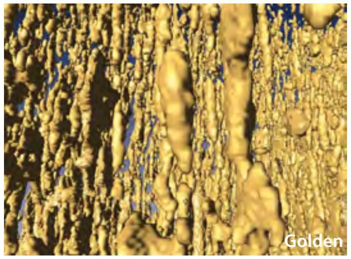
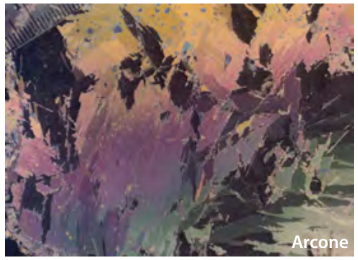
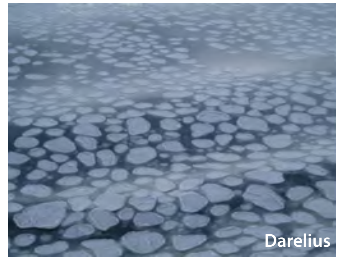
millimeters



centimeters



meters



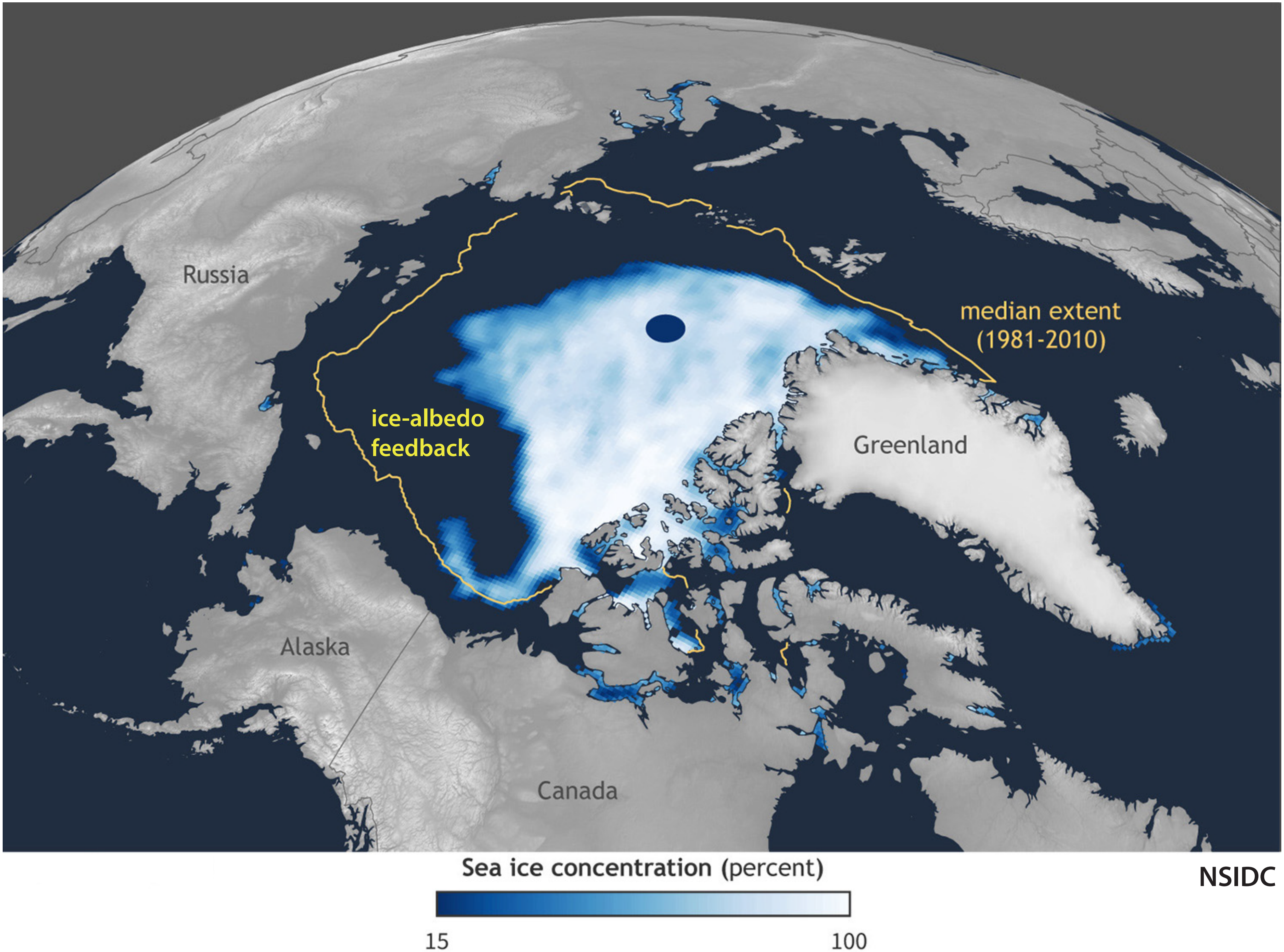
kilometers

10^3 kilometers

Fractal Geometry of Sea Ice Structures

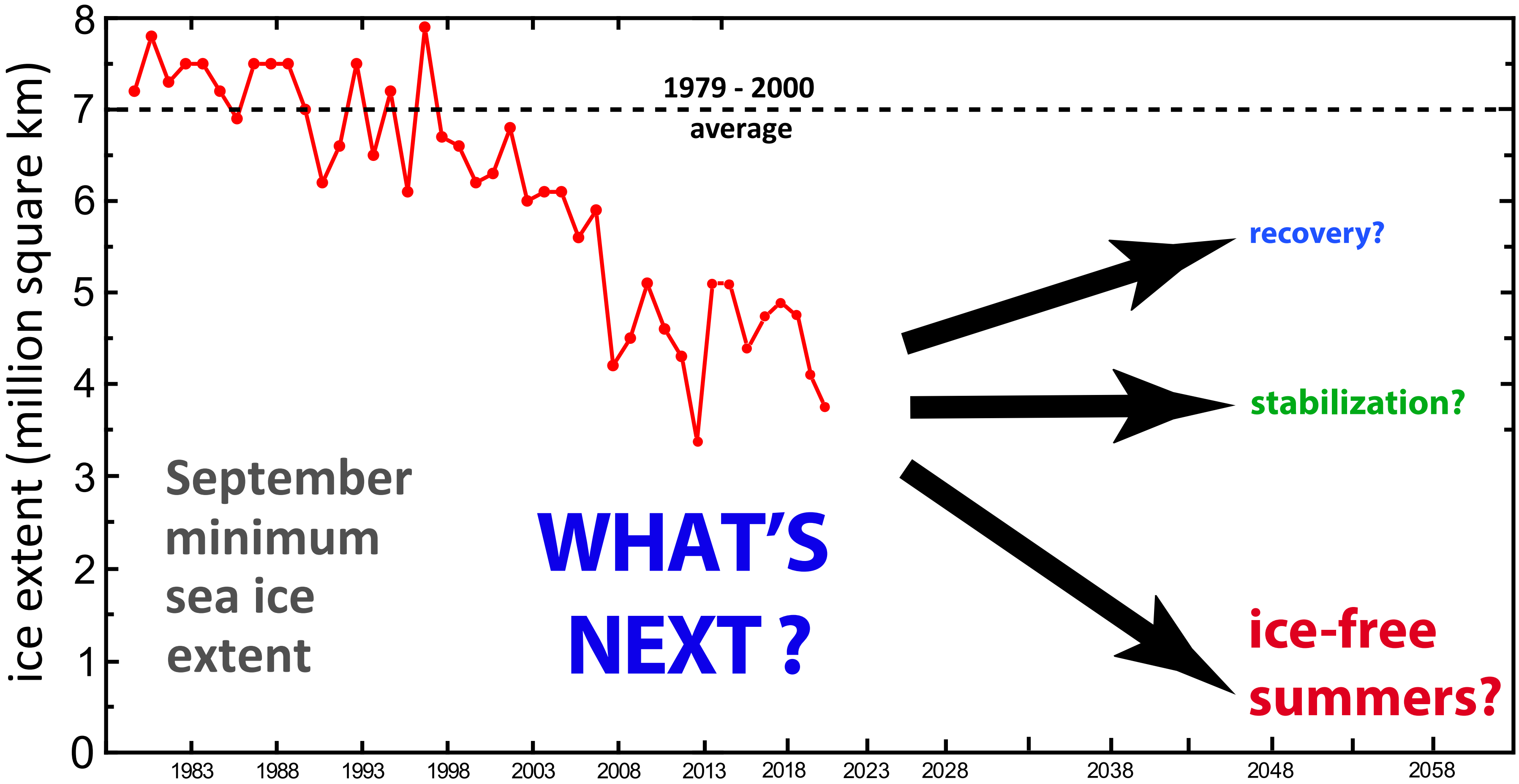
Ken Golden, University of Utah

Arctic sea ice extent September 15, 2020



NSIDC

Predicting what may come next requires lots of math modeling.



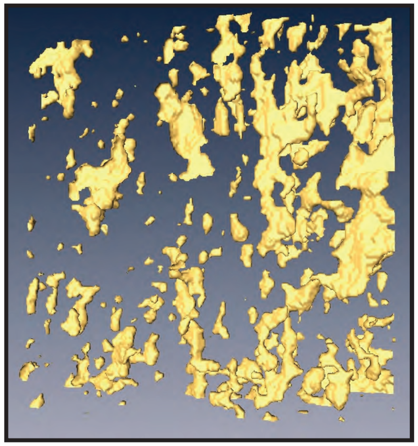
National Snow and Ice Data Center (NSIDC)

Sea Ice is a Multiscale Composite Material

microscale

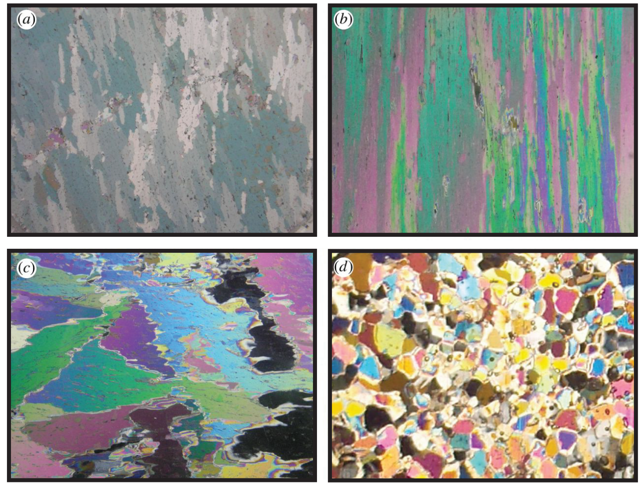
brine inclusions

Weeks & Assur 1969



H. Eicken
Golden *et al. GRL* 2007

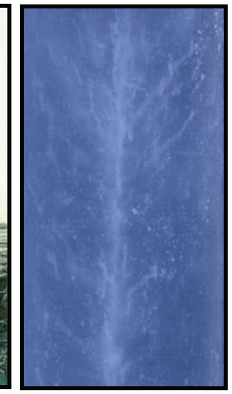
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

millimeters

centimeters

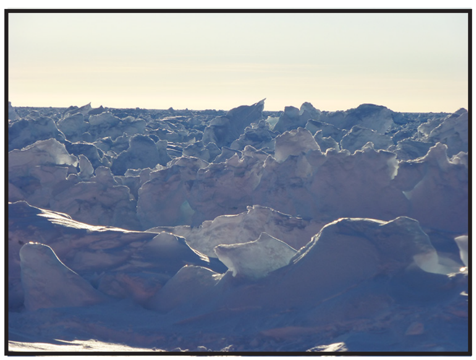
---

mesoscale

Arctic melt ponds

K. Frey

Antarctic pressure ridges



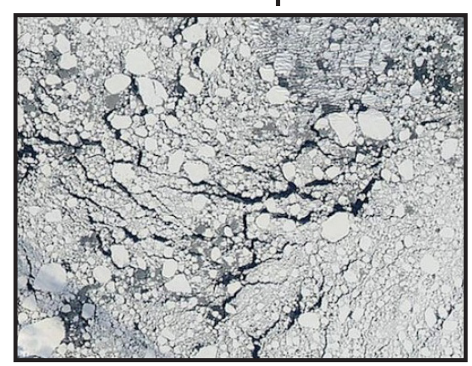
K. Golden

macroscale

sea ice floes

J. Weller

sea ice pack

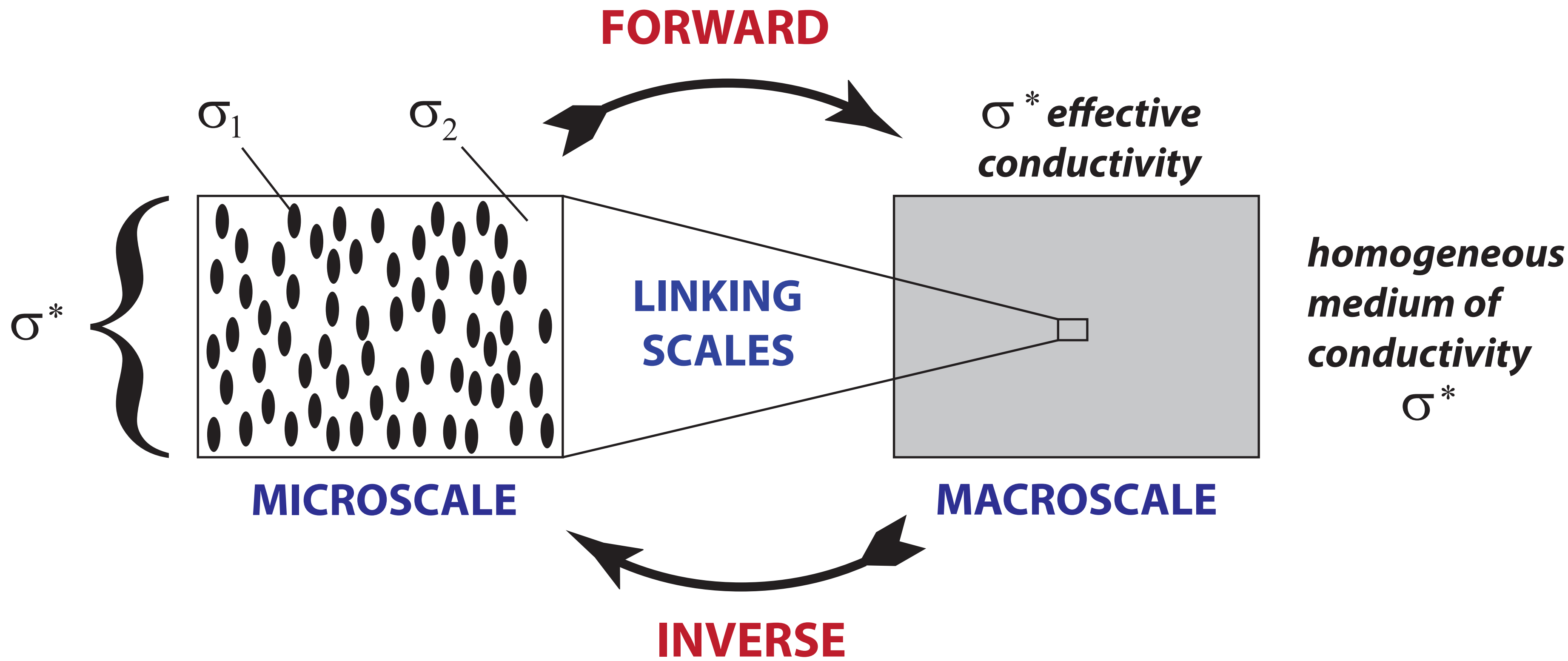


NASA

meters

kilometers

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is this talk about?

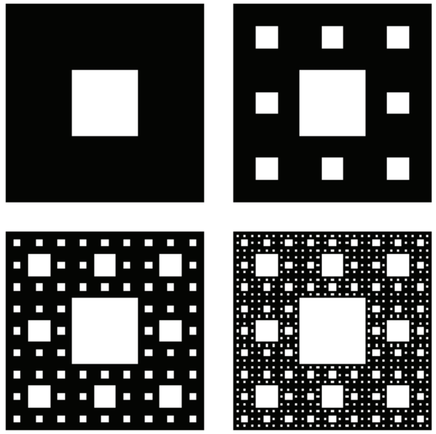
A tour of recent results on multiscale modeling of physical and ecological processes in the sea ice system, with a focus on novel mathematics.

fractal geometry

microscale

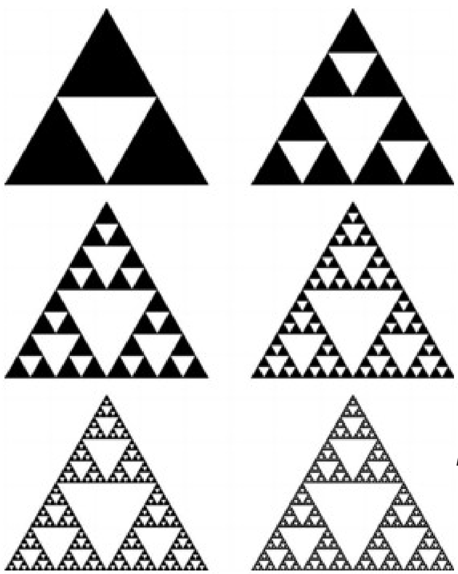
mesoscale

macroscale

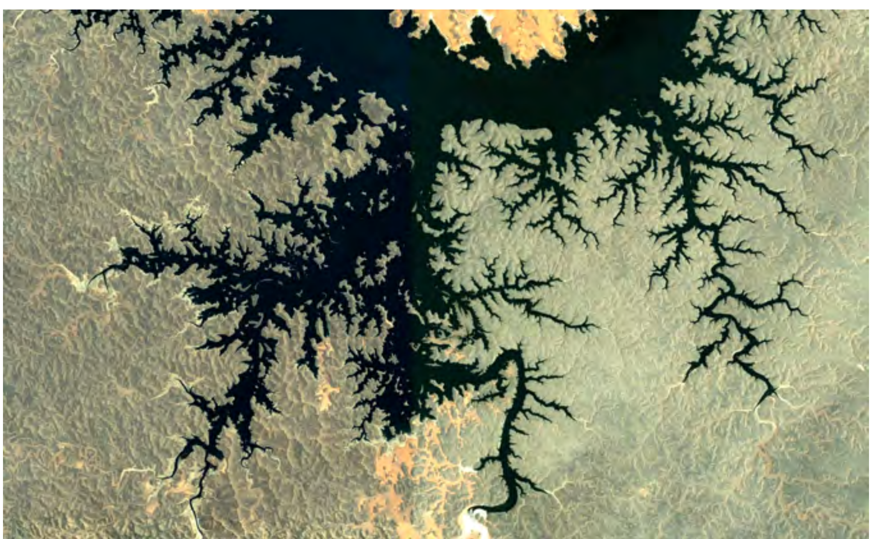


fractals

self-similar structure
non-integer dimension

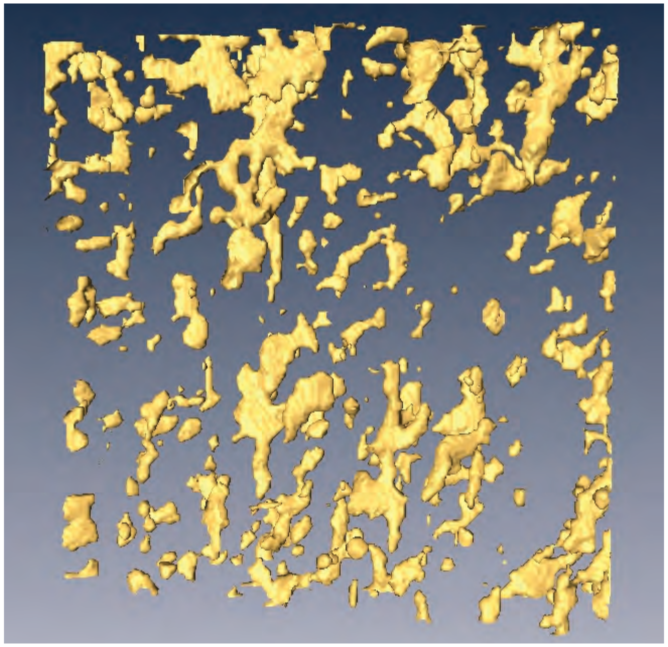


$$D = \frac{\log 3}{\log 2} = 1.585\ldots$$

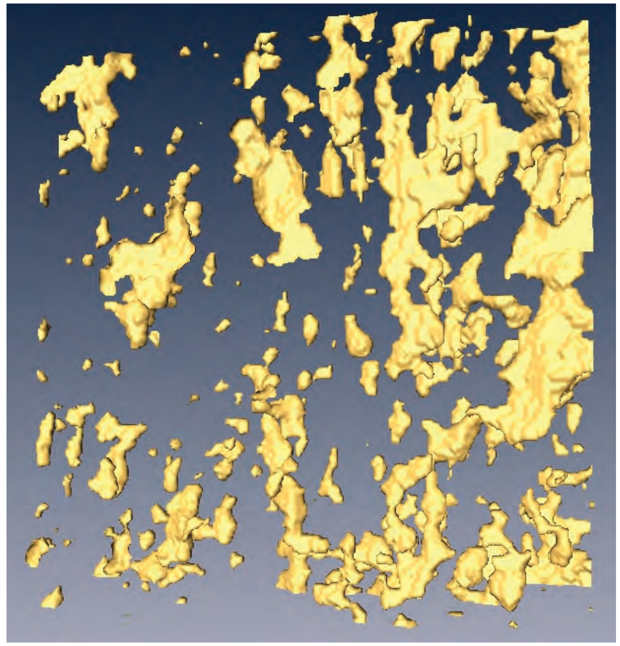


microscale

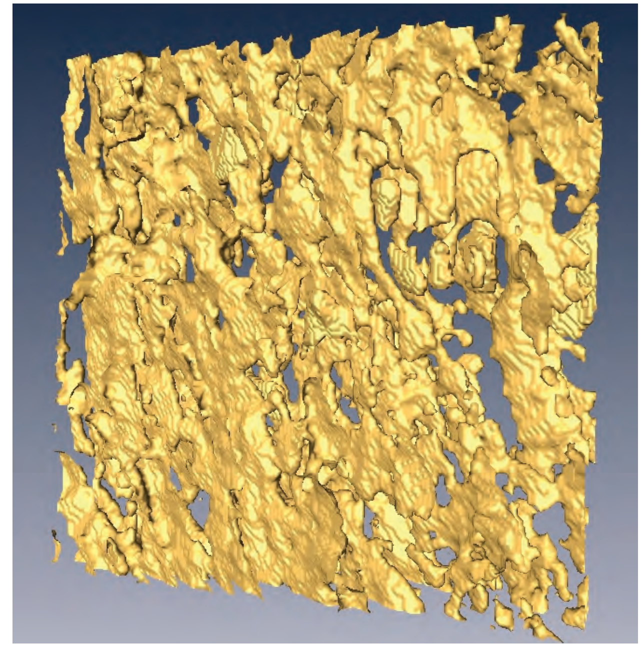
brine volume fraction and ***connectivity*** increase with temperature



