Introduction to Mathematics of Energy

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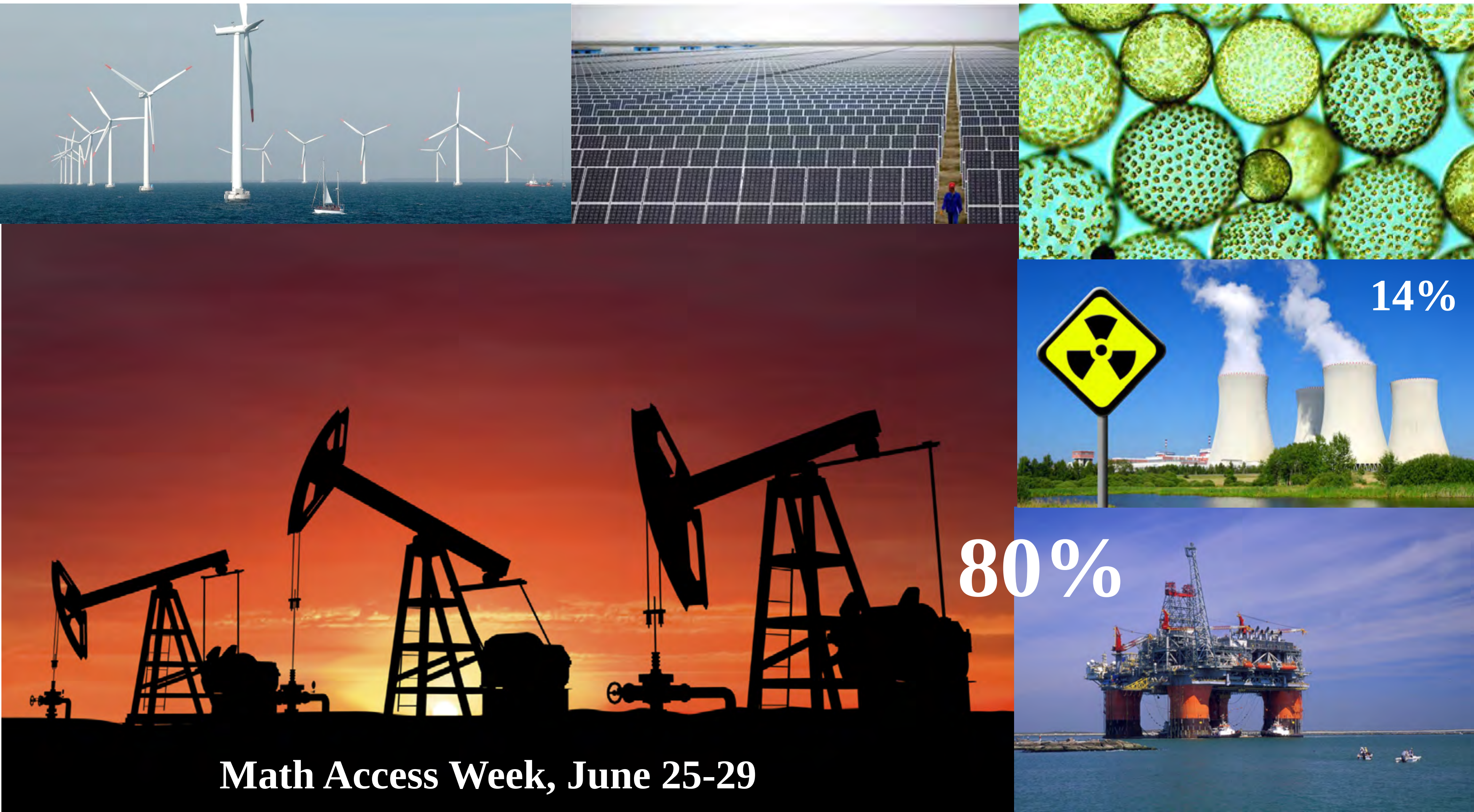
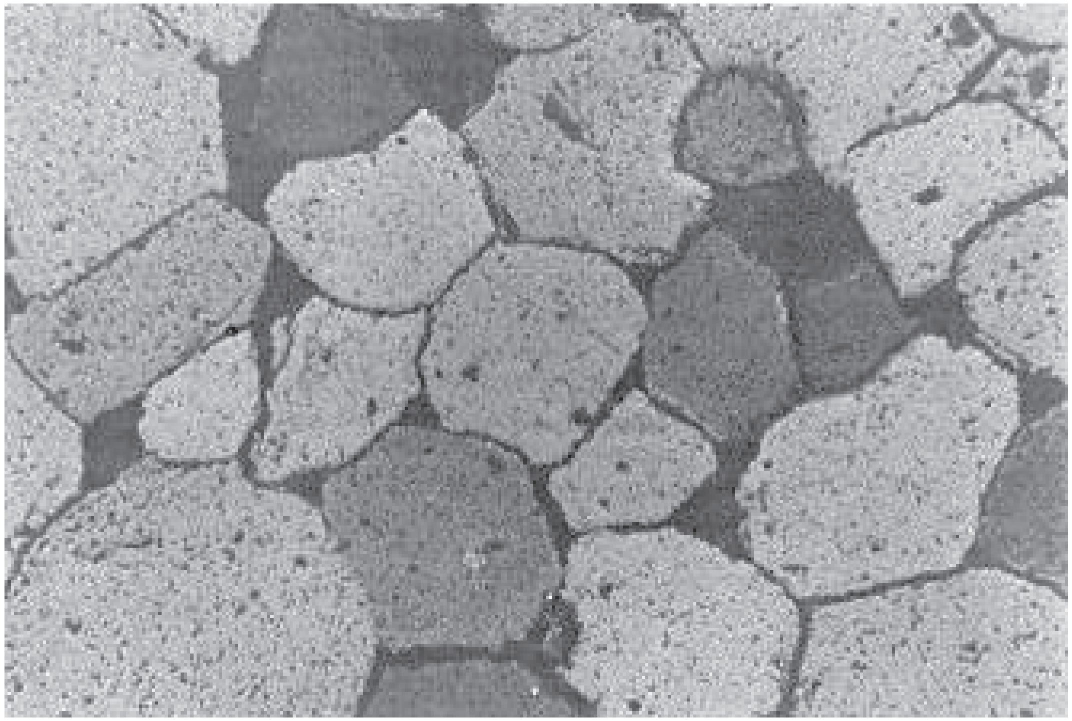
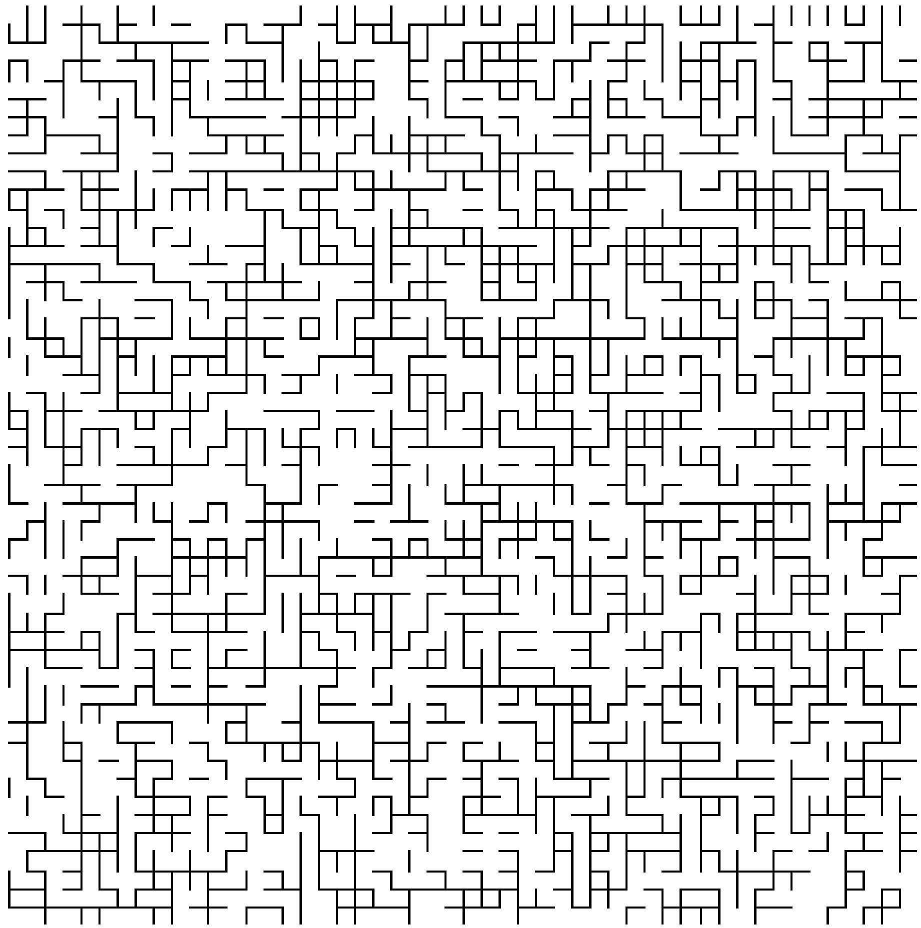




Cemented Devonian sandstone from Illinois.
(Roberts and Schwartz, 1985)

Transport of oil or gas through rocks depends strongly on the connectedness of the pores. Below a critical porosity ϕ_c fluid cannot flow.

