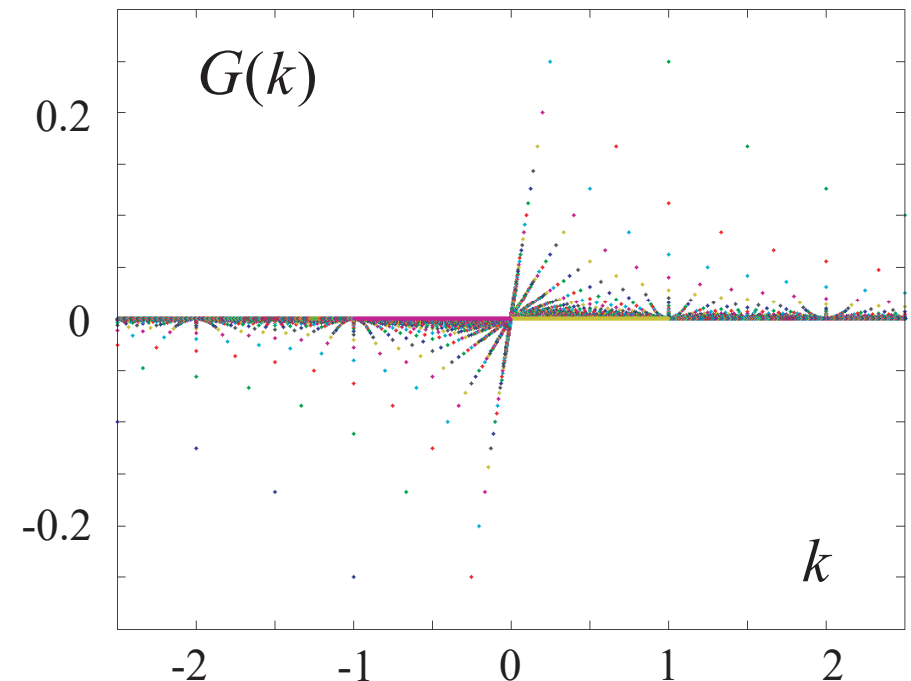
line of slope k through
an infinite checkerboard

effective conductivity $\sigma^*(k)$

effective resistivity $1/\sigma^*(k) = 1 - G(k)$

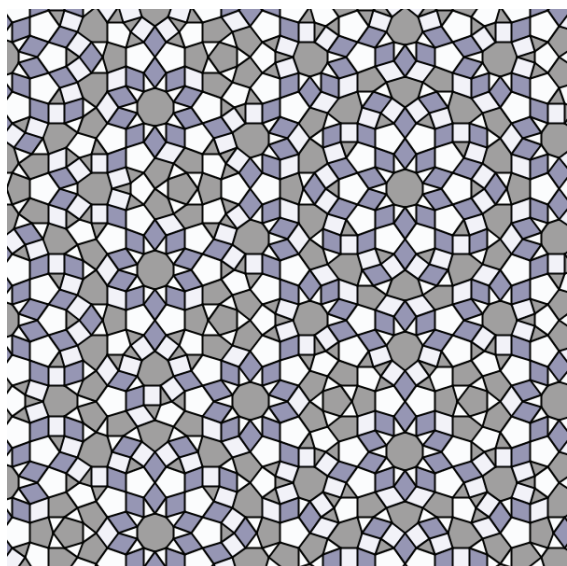
$$G(k) = \begin{cases} 0, & k \text{ irrational} \\ 1/pq, & k = p/q \text{ rational} \end{cases}$$

continuous at k irrational
discontinuous at k rational



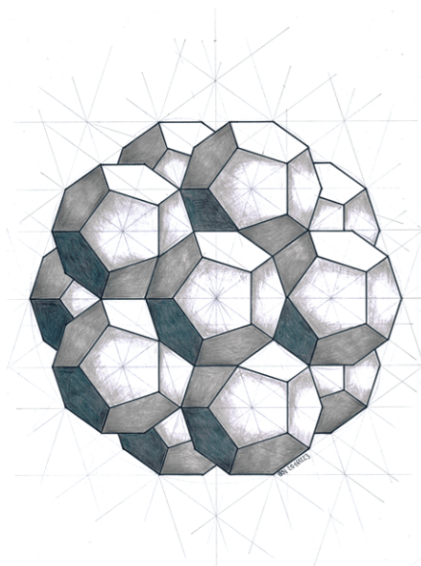
Order to Disorder in Quasiperiodic Composites