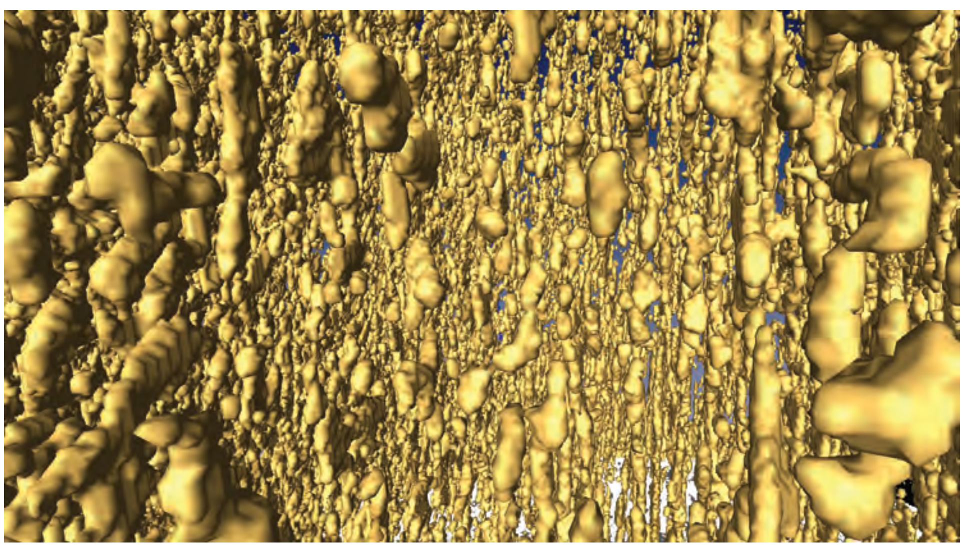
T = -15 °C, $\phi = 0.033$



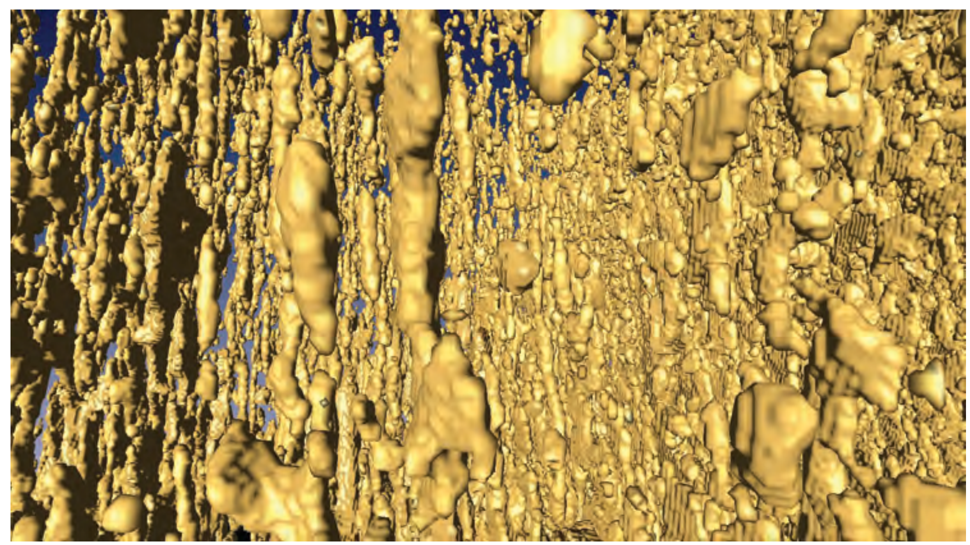
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$



T = -8° C, $\phi = 0.057$



T = -4° C, $\phi = 0.113$

X-ray tomography for brine in sea ice

Golden *et al.*, *Geophysical Research Letters*, 2007

Critical behavior of fluid transport in sea ice

impermeable permeable

"on - off" switch for bulk fluid flow

Arctic field data

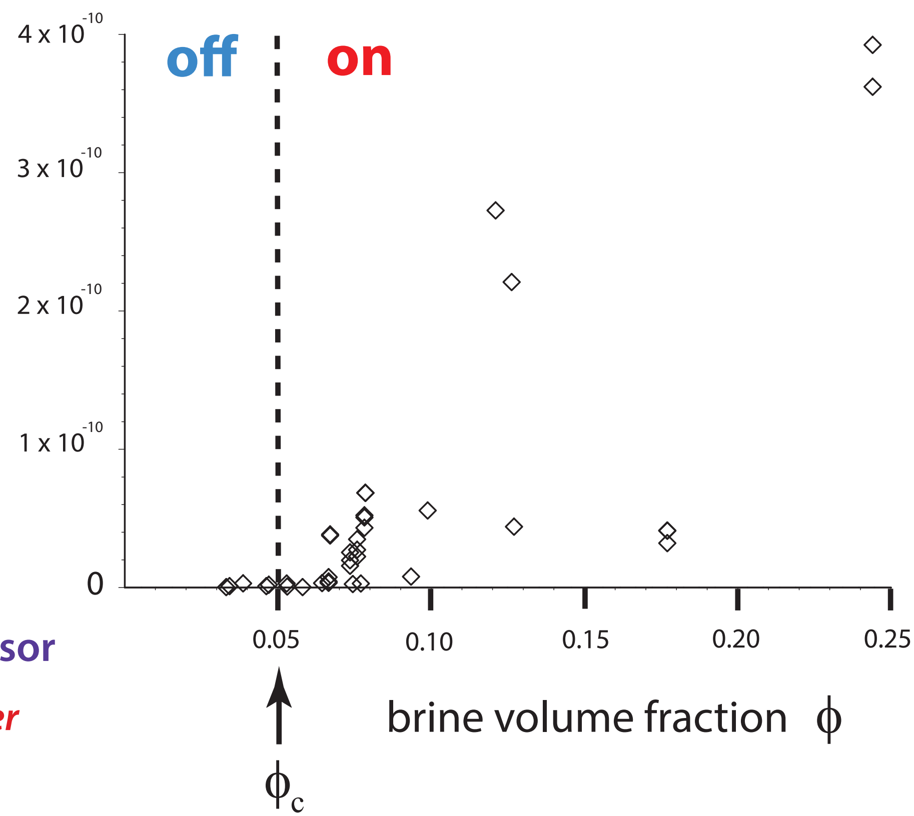
vertical fluid permeability k (m^2)

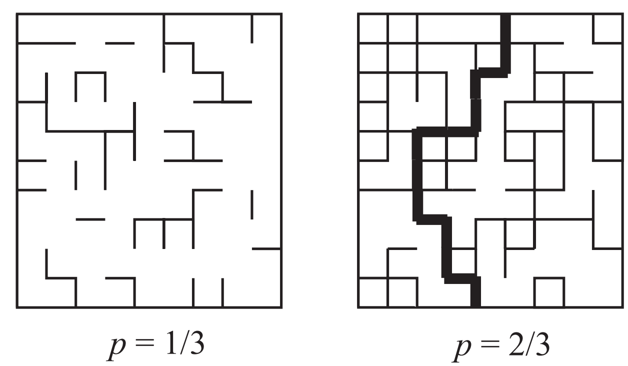
Darcy's Law

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta}\nabla p$$

$\mathbf{k}$ = fluid permeability tensor

homogenized parameter





lattice percolation

FRACTAL percolation clusters

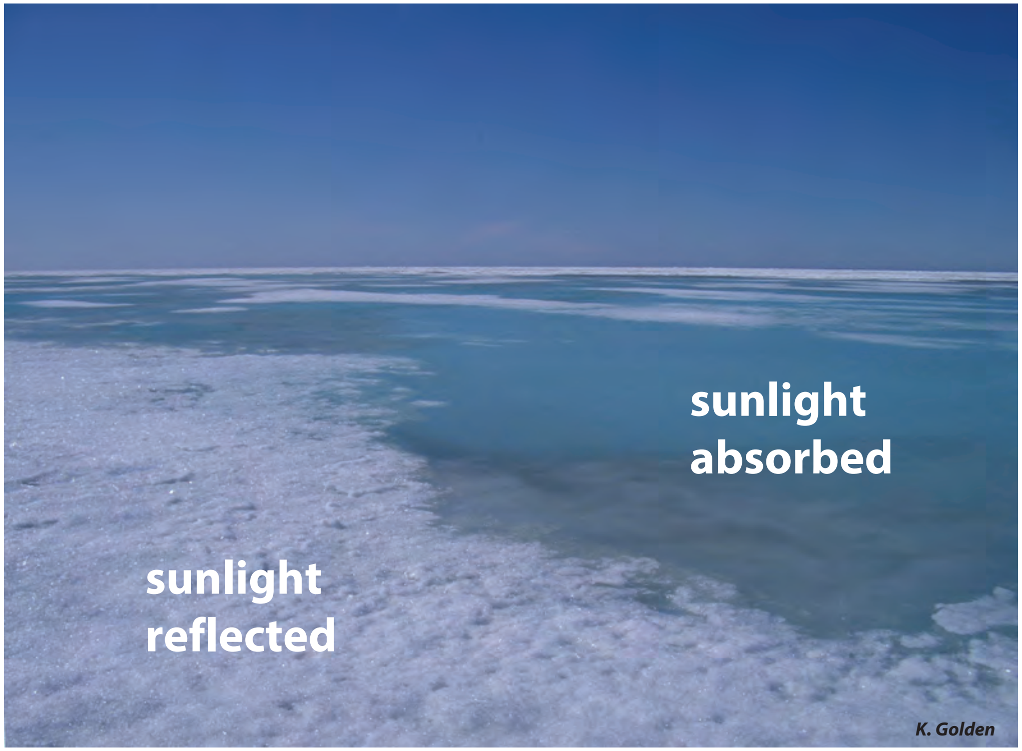
PERCOLATION THRESHOLD $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ C,\ S \approx 5$ ppt

RULE OF FIVES

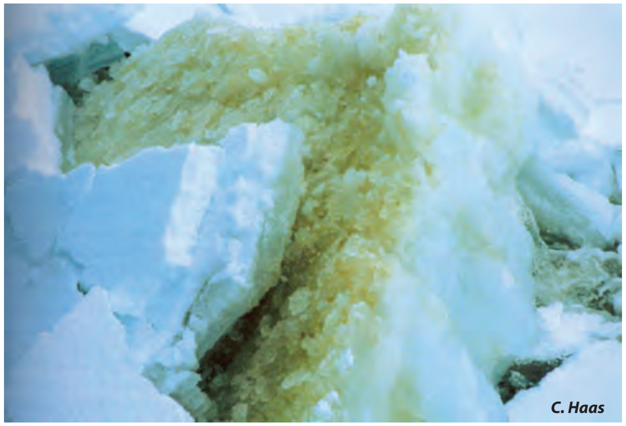
Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

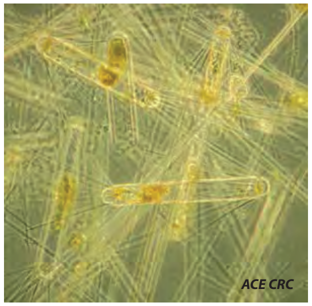
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

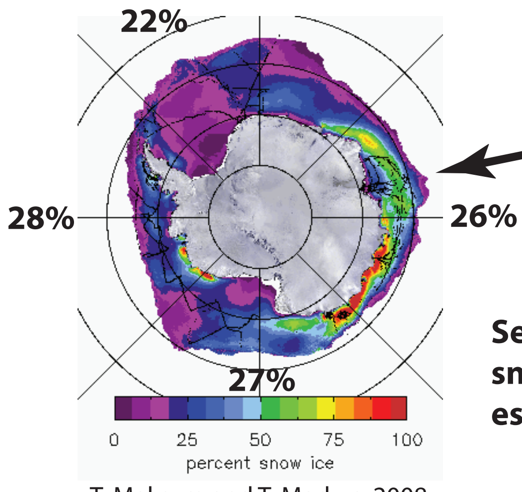
evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







T. Maksym and T. Markus, 2008

September snow-ice estimates

Antarctic surface flooding and snow-ice formation

- evolution of salinity profiles

- ocean-ice-air exchanges of heat, CO_2

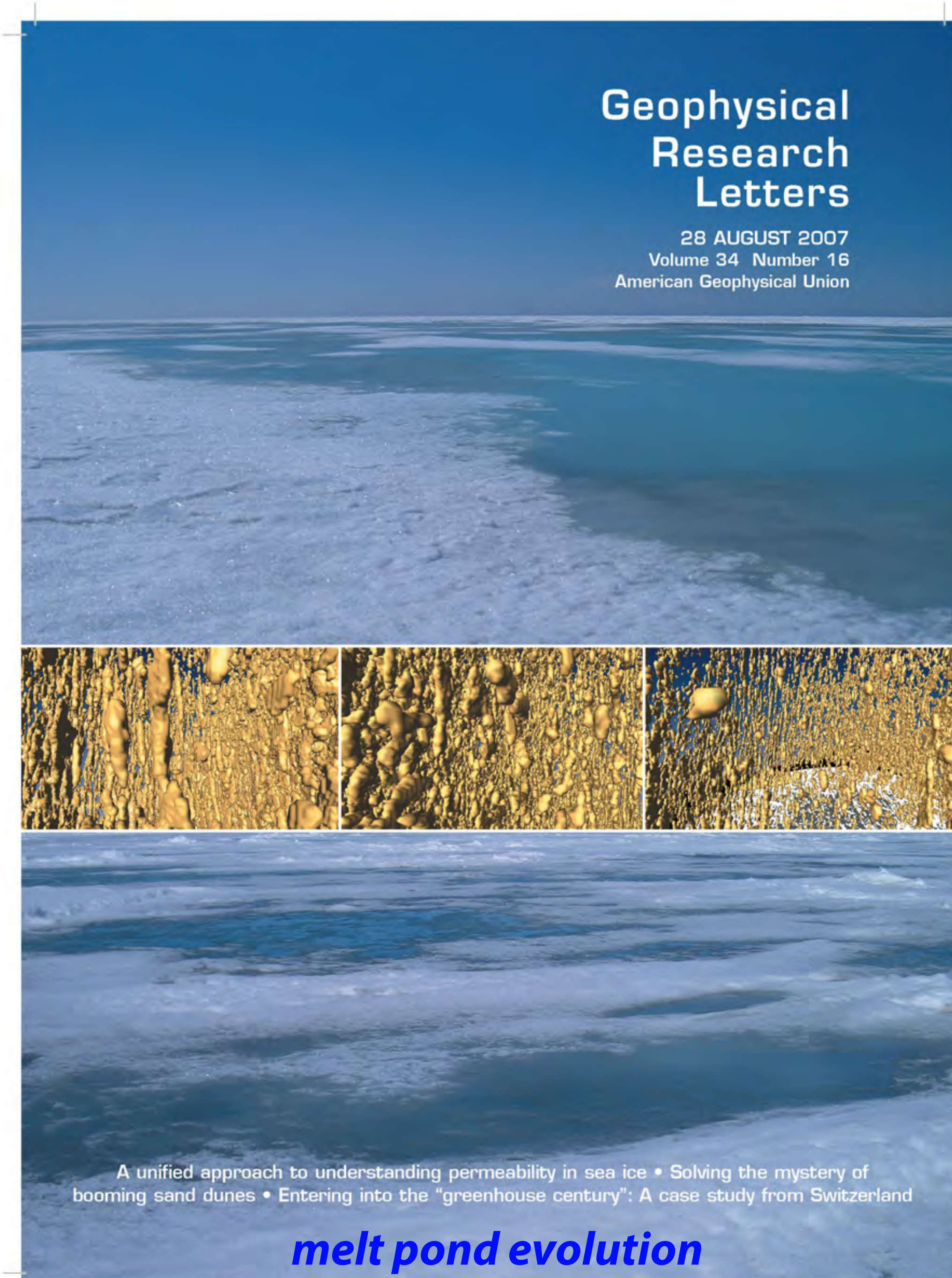
Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007

microscale

governs

mesoscale processes



melt pond evolution

percolation theory for fluid permeability

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8}\ \mathrm{m}^2$$

from critical path analysis in hopping conduction

hierarchical model

rock physics

network model

rigorous bounds

X-ray tomography for brine inclusions

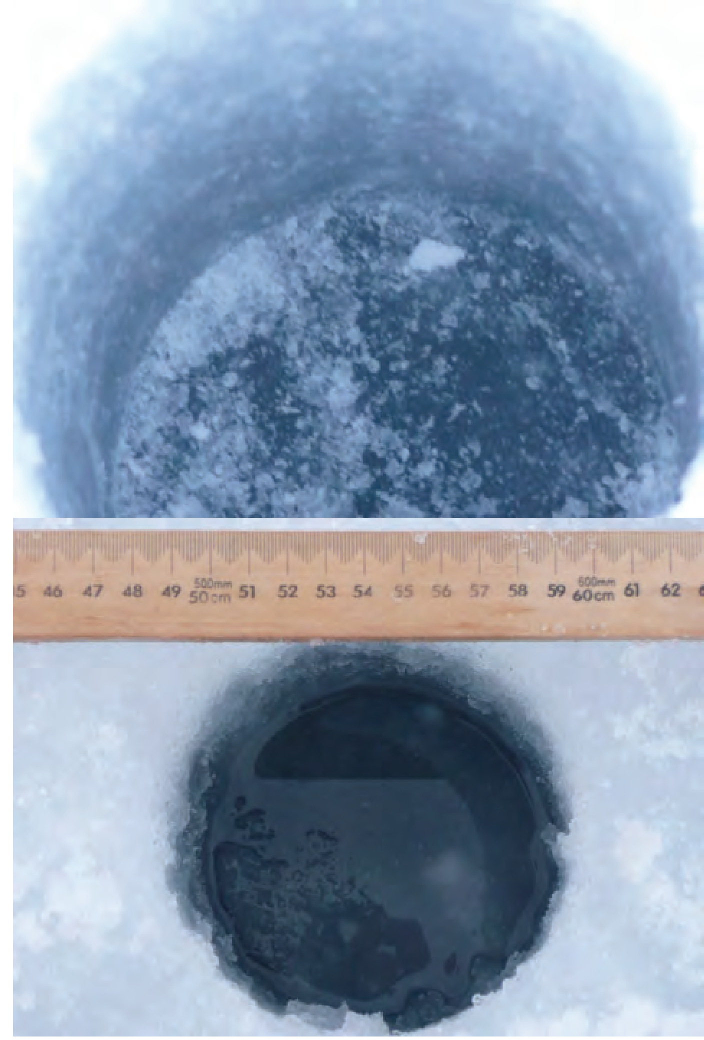
confirms rule of fives

brine percolation threshold of $\phi = 5\%$ for bulk fluid flow

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

theories agree closely with field data

photo by Jan Lieser



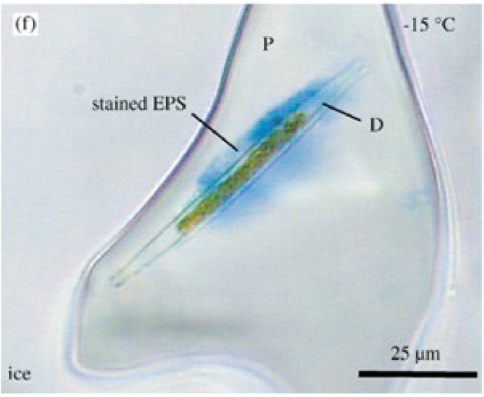
measuring
fluid permeability
of Antarctic sea ice

SIPEX 2007

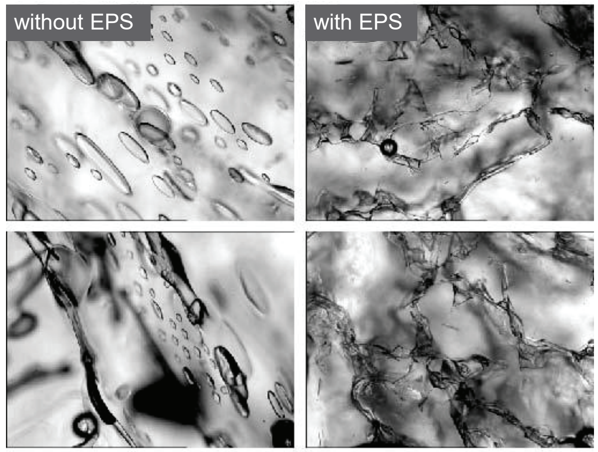
Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport? **How does the biology affect the physics?**

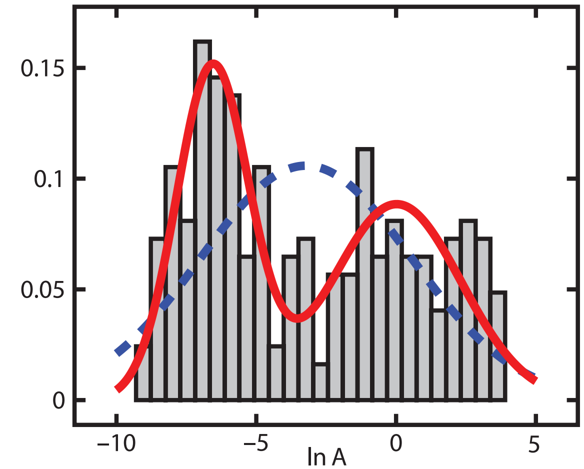
FRACTAL



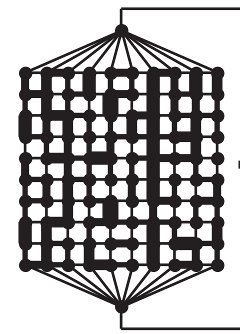
Krembs

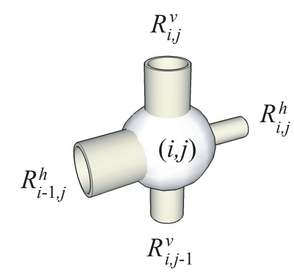


Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL





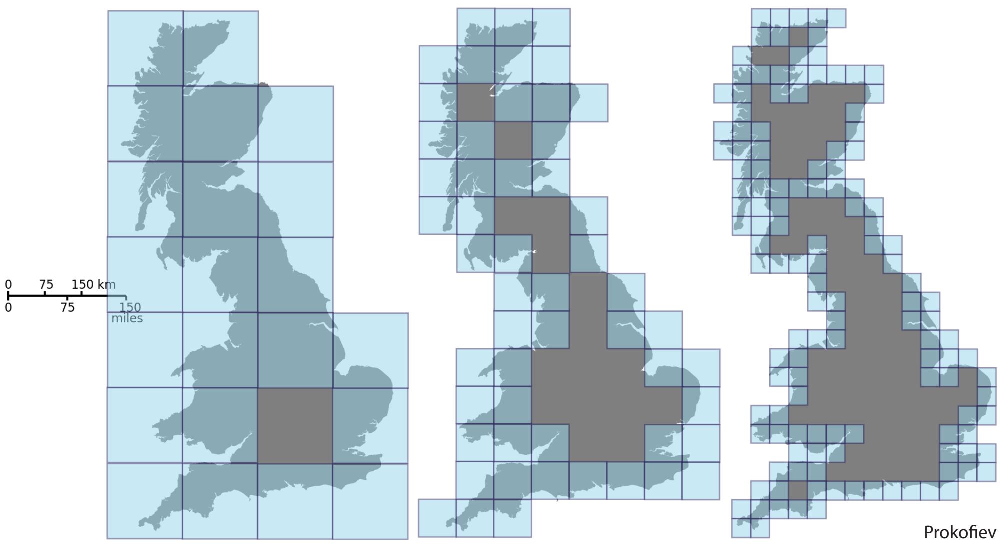
- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability *k;* results predict observed drop in *k*

