sea ice components of GCM's

What are the key ingredients -- or ***governing equations*** that need to be solved on grids using powerful computers?

1. Ice thickness distribution $g(x, y, h, t)$ evolution equation

dynamics + **thermodynamics**

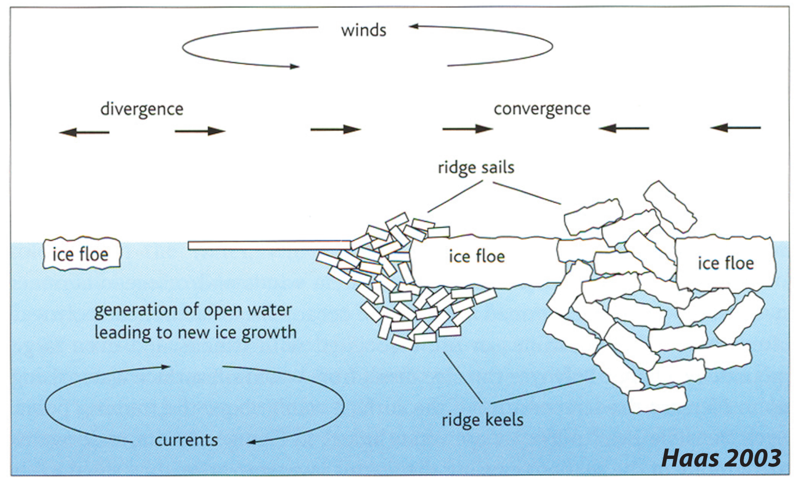
(Thorndike *et al.* 1975)

$$\frac{Dg}{Dt} = -g\nabla \cdot \mathbf{u} + \Psi(g) - \frac{\partial}{\partial h}(\tau g) + \mathcal{L}$$

nonlinear PDE with ice velocity field

mechanical redistribution - ridging and opening

ice growth ice melting



2. Conservation of momentum, stress *vs.* strain relation (Hibler 1979)

$$m\frac{D\mathbf{u}}{Dt} = -mf\mathbf{k}\times\mathbf{u} + \boldsymbol{\tau}_a + \boldsymbol{\tau}_o - mg\nabla H + \mathbf{F}_{int}$$

F = ma **for sea ice** ***dynamics***

3. Heat equation of sea ice and snow

(Maykut and Untersteiner 1971)

$$\frac{\partial T}{\partial t} + \mathbf{u}_{br}\cdot\nabla T = \nabla \cdot k(T)\,\nabla T$$

thermodynamics

+ balance of radiative and thermal fluxes on interfaces

transform ice thickness distribution equation to Fokker-Planck type equation; Boltzmann framework

Toppaladoddi and Wettlaufer, *PRL*, 2015

thickness h is a diffusion process with probability density g(h,t)

"microscopic" mechanical processes that influence ice thickness distribution— rafting, ridging, and open water formation occur over very rapid time scales relative to geophysical-scale changes of *g(h)*

$$\Psi = \int_0^\infty [g(h+h')w(h+h',h') - g(h)w(h,h')]dh'$$

w = ***transition probability***

moments k_1 , k_2

Fokker-Planck

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial h}\left[\left(\frac{\epsilon}{h} - k_1\right) g\right] + \frac{\partial^2}{\partial h^2}(k_2 g)$$

Langevin

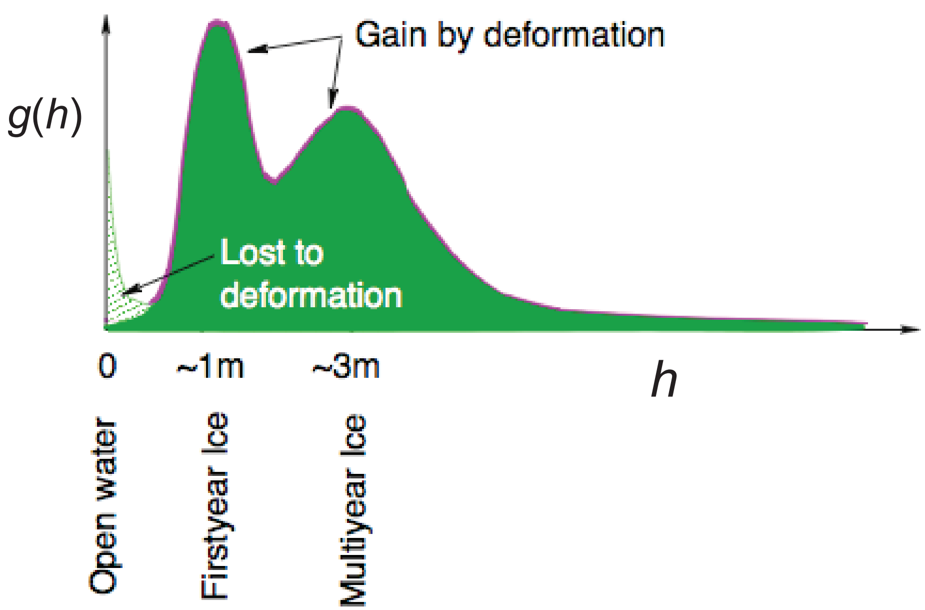
$$\frac{dh}{dt} = \left(\frac{\epsilon}{h} - k_1\right) + \sqrt{2k_2}\,\xi(t)$$