D. Morison (Physics), N. B. Murphy, E. Cherkaev, K. M. Golden, *Communications Physics* 2022



quasiperiodic checkerboard

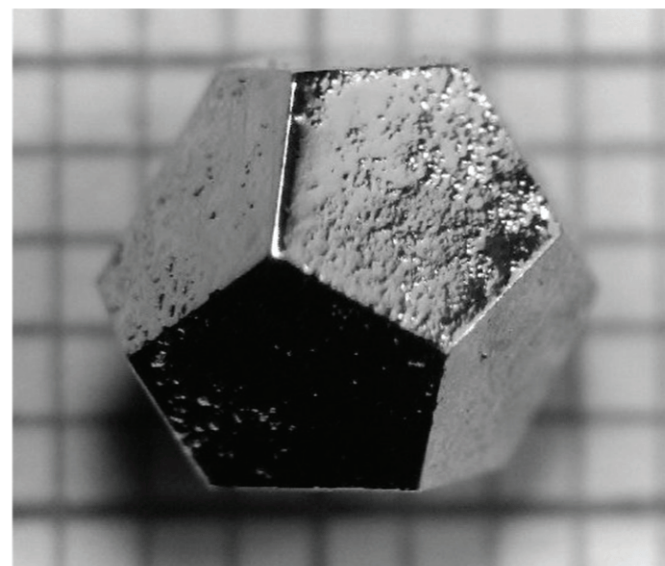
Stampfli, 2013



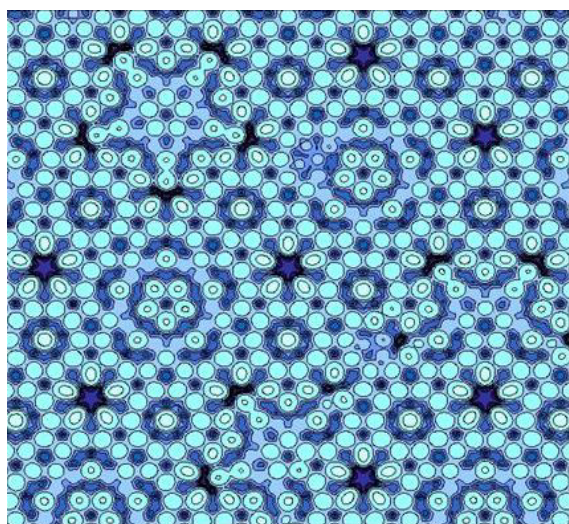
dense packing of dodecahedra

3D Penrose tiling

Tripkovic, 2019



Holmium–magnesium–zinc quasicrystal



energy surface Al-Pd-Mn quasicrystal

Unal et al., 2007

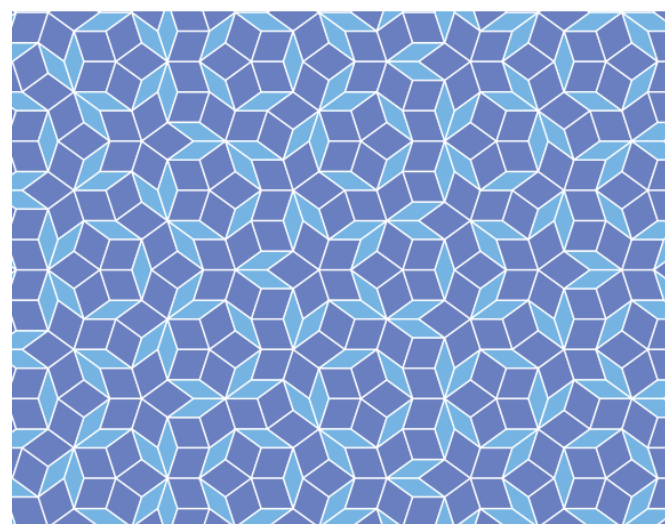
quasiperiodic crystal
quasicrystal

ordered but aperiodic

lacks translational symmetry