Lattice Percolation Models



Bond percolation was originally introduced by Broadbent and Hammersly (1957), the former of which was interested in the design of gas masks for coal miners.

Basic, yet interesting, application: Consider a porous rock placed in a bucket of water. Does the center of the stone become wet?

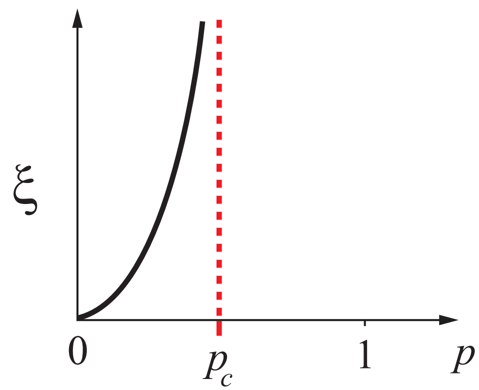
DISCUSS!

order parameters in percolation theory

geometry

correlation length

characteristic scale
of connectedness

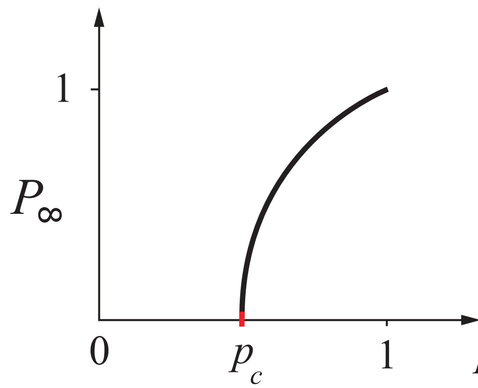


$$\xi\,(p) \sim |\,p - p_c|^{-\nu}$$

$$p \longrightarrow p_c$$

infinite cluster density

probability the origin
belongs to infinte cluster

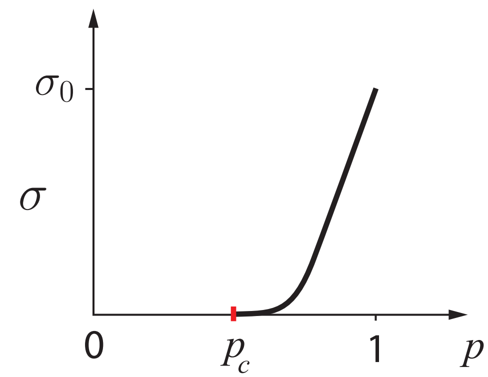


$$P_\infty(p) \sim (\,p - p_c)^{\beta}$$

$$p \longrightarrow p_c^+$$

transport

effective conductivity
or fluid permeability



$$\sigma\,(p) \sim \sigma_0\,(p - p_c)^t$$

$$p \longrightarrow p_c^+$$

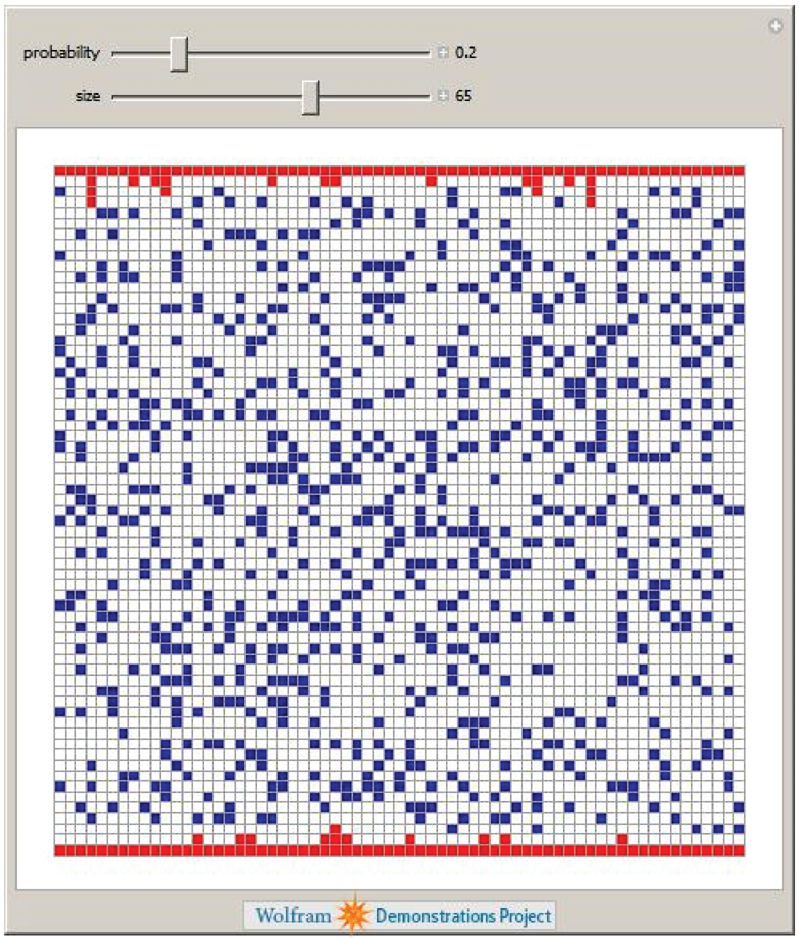
UNIVERSAL critical exponents for lattices -- depend only on dimension

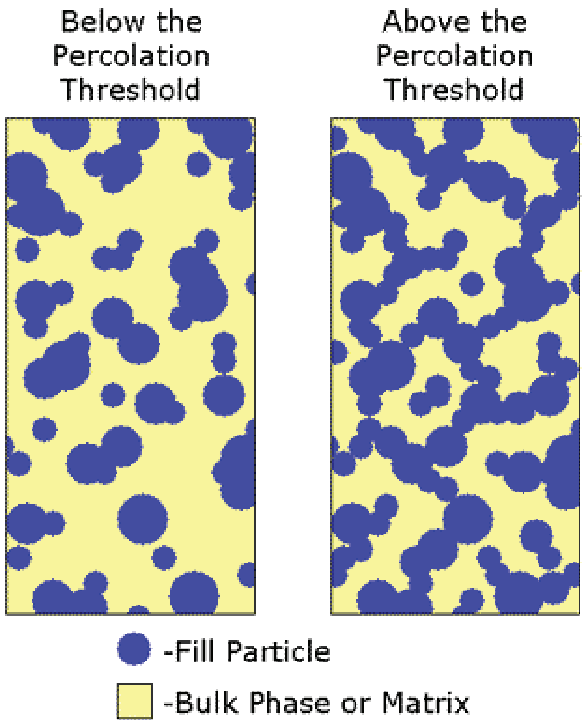
$1 \leq t \leq 2$ (for idealized model), Golden, *Phys. Rev. Lett.* 1990 ; *Comm. Math. Phys.* 1992

