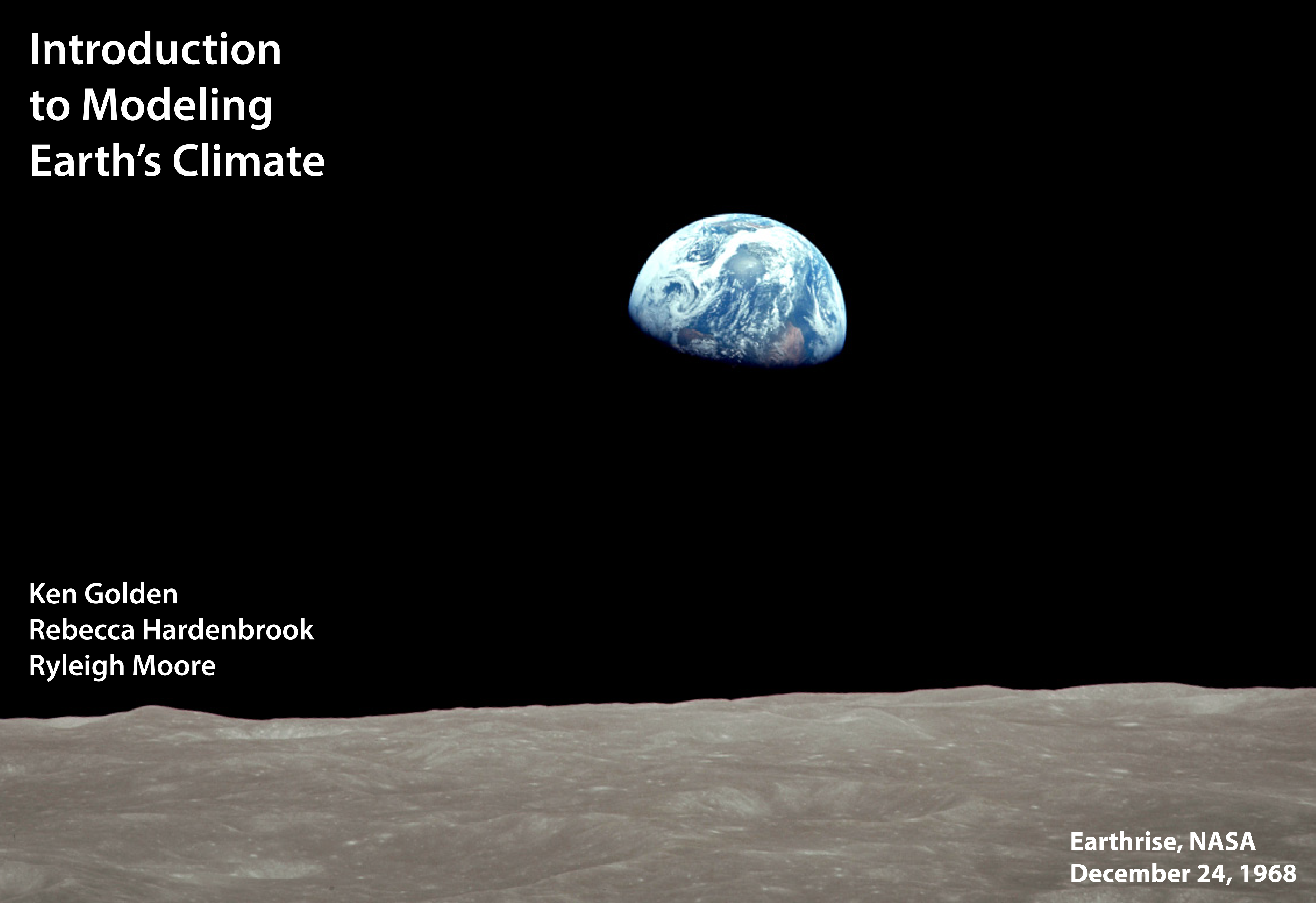
Introduction to Modeling Earth's Climate