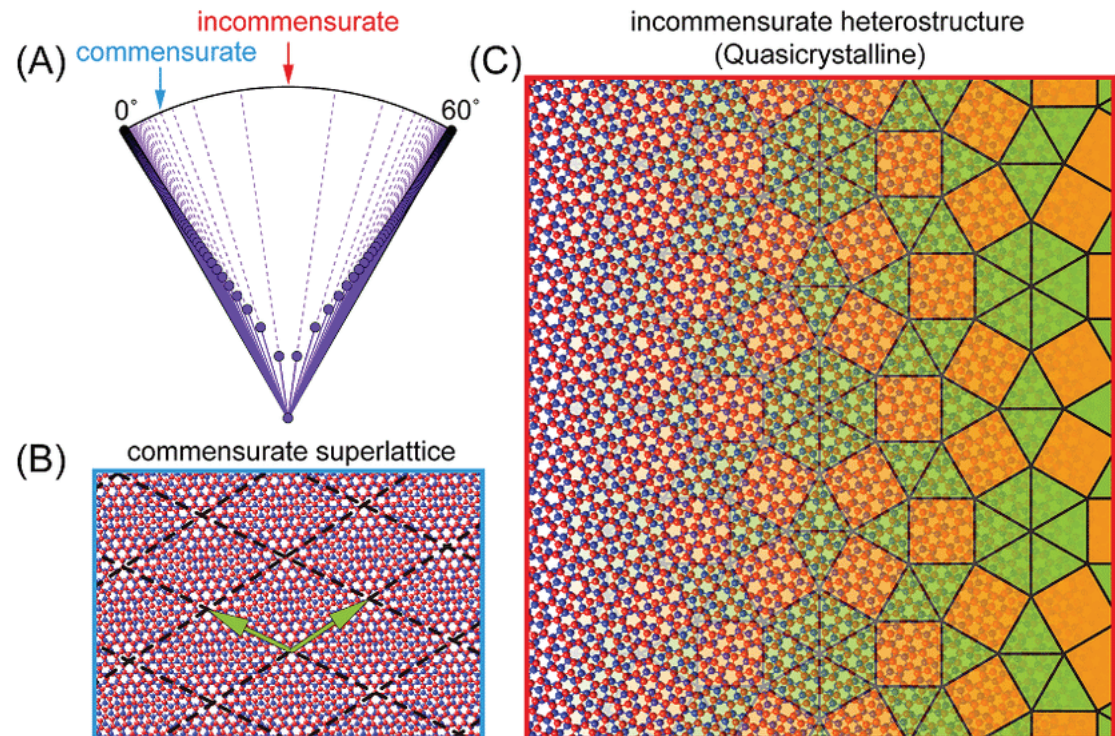
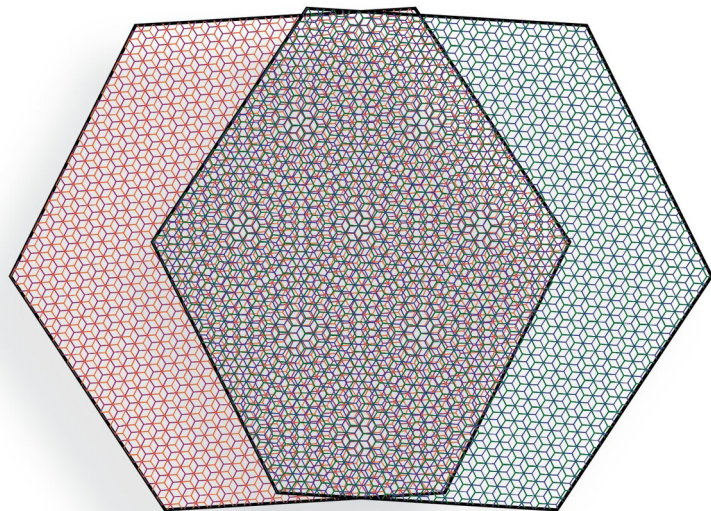
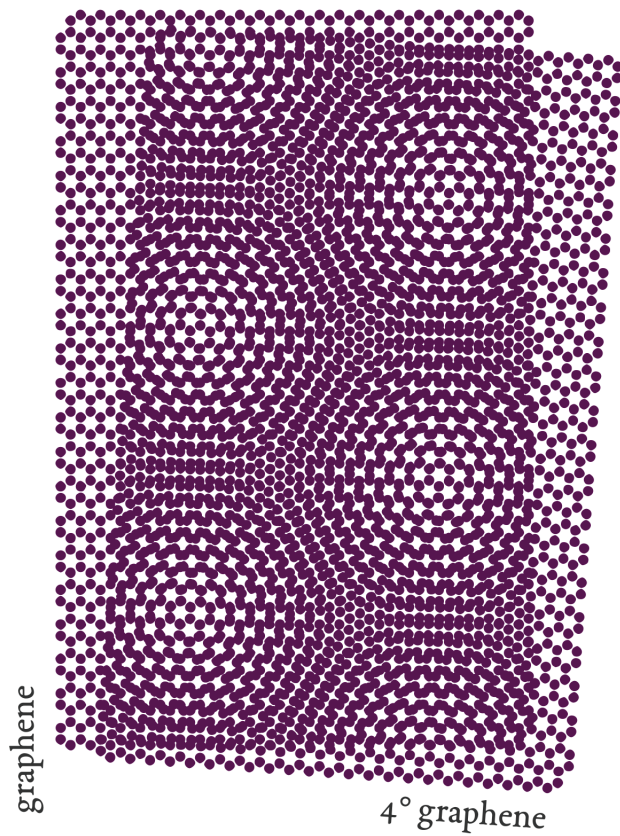
Schechtman et al., 1984

Levine & Steinhardt, 1984

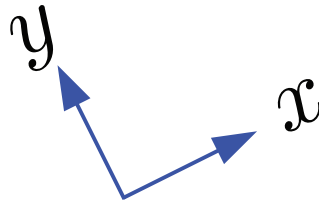
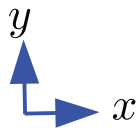


aperiodic tiling of the plane - R. Penrose 1970s

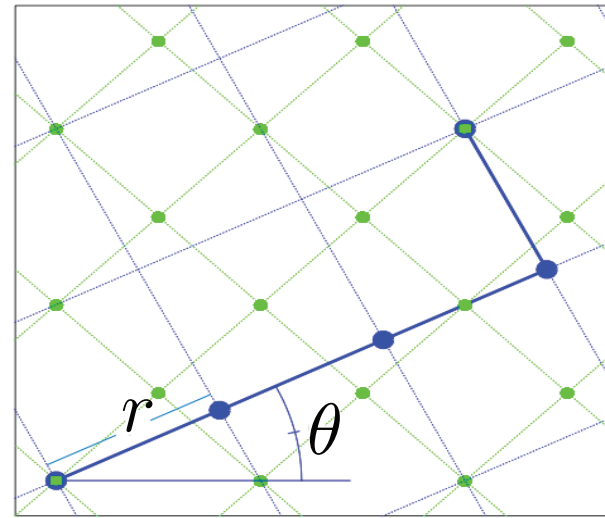
twisted bilayer graphene



Moiré patterns generate two component composites



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = r \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