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
***Multiscale Modeling and Simulation*, 2018**

Zhu, Jabini, Golden, Eicken, Morris
Ann. Glac. 2006

Thermal Evolution of Brine Fractal Geometry in Sea Ice

Nash Ward, Daniel Hallman, Benjamin Murphy, Jody Reimer,
Marc Oggier, Megan O'Sadnick, Elena Cherkaev and Kenneth Golden, 2022

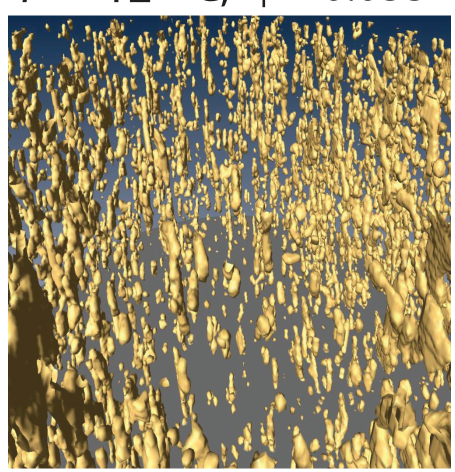


fractal dimension of the coastline of Great Britain by box counting

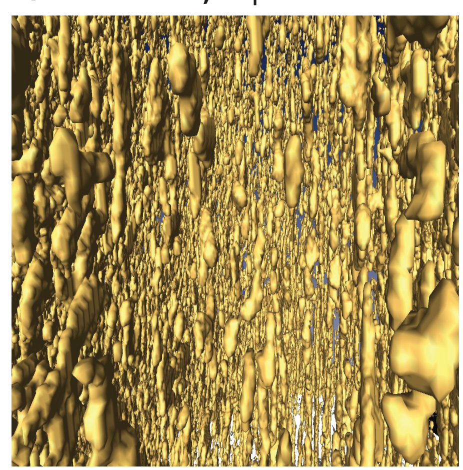
$$N(\epsilon) \sim \epsilon^{-D}$$

brine channels and inclusions "look" like fractals (from 30 yrs ago)

$T =$ -12° C, $\phi = 0.033$



$T =$ -8° C, $\phi = 0.057$

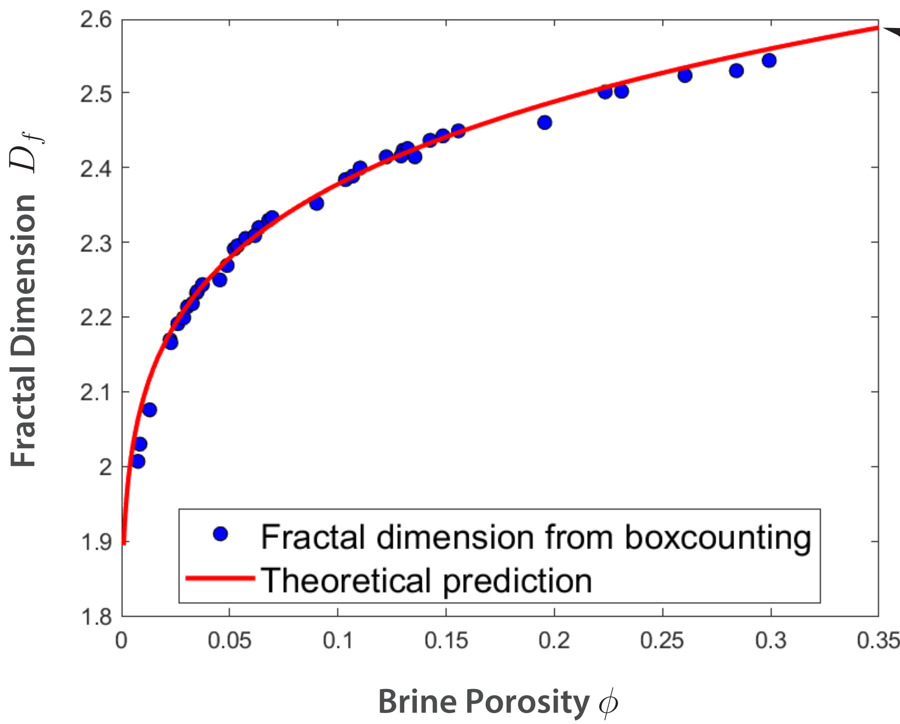


Golden, Eicken, et al. *GRL*, 2007

X-ray computed tomography of brine in sea ice

columnar and granular

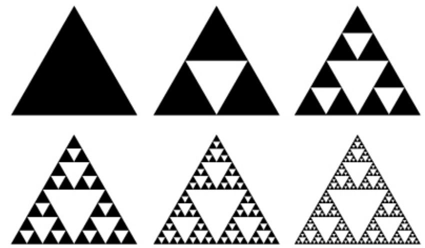
The first comprehensive, quantitative study of the fractal dimension of brine in sea ice and its strong dependence on temperature and porosity.

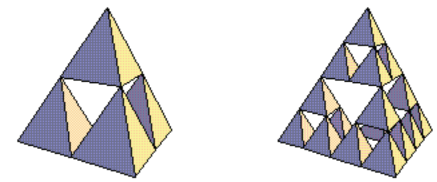


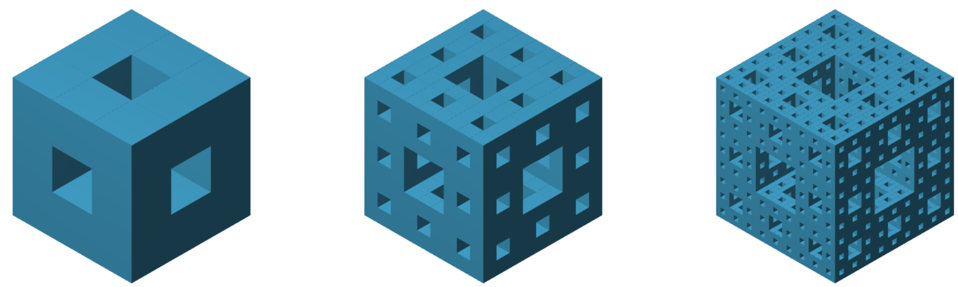
$$D_f = 3 - \frac{\ln \phi}{\ln(\lambda_{min}/\lambda_{max})}$$

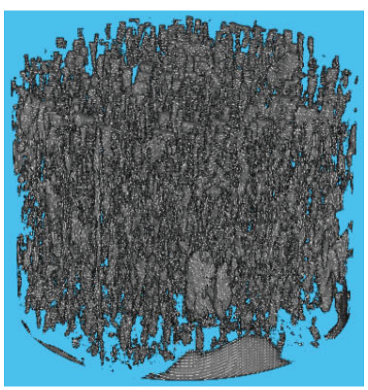
The red curve is exact for the Sierpinski pyramid (an exactly self-similar geometry); discovered for sandstones - statistically self-similar porous media like sea ice.

Katz and Thompson, 1985; Yu and Li, 2001

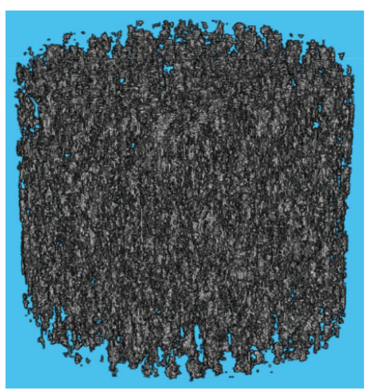




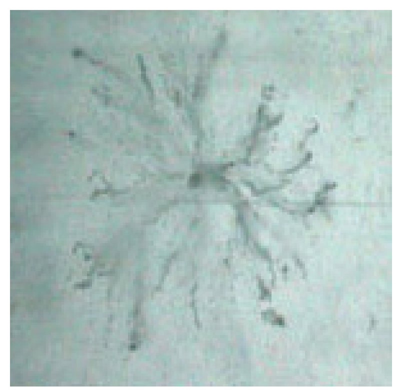




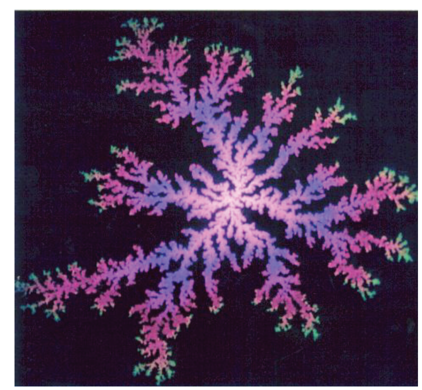
X-ray tomography



DLA model

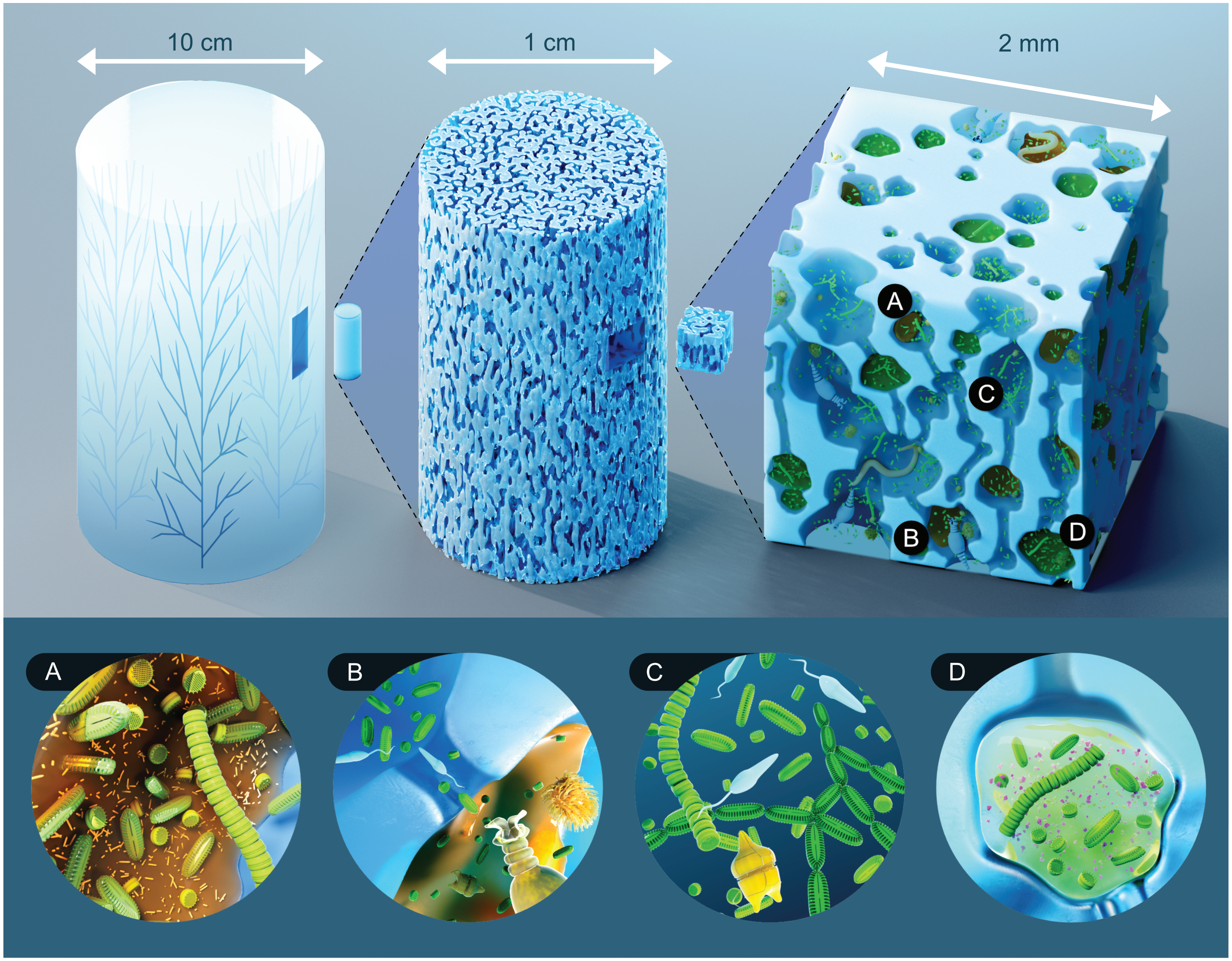


brine channel in sea ice



diffusion limited aggregation

Implications of brine fractal geometry on sea ice ecology and biogeochemistry



Brine inclusions are home to ice endemic organisms, e.g., bacteria, diatoms, flagellates, rotifers, nematodes.

The habitability of sea ice for these organisms is inextricably linked to its complex brine geometry.

(A) Many sea ice organisms attach themselves to inclusion walls; inclusions with a higher fractal dimension have greater surface area for colonization.

(B) Narrow channels prevent the passage of larger organisms, leading to refuges where smaller organisms can multiply without being grazed, as in (C).

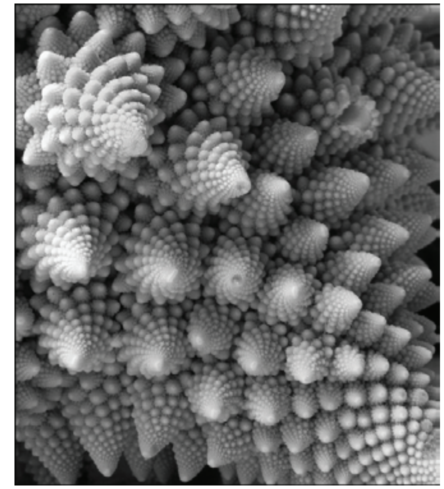
(D) Ice algae secrete extracellular polymeric substances (EPS) which alter incusion geometry and may further increase the fractal dimension.

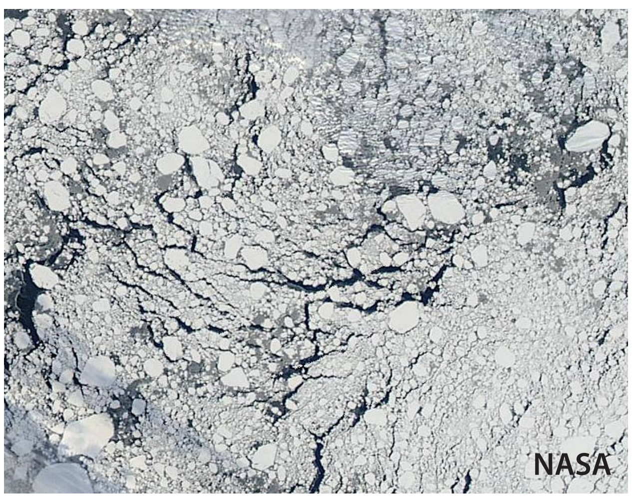
Golden

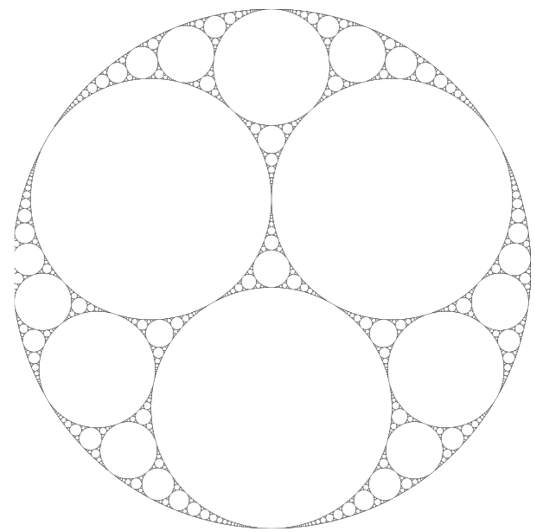
mesoscale

the sea ice pack is a ***fractal***

displaying self-similar structure on many scales







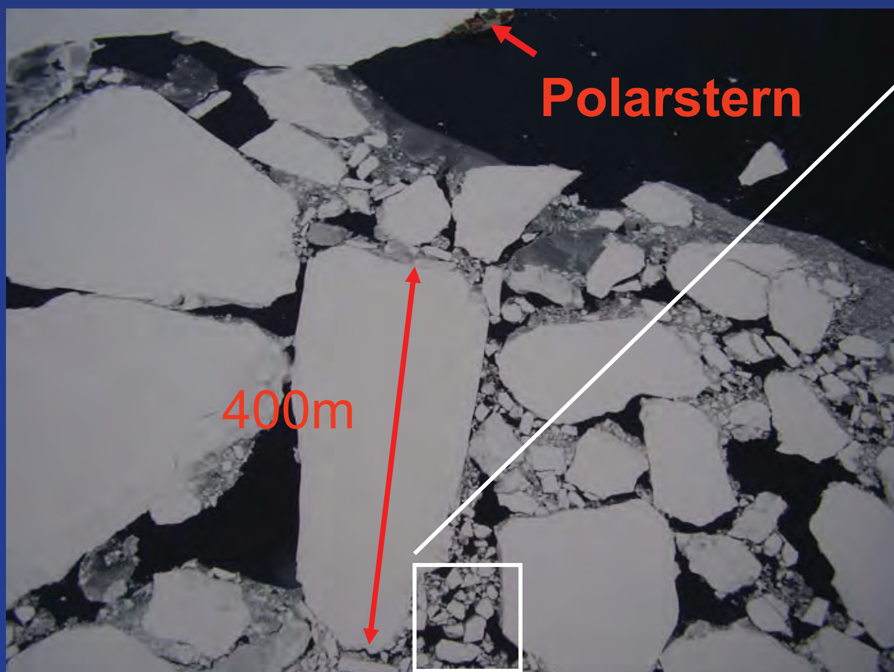
floe size distribution, area-perimeter relations, etc. important in dynamics (fracture), thermodynamics (melting)

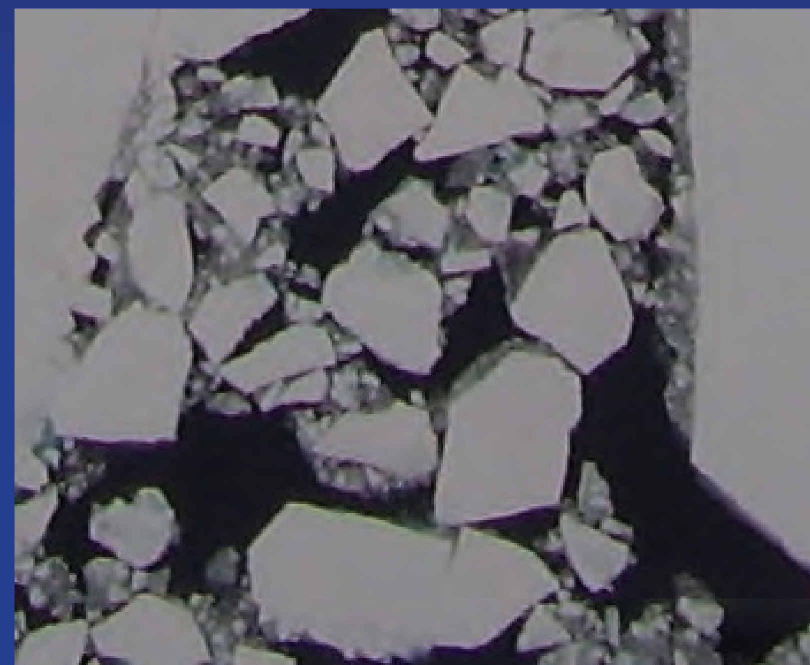
Toyota, et al. Geophys. Res. Lett. 2006
Rothrock and Thorndike, J. Geophys. Res. 1984

The sea ice pack has fractal structure.