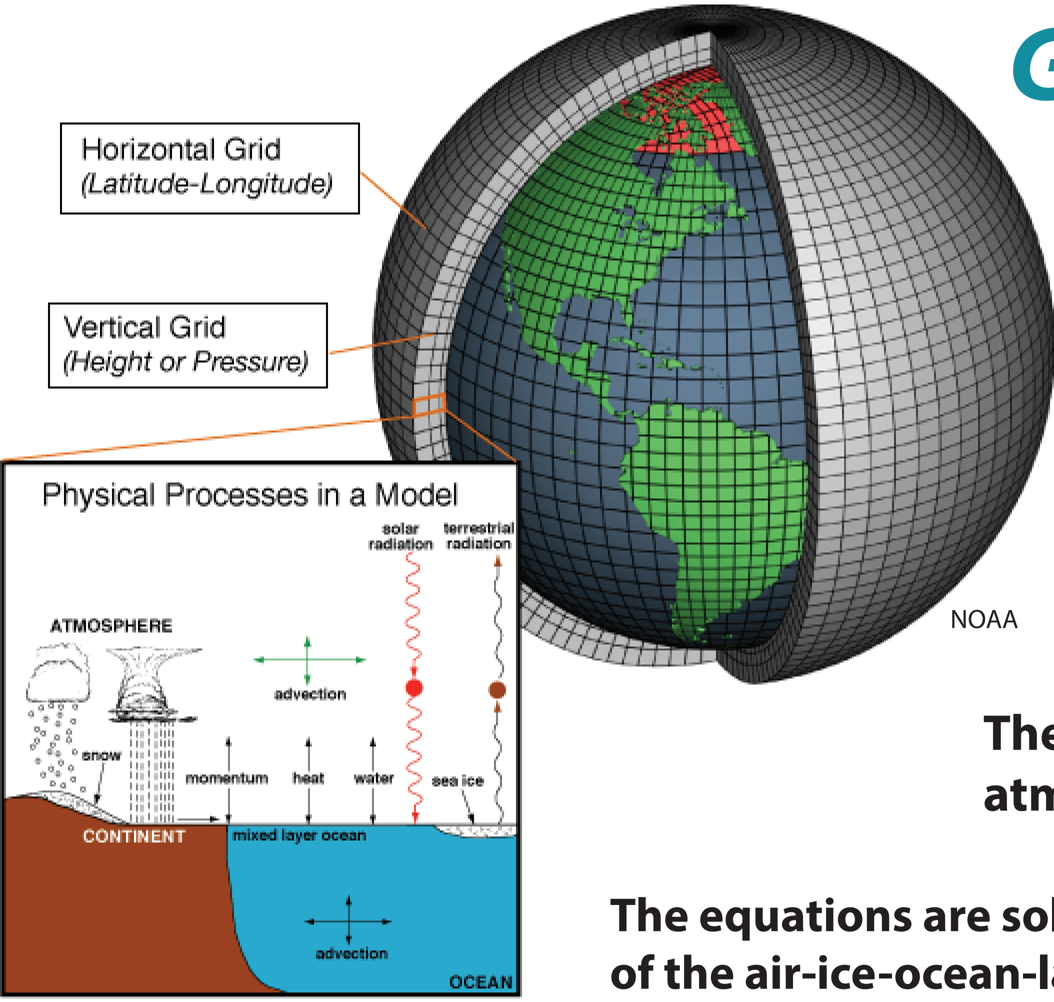


Ken Golden
Rebecca Hardenbrook
Ryleigh Moore

Earthrise, NASA
December 24, 1968

Math ACCESS Week, June 25 - 29, 2018

Global Climate Models



NOAA

climate fueled by the nonuniform spatial distribution of incoming solar radiation.

Stute et al., PNAS 2001

Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

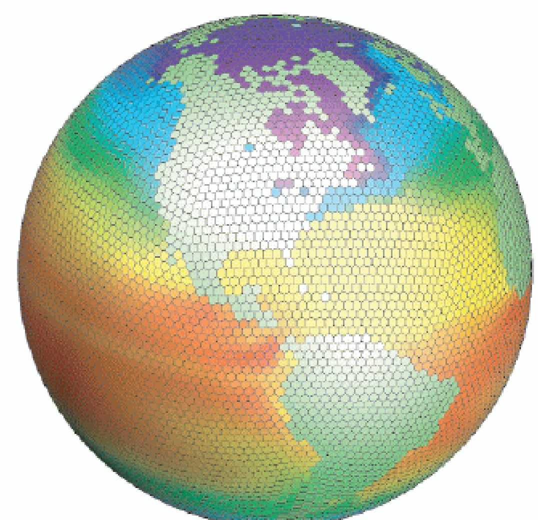
They describe the state of the ocean, land, ice atmosphere, biosphere, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 100 km), using very powerful computers.