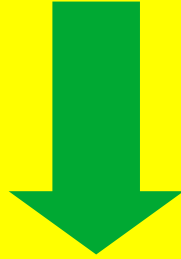


$$\psi(x', y') = \cos 2\pi x' \cos 2\pi y'$$

$$\chi = \begin{cases} 1, & \psi \geq 0 \\ 0, & \psi < 0 \end{cases}$$

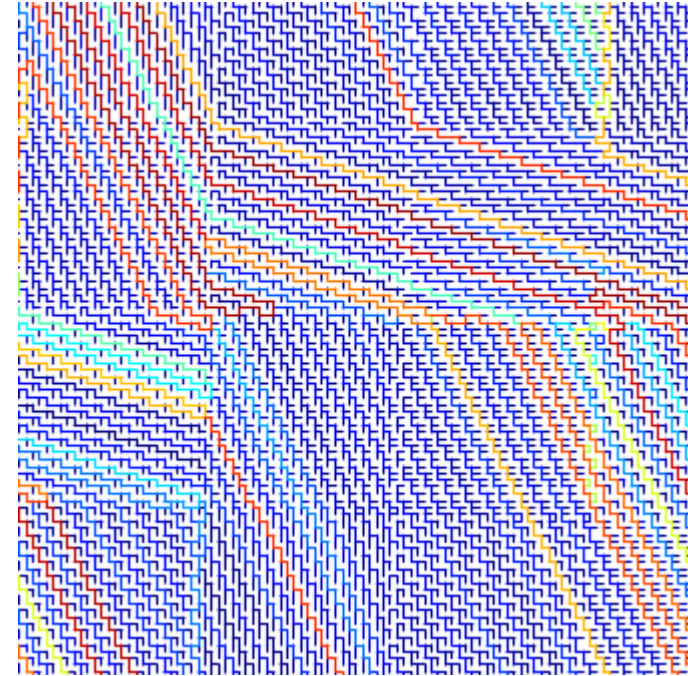
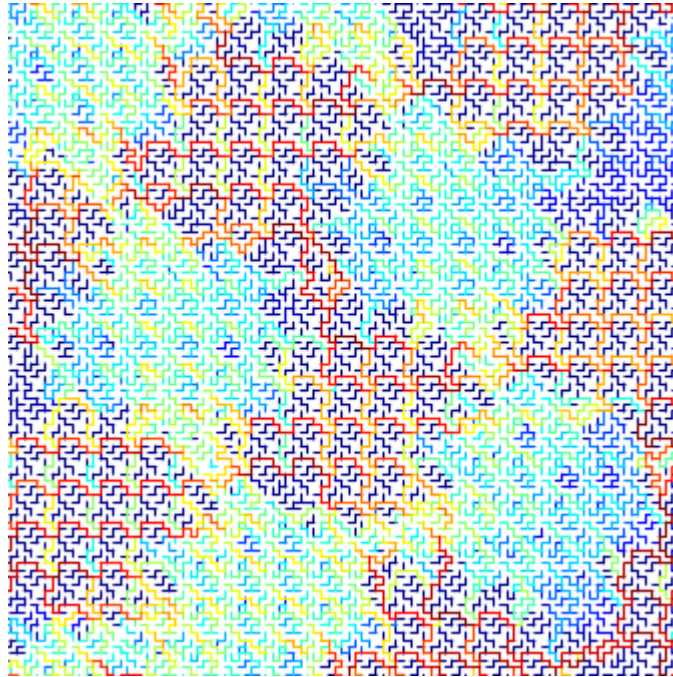
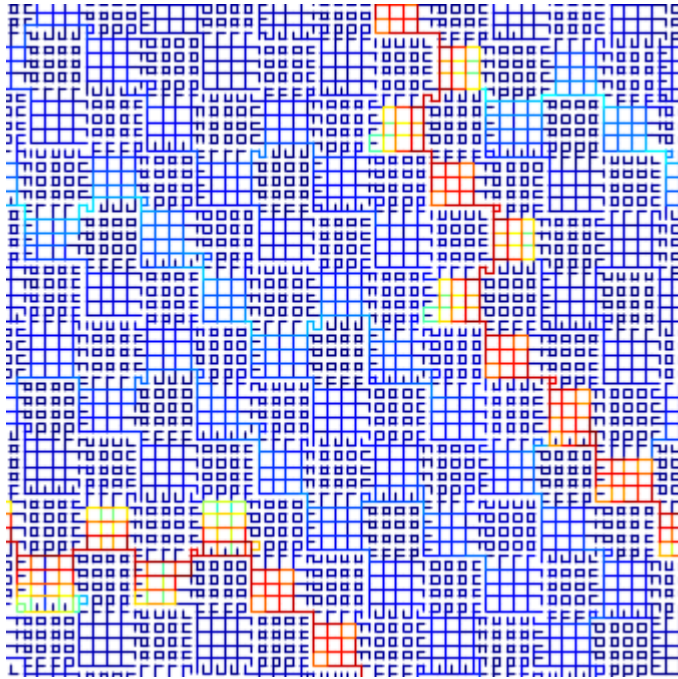
rotation and dilation