$\xi(t)$ = ***Gaussian white noise***

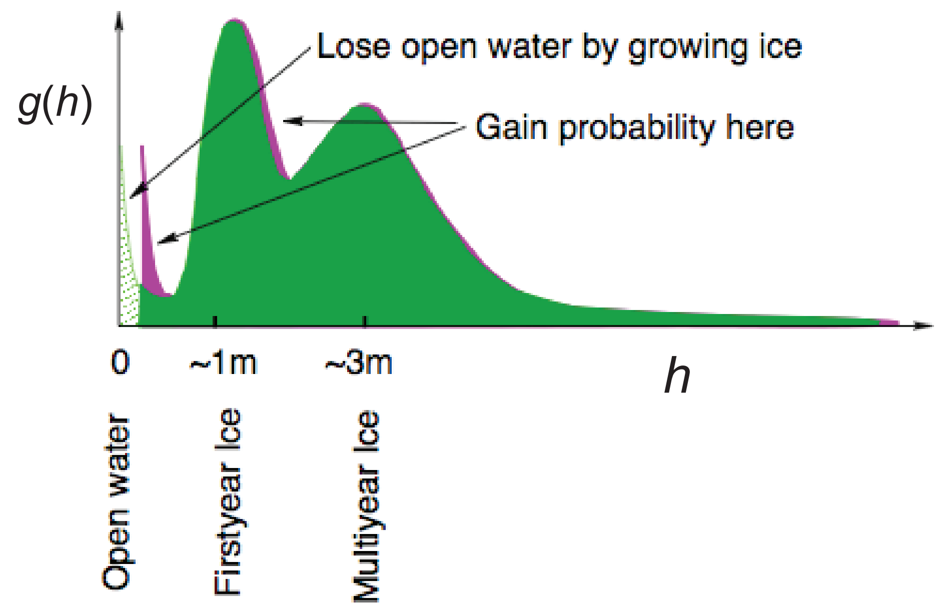
ice thickness distribution function $g(x,y,h,t)$

h = ice thickness

mechanical redistribution



advection in thickness space from growth



Cecilia Bitz

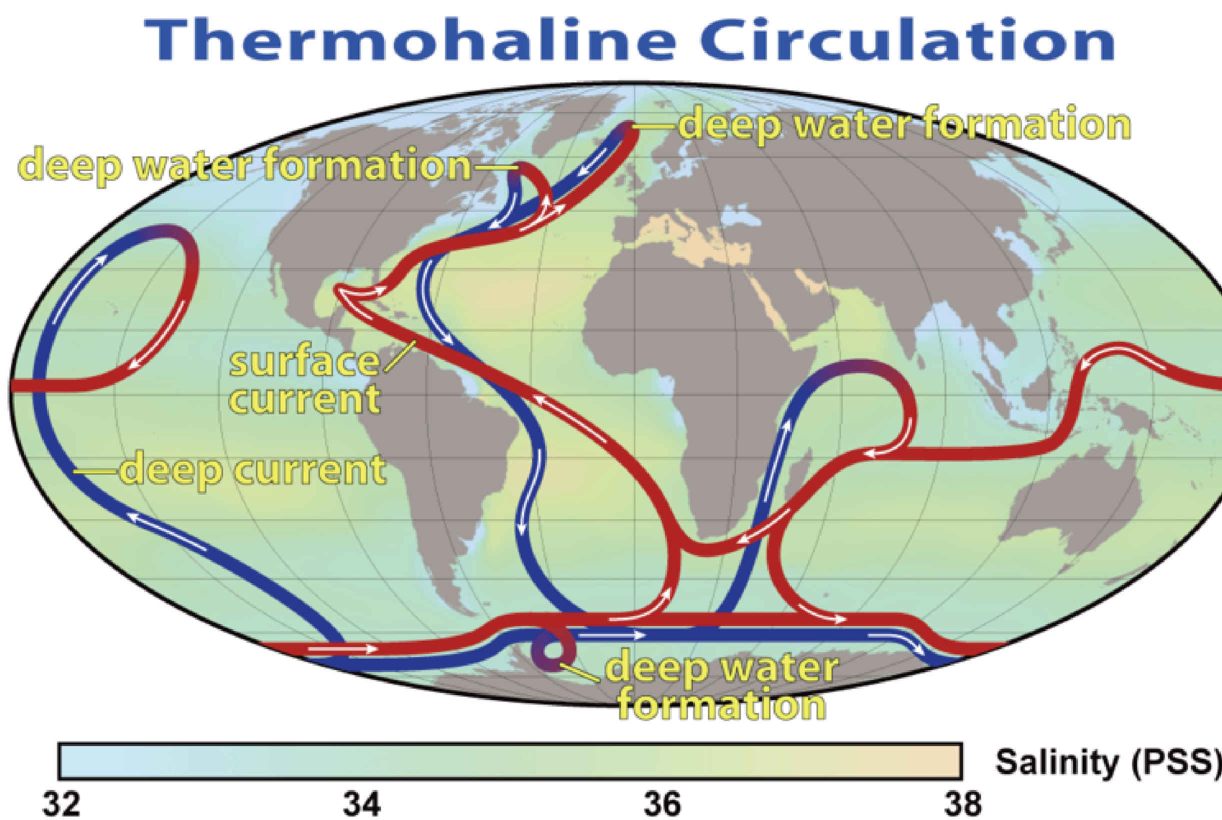
sea ice and global ocean circulation

brine expulsion from freezing sea water results in sinking of dense, saltier water to great depths