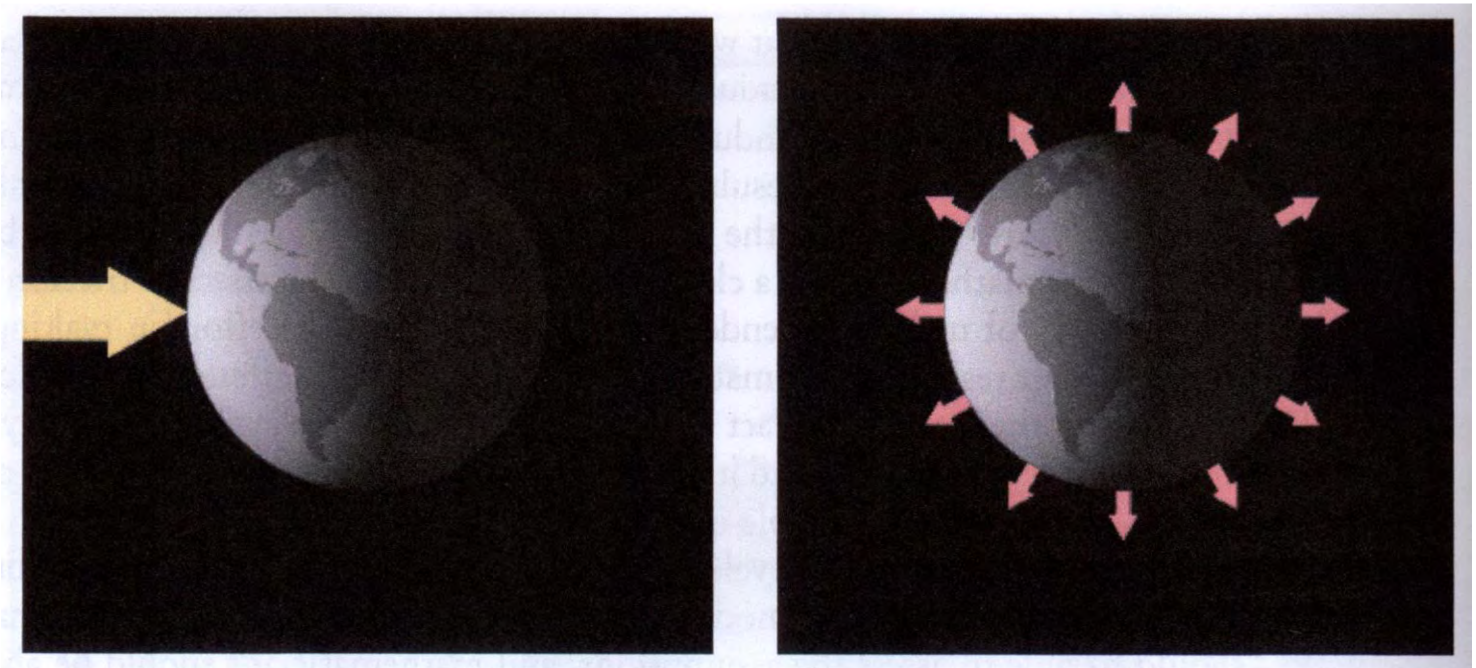
key challenge :