Small Difference in Moiré Parameters



Big Difference in Material Properties

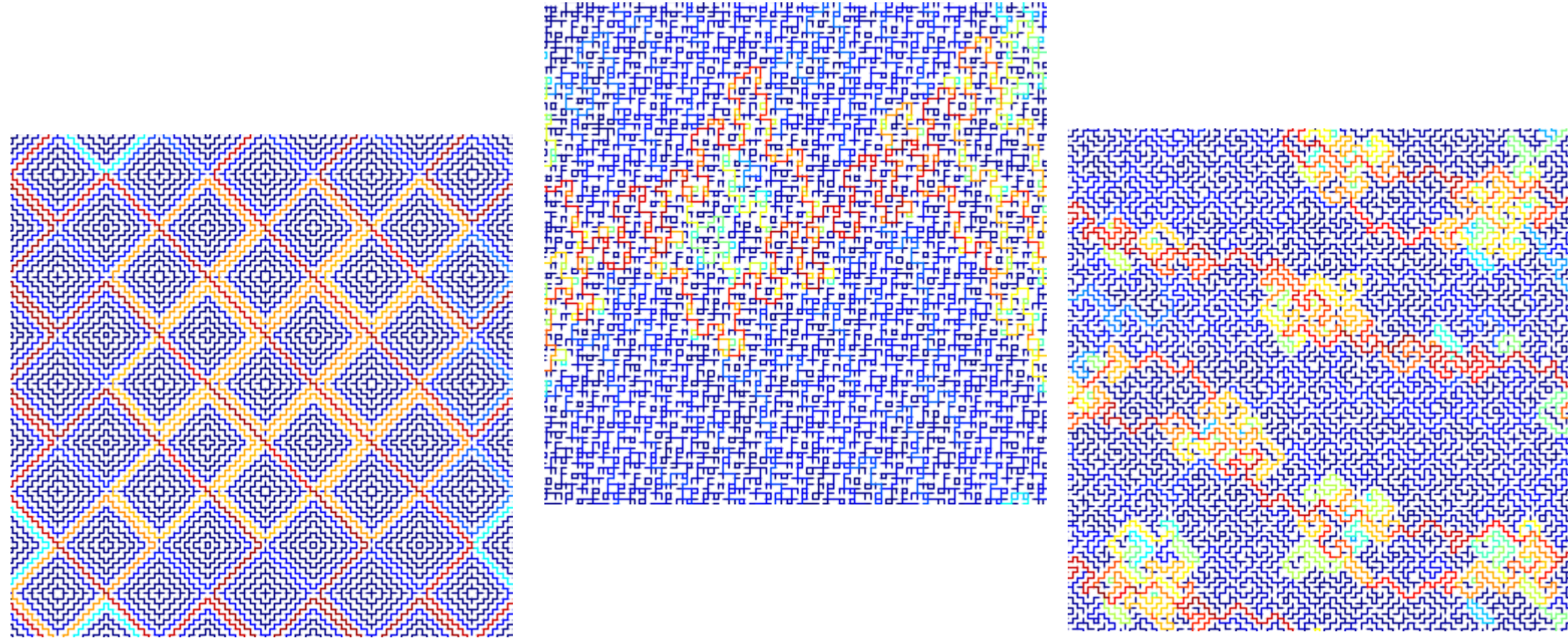
Wide Variety of Microgeometries



E



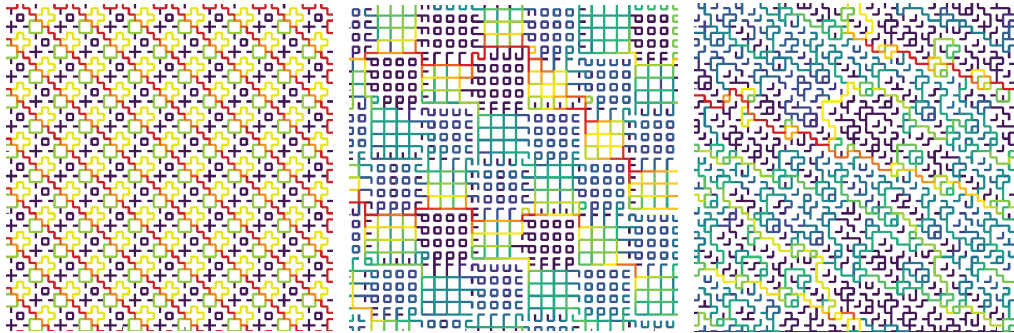
Wide Variety of Microgeometries



Order to disorder in quasiperiodic composites

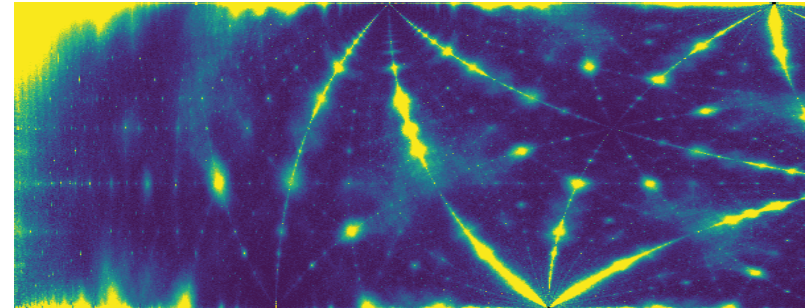
Morison, Murphy, Cherkaev, Golden, *Commun. Phys.* 2022