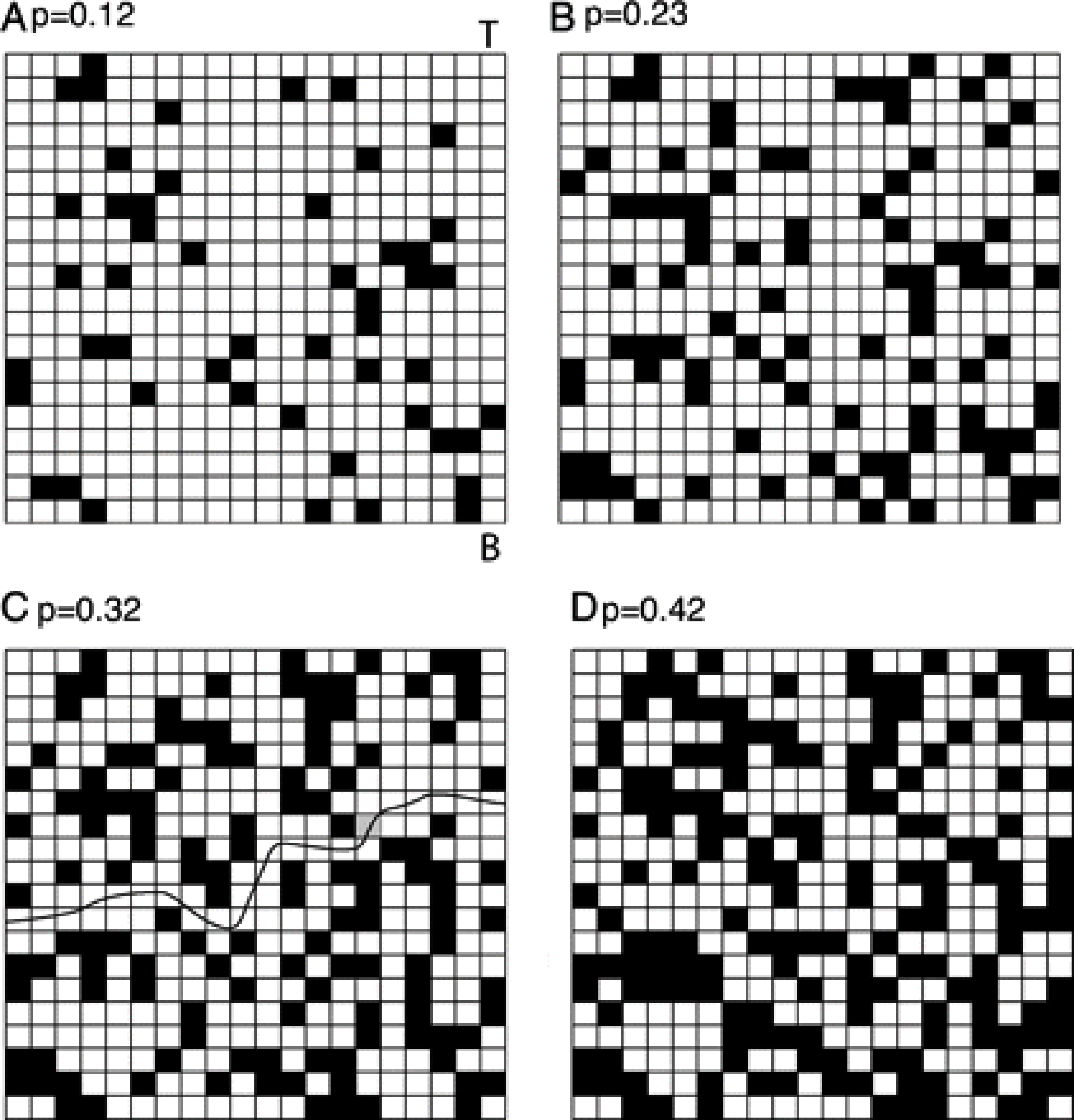
non-universal behavior in continuum

percolation

theory of connectedness






A p=0.12
T
B
B p=0.23
C p=0.32
D p=0.42

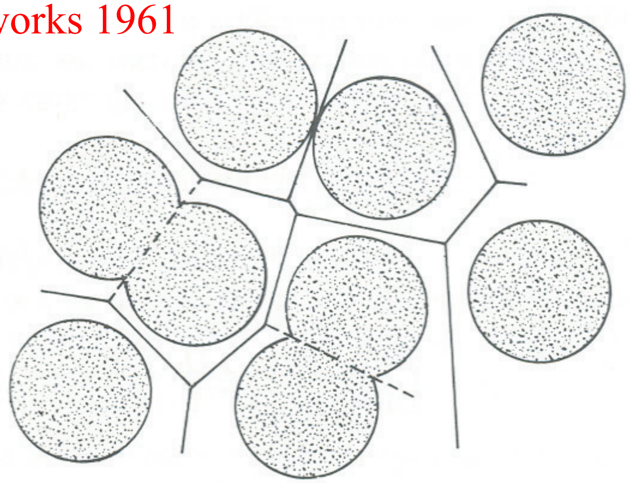
Non-universal behavior in the continuum:

critical exponents for transport in Swiss cheese model take values different than for lattices, e.g. $t > 2$

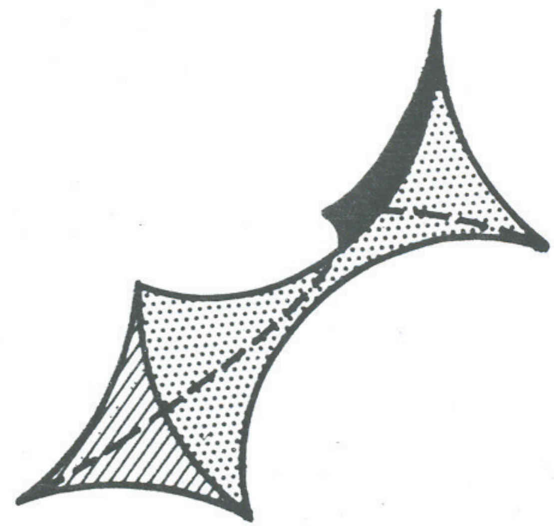
$e \neq t$

Halperin, Feng, Sen, *Phys. Rev. Lett.* 1985

Gilbert disc model for wireless networks 1961



Swiss cheese model
$d = 2$



conducting neck in $d = 3$
Swiss cheese model

in general, non-universal exponents arise from a singular distribution of local conductances

In sea ice, this distribution is lognormal.

(excluding inclusions below cutoff)

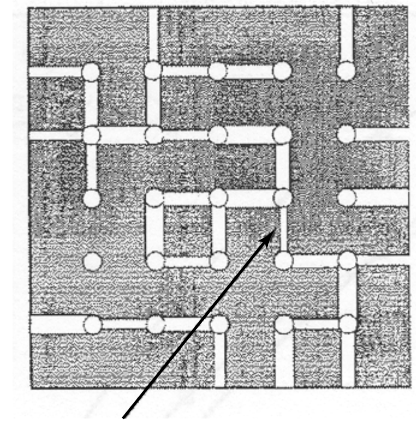
Thus, the permeability exponent for sea ice is **2**, the universal lattice value.

ESTIMATE fluid conductivity scaling factor $k_0 = r^2 / 8$

for media with broad range of conductances

CRITICAL PATH ANALYSIS

bottlenecks control flow


critical pore

Ambegaokar, Halperin, Langer 1971: CPA in electronic hopping conduction
Friedman, Seaton 1998: CPA in fluid and electrical networks
Golden, Kozlov 1999: rigorous CPA on long-range checkerboard model

$k_0 \approx r_c^2 / 8$ critical fluid conductivity

Microstructural analyses yield $r_c \approx$ 0.5 mm

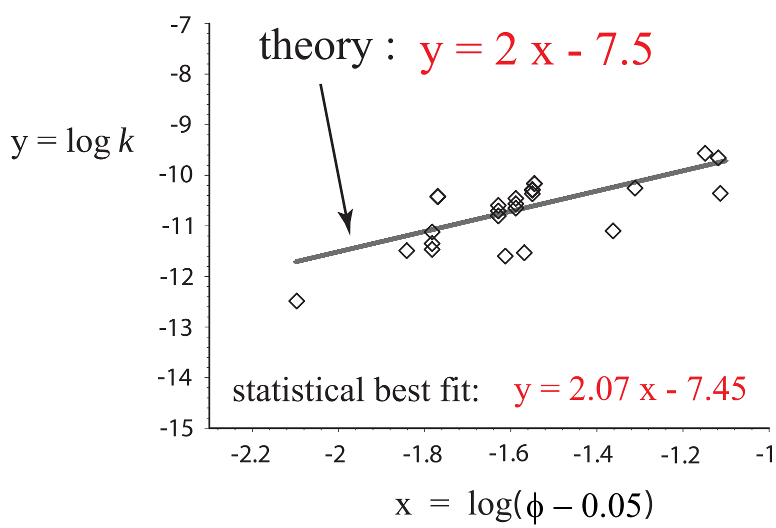
lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

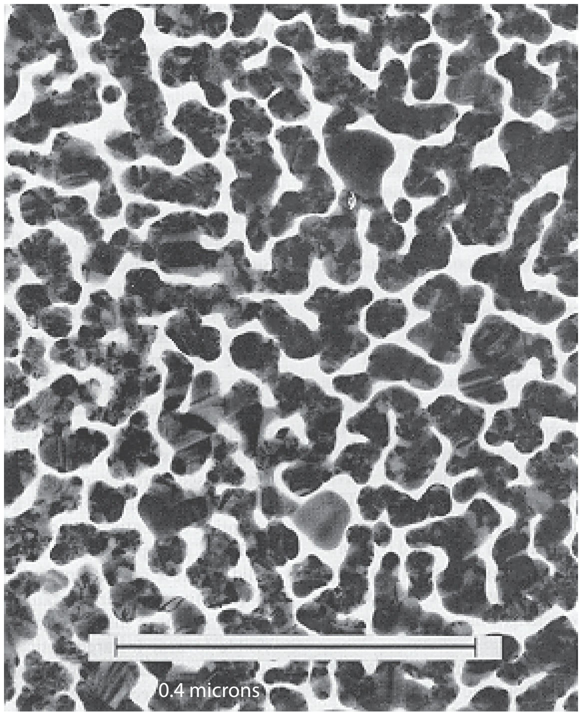
$$k_0 = 3 \times 10^{-8}\ \mathrm{m}^2$$