melting sea ice inputs fresher water into the upper ocean

↓

deep-water formation drives circulation in the world's oceans



GLOBAL THERMOHALINE CONVEYOR BELT

Bifurcation Diagram

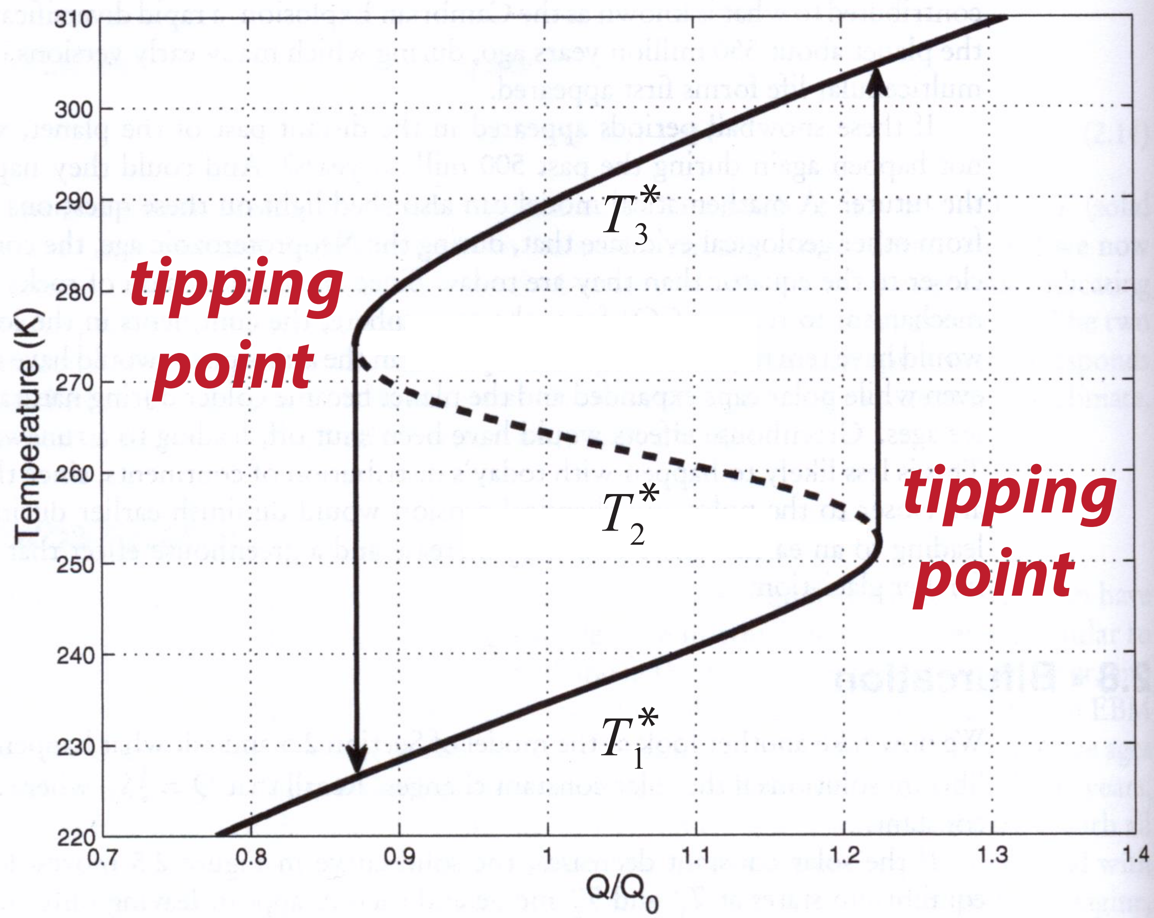


Figure 2.6. *Mean surface temperatures at equilibrium as a function of the solar constant (in units of its present value).*

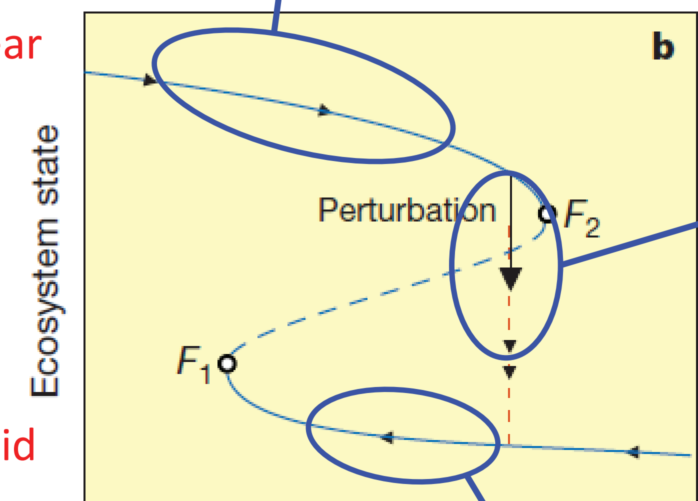
tipping points in ecological dynamics

lake eutrophication

water clarity little affected by increasing nutrient load

clear

turbid



R. Law

nutrients + fish can push lake over basin boundary to turbidity

nutrient load

reducing nutrient load not enough to restore clear water

Scheffer et al., *Nature* 2001

Lake Paul and Pete

Carpenter et al., *Science* 2011

desertification from grazing

(box models in physical oceanography)