Parameterized Moiré Pattern Creates Tunable Microgeometry



constellation of periodic systems in a sea of randomness

Poisson
Wigner-Dyson



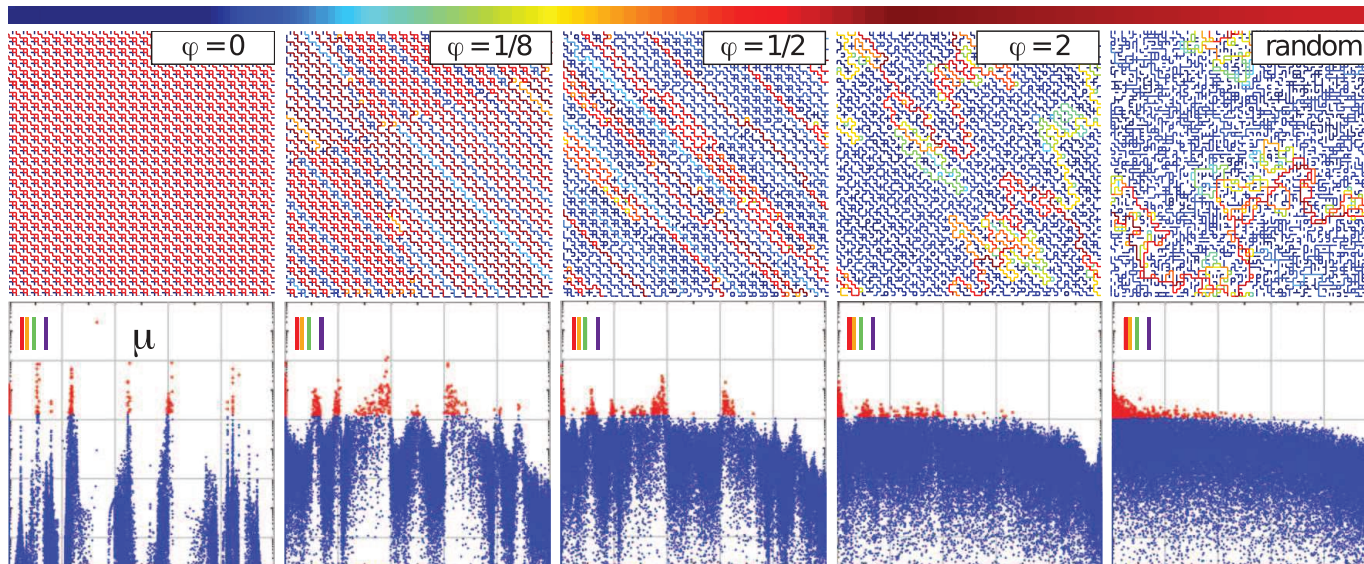
parameter space

periodic



quasiperiodic

electric field
strength

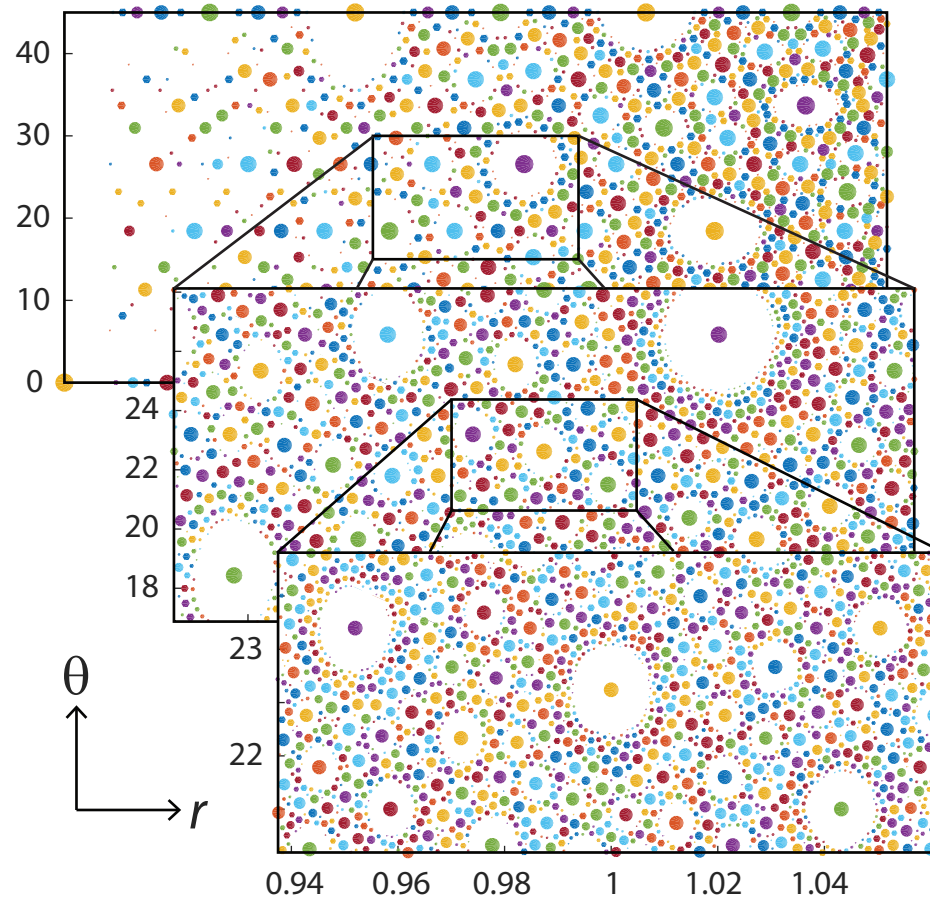


RRN at
percolation
threshold

we bring the framework of solid state physics of electronic transport and band gaps in semiconductors to classical transport in periodic and quasiperiodic composites