incorporating sub - grid scale processes

linking scales


Randall *et al.*, 2002

Earth's energy balance



E_{in}

incoming shortwave radiation
sunlight

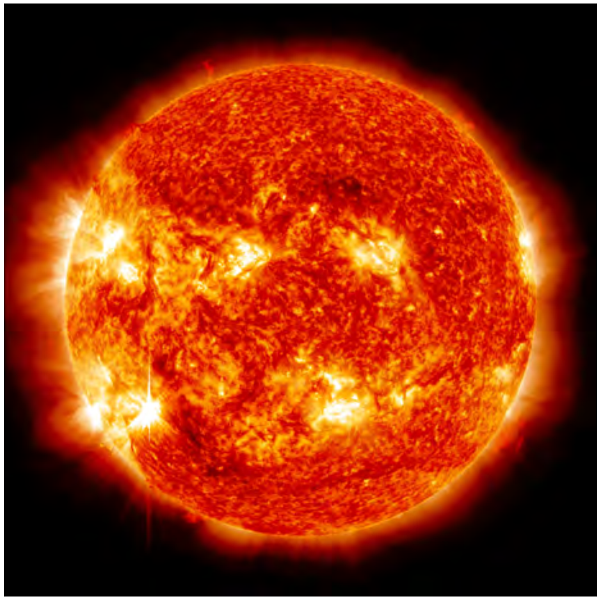
E_{out}

outgoing longwave radiation
heat

if $E_{out} > E_{in}$ **planet cools**

if $E_{out} < E_{in}$ **planet warms**

climate system parameters



$$S_0 = 1,368\,\mathrm{Wm}^{-2}$$

solar enery flux per unit area

solar "constant"

depends on sun's strength and average distance from Earth to Sun

MODEL INPUT

$$T(t) = \langle \mathrm{Temperature}(x,t) \rangle$$

global mean surface temperature

How to predict ??

(How to define and measure??)

MODEL OUTPUT

most important climate parameter:
how much energy reflected *vs.* how much energy absorbed

reflection

albedo $\alpha = \dfrac{\text{reflected sunlight}}{\text{incident sunlight}}$

absorption

co- albedo $1-\alpha$

fraction of energy that reaches Earth's surface

amount of solar energy intercepted by Earth per unit time (as a flat disk)