Self-similarity of sea ice floes

Weddell Sea, Antarctica





fractal dimensions of Okhotsk Sea ice pack
smaller scales D~1.2, larger scales D~1.9

Toyota, *et al. Geophys. Res. Lett.* 2006

Rothrock and Thorndike, *J. Geophys. Res. 1984*

wave propagation in the marginal ice zone (MIZ)

Stieltjes integral representation and bounds for the complex viscoelasticity of the ice - ocean layer

Sampson, Murphy, Cherkaev, Golden 2023

first theory of key parameter in wave-ice interactions only fitted to wave data before

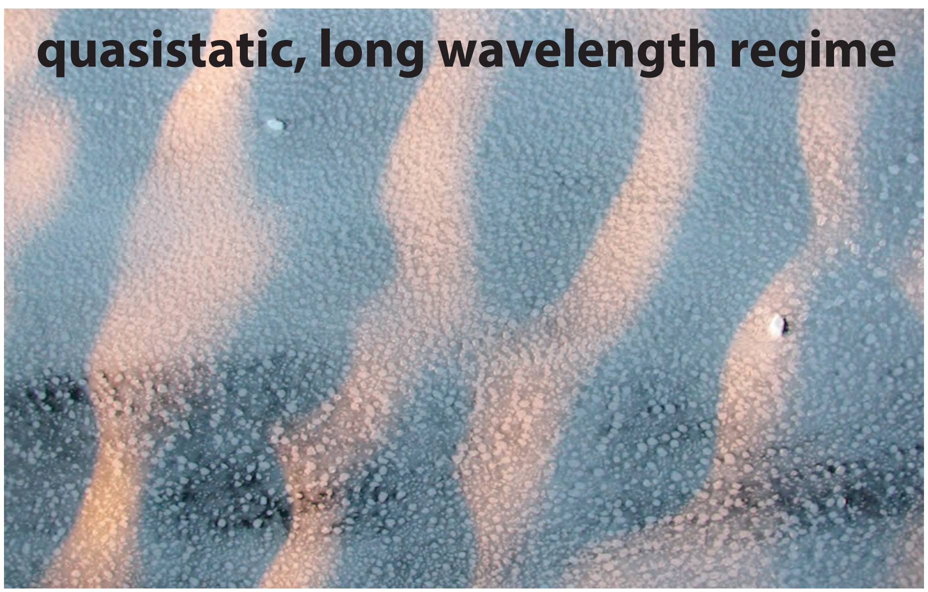
Keller, 1998
Mosig, Montiel, Squire, 2015
Wang, Shen, 2012

Analytic Continuation Method

Bergman (78) - Milton (79) integral representation for ε^*

Golden and Papanicolaou (83)

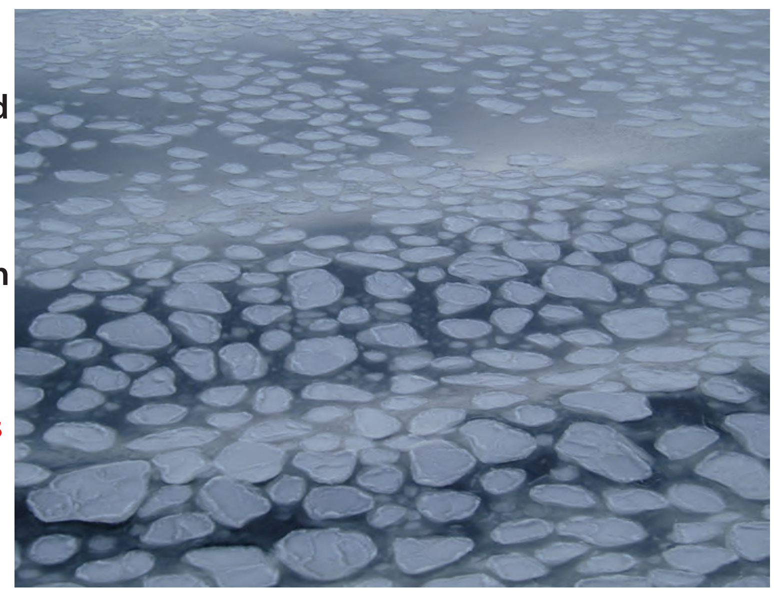
Milton, *Theory of Composites* (02)



quasistatic, long wavelength regime

homogenized parameter depends on sea ice concentration and ice floe geometry

like EM waves



melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

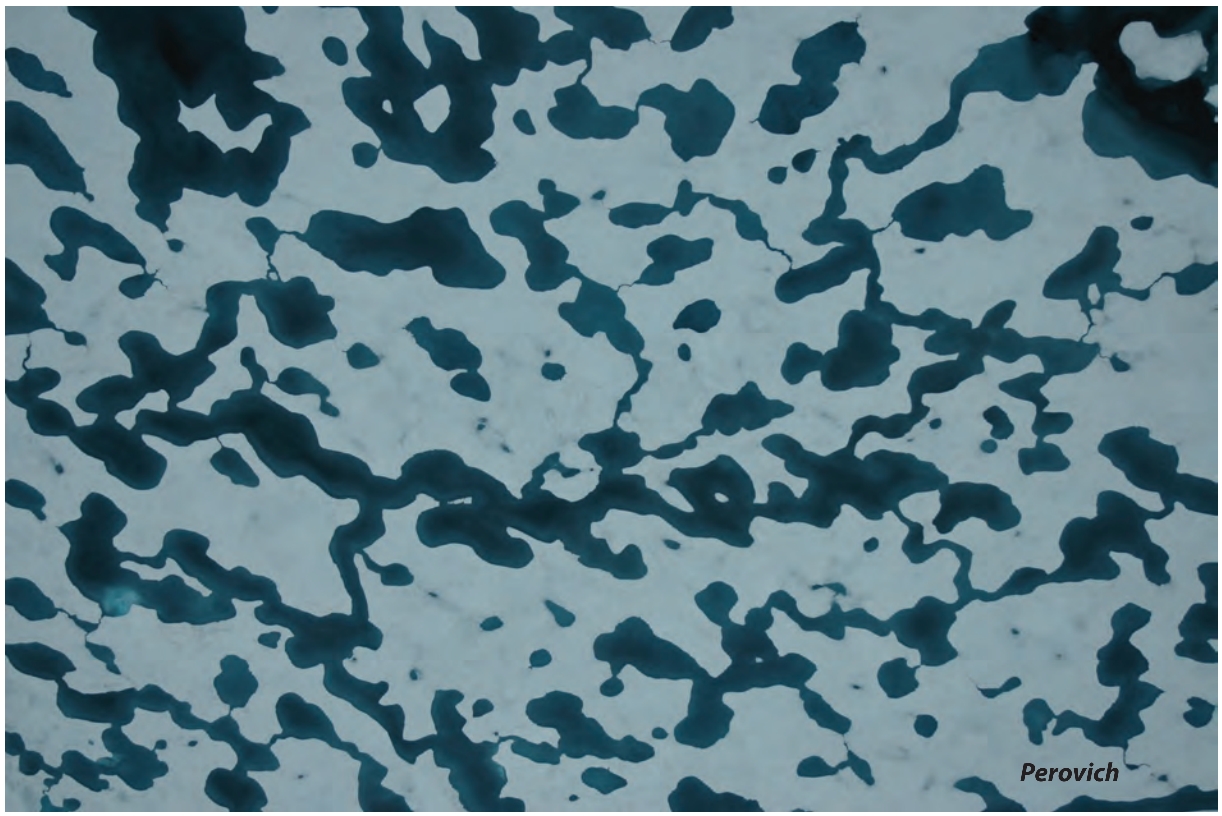
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

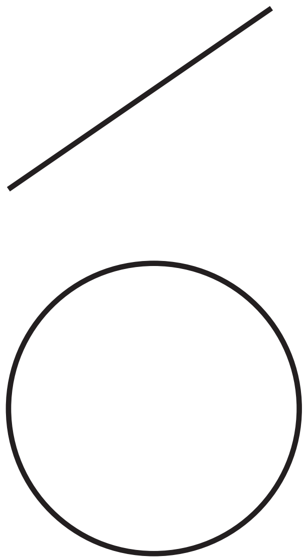
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

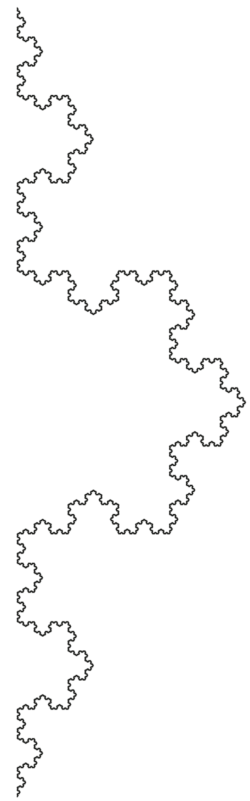
fractal curves in the plane

they wiggle so much that their dimension is >1



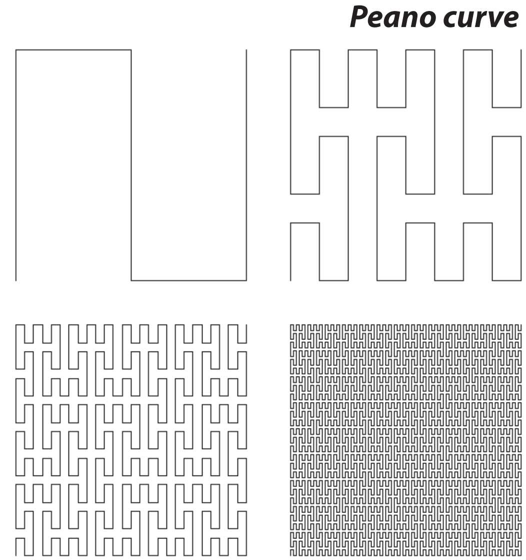
simple curves

$D = 1$



Koch snowflake

$D = 1.26$



space filling curves

$D = 2$

clouds exhibit fractal behavior from 1 to 1000 km

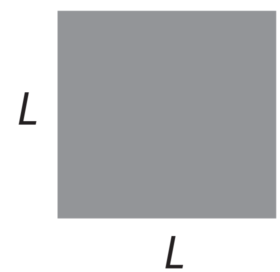
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

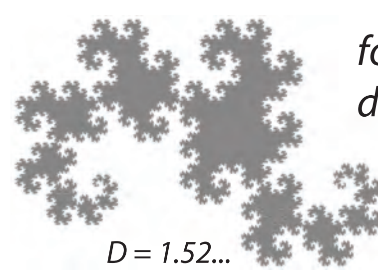
simple shapes



$A = L^2$

$P = 4L = 4\sqrt{A}$

$$P \sim \sqrt{A}^{\,D}$$

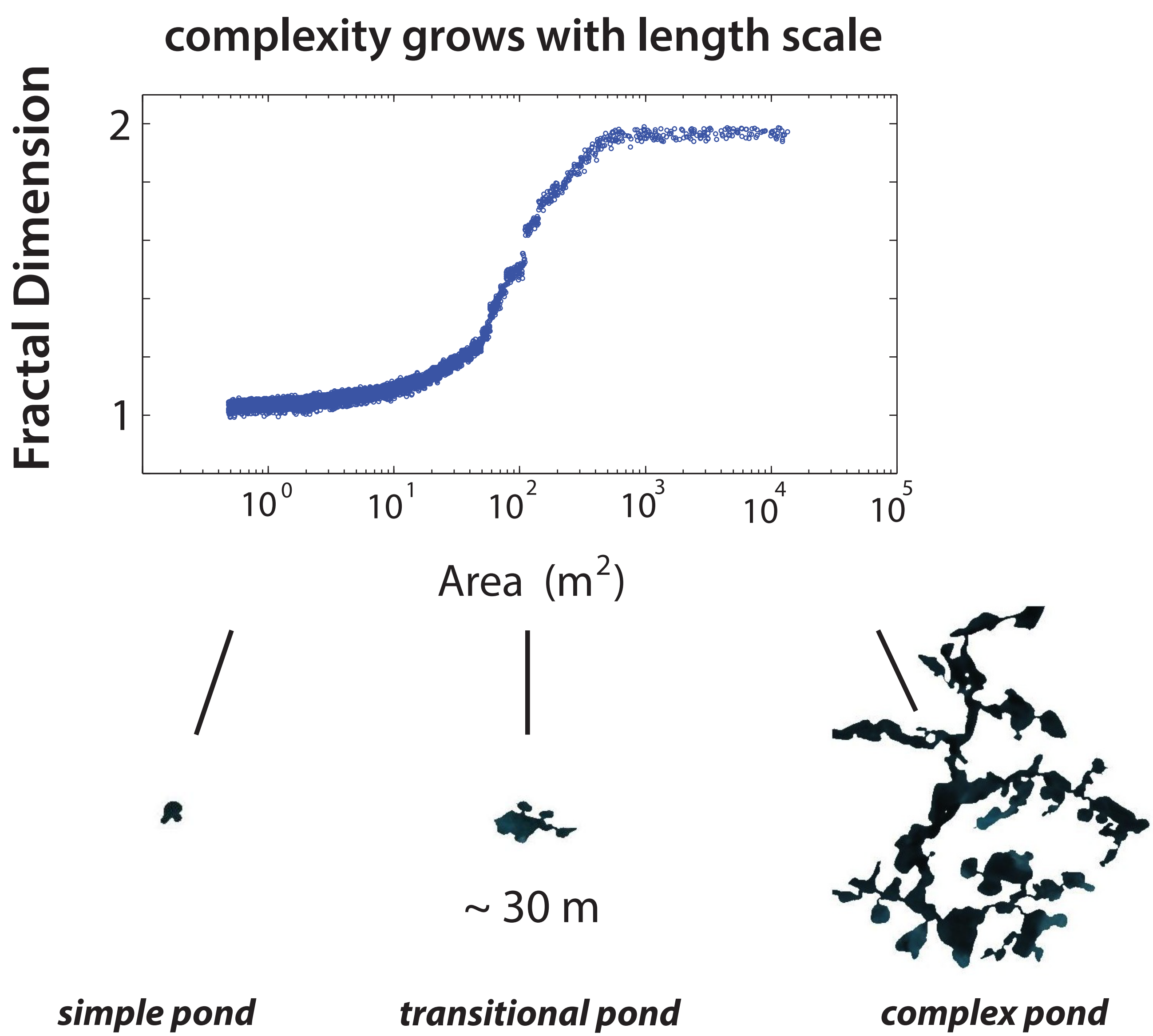


for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

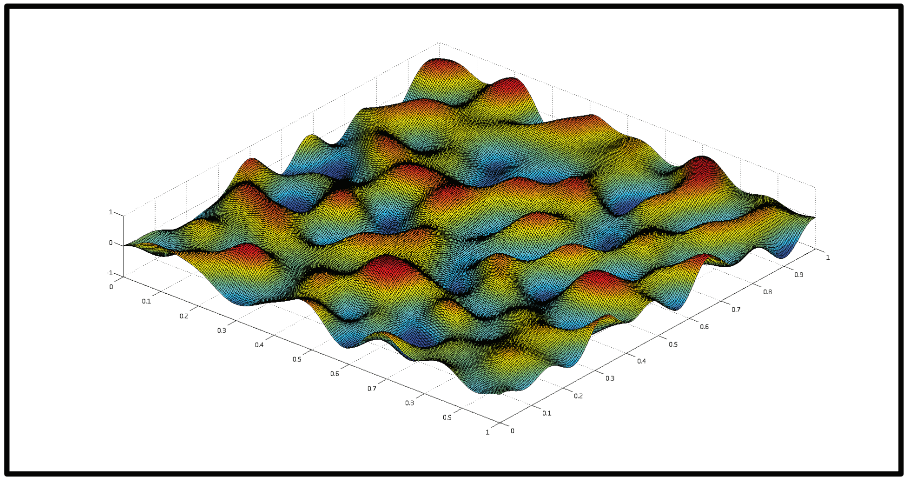
The Cryosphere, 2012



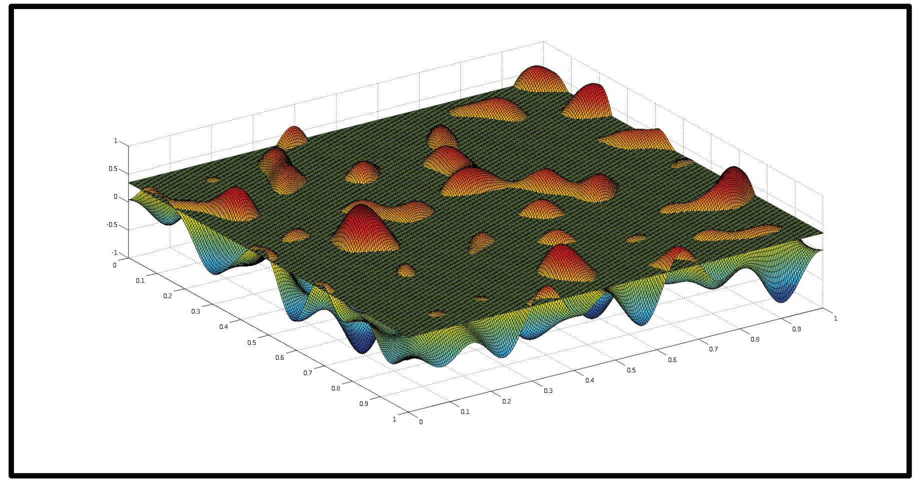
Continuum percolation model for melt pond evolution

level sets of random surfaces

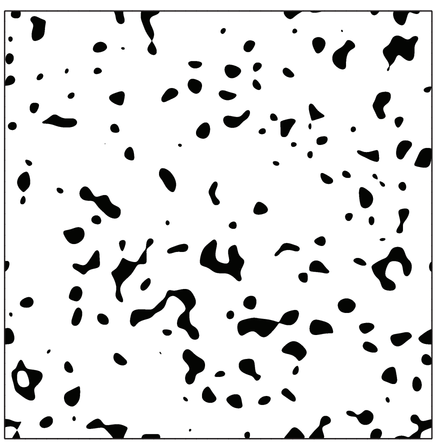
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018

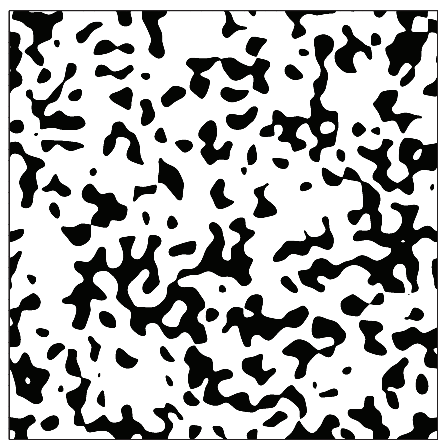


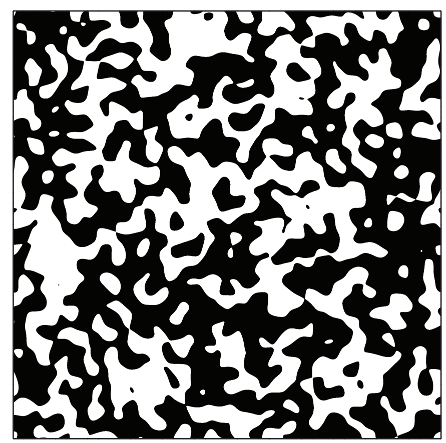
random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds

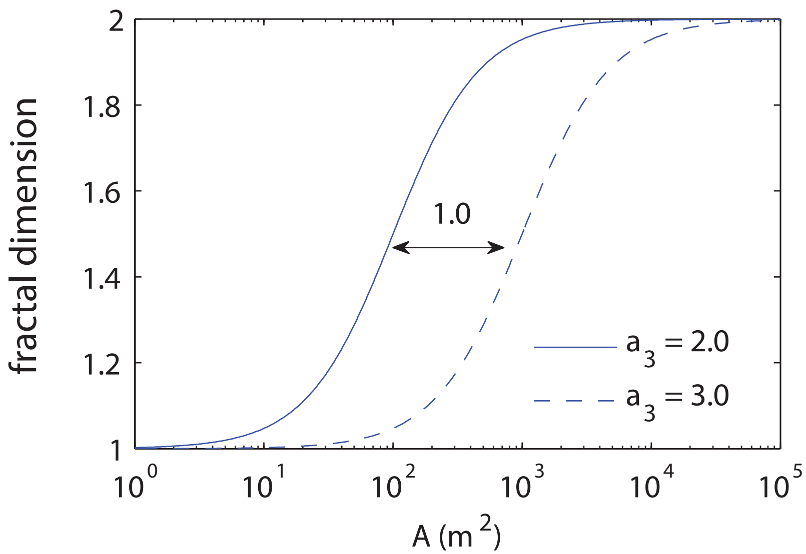


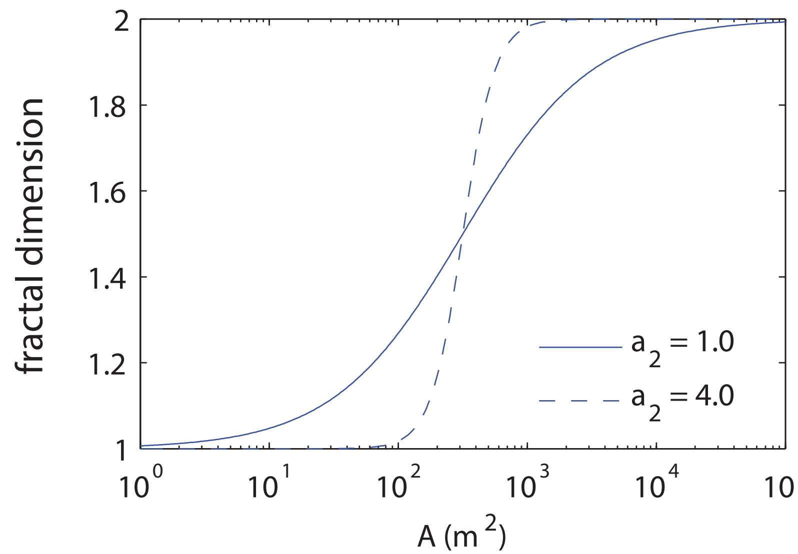




electronic transport in disordered media   *diffusion in turbulent plasmas*   *Isichenko, Rev. Mod. Phys., 1992*

fractal dimension curves depend on statistical parameters defining random surface





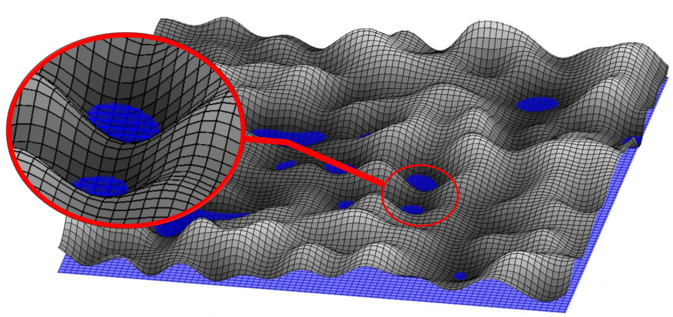
Topology of the sea ice surface and the fractal geometry of Arctic melt ponds

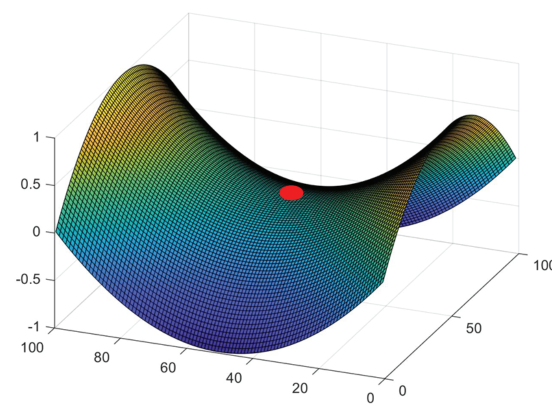
Physical Review Research (invited, under revision)

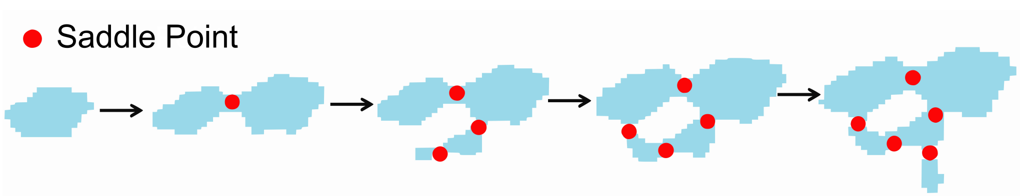
Ryleigh Moore, Jacob Jones, Dane Gollero,
Court Strong, Ken Golden

Several models replicate the transition in fractal dimension, but none explain how it arises.