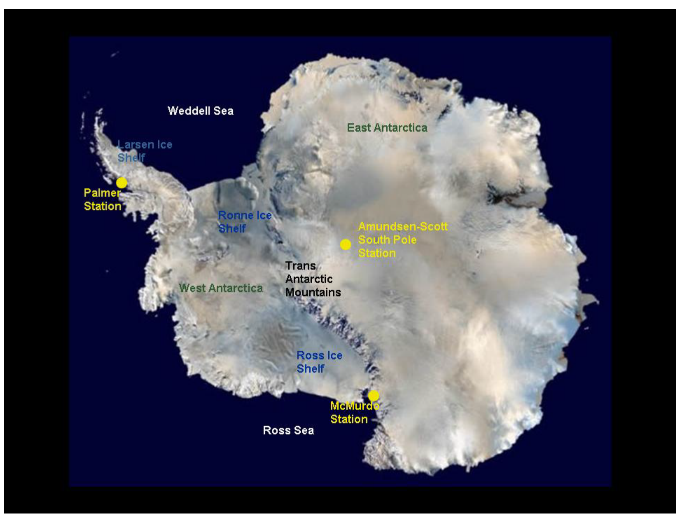
under ice algal blooms ??

melting sea ice *vs.* glacial ice (ice sheets, shelves, bergs)

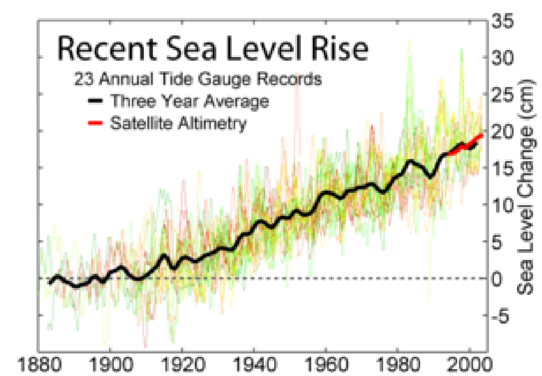
glacial ice
(iceberg)

sea ice

melting ice sheets covering Antarctica and Greenland → sea level rises



Glaciers store about 75% of the world's fresh water. If all land ice melted the seas would rise about 70 meters (~ 230 feet).

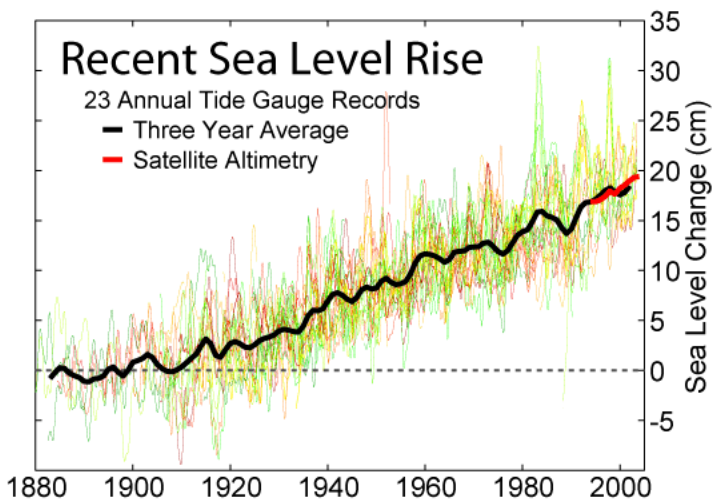


Does melting sea ice contribute to sea level rise? - not directly

sea ice and icebergs are in isostatic balance with the ocean
when they melt, sea level doesn't change

... but indirect effects and feedbacks can influence sea level rise

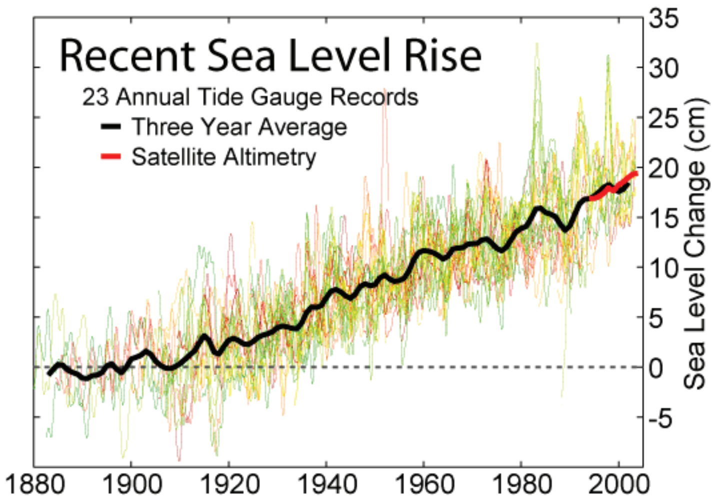
The interaction of warm waters with the periphery of the large ice sheets represents one of the most significant possibilities for abrupt change in the climate sysytem.

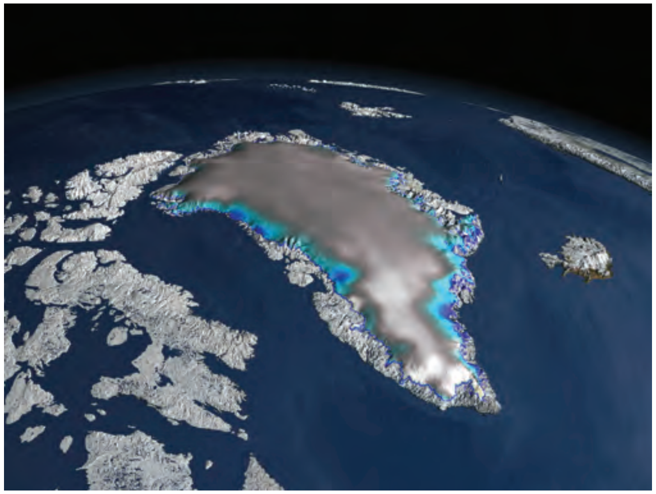


As Earth's climate warms, why does sea level rise?