$$\pi R^2 S_0$$

amount of solar energy reaching Earth's surface per unit time

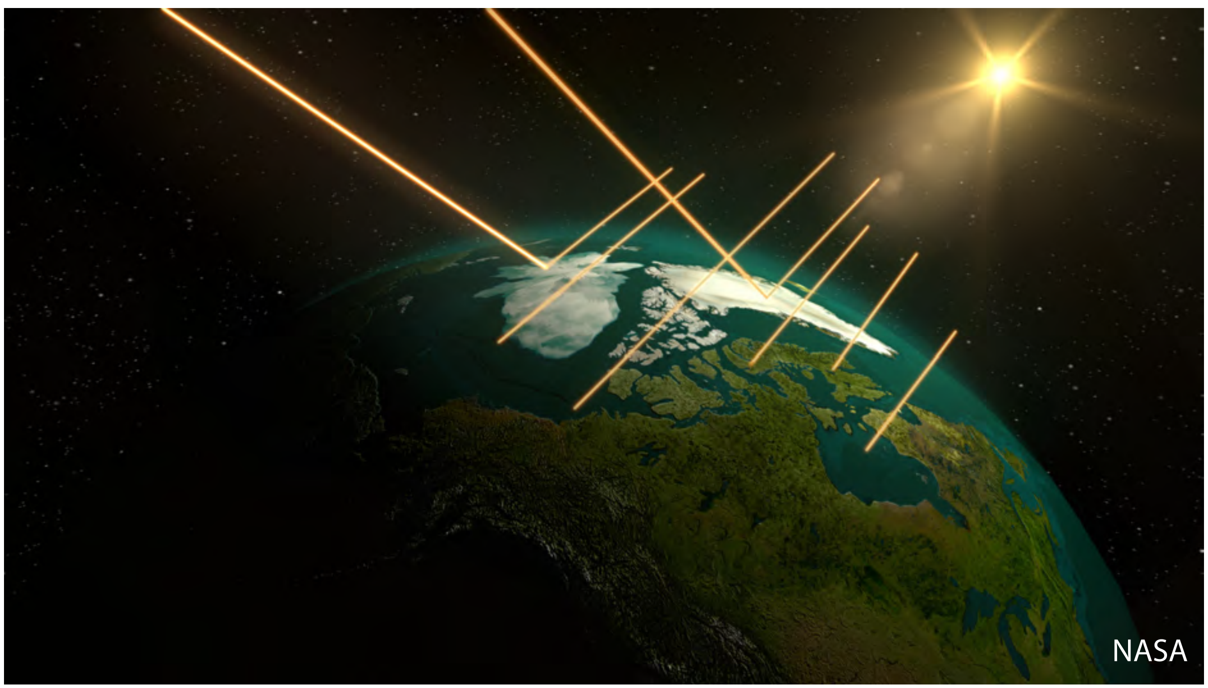
$$(1-\alpha)\pi R^2 S_0$$

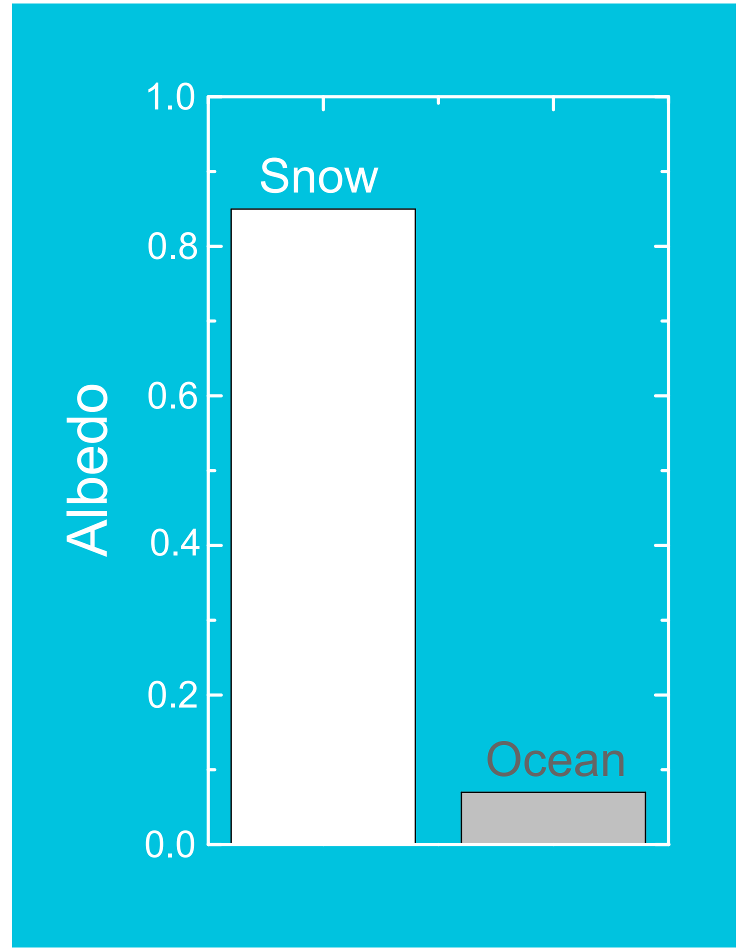
amount of solar energy reaching Earth's surface per unit time / unit area

$$E_{in} = \frac{(1-\alpha)\pi R^2 S_0}{4\pi R^2} = \frac{(1-\alpha)S_0}{4} = (1-\alpha)Q$$