We use Morse theory applied to the random surface model to show that saddle points play the critical role in the fractal transition.



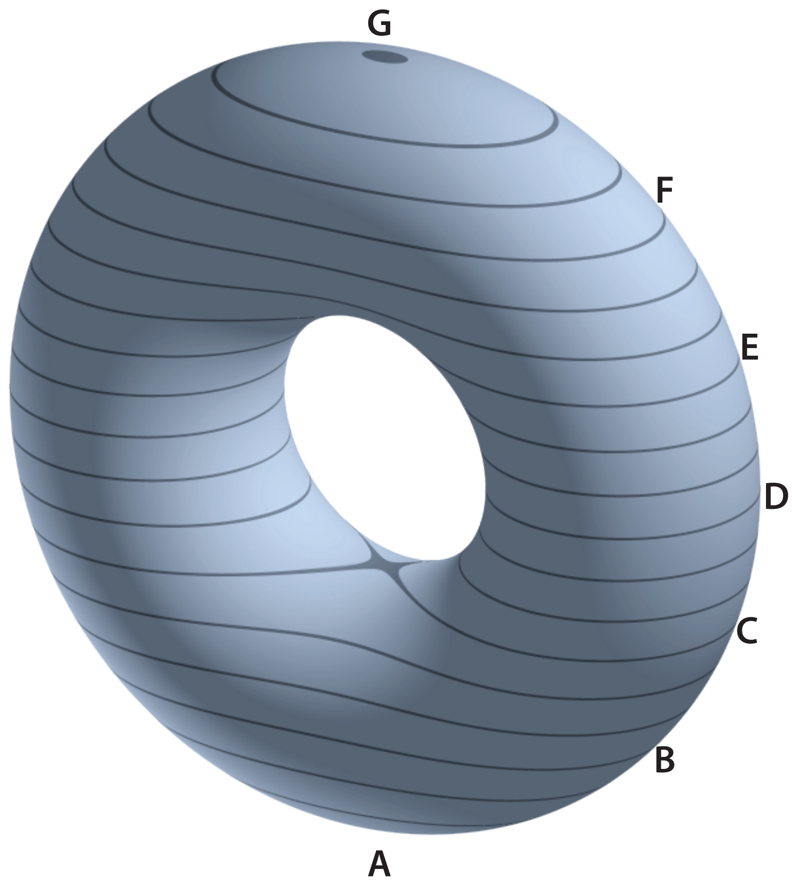




saddles drive the fractal transition

ponds coalesce
(change topology) and
complexify at saddle points

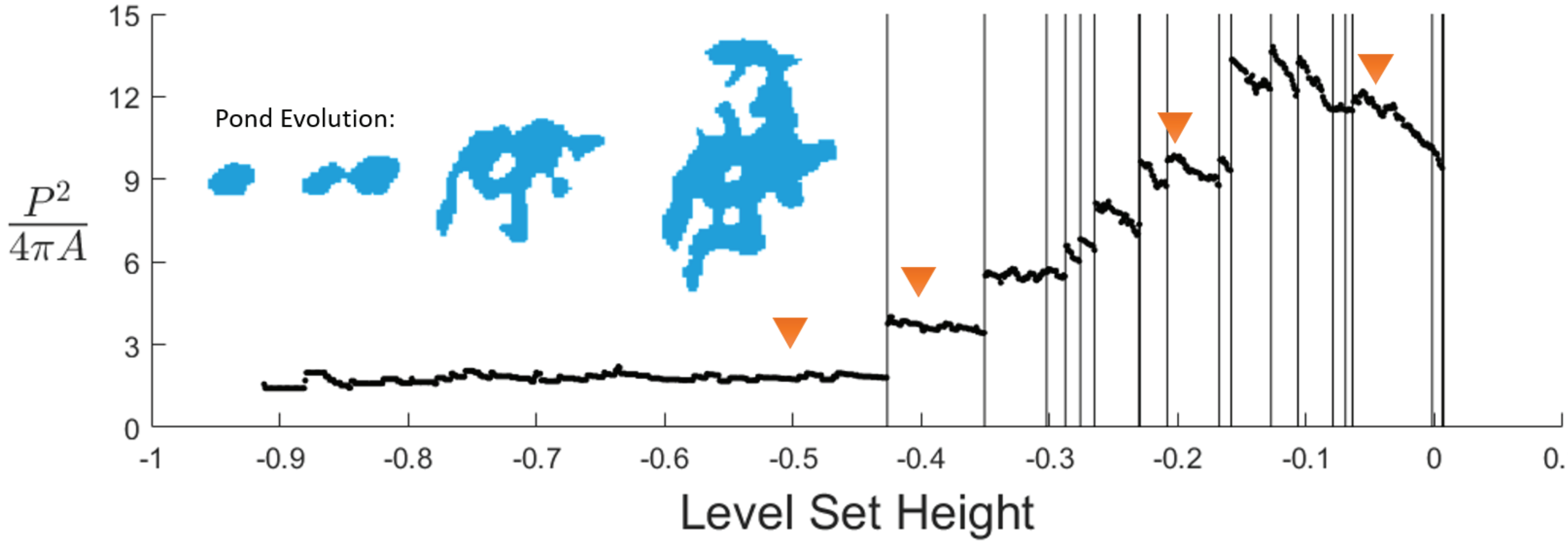
Morse theory



Morse theory tells us that changes in the topology of a surface occur at critical points of smooth functions on the surface: maxima, minima, and saddles.

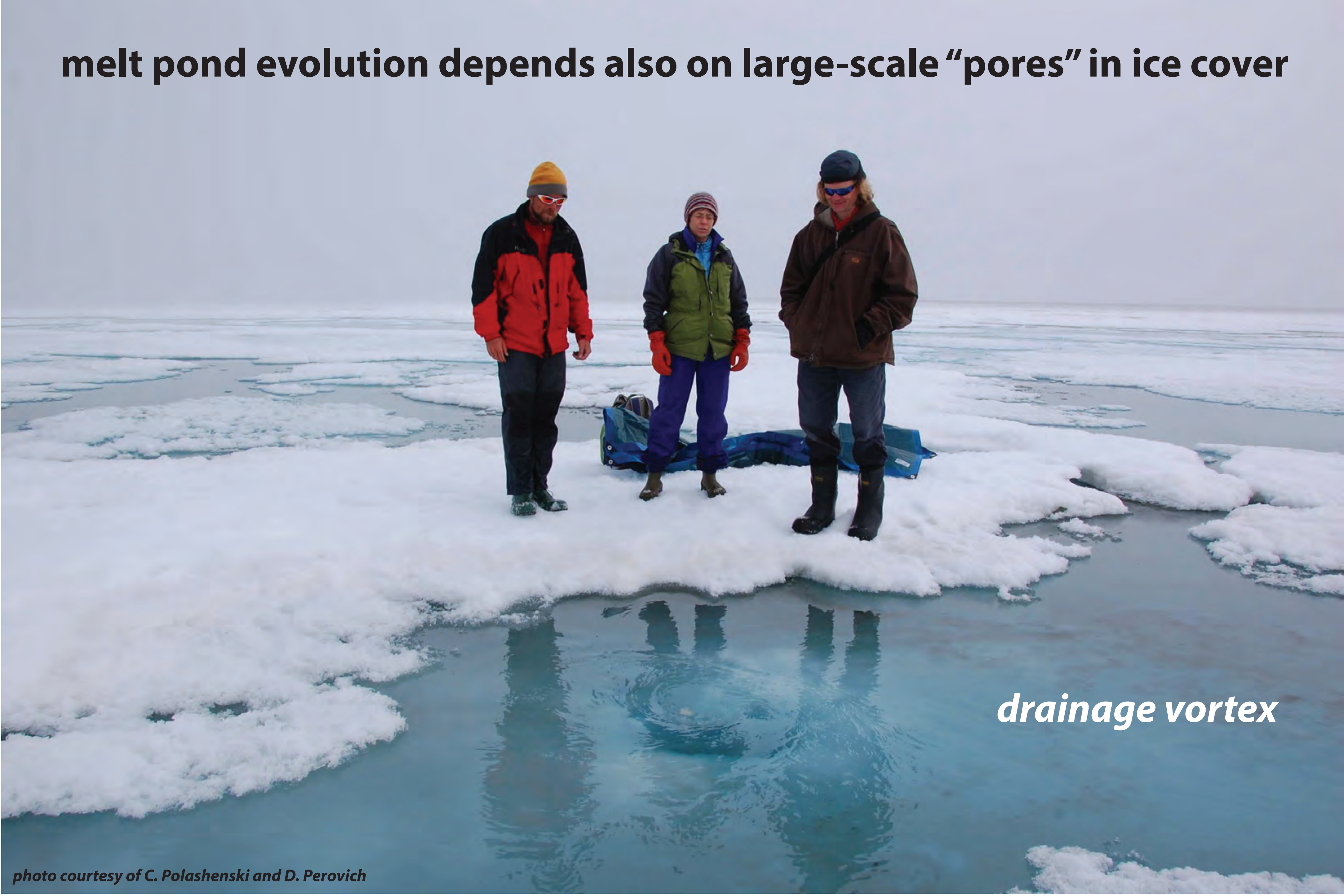
Main results

Isoperimetric quotient - as a proxy for fractal dimension - increases in discrete jumps when ponds coalesce at saddle points.



Horizontal fluid permeability "controlled" by saddles ~ electronic transport in 2D random potential.

drainage processes, seal holes



Melt pond connectivity enables vast expanses of melt water to drain down seal holes, thaw holes, and leads in the ice.

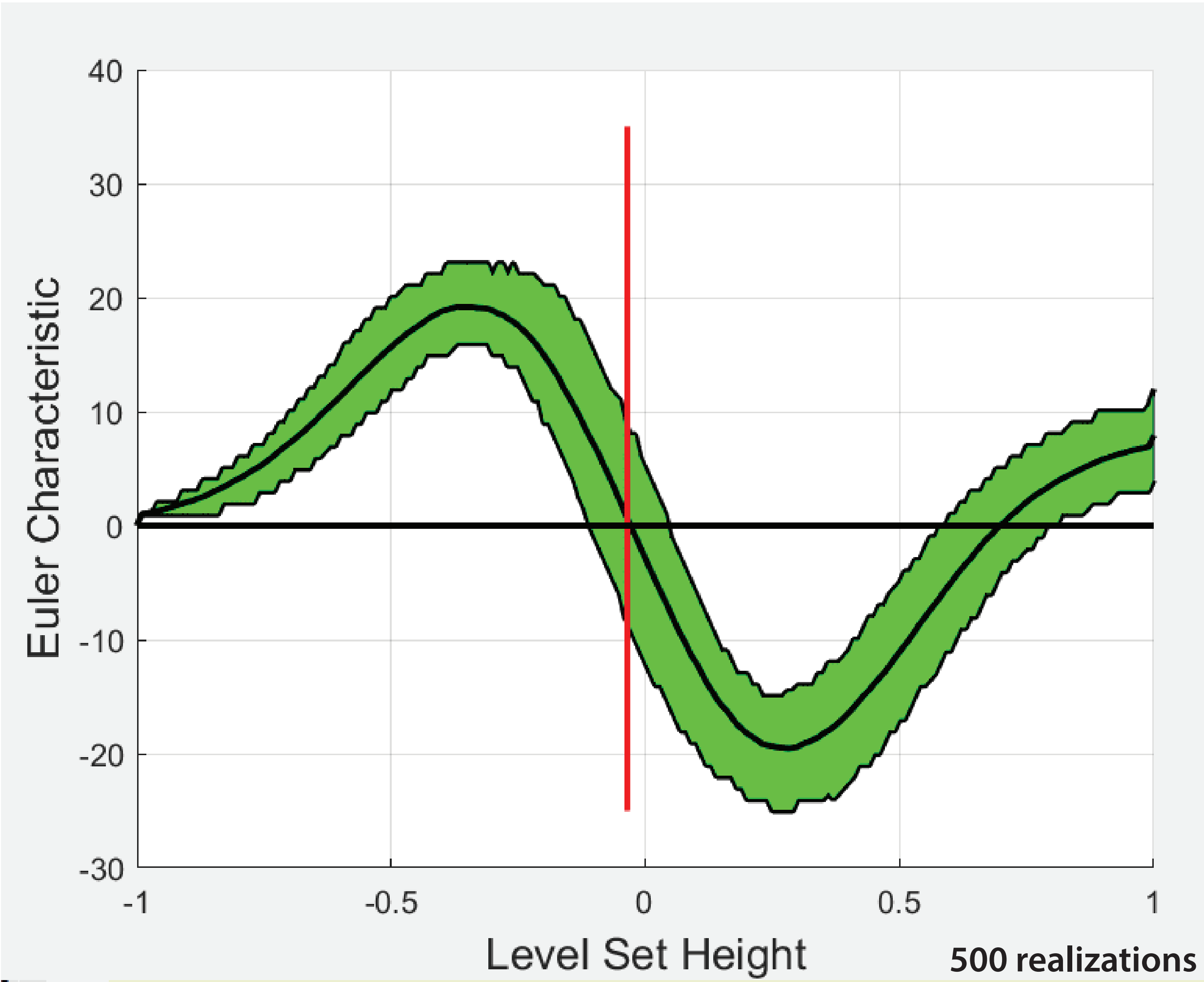
Topological Data Analysis

Euler characteristic = # maxima + # minima - # saddles

topological invariant

persistent homology

filtration - sequence of nested topological spaces, indexed by water level



Expected
Euler Characteristic Curve (ECC)

tracks the evolution of the EC of the flooded surface as water rises

zero of ECC ~ percolation

percolation on a torus
creates a giant cycle

Bobrowski & Skraba, 2020

Carlsson, 2009

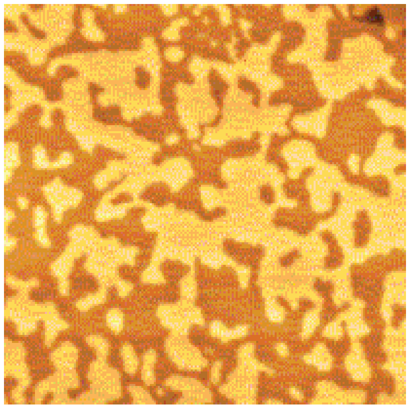
Vogel, 2002 GRF

image analysis
porous media
cosmology
brain activity

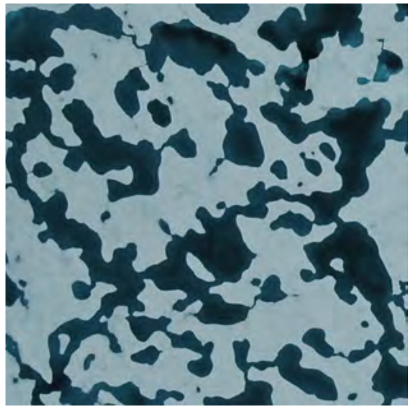
melt pond donuts

From magnets to melt ponds

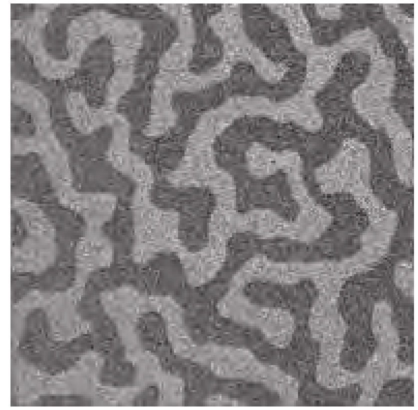
100 year old model for magnetic materials used to explain melt pond fractal geometry



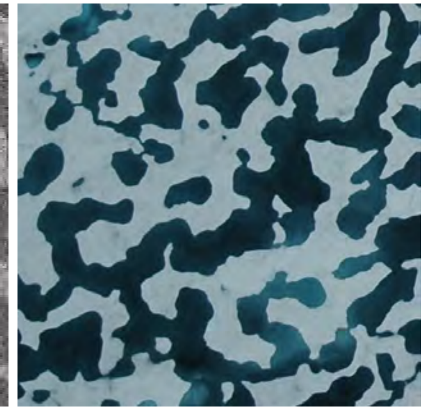
magnetic domains
cobalt



Arctic melt ponds

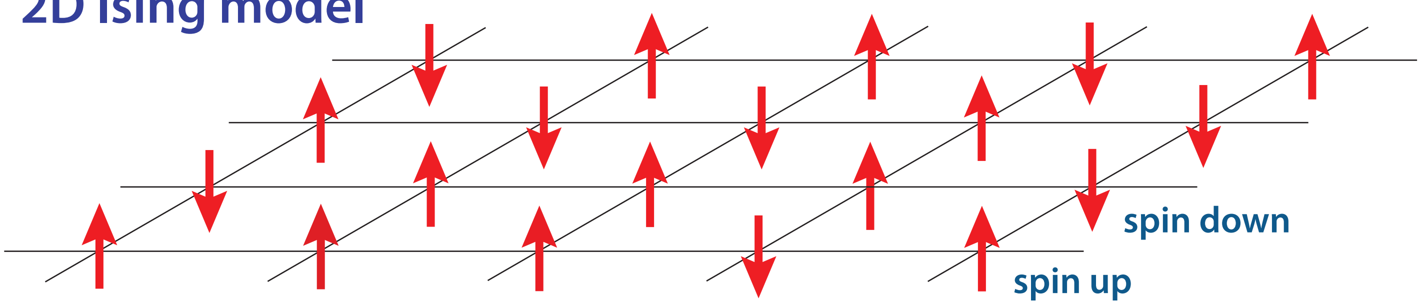


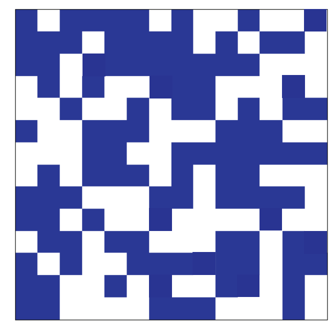
magnetic domains
cobalt-iron-boron



Arctic melt ponds

2D Ising model

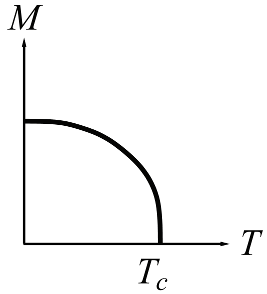




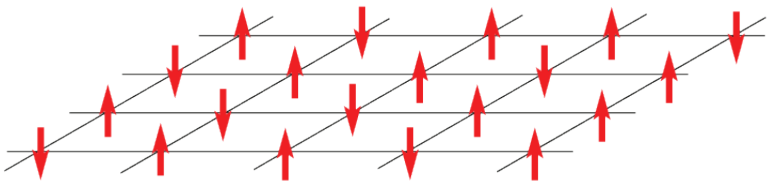
Ma, Sudakov, Strong, Golden, *New J. Phys.* 2019

Golden, Ma, Strong, Sudakov, *SIAM News* 2020

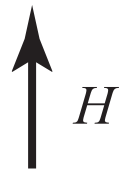
Ising Model for a Ferromagnet



Curie point
critical temperature



applied magnetic field



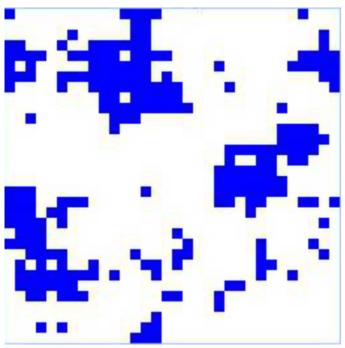
$$s_i = \begin{cases} +1 & \text{spin up} \\ -1 & \text{spin down} \end{cases}$$

blue
white

$$\mathcal{H} = -H\sum_i s_i - J\sum_{<i,j>} s_i s_j$$

nearest neighbor Ising Hamiltonian

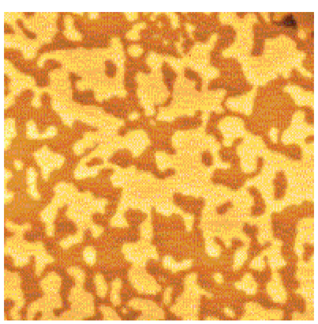
islands of like spins



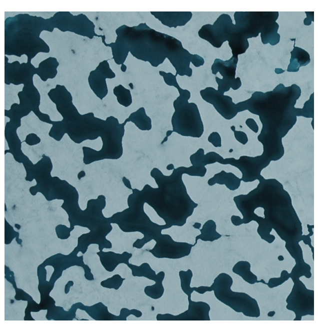
$$M(T,H) = \lim_{N\to\infty} \frac{1}{N}\left\langle \sum_j s_j \right\rangle$$

effective magnetization

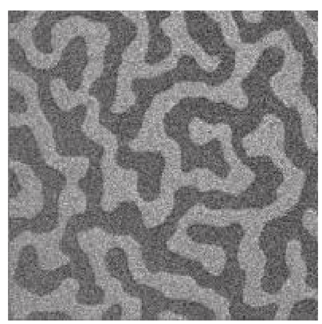
energy is lowered when nearby spins align with each other, forming **magnetic domains**



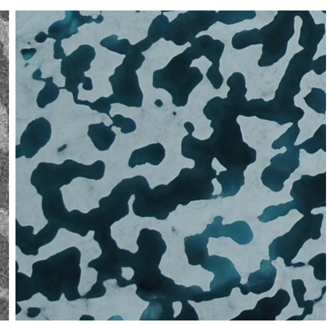
magnetic domains in cobalt



melt ponds (Perovich)



magnetic domains in cobalt-iron-boron



melt ponds (Perovich)

Ising model for ferromagnets ⟶ Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

$$\mathcal{H} = -\sum_{i}^{N} H_i\, s_i - J \sum_{<i,j>}^{N} s_i s_j \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow & -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

random magnetic field represents snow topography

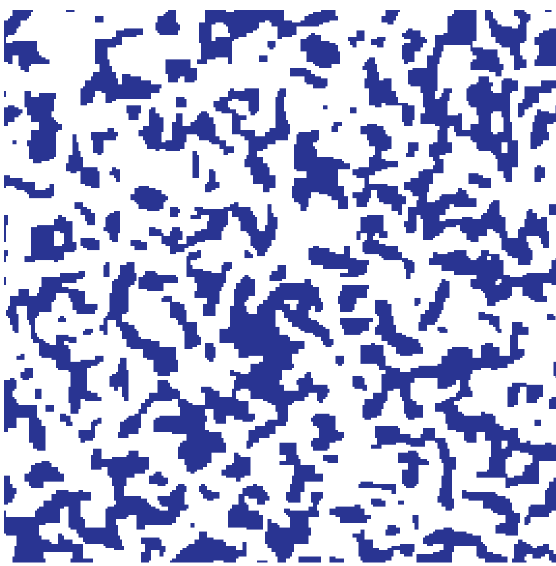
magnetization M

pond area fraction ***~ albedo*** $F = \dfrac{(M+1)}{2}$

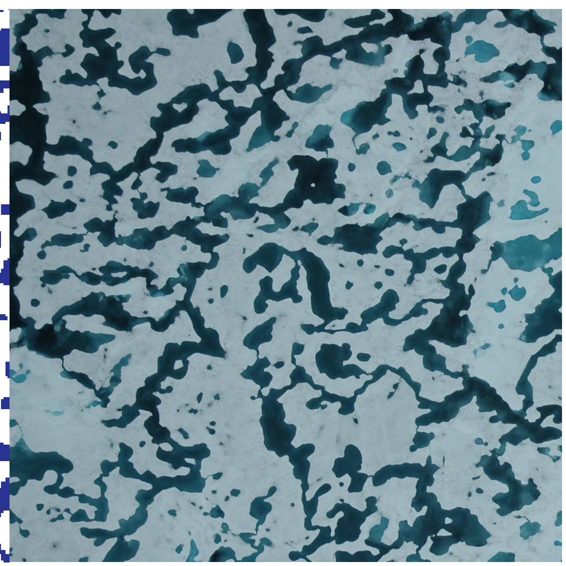
only nearest neighbor patches interact

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system "flows" toward metastable equilibria.

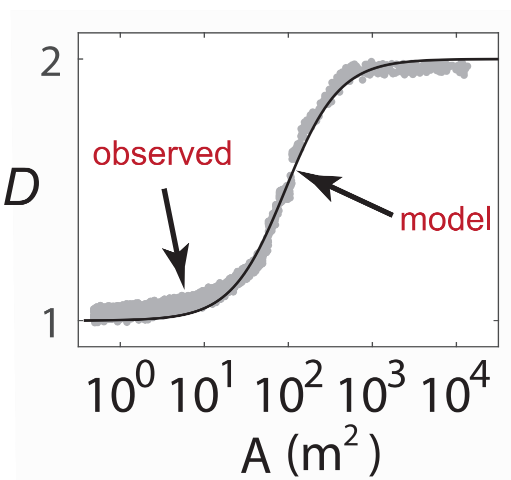
Order from Disorder



Ising model



melt pond photo (Perovich)



pond size distribution exponent

observed -1.5

(Perovich, *et al.* 2002)

model -1.58

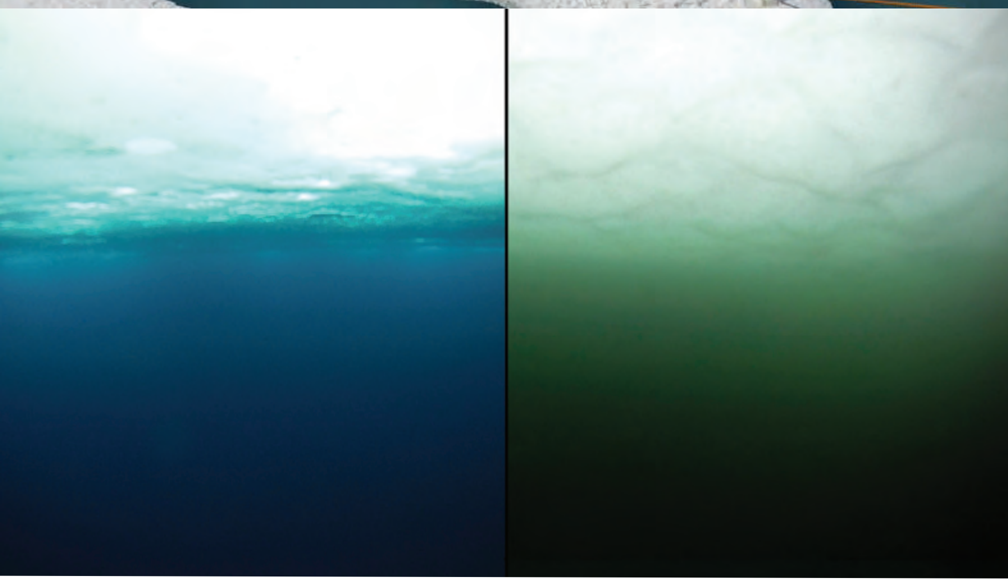
Scientific American
EOS, PhysicsWorld, ...

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data

Perovich

Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

WINDOWS



no bloom bloom

massive under-ice algal bloom

Arrigo et al., *Science* 2012

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

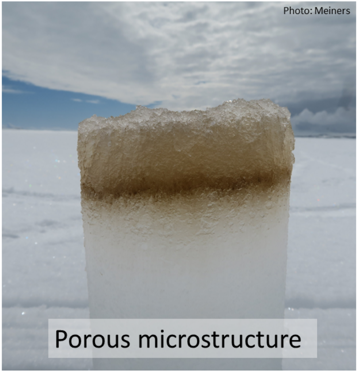
Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, ***Science Advances*** 2017

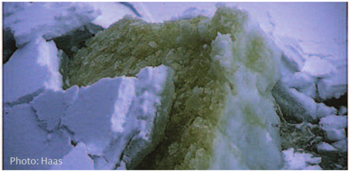
The effect of melt pond geometry on the distribution of solar energy under first year sea ice

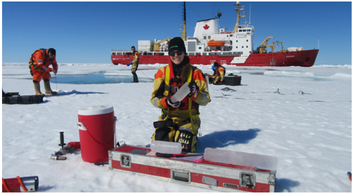
Horvat, Flocco, Rees Jones, Roach, Golden
Geophys. Res. Lett. 2019

(2015 AMS MRC)

Sea ice algae







Can we improve agreement between algae models and data?

80% of polar bear diet can be traced to ice algae*.

---

*Brown TA, et al. (2018). *PloS one*, 13(1), e0191631

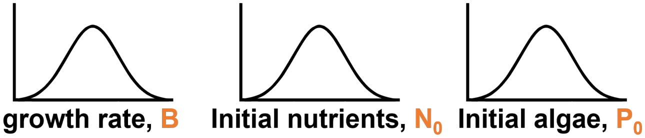
Heterogeneity in initial conditions

At each location within a larger region, we could consider

Nutrients $$\frac{dN}{dt} = \alpha - BNP - \eta N$$

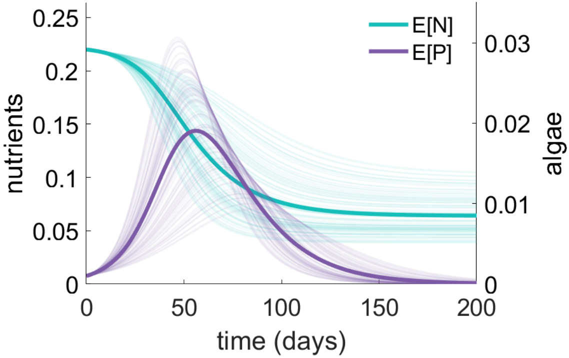
Algae $$\frac{dP}{dt} = \gamma BNP - \delta P$$

$$N(0) = N_0, \qquad P(0) = P_0$$



How do we analyze this model?

Monte Carlo simulations?



Too slow! Full algae model takes **8 hours** (cloud computing).

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DOI: 10.1111/ele.14095

METHOD

ECOLOGY LETTERS WILEY

Uncertainty quantification for ecological models with random parameters

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[2]School of Biological Sciences, University of Utah, Salt Lake City, Utah, USA

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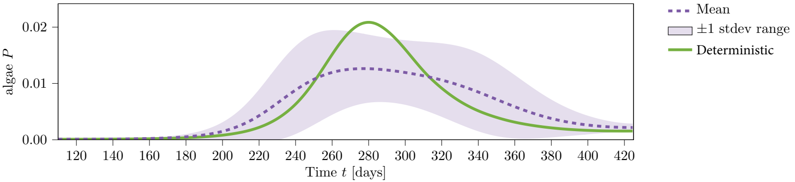
Abstract

There is often considerable uncertainty in parameters in ecological models. This uncertainty can be incorporated into models by treating parameters as random variables with distributions, rather than fixed quantities. Recent advances in uncertainty quantification methods, such as polynomial chaos approaches, allow for the analysis of models with random parameters. We introduce these methods with a motivating case study of sea ice algal blooms in heterogeneous environments. We compare Monte Carlo methods with polynomial chaos techniques to help understand the dynamics of an algal bloom model with random parameters.

Introduce polynomial chaos approach to widely used ecological ODE models, but with random parameters.

Ecological insights



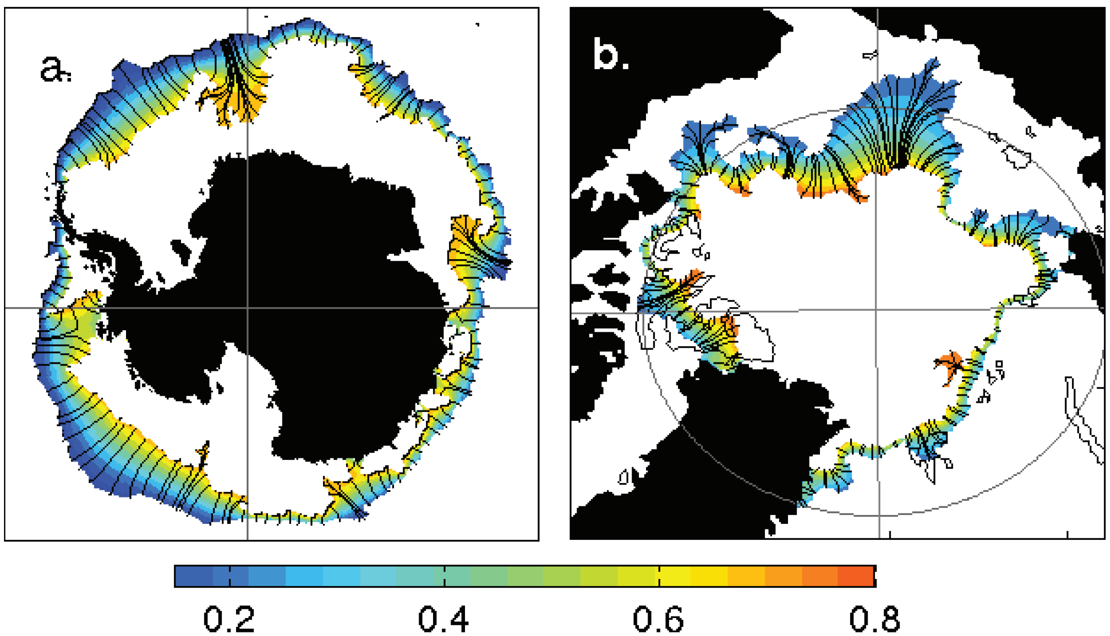
- lower peak bloom intensity
- longer bloom duration
- able to compare variance to data

macroscale

Marginal Ice Zone

MIZ

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



transitional region between
dense interior pack ($c > 80\%$)
sparse outer fringes ($c < 15\%$)

MIZ WIDTH
fundamental length scale of
ecological and climate dynamics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

How to objectively measure the "width" of this complex, non-convex region?

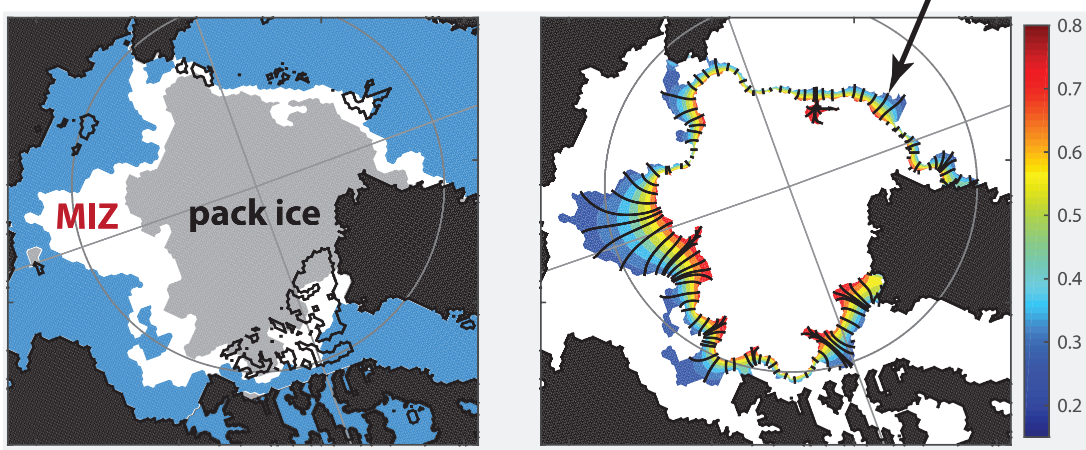
Objective method for measuring MIZ width motivated by medical imaging and diagnostics

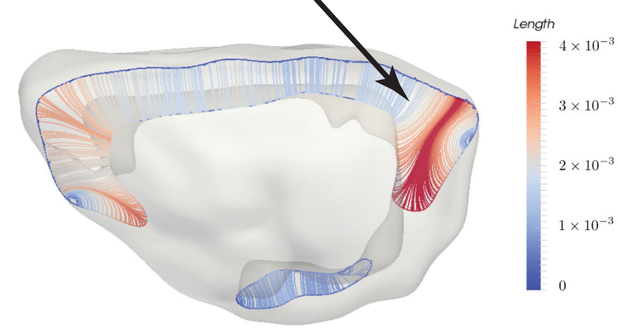
Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

39% widening
1979 - 2012

streamlines of a solution to Laplace's equation

"average" lengths of streamlines





Arctic Marginal Ice Zone

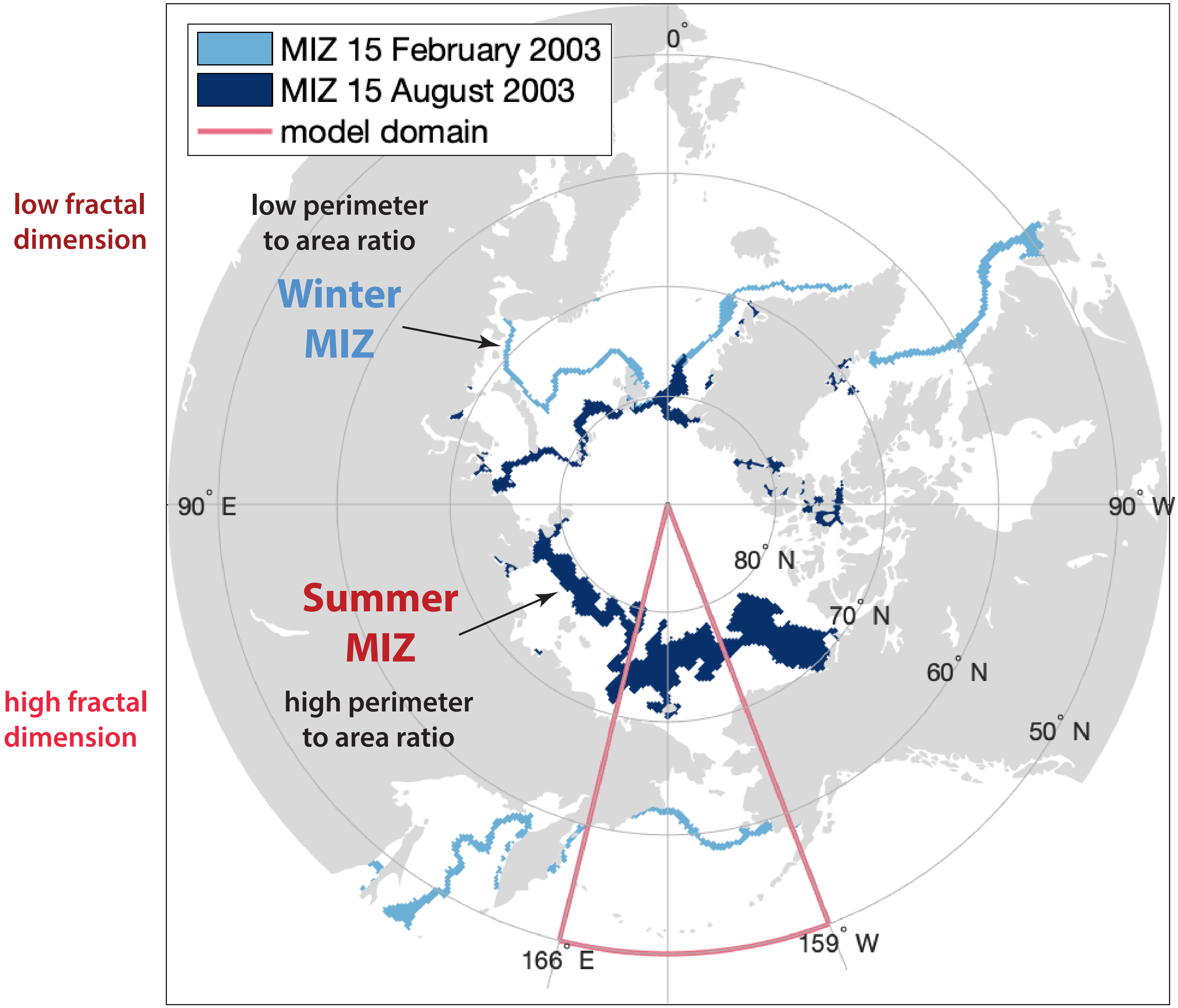
crossection of the cerebral cortex of a rodent brain

analysis of different MIZ WIDTH definitions

Strong, Foster, Cherkaev, Eisenman, Golden
J. Atmos. Oceanic Tech. 2017

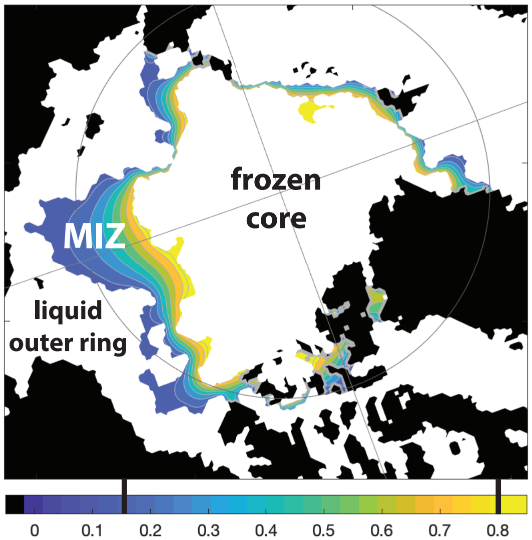
Strong and Golden
Society for Industrial and Applied Mathematics News, April 2017

Observed Arctic MIZ



Model larger scale effective behavior with partial differential equations that *homogenize* complex local structure and dynamics.

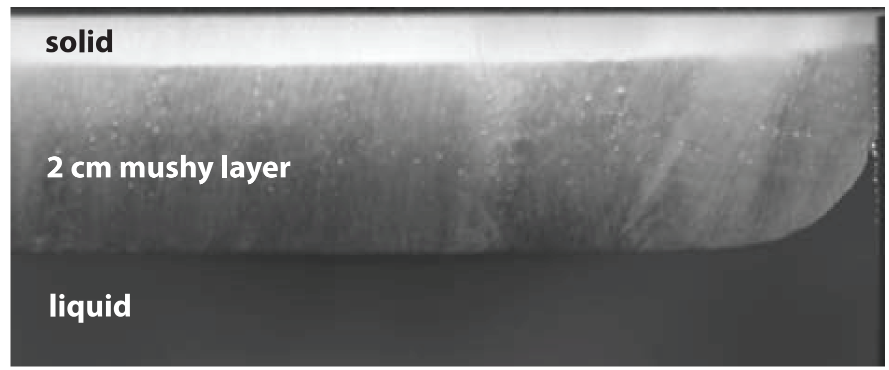
Arctic MIZ



sea ice concentration ψ

Predict MIZ width and location with basin-scale phase change model.

seasonal and long term trends



NaCl-H2O in lab
(Peppin et al., 2007;, J. Fluid Mech.)

Partial differential equation models and deep learning for the sea ice concentration field, 2023

Delaney Mosier, Eric Brown, Court Strong, Jingyi Zhu, Bao Wang, Ken Golden

Annual cycle of Arctic marginal ice zone location and width explained by macroscale mushy layer model, 2023

C. Strong, E. Cherkaev, and K. M. Golden

MIZ as a moving phase transition region

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + S$$

$$S = [\rho(c_l - c_s)T + \rho L] \frac{\partial \psi}{\partial t}$$

$$\psi = 1 - \left(\frac{T - T_s}{T_l - T_s} \right)^{\alpha}$$

$$k_x = \left(\frac{\psi}{k_s} + \frac{1 - \psi}{k_l} \right)^{-1}$$

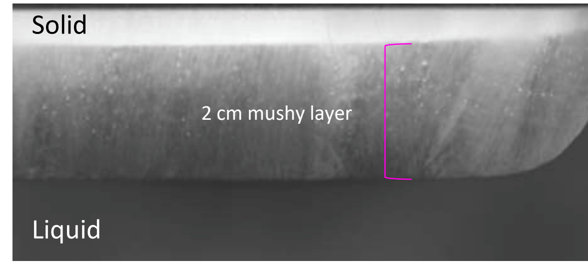
homogenization

$$k_z = \psi k_s + (1 - \psi) k_l$$

ρ effective density
T temperature
c specific heat
L latent heat of fusion

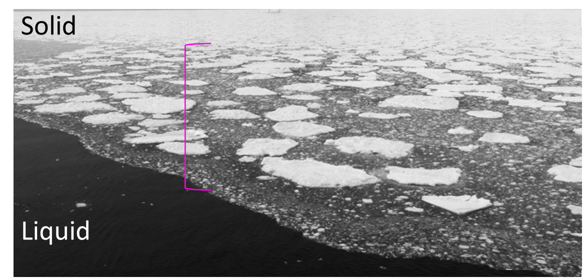
S models nonlinear phase change
ψ sea ice concentration
k effective diffusivity
l liquid, s solid

Classical small-scale application



NaCl-H_2O in lab
(Peppin et al., 2007;, J. Fluid Mech.)