- **melting land ice sheets**
- **thermal expansion of ocean**

As Earth's climate warms, why does sea level rise?





- ***melting land ice: Antarctica, Greenland, mountain glaciers***
- ***thermal expansion of warming ocean***

  accounts for about 25% of rise in last half of 20th century, rate ~ tripled in 21st century
- ***continental rebound***

  rise of land masses that were depressed by the huge weight of ice sheets

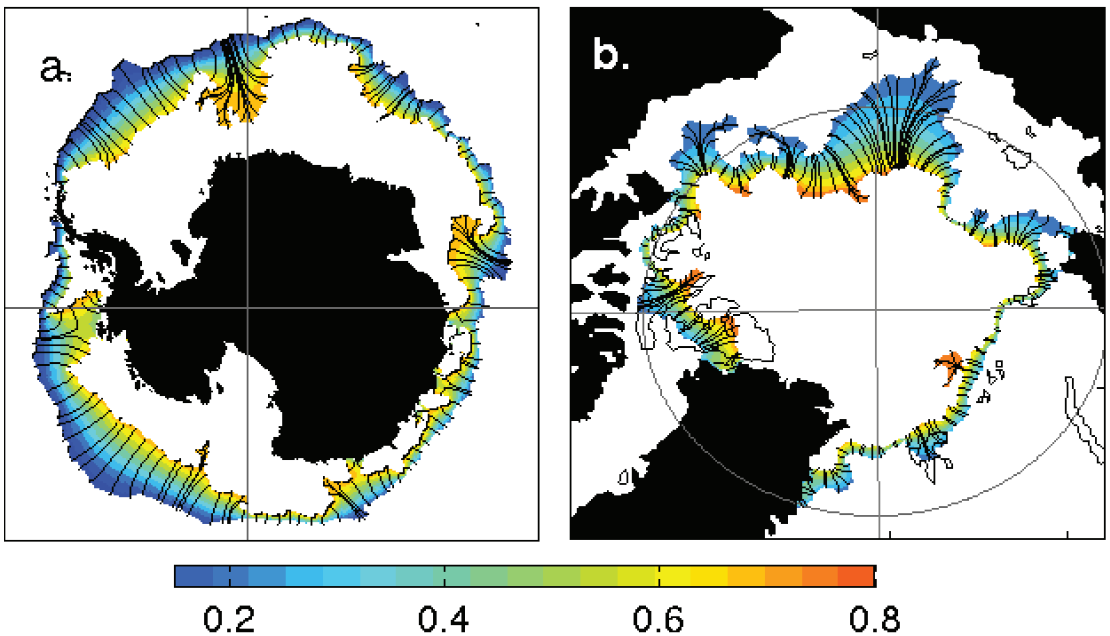
The interaction of warm waters with the periphery of the large ice sheets represents one of the most significant possibilities for abrupt change in the climate sysytem.

no coupling yet of ice sheets and ocean in climate models - no feedback effects

Marginal Ice Zone

MIZ

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



transitional region between
dense interior pack ($c > 80\%$)
sparse outer fringes ($c < 15\%$)

MIZ WIDTH
fundamental length scale of
ecological and climate dynamics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

How to objectively measure the "width" of this complex, non-convex region?

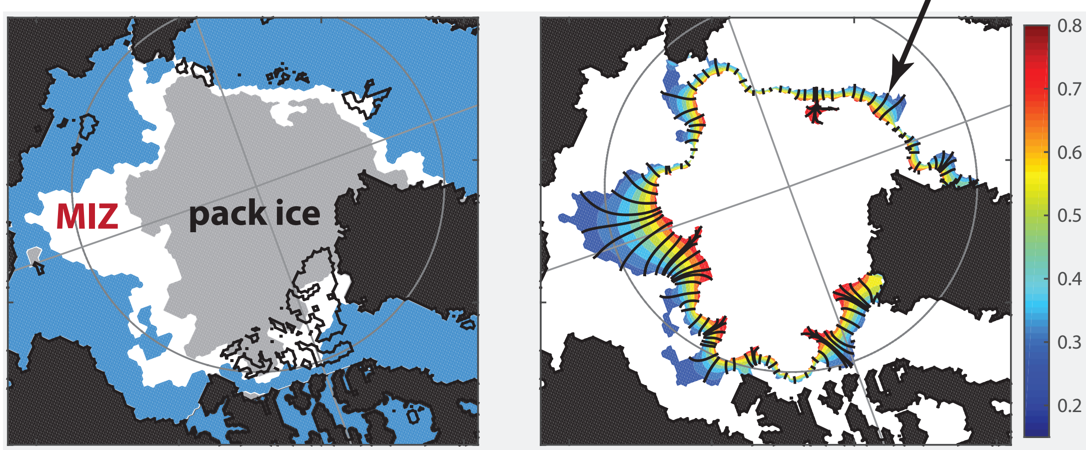
Objective method for measuring MIZ width motivated by medical imaging and diagnostics

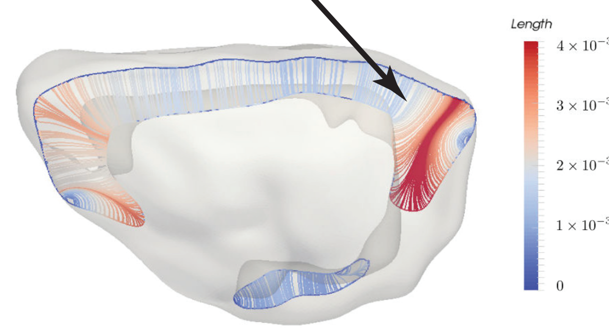
Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