- exponent is UNIVERSAL lattice value $t \approx 2.0$
- sedimentary rocks like sandstones also exhibit universality
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0



thin silver film

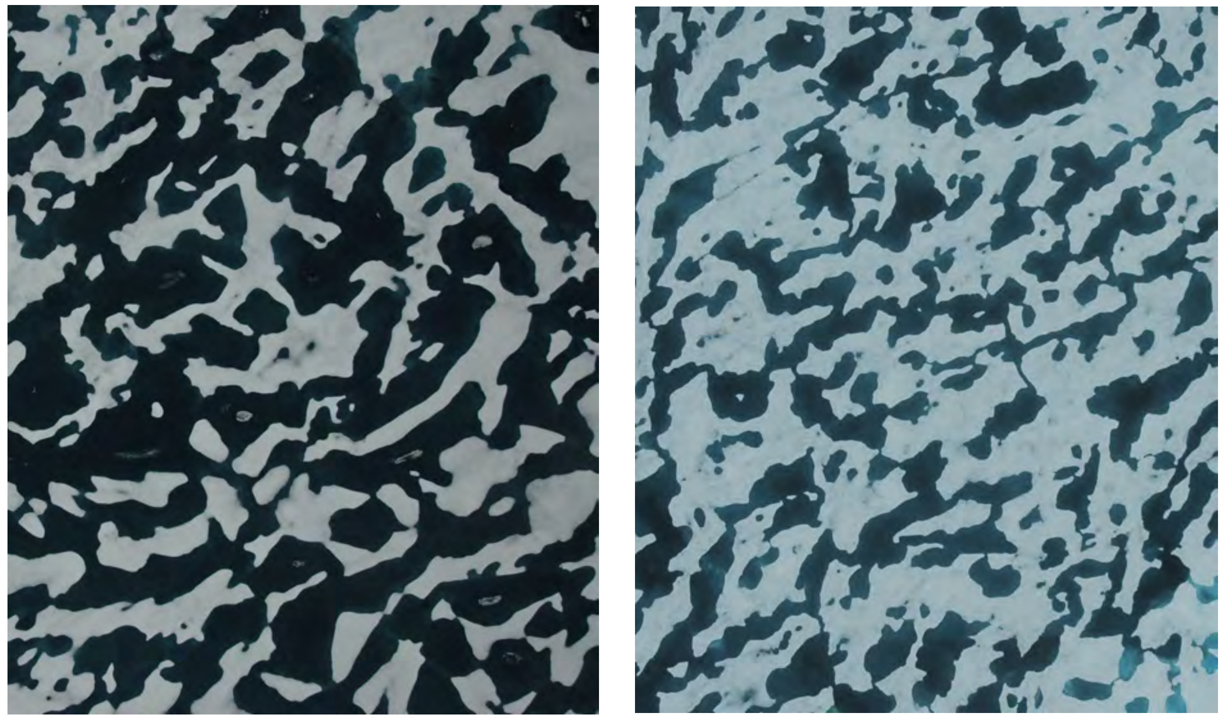
microns



(Davis, McKenzie, McPhedran, 1991)

Arctic melt ponds

kilometers



(Perovich, 2005)

optical properties

composite geometry -- area fraction of phases, connectedness, necks

Invasion Percolation

In 1983, Wilkinson and Willemson introduced the invasion percolation model describing fluid distributions for slow immiscible fluid invasion in porous media.

They found that the mass of the invaded fluid M grew as a function of the length L of the lattice like L^D where D is the fractal dimension of the percolation cluster taking account for the trapping.

It was found in their 1983 paper and confirmed in subsequent experiments by Lenormand and Zarcone that:

$$D \simeq 1.82$$

invasion percolation

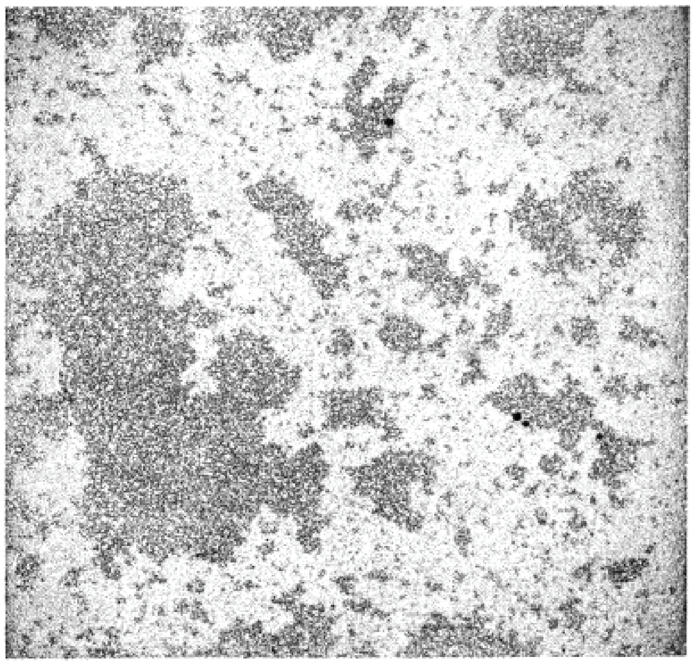
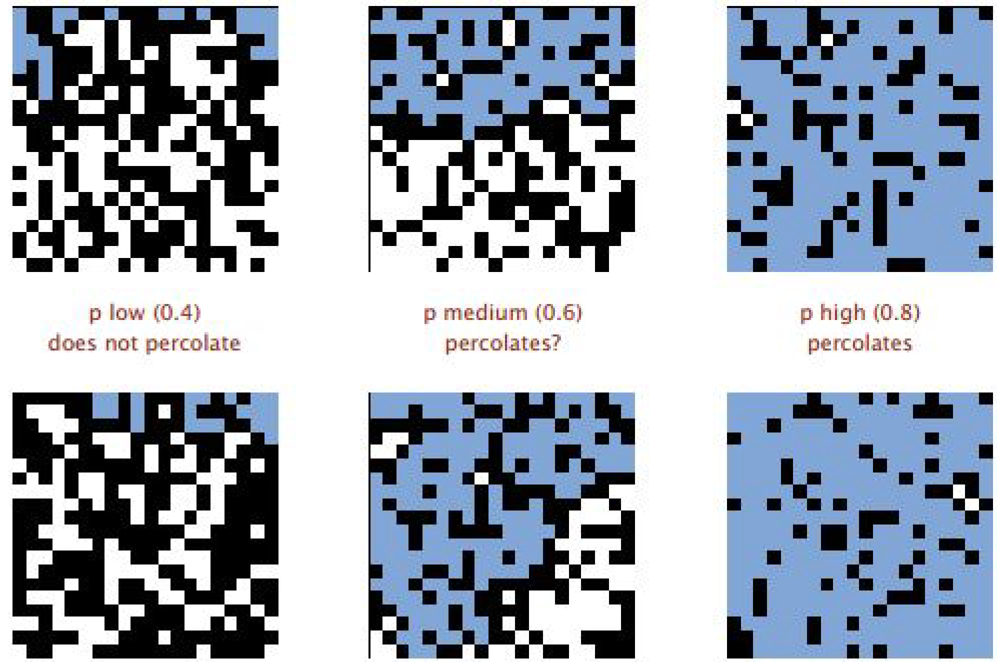


Image Credit: Lenormand, R. and Zarcone, C. (1985)



Displacement of the invaded fluid (think oil) by the slow invasion of the invading fluid (think water) from the left. The cluster size varies from the pore size to the network size, the latter of which is pictured above.

forest fires
invading species, ...

Carbon Capture and Sequestration (CCS)

The process of collecting carbon emissions from large sources such as fossil fuel plants, and storing them.

Captured carbon can be stored into underground geo-logical formations, e.g. depleted oil reservoirs and un-mineable coal beds.

Carbon could also be sequestered in the ocean, either by injection into water depths of 1000-2000 m, the release of solid carbon in deep water, or other methods.

The technology to utilize these methods is already available to humans!

This application of the theory created by Broadbent and Hammersly became of great interest!