polar ice caps critical to global climate in reflecting incoming solar radiation

white snow and ice reflect





dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

albedo of Earth's surface

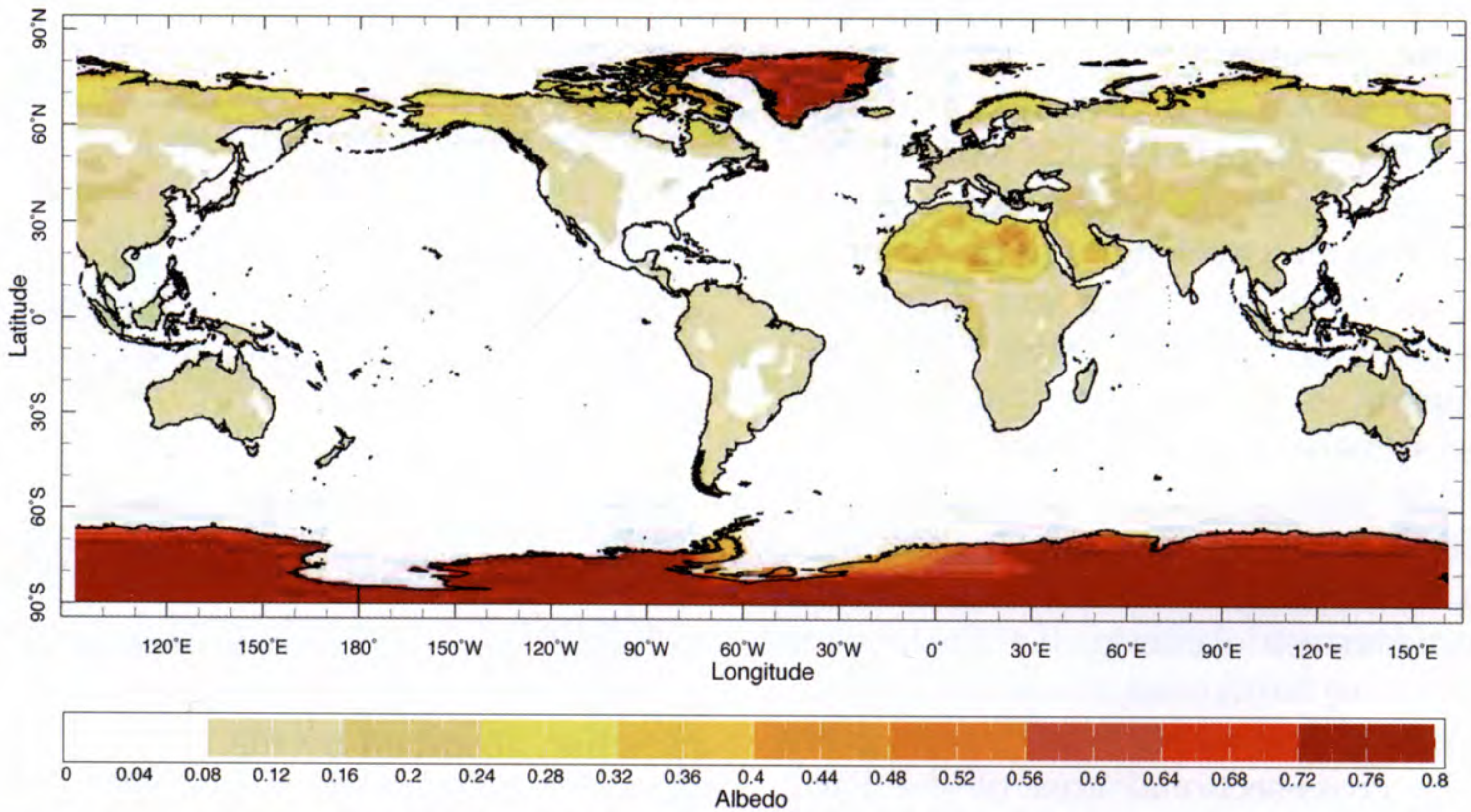


FIGURE 2.5. The albedo of the Earth's surface. Over the ocean the albedo is small (2–10%). It is larger over the land (typically 35–45% over desert regions) and is particularly high over snow and ice (~80%) (see Table 2.2).

average albedo ~ 0.3

Marshall and Plumb 2008

How much energy radiates out into space from a ball at temperature T?

Earth radiates mostly in infrared spectrum, energy radiated depends on T
(approximate as a "black body")

$$E_{out} = \sigma T^4$$

$$\sigma = 5.67 * 10^{-8}\,\mathrm{Wm}^{-2}\mathrm{K}^{-4}$$

Stefan - Boltzmann constant

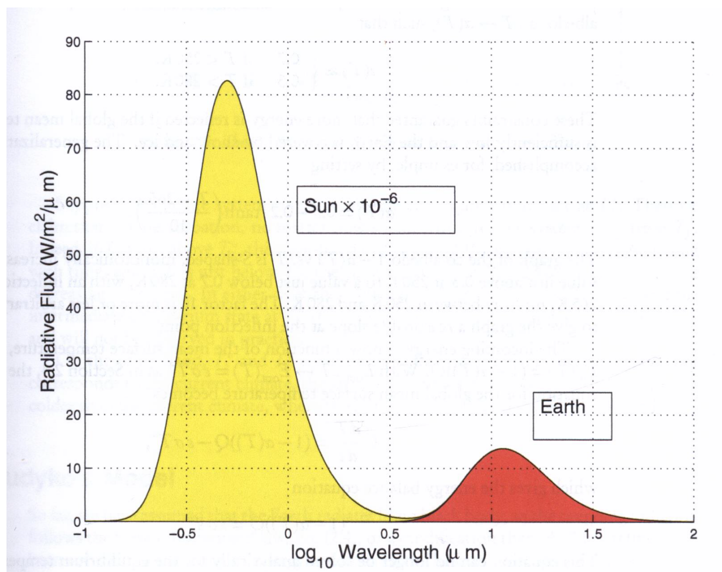


Figure 2.4. *Black-body emission curves of the Sun and the Earth. Reprinted courtesy of NASA.*