photonic crystals and quasicrystals

Fractal arrangement of periodic systems



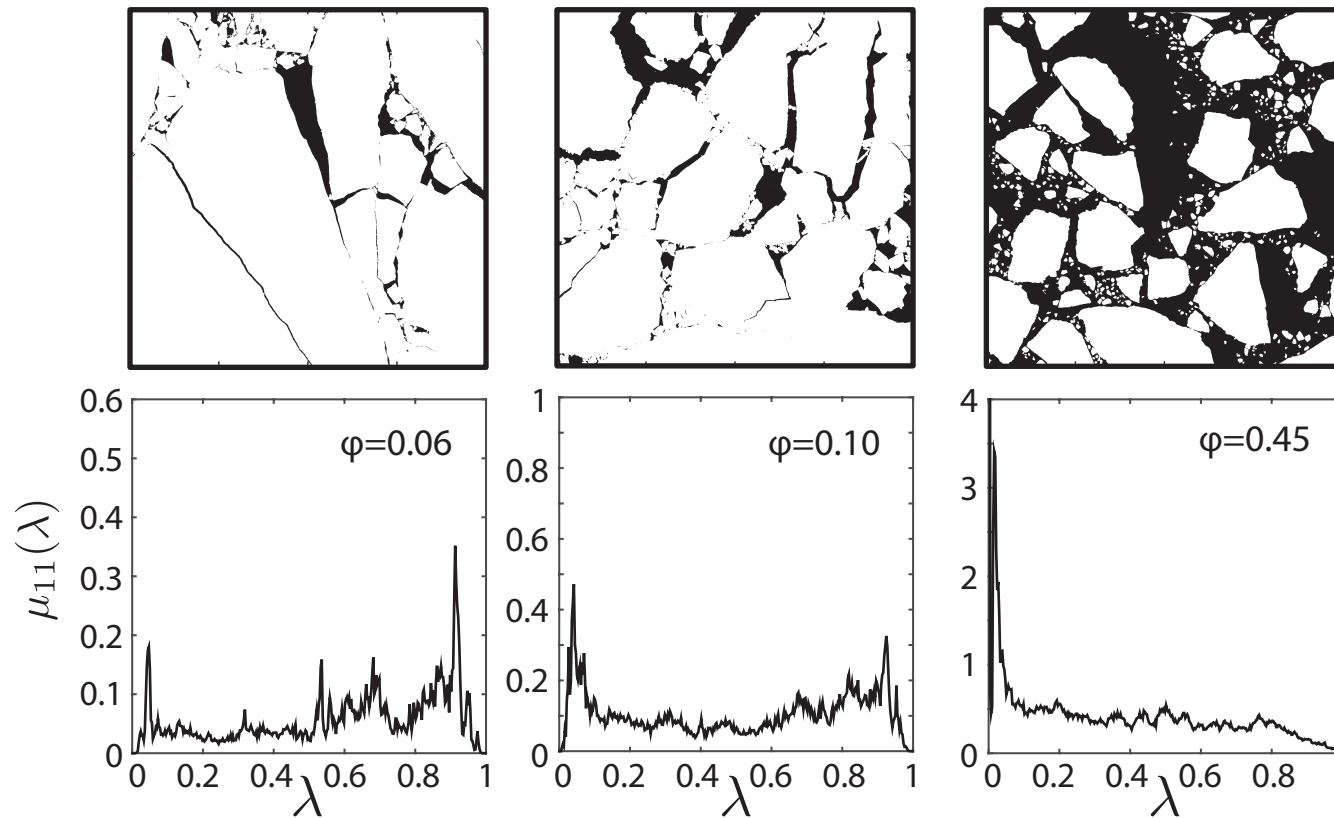
Sequential insets zooming into smaller regions of parameter space.

size of the dots $\sim$ length of period

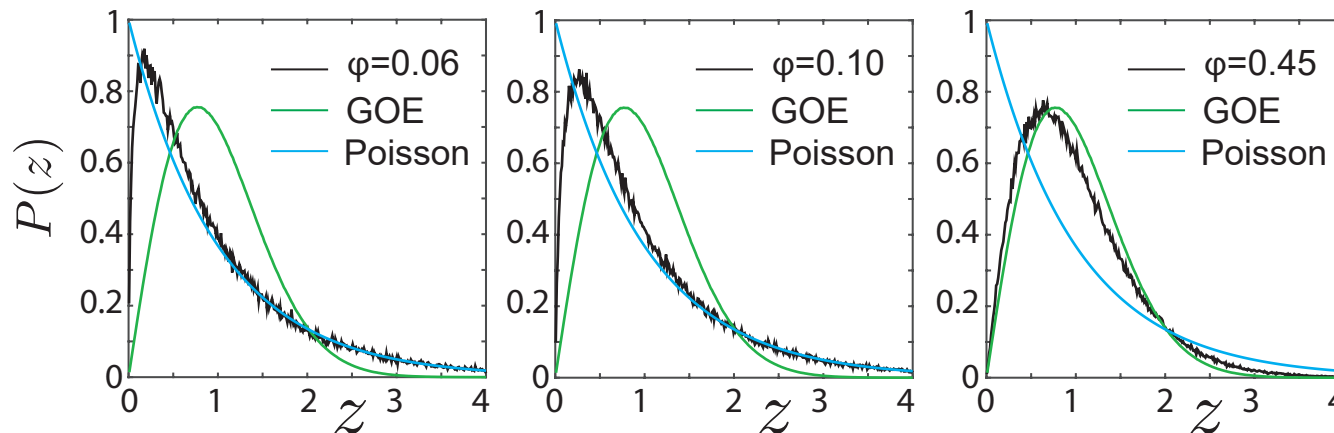
(large dot $\sim$ small period; small dot $\sim$ large period; white space $\sim$ "infinite" period)