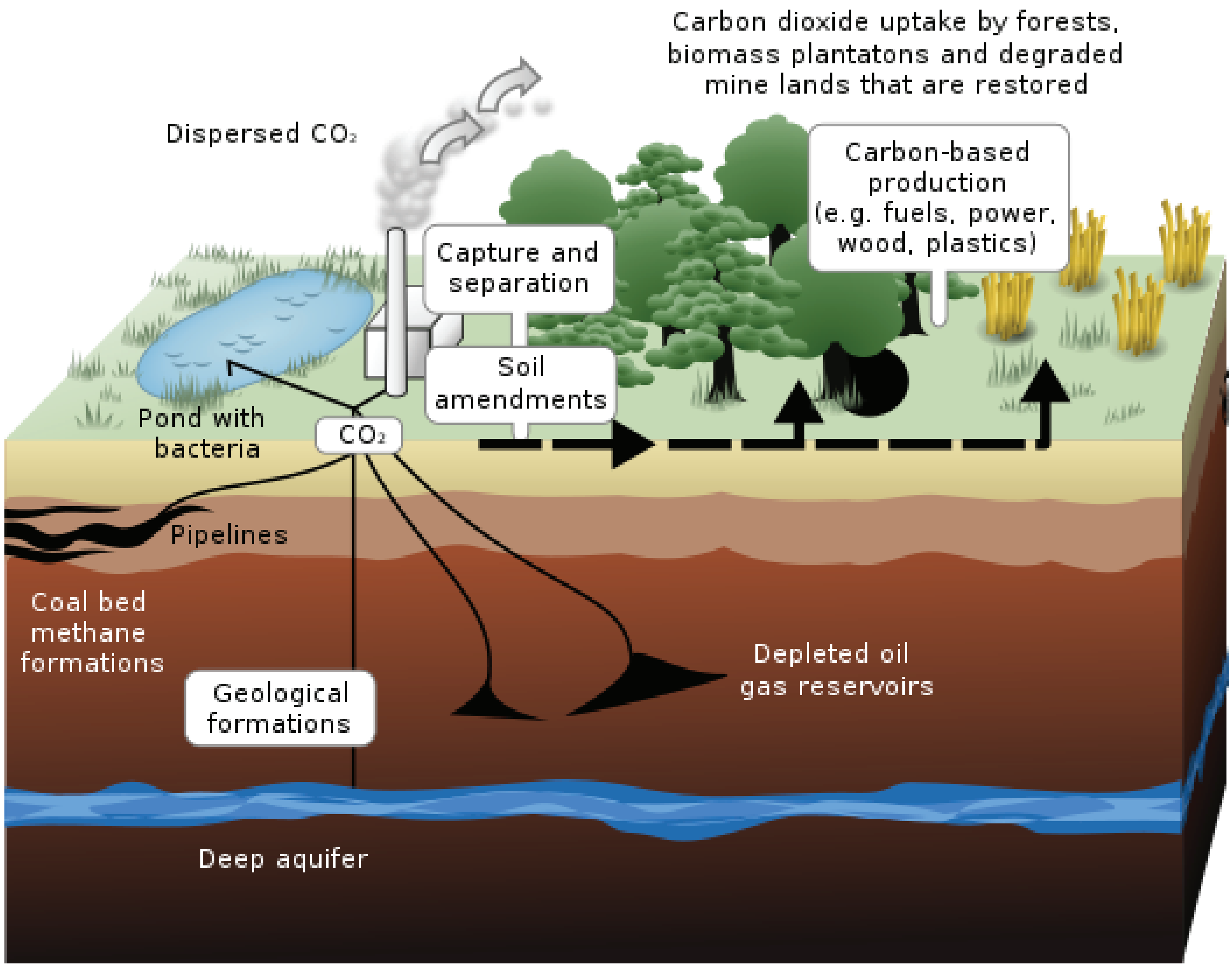
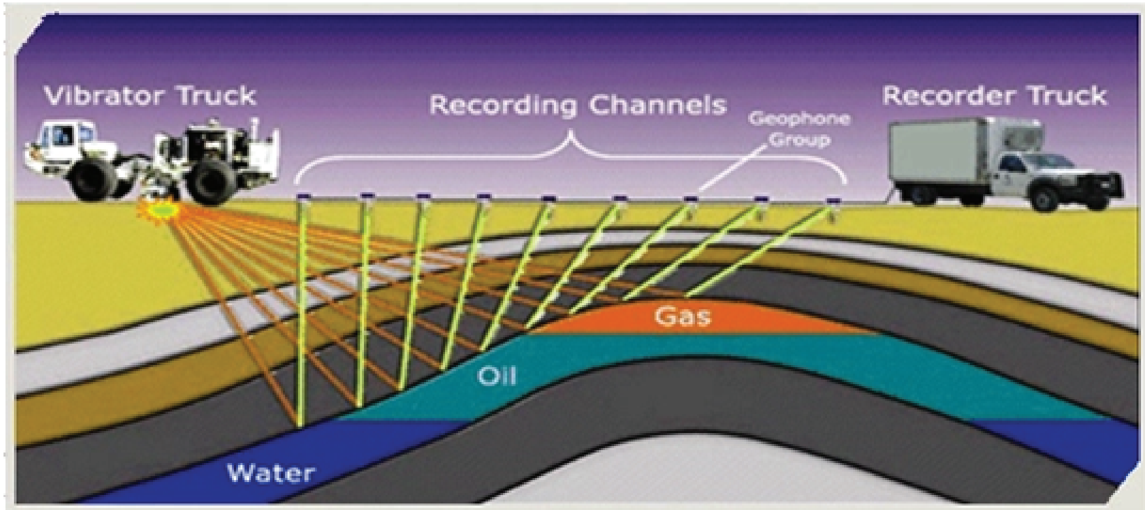
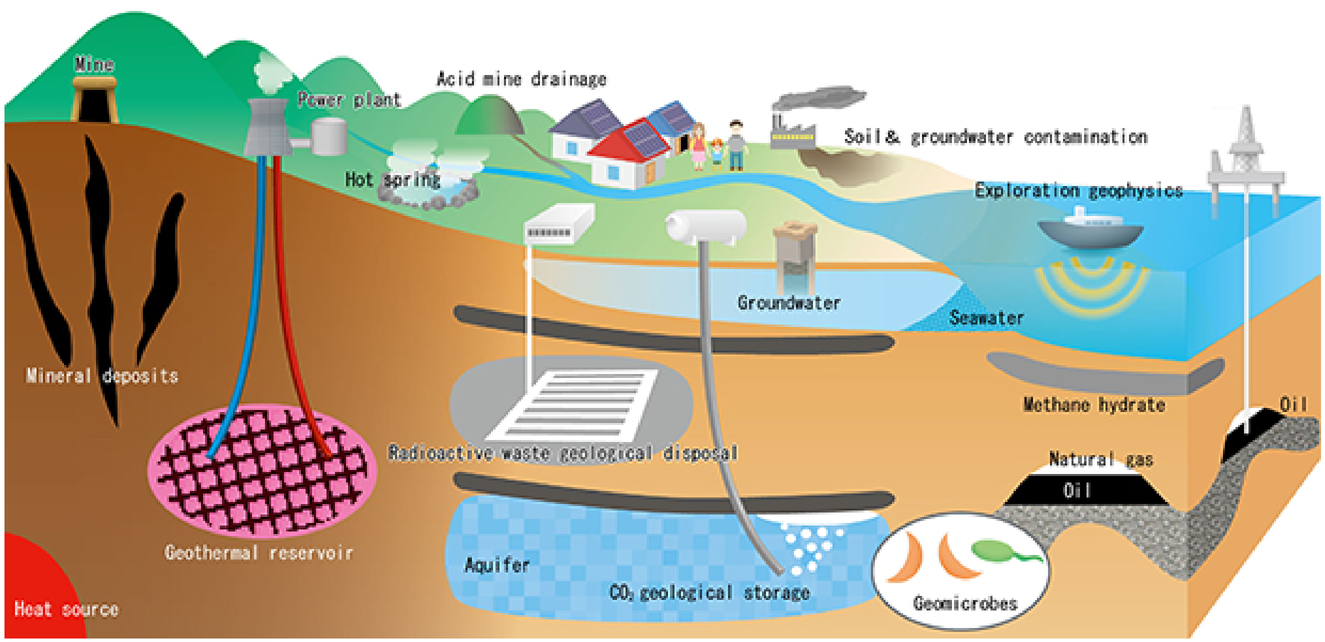