Macroscale application



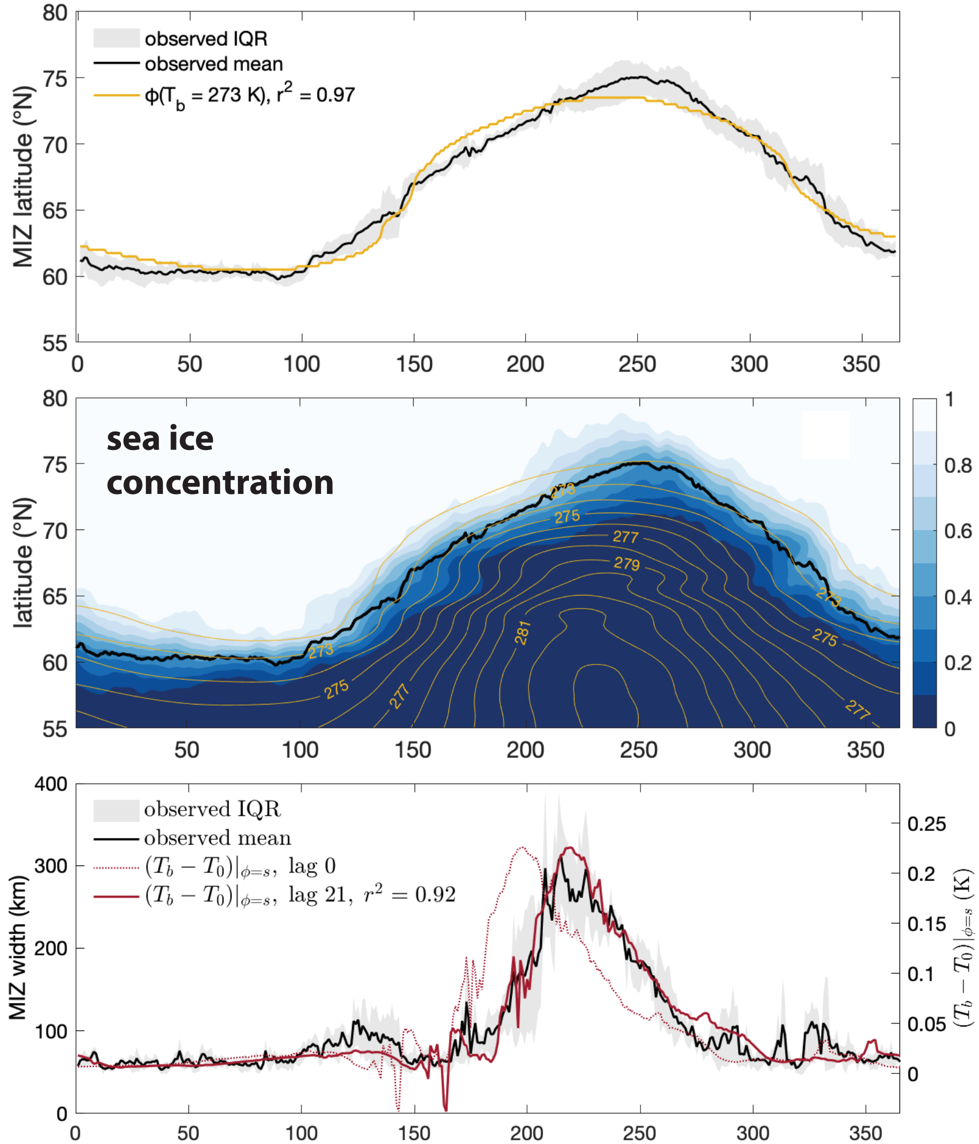
- Develop multiscale PDE model for simulating phase transition fronts to predict MIZ seasonal cycles and decadal trends
- Model simulates MIZ as a large-scale mushy layer with effective thermal conductivity derived from physics of composite materials

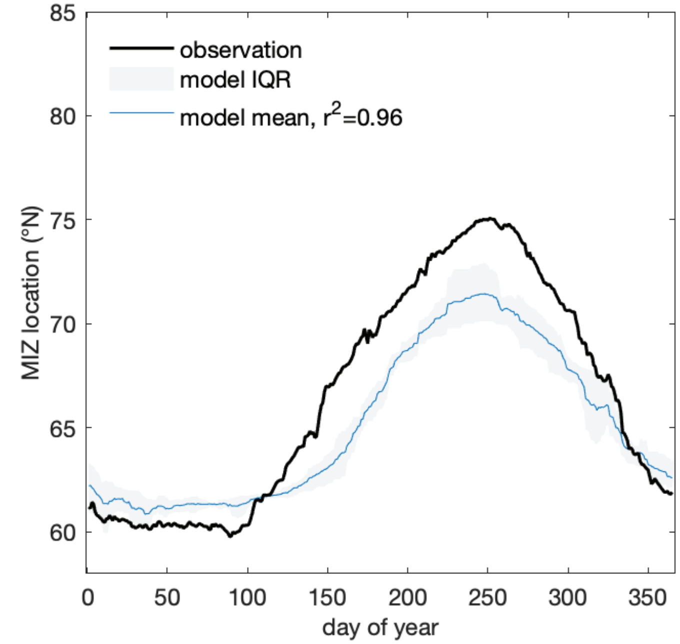
MIZ observations

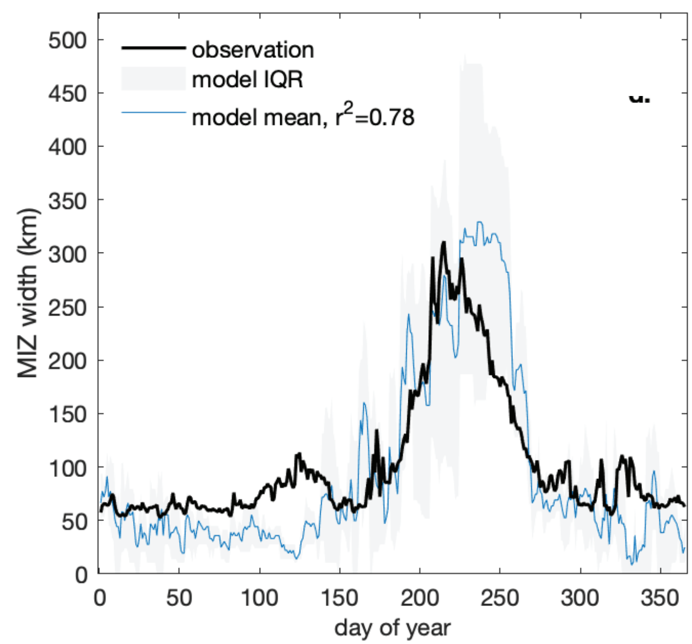
MIZ model vs. observations

location

width





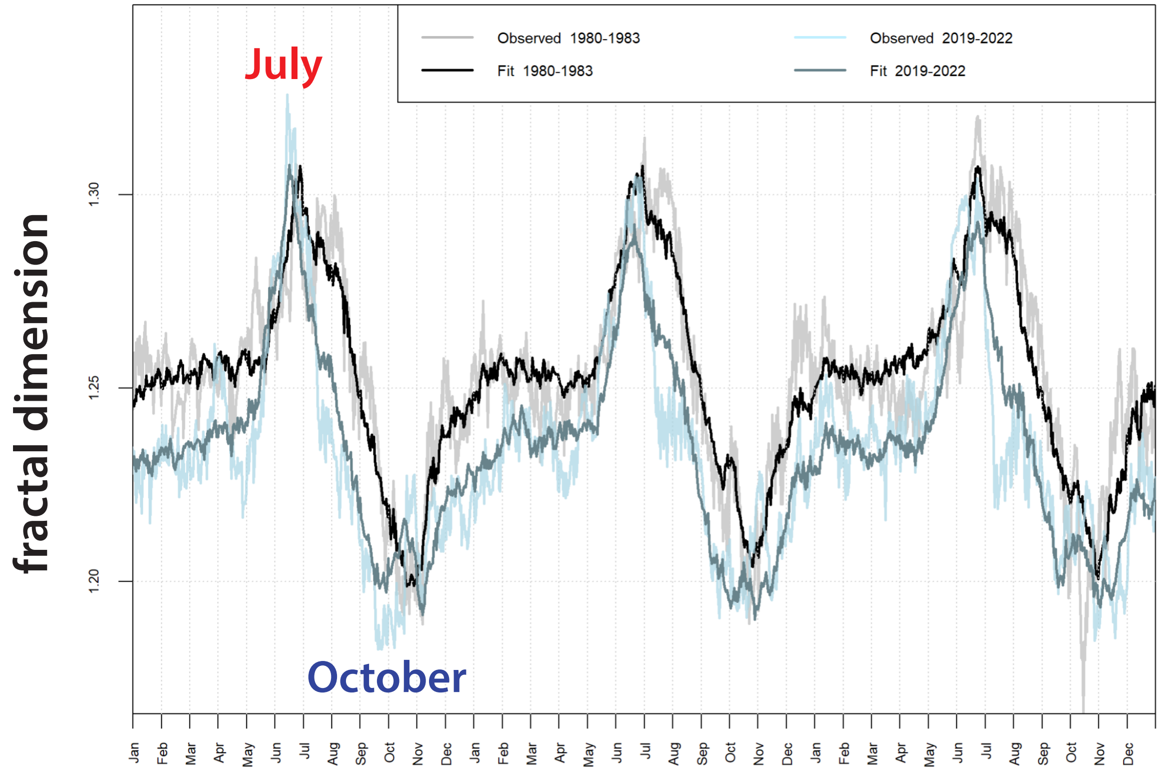


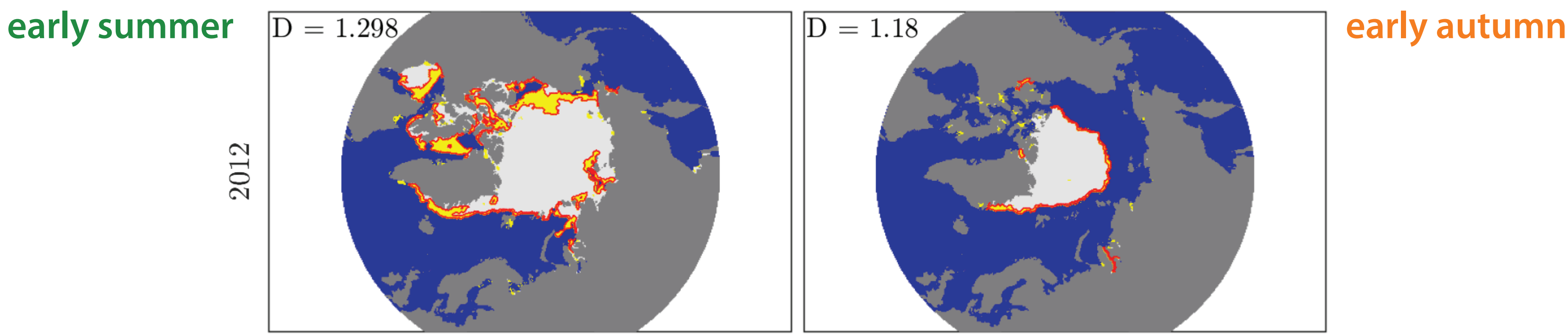
Model captures basic physics of MIZ dynamics.

Evolution of the Fractal Geometry of the Arctic Marginal Ice Zone

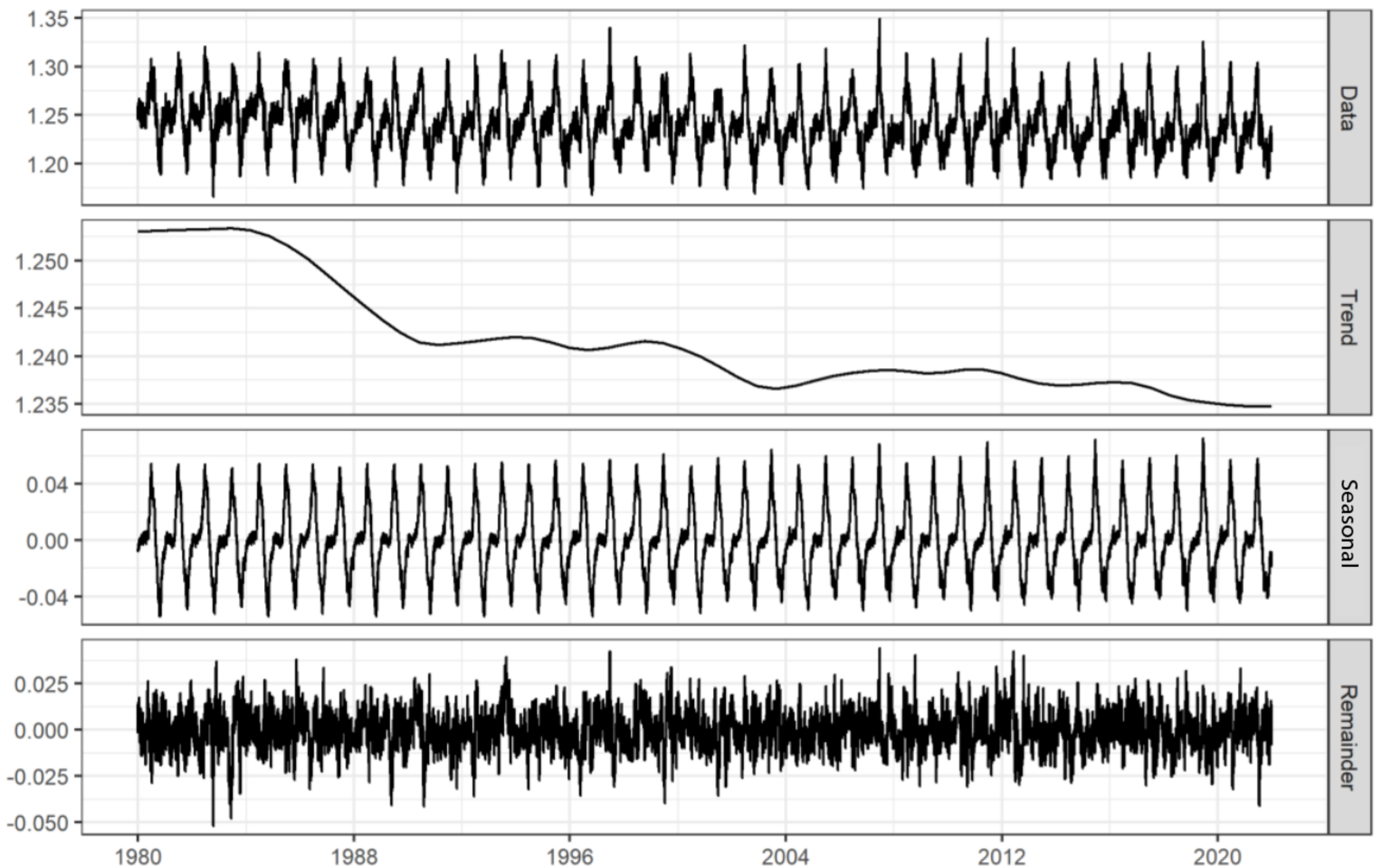
Julie Sherman, Court Strong, Ken Golden, submitted 2023

Compute the fractal dimension of the boundary of the Arctic MIZ by boxcounting methods; analyze seasonal cycle and long term trends.

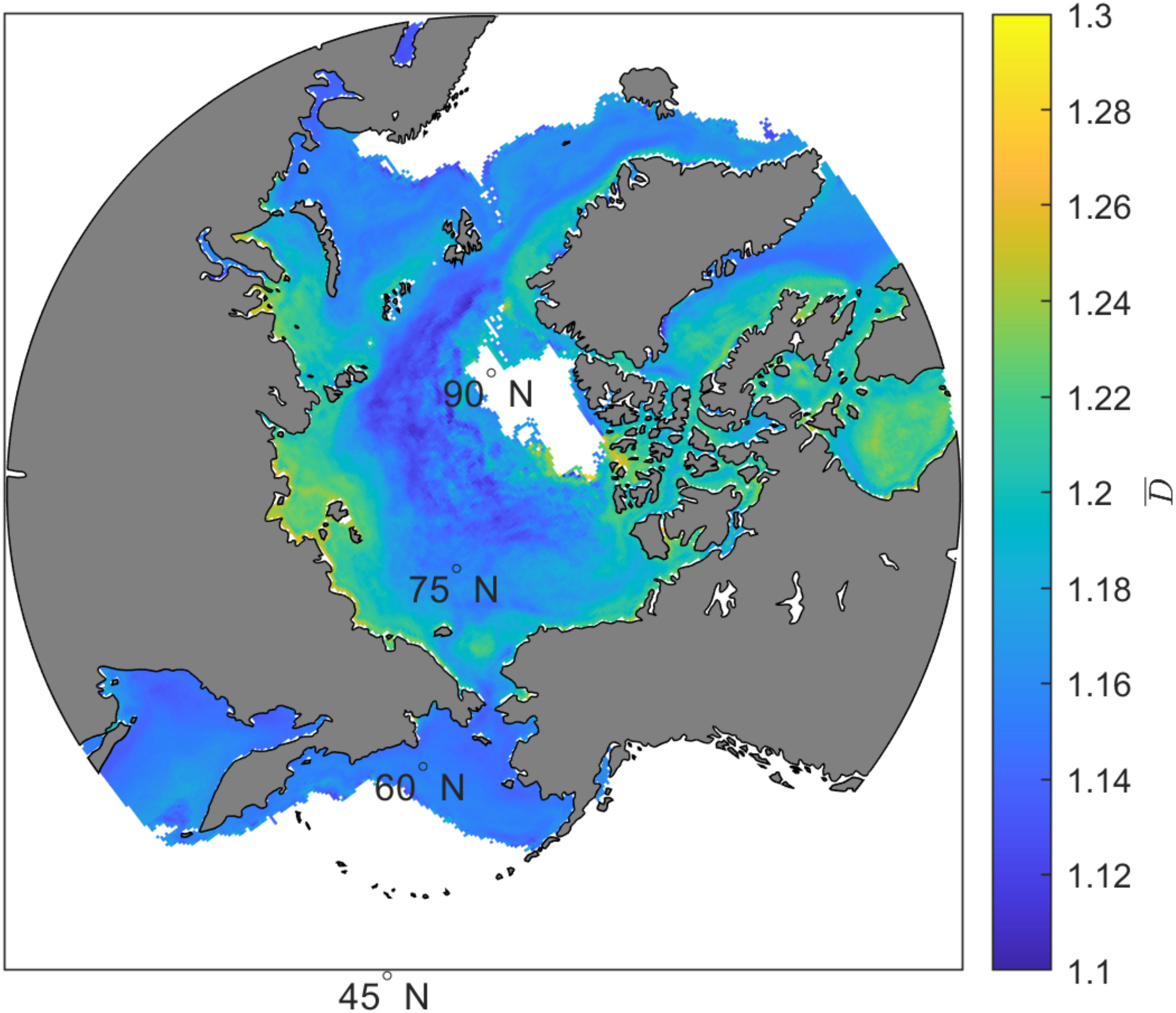




Arctic MIZ fractal dimension from 1980 to 2021



Geographical distribution of average fractal dimension



Conclusions

Fractals appear naturally in the sea ice system.

Mathematics of sea ice advances the theory of composites, inverse problems, and other areas of science and engineering - like fractal geometry of natural structures

Our research is helping to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

Modeling sea ice leads to unexpected areas of math and physics!

University of Utah Sea Ice Modeling Group (2017-2023)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics
Elena Cherkaev, Professor of Mathematics
Court Strong, Associate Professor of Atmospheric Sciences
Ben Murphy, Adjunct Assistant Professor of Mathematics

Postdoctoral Researchers: Noa Kraitzman, Jody Reimer, Bohyun Kim

Graduate Students: Kyle Steffen (now at UT Austin)
Christian Sampson (now at NCAR)
Huy Dinh (MURI sea ice Postdoc at NYU/Courant)
Rebecca Hardenbrook (-> Dartmouth Postdoc)
David Morison (Physics Department)
Ryleigh Moore
Delaney Mosier, Daniel Hallman, Julie Sherman

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Dane Gollero, Samir Suthar, Anna Hyde,
Kitsel Lusted, Ruby Bowers, Kimball Johnston,
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The cover is based on "Modeling Sea Ice,"
page 1535.

THANK YOU

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ace CRC · Antarctic Climate & Ecosystems Cooperative Research Centre

NSF

Antarctica New Zealand

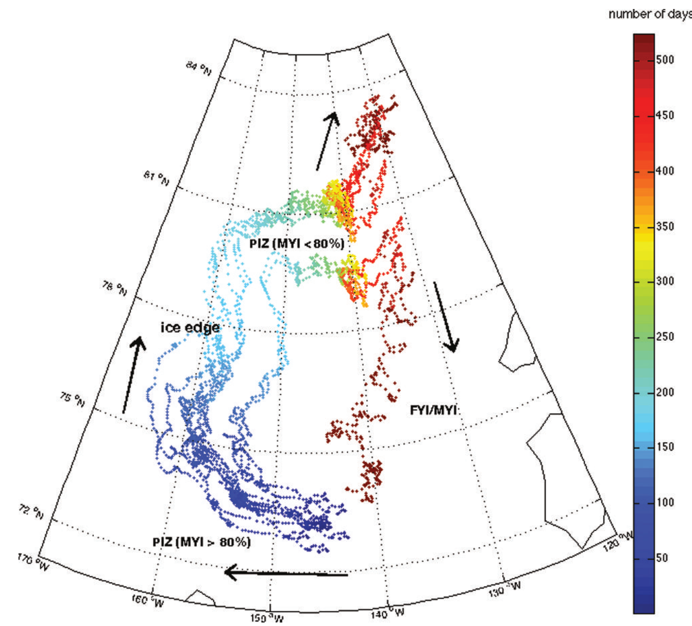
UAF · University of Alaska Fairbanks

The University of Utah

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999

Anomalous diffusion in sea ice dynamics

Ice floe diffusion in winds and currents



observations from GPS data:

Jennifer Lukovich, Jennifer Hutchings, David Barber, *Ann. Glac.* 2015

- On short time scales floes observed (buoy data) to exhibit Brownian-like behavior, but they are also being advected by winds and currents.
- Effective behavior is purely diffusive, sub-diffusive or super-diffusive depending on ice pack and advective conditions - Hurst exponent.

modeling:

Huy Dinh, Ben Murphy, Elena Cherkaev, Court Strong, Ken Golden 2022

floe scale model to analyze transport regimes in terms of ice pack crowding, advective conditions

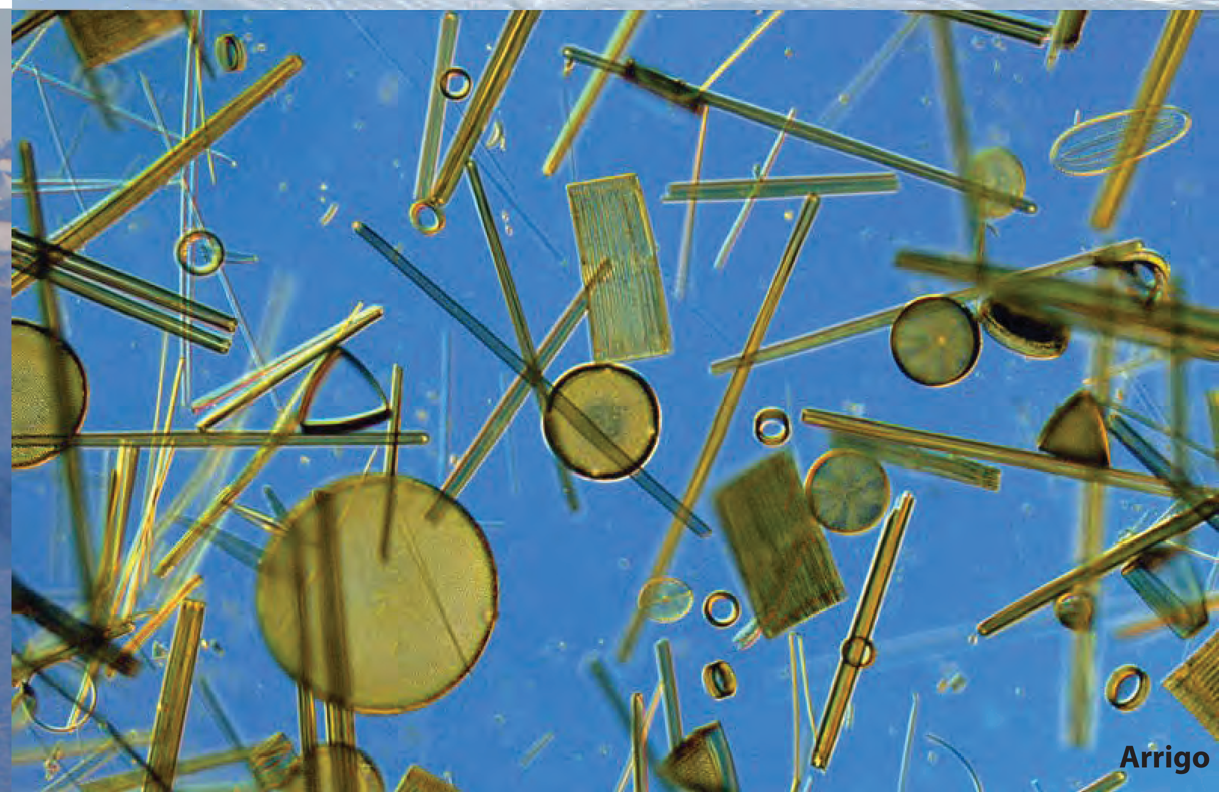
Delaney Mosier, Jennifer Hutchings, Jennifer Lukovich, Marta D'Elia, George Karniadakis, Ken Golden 2022

learning fractional PDE governing diffusion from data

From Microbes to Megafauna: How they impact and are impacted by the physics of sea ice

How do the physical properties of sea ice affect the communities it hosts?

How does the presence of life in and on sea ice affect its physical properties?

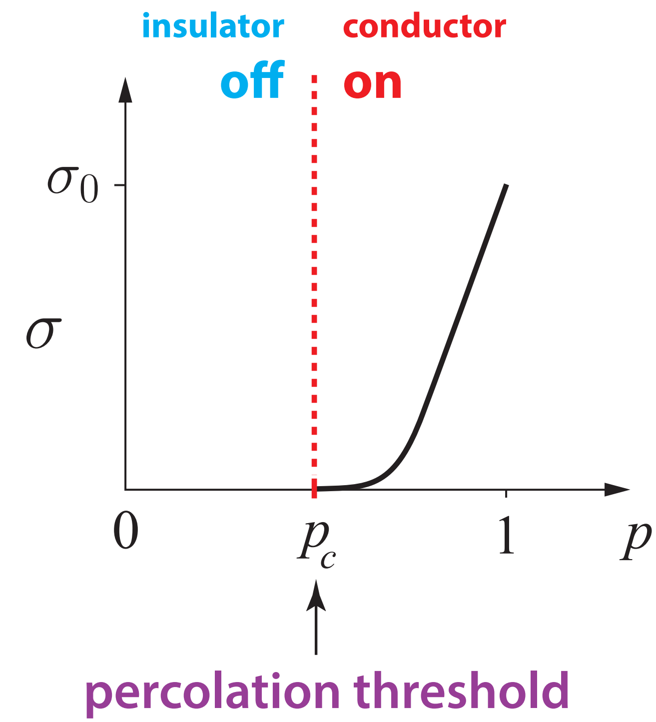


transport in percolation theory

MICRO —— lattice homogenization ——→ **MACRO**

local conductivity (electrical or fluid) **effective** conductivity or fluid permeability

$$\textbf{bond} \longrightarrow \begin{cases} \sigma_0 & \text{probability } p \\ 0 & \text{probability } 1 - p \end{cases}$$



consider local conductivities

1 and *h* > 0

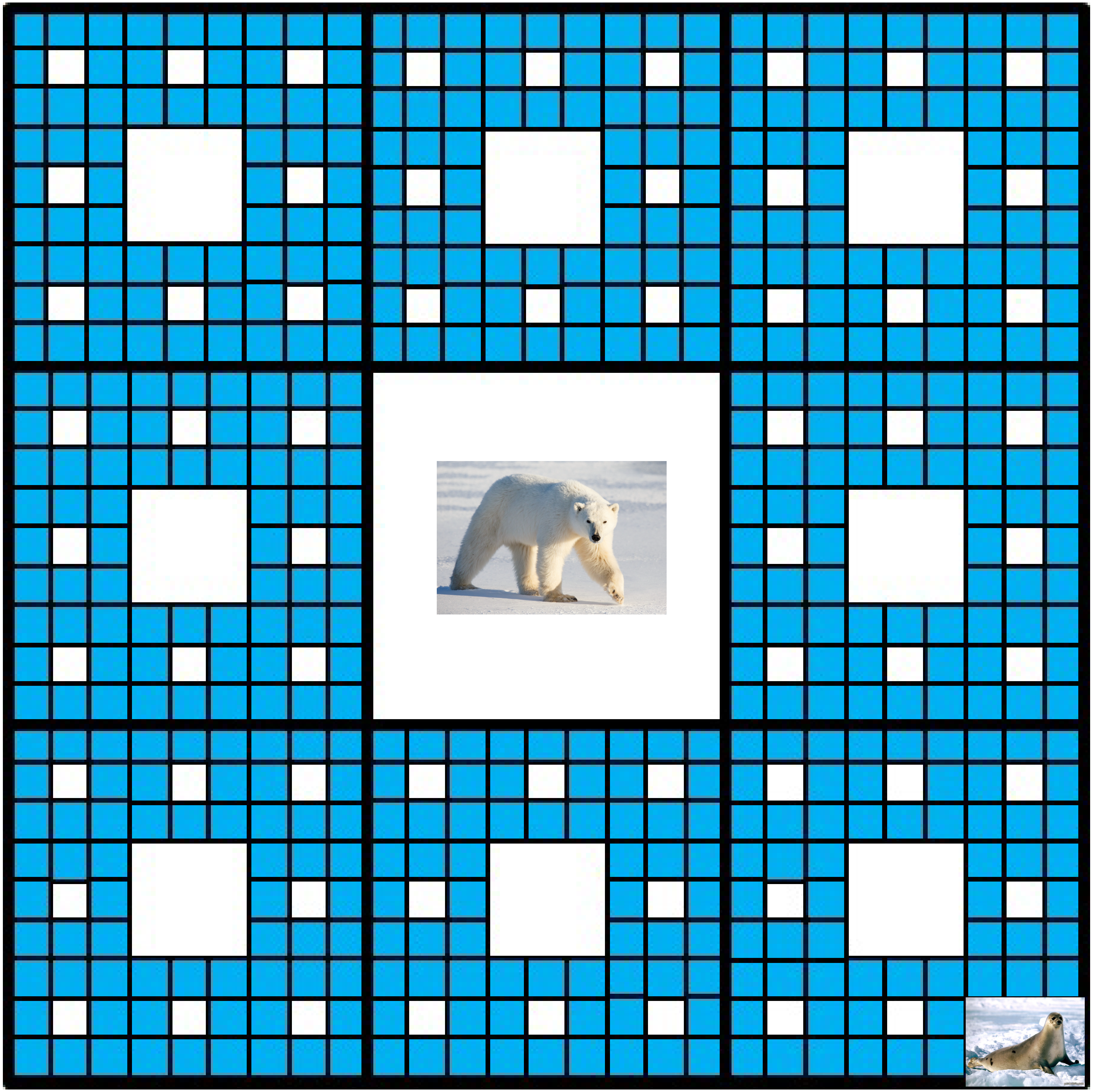
smooths, softens transition

$$\sigma(p) \sim \sigma_0 (p - p_c)^t \quad p \rightarrow p_c^+$$

UNIVERSAL critical exponents for lattices -- depend only on dimension

1 ≤ *t* ≤ 2 (for idealized model), Golden, *Phys. Rev. Lett.* 1990 ; *Comm. Math. Phys.* 1992

non-universal behavior in continuum



polar bear foraging in a fractal icescape

Nicole Forrester
Jody Reimer
Ken Golden

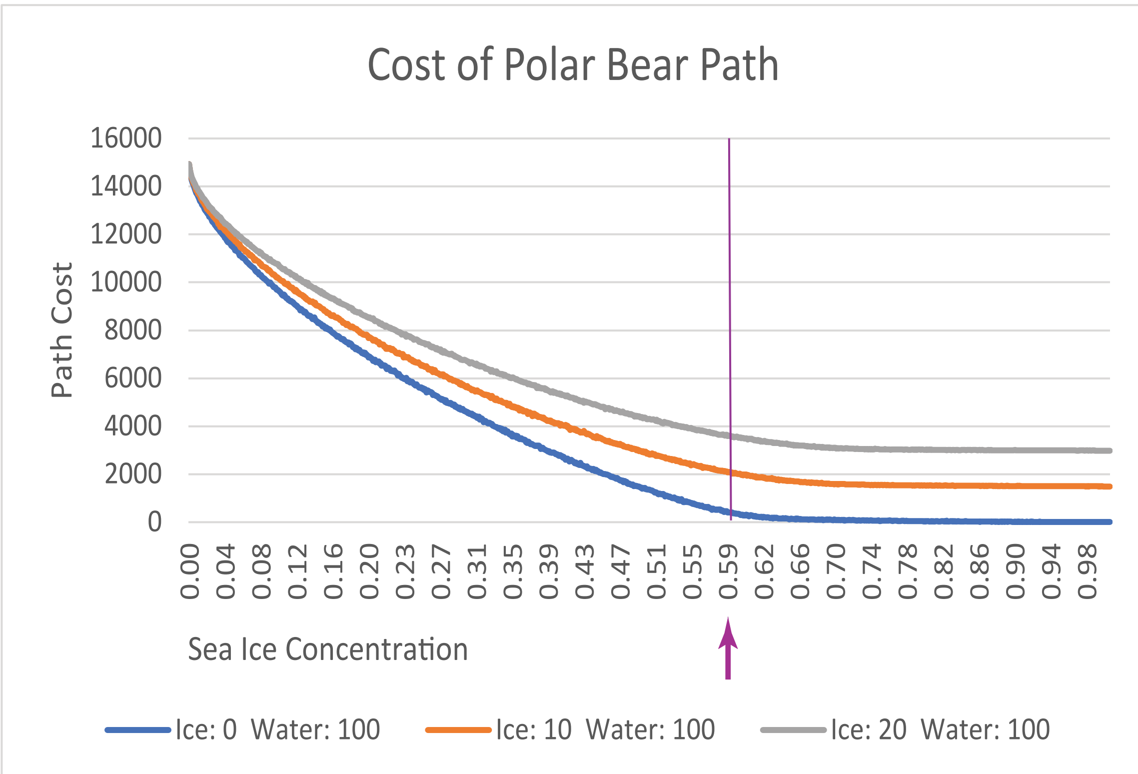
It costs the polar bear 5 times the energy to swim through water than to walk on sea ice.

What pathway to a seal minimizes energy spent?

Polar Bear Percolation

Optimal Movement of a Polar Bear in a Heterogenous Icescape

$C(p)$



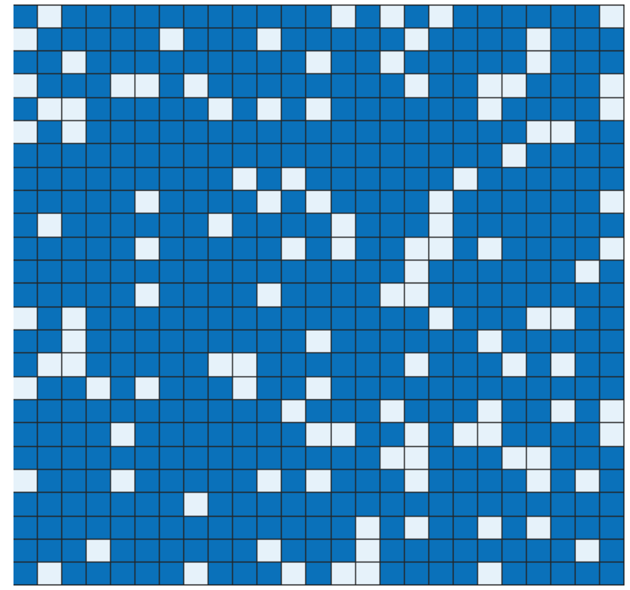
← $h = 0.2$

← $h = 0.1$

← $h = 0$

$$h = \frac{c_i}{c_w}$$

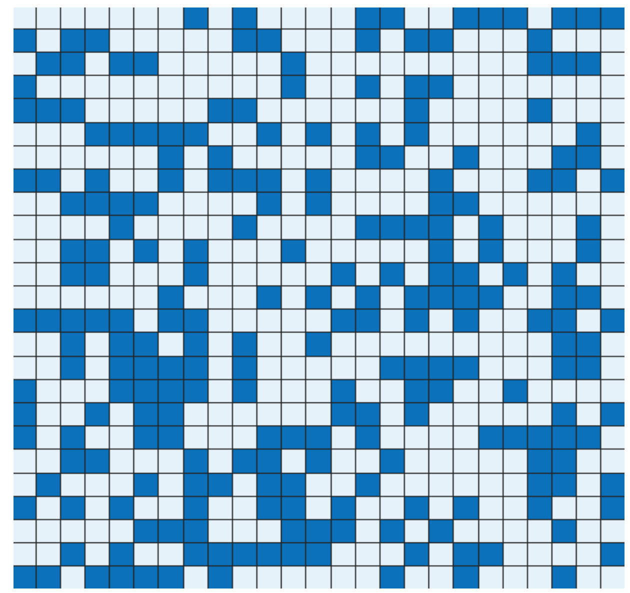
ratio of local "conductivities"



20% Ice

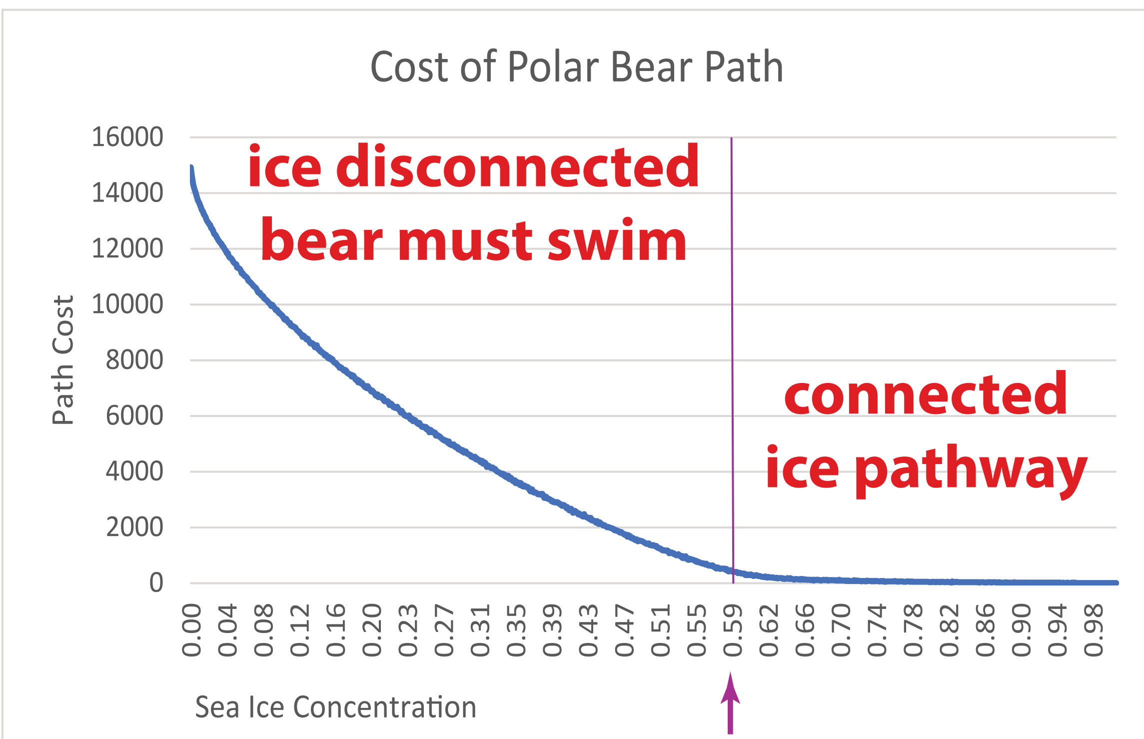
site percolation threshold

$p_c = 0.59$ for $d = 2$

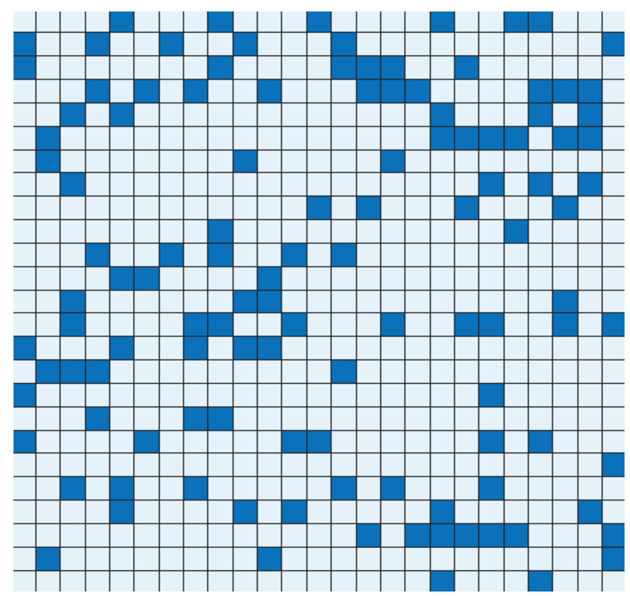


60% Ice

$C(p)$



← $h = 0$



80% Ice

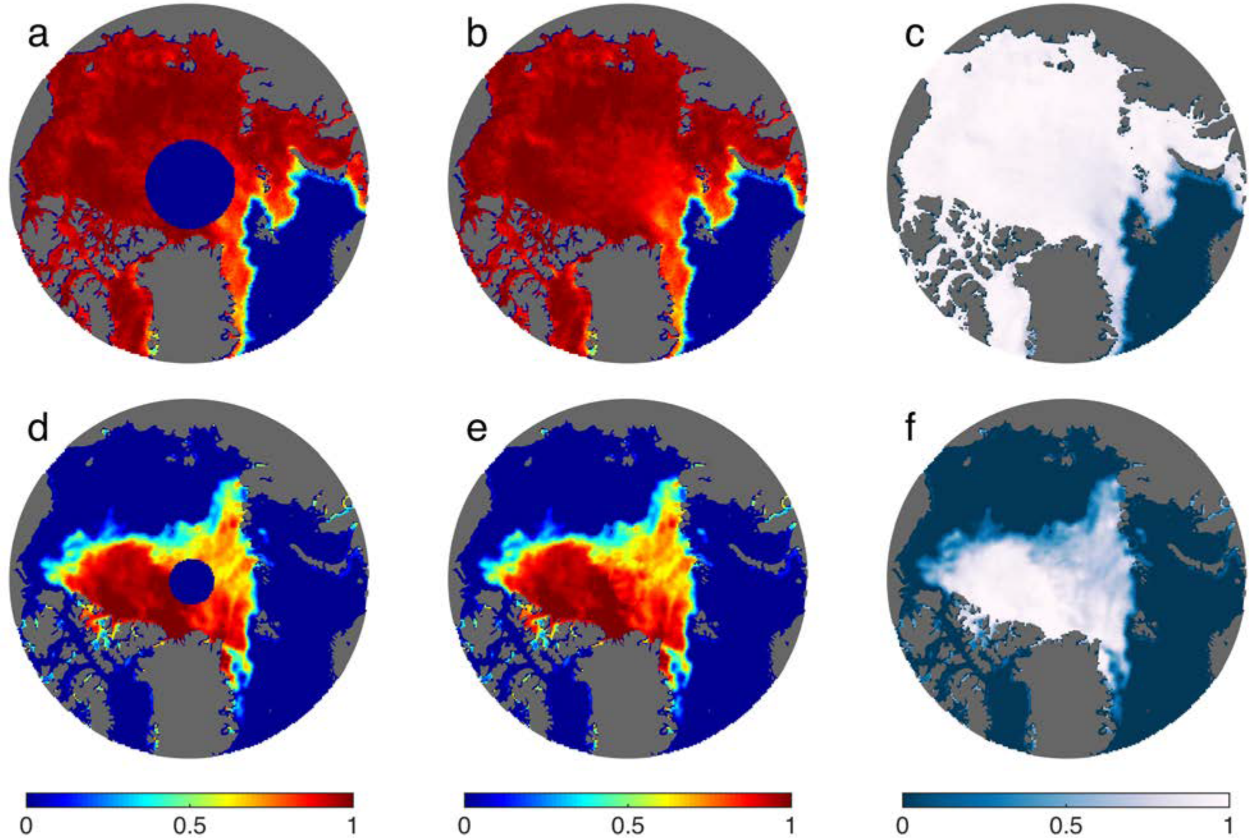
Filling the polar data gap with partial differential equations

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007



$$\Delta\psi=0$$

fill = harmonic function with learned stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

NOAA/NSIDC Sea Ice Concentration CDR product update will use our PDE method.