Spectral computations for sea ice floe configurations

spectral
measures



eigenvalue
spacing
distributions



uncorrelated



level repulsion

UNIVERSAL
Wigner-Dyson
distribution

Eigenvalue Statistics of Random Matrix Theory

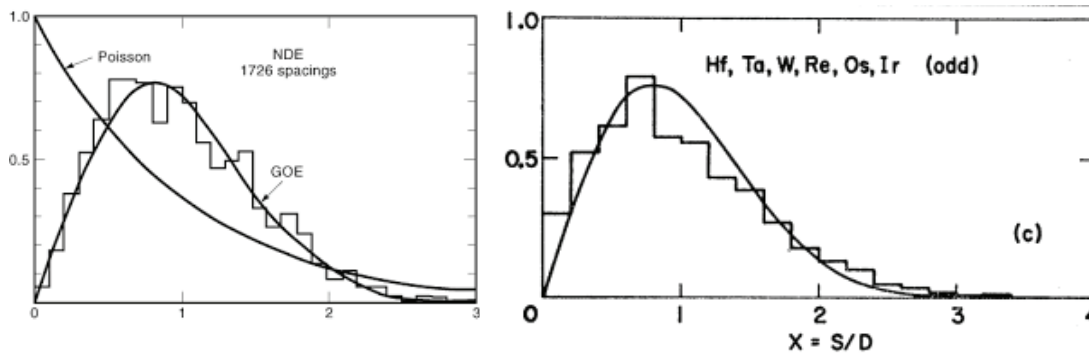
Wigner (1951) and Dyson (1953) first used random matrix theory (RMT) to describe quantized energy levels of heavy atomic nuclei.

$[N]_{ij} \sim N(0,1), \quad A = (N + N^T)/2 \quad \text{Gaussian orthogonal ensemble (GOE)}$

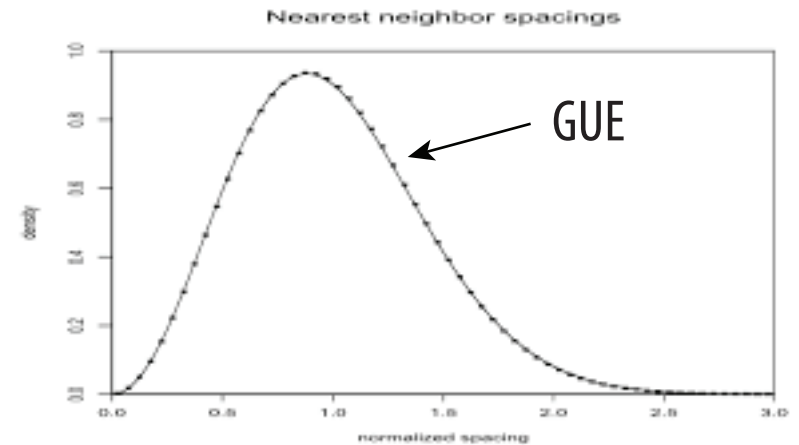
$[N]_{ij} \sim N(0,1) + iN(0,1), \quad A = (N + N^\dagger)/2 \quad \text{Gaussian unitary ensemble (GUE)}$

Short range and long range correlations of eigenvalues are measured by various eigenvalue statistics.

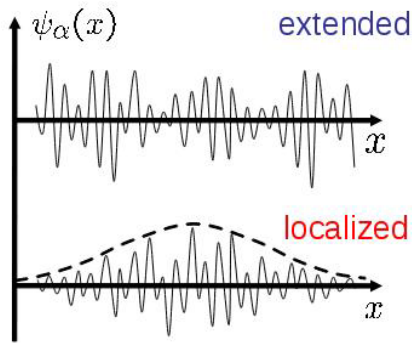
Spacing distributions of energy levels for heavy atomic nuclei



Spacing distributions of the first billion zeros of the Riemann zeta function



Universal eigenvalue statistics arise in a broad range of “unrelated” problems!



electronic transport in semiconductors

metal / insulator transition

localization

Anderson 1958
Mott 1949
Shklovshii et al 1993
Evangelou 1992

**Anderson transition in wave physics:
 quantum, optics, acoustics, water waves, ...**

from analysis of spectral measures for brine, melt ponds, ice floes

we find percolation-driven

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017

**PERCOLATION
 TRANSITION**



**universal eigenvalue statistics (GOE)
 extended states, mobility edges**

-- but with NO wave interference or scattering effects ! --