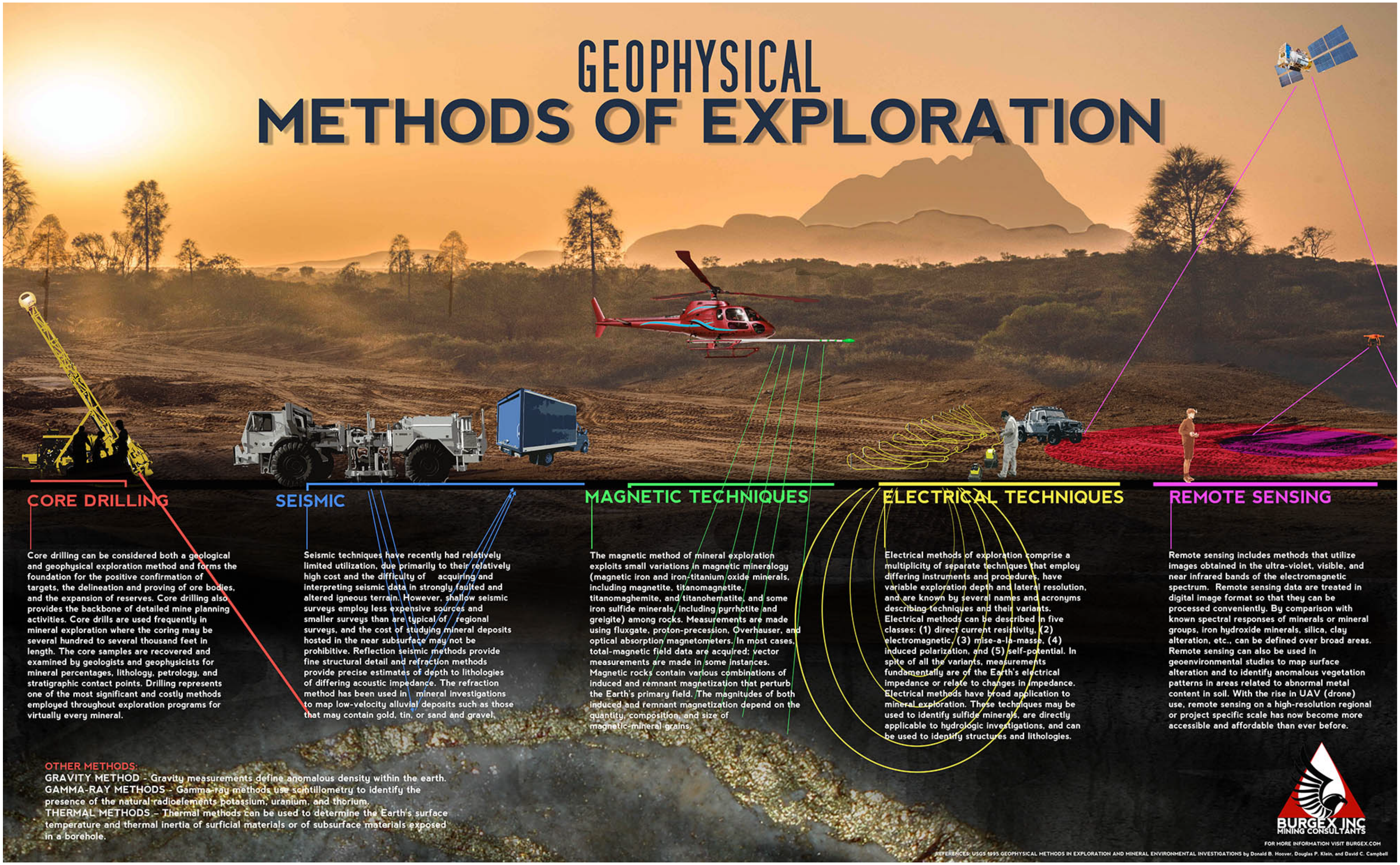
Image Credit: Wikipedia

Create a strategy to store carbon dioxide underground.

Exploration geophysics






GEOPHYSICAL
METHODS OF EXPLORATION
CORE DRILLING
Core drilling can be considered both a geological and geophysical exploration method and forms the foundation for the positive confirmation of targets, the delineation and proving of ore bodies, and the expansion of reserves. Core drilling also provides the backbone of detailed mine planning activities. Core drills are used frequently in mineral exploration where the coring may be several hundred to several thousand feet in length. The core samples are recovered and examined by geologists and geophysicists for mineral percentages, lithology, petrology, and stratigraphic contact points. Drilling represents one of the most significant and costly methods employed throughout exploration programs for virtually every mineral.
SEISMIC
Seismic techniques have recently had relatively limited utilization, due primarily to their relatively high cost and the difficulty of acquiring and interpreting seismic data in strongly faulted and altered igneous terrain. However, shallow seismic surveys employ less expensive sources and smaller surveys than are typical of regional surveys, and the cost of studying mineral deposits hosted in the near subsurface may not be prohibitive. Reflection seismic methods provide fine structural detail and refraction methods provide precise estimates of depth to lithologies of differing acoustic impedance. The refraction method has been used in mineral investigations to map low-velocity alluvial deposits such as those that may contain gold, tin, or sand and gravel.
MAGNETIC TECHNIQUES
The magnetic method of mineral exploration exploits small variations in magnetic mineralogy (magnetic iron and iron-titanium oxide minerals, including magnetite, titanomagnetite, titanomaghemite, and titanohematite, and some iron sulfide minerals, including pyrrhotite and greigite) among rocks. Measurements are made using fluxgate, proton-precession, Overhauser, and optical absorption magnetometers. In most cases, total-magnetic field data are acquired; vector measurements are made in some instances. Magnetic rocks contain various combinations of induced and remnant magnetization that perturb the Earth's primary field. The magnitudes of both induced and remnant magnetization depend on the quantity, composition, and size of magnetic-mineral grains.
ELECTRICAL TECHNIQUES
Electrical methods of exploration comprise a multiplicity of separate techniques that employ differing instruments and procedures, have variable exploration depth and lateral resolution, and are known by several names and acronyms describing techniques and their variants. Electrical methods can be described in five classes: (1) direct current resistivity, (2) electromagnetic, (3) mise-a-la-masse, (4) induced polarization, and (5) self-potential. In spite of all the variants, measurements fundamentally are of the Earth's electrical impedance or relate to changes in impedance. Electrical methods have broad application to mineral exploration. These techniques may be used to identify sulfide minerals, are directly applicable to hydrologic investigations, and can be used to identify structures and lithologies.
REMOTE SENSING
Remote sensing includes methods that utilize images obtained in the ultra-violet, visible, and near infrared bands of the electromagnetic spectrum. Remote sensing data are treated in digital image format so that they can be processed conveniently. By comparison with known spectral responses of minerals or mineral groups, iron hydroxide minerals, silica, clay alteration, etc., can be defined over broad areas. Remote sensing can also be used in geoenvironmental studies to map surface alteration and to identify anomalous vegetation patterns in areas related to abnormal metal content in soil. With the rise in UAV (drone) use, remote sensing on a high-resolution regional or project specific scale has now become more accessible and affordable than ever before.
OTHER METHODS:
GRAVITY METHOD - Gravity measurements define anomalous density within the earth.
GAMMA-RAY METHODS - Gamma-ray methods use scintillometry to identify the presence of the natural radioelements potassium, uranium, and thorium.
THERMAL METHODS – Thermal methods can be used to determine the Earth's surface temperature and thermal inertia of surficial materials or of subsurface materials exposed in a borehole.
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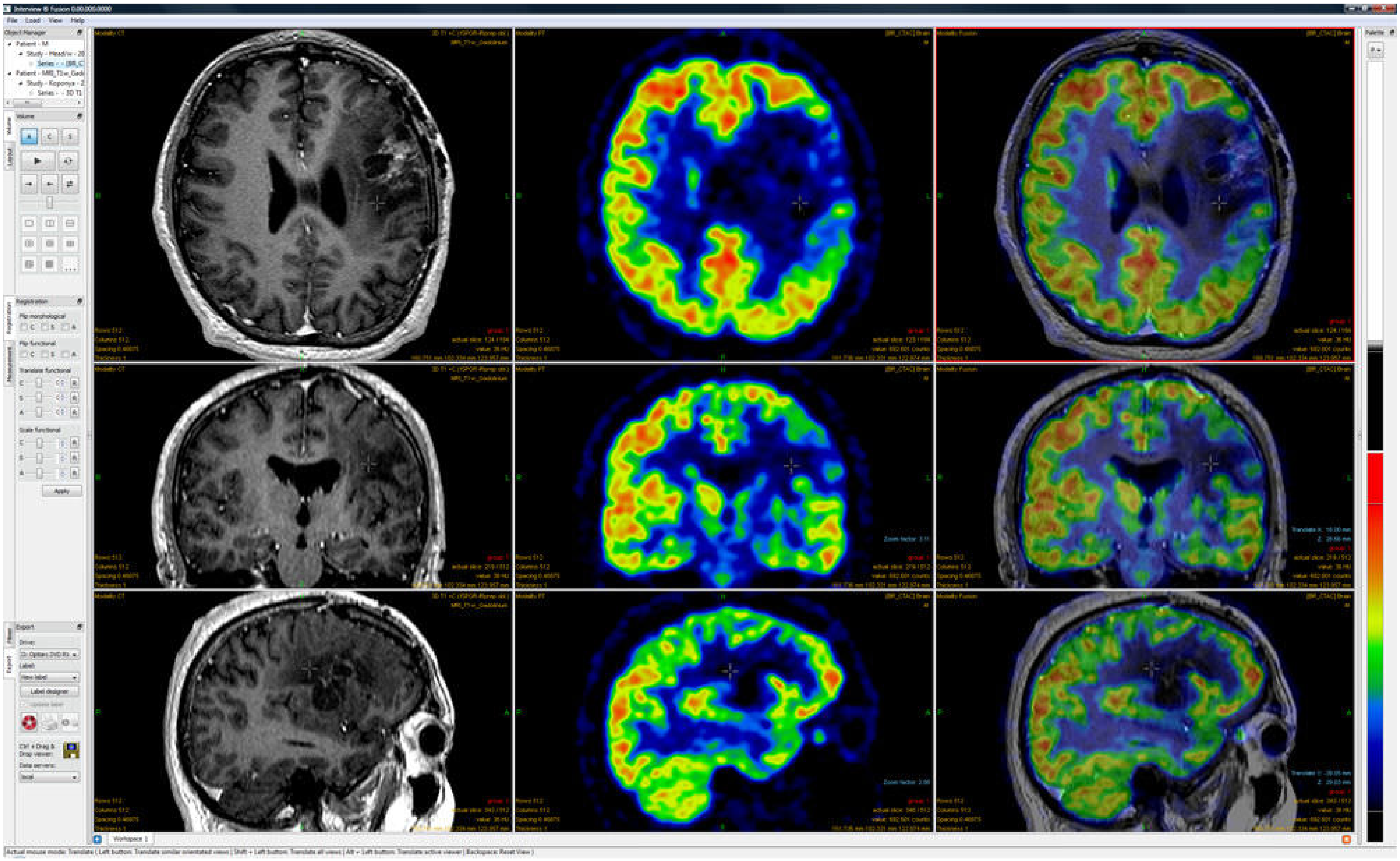
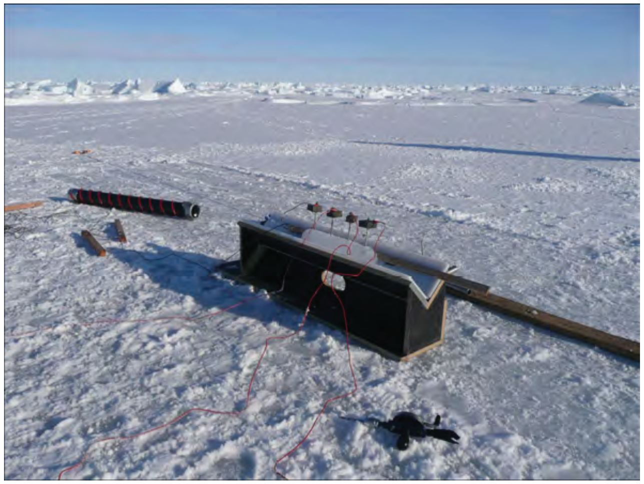
REFERENCES: USGS 1995 GEOPHYSICAL METHODS IN EXPLORATION AND MINERAL ENVIRONMENTAL INVESTIGATIONS by Donald B. Hoover, Douglas P. Klein, and David C. Campbell