39% widening
1979 - 2012

streamlines of a solution to Laplace's equation

"average" lengths of streamlines





Arctic Marginal Ice Zone

crossection of the cerebral cortex of a rodent brain

analysis of different MIZ WIDTH definitions

Strong, Foster, Cherkaev, Eisenman, Golden
J. Atmos. Oceanic Tech. 2017

Strong and Golden
Society for Industrial and Applied Mathematics News, April 2017

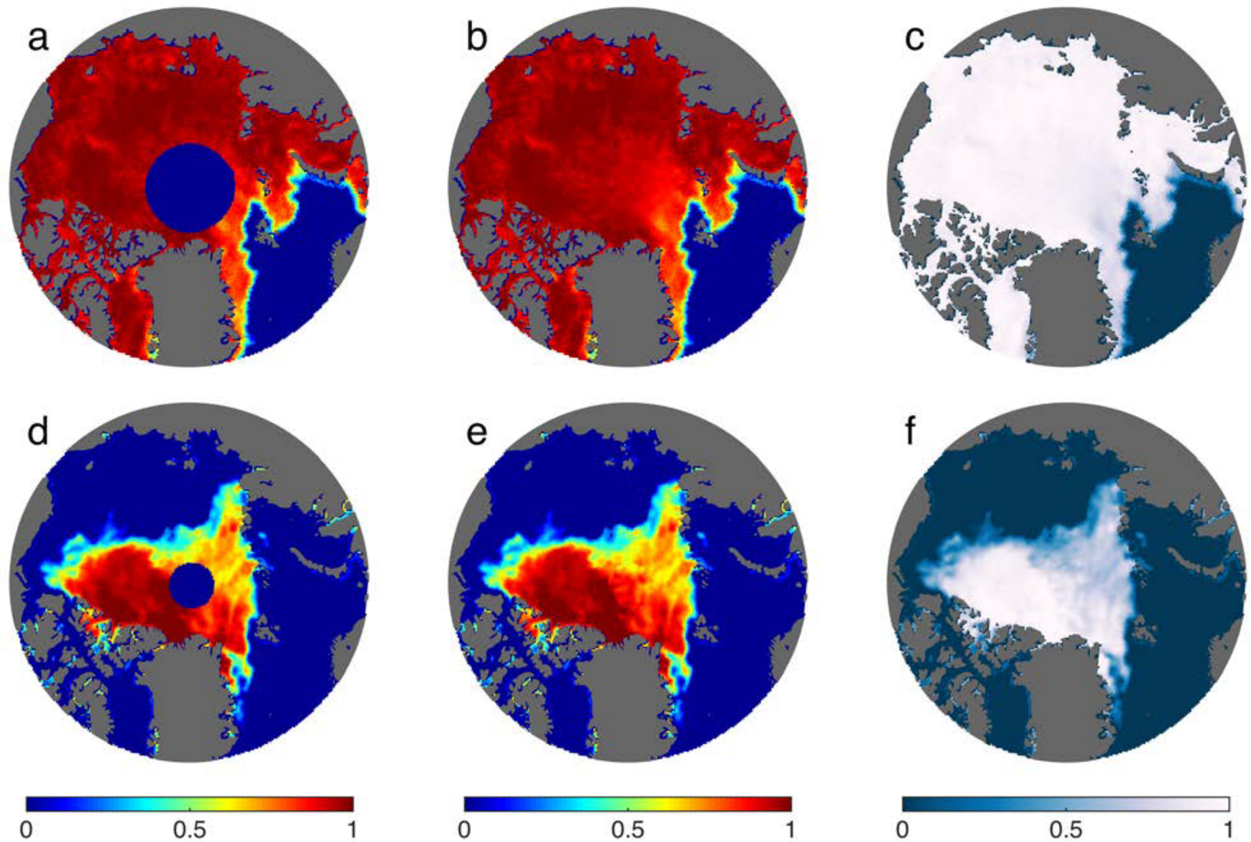
Filling the polar data gap

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985

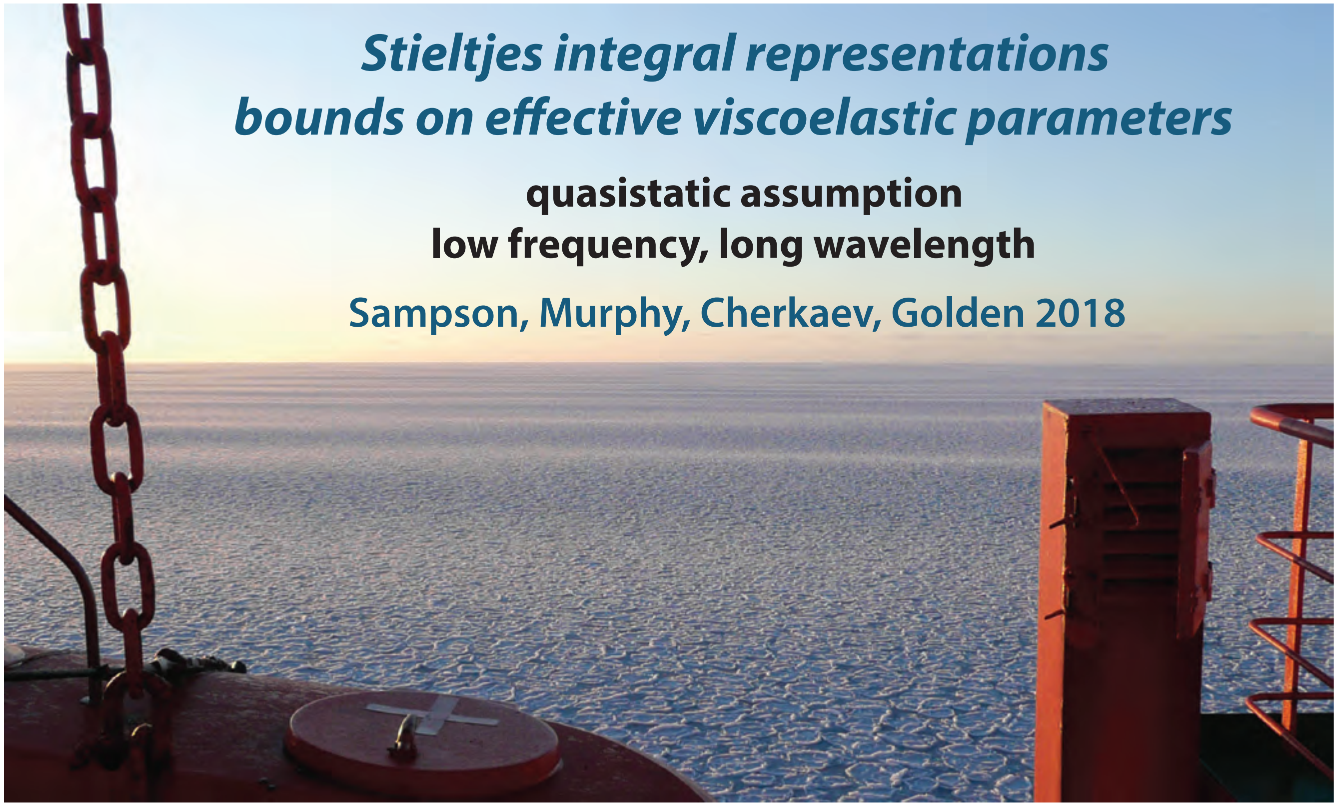