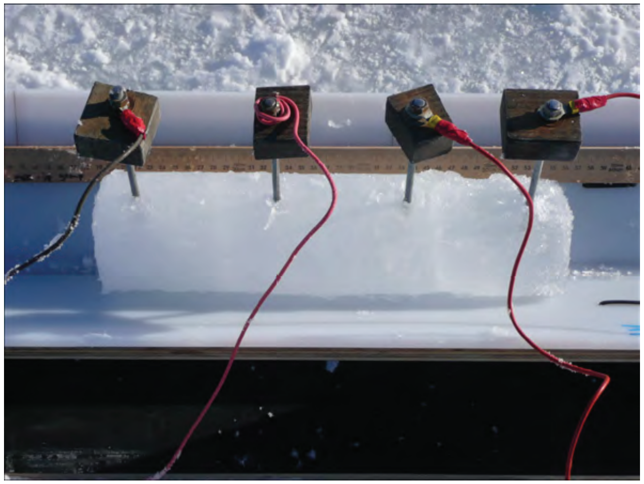
medical imaging

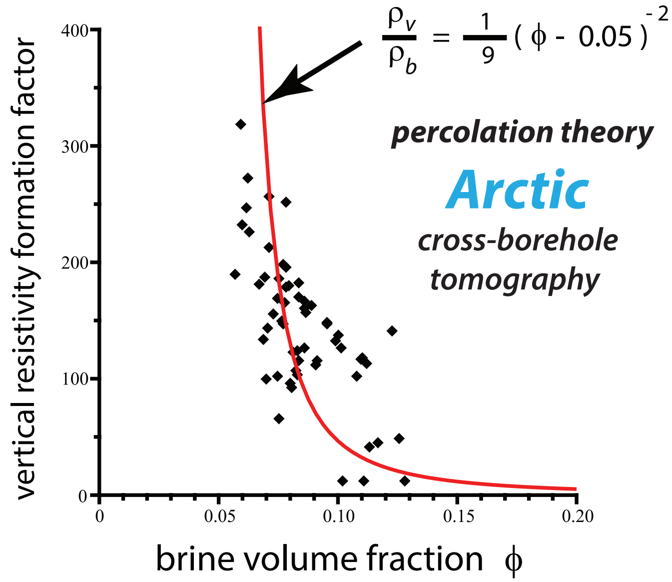


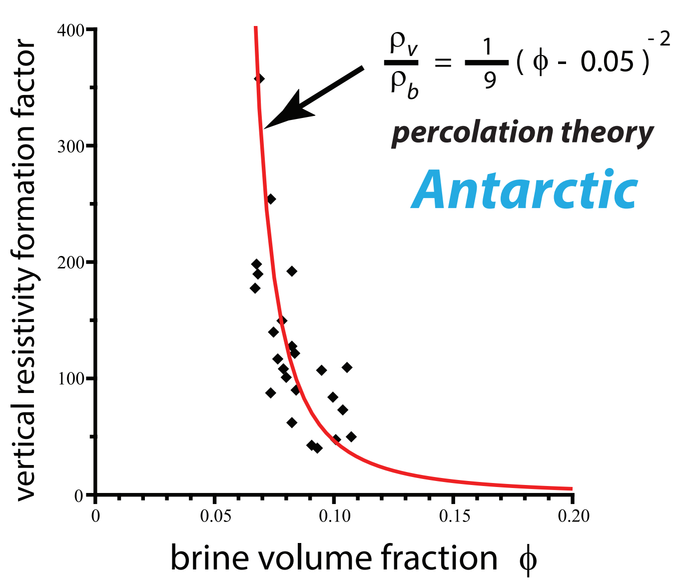
critical behavior of electrical transport in sea ice

electrical signature of the on-off switch for fluid flow





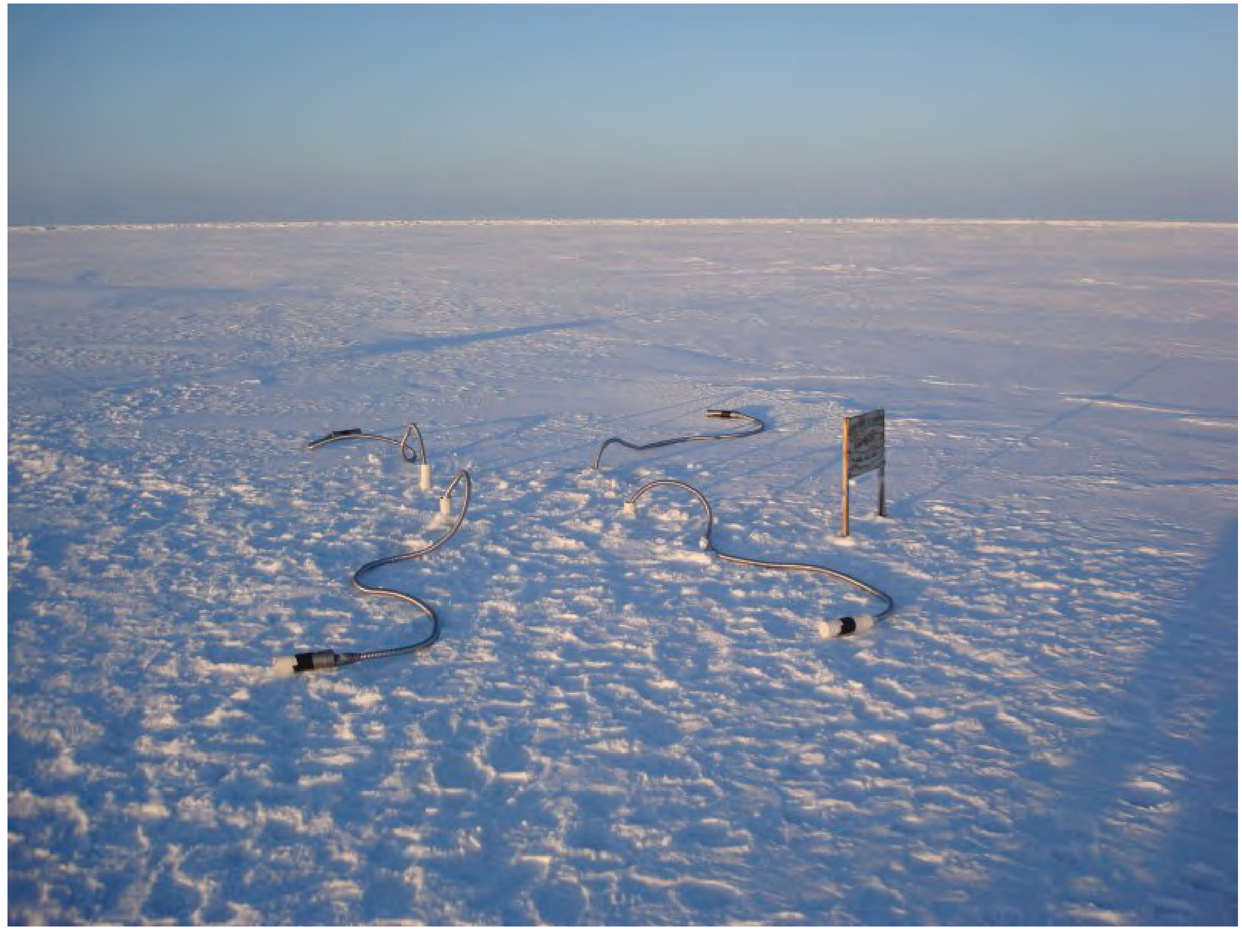




cross-borehole tomography - electrical classification of sea ice layers

Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2016

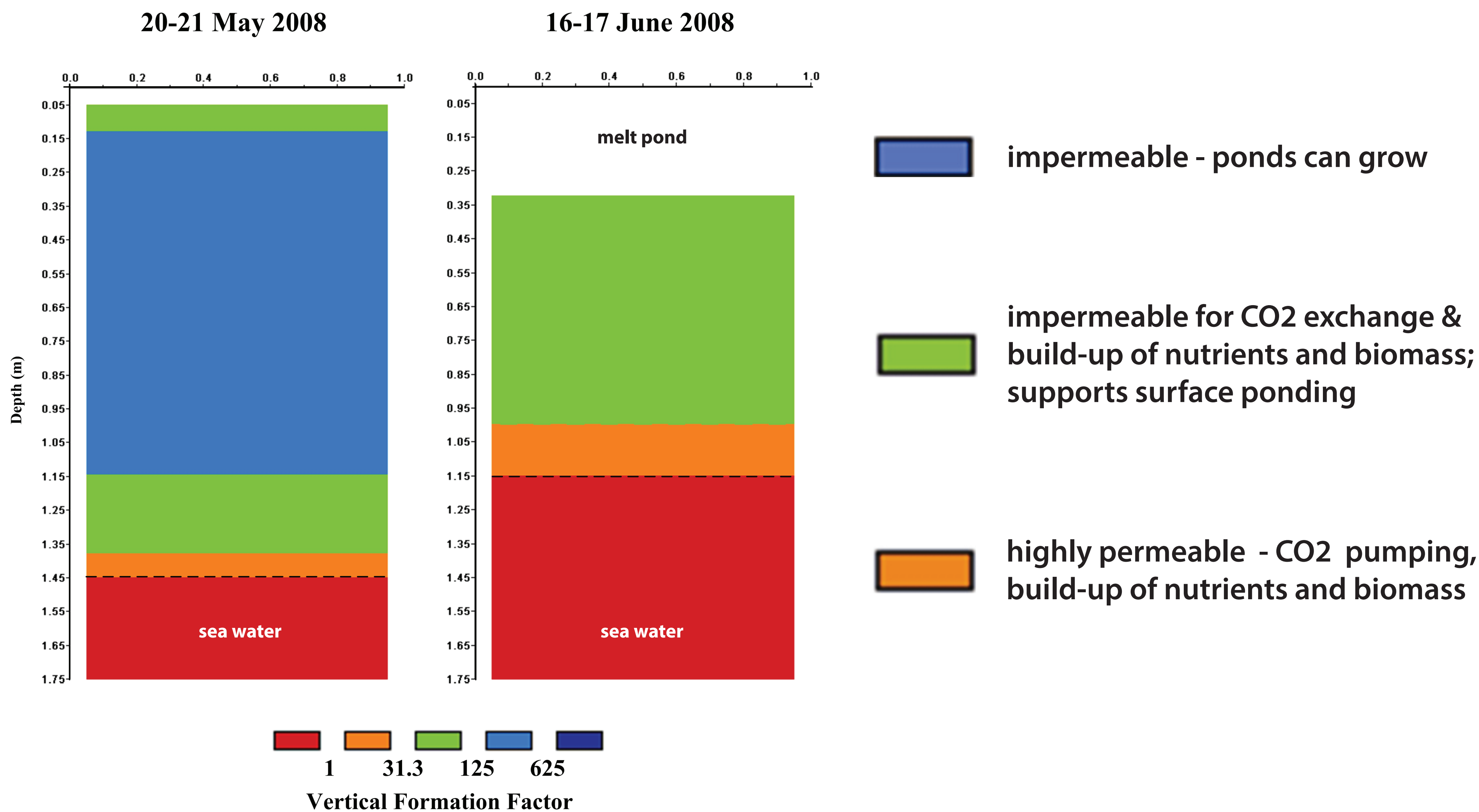
cross borehole tomography



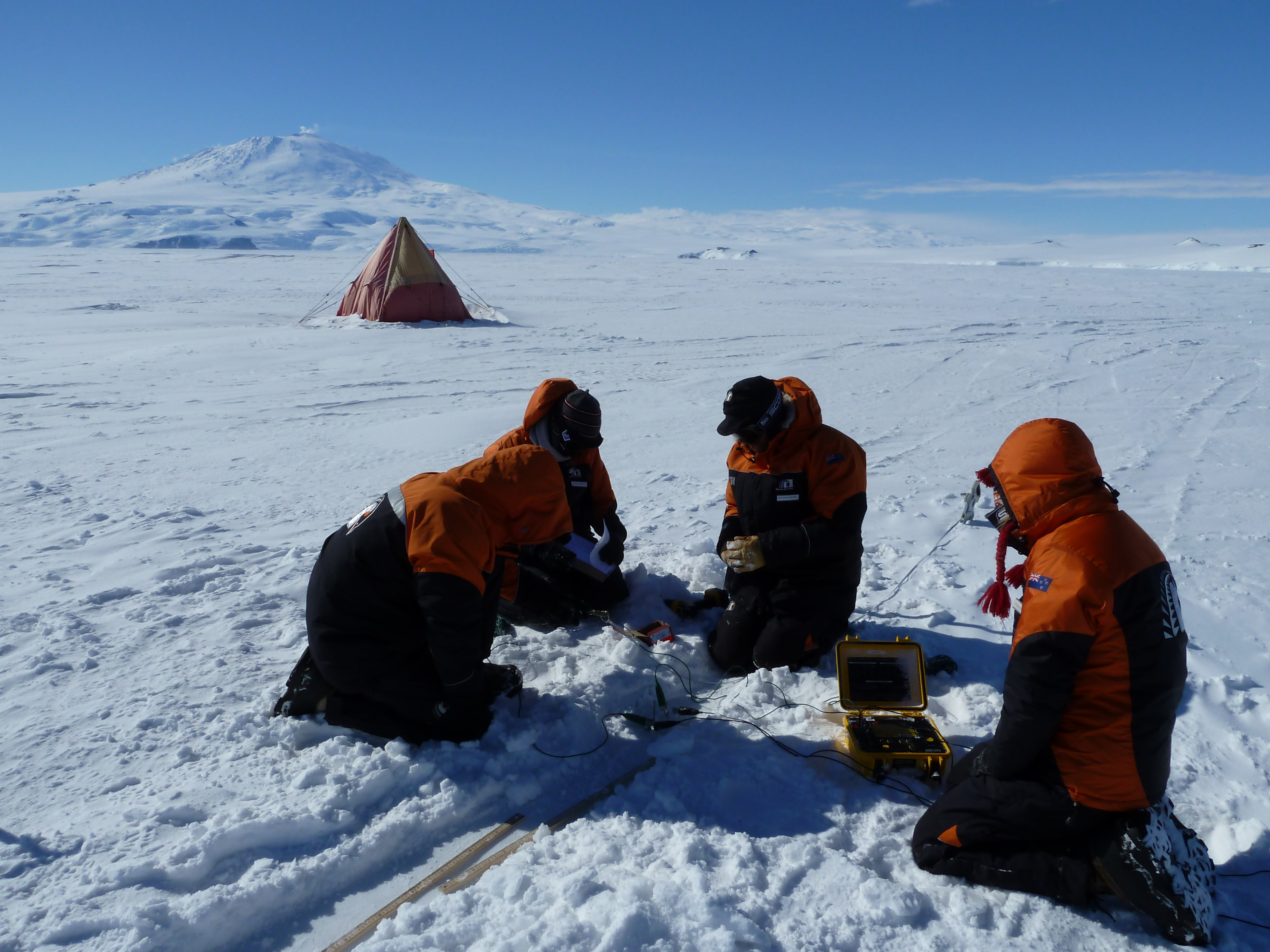
Ingham, Jones, Buchanan
Victoria University, Wellington, NZ

Cross-borehole tomographic reconstructions of sea ice resistivity

before and after melt pond formation



Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, and Worby 2016

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