Gap radius: 311 km
30 August 2007

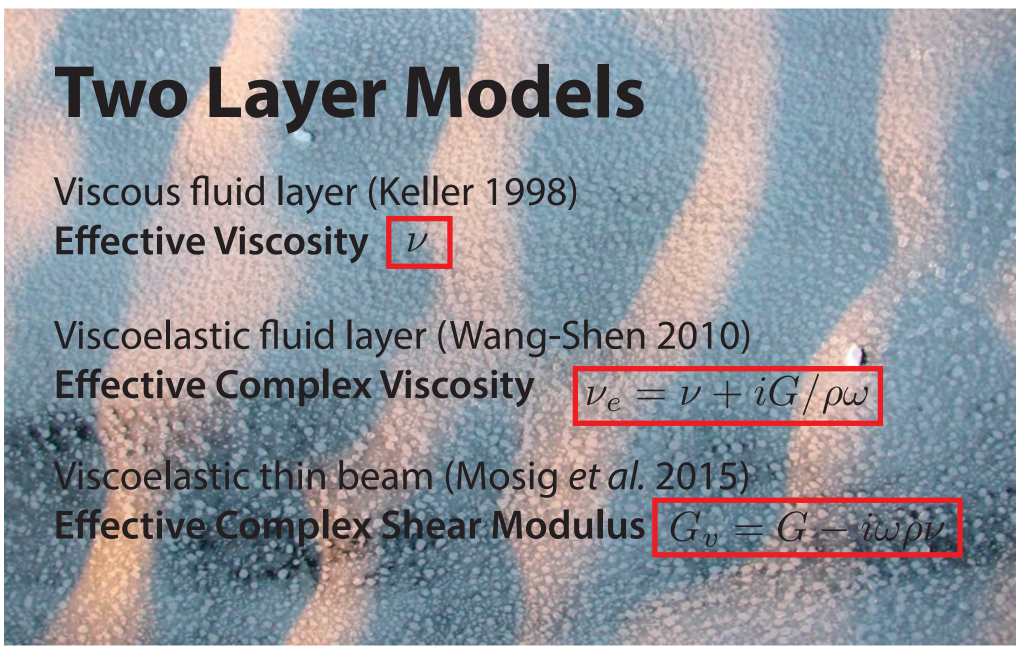


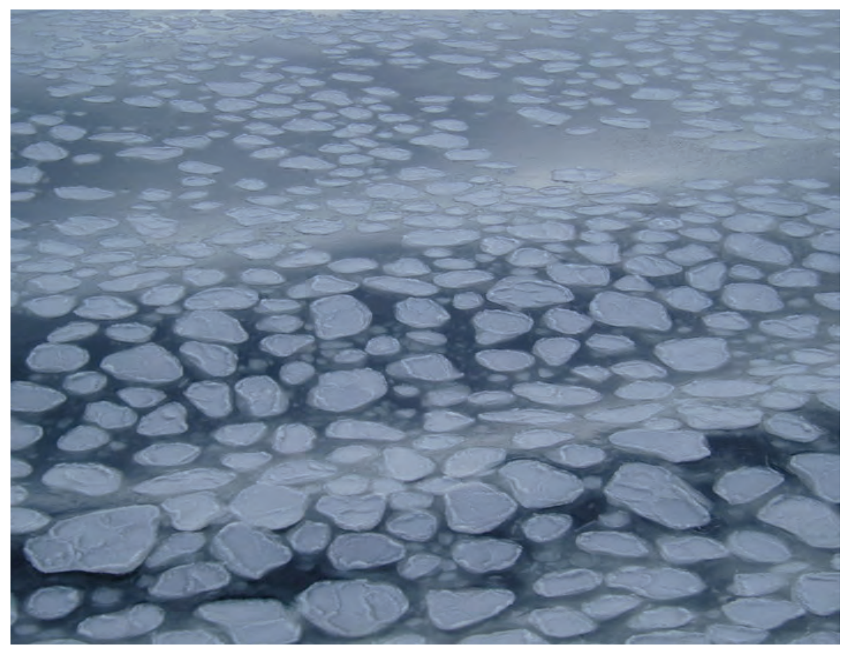
fill with harmonic function satisfying satellite BC's plus stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

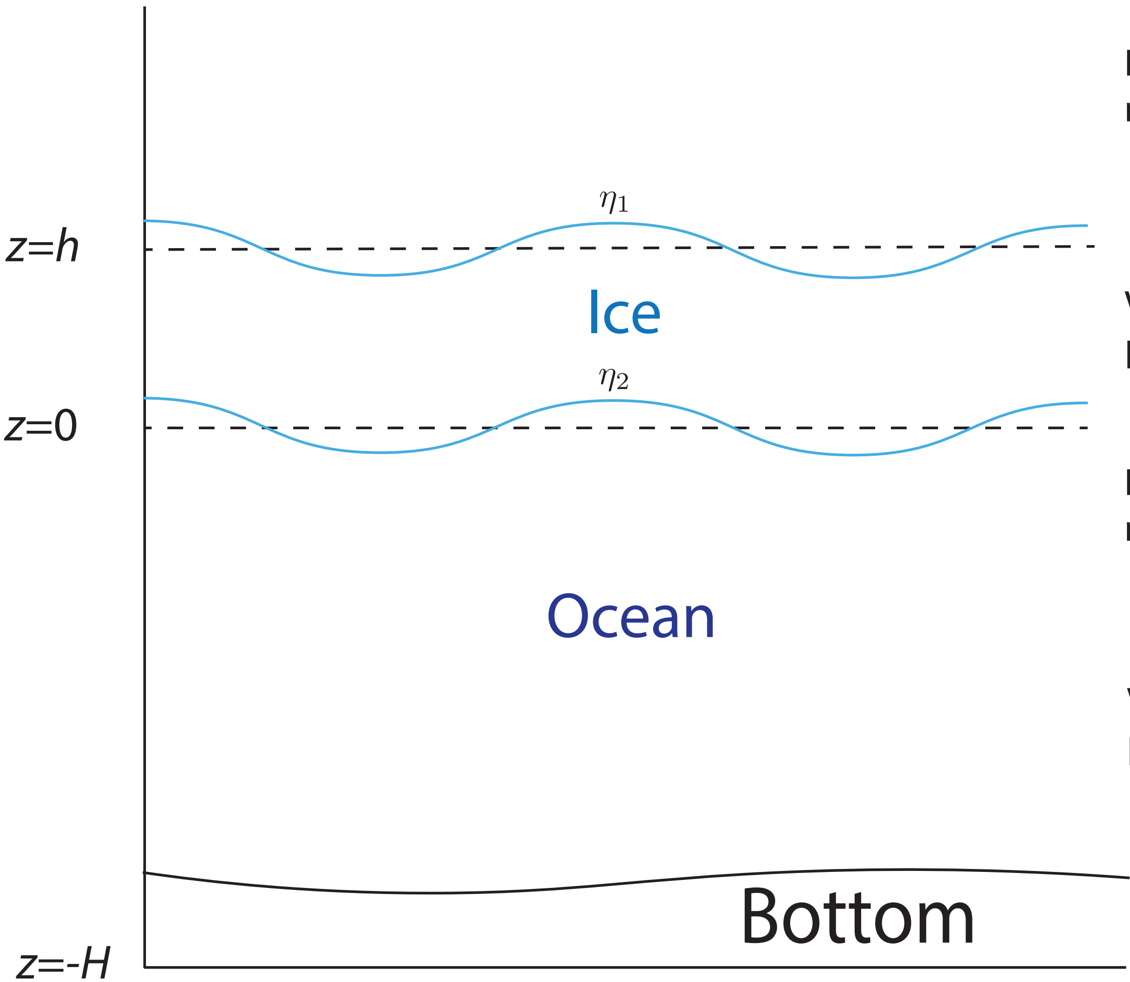
wave propagation in the marginal ice zone







Two Layer Models and Effective Parameters



Viscous fluid layer (Keller 1998)
Effective Viscosity ν

Equations of motion:

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 U + g$$

Viscoelastic fluid layer (Wang-Shen 2010)
Effective Complex Viscosity $\nu_e = \nu + iG/\rho\omega$

Equations of motion

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho}\nabla P + \nu_e \nabla^2 U + g$$

Viscoelastic thin beam (Mosig *et al.* 2015)
Effective Complex Shear Modulus $G_v = G - i\omega\rho\nu$

G shear modulus

P pressure

ω angular frequency

U velocity field

ν viscosity

λ Poission ratio

ρ density

g gravity

Stieltjes integral representation for effective complex viscoelastic parameter; bounds

Sampson, Murphy, Cherkaev, Golden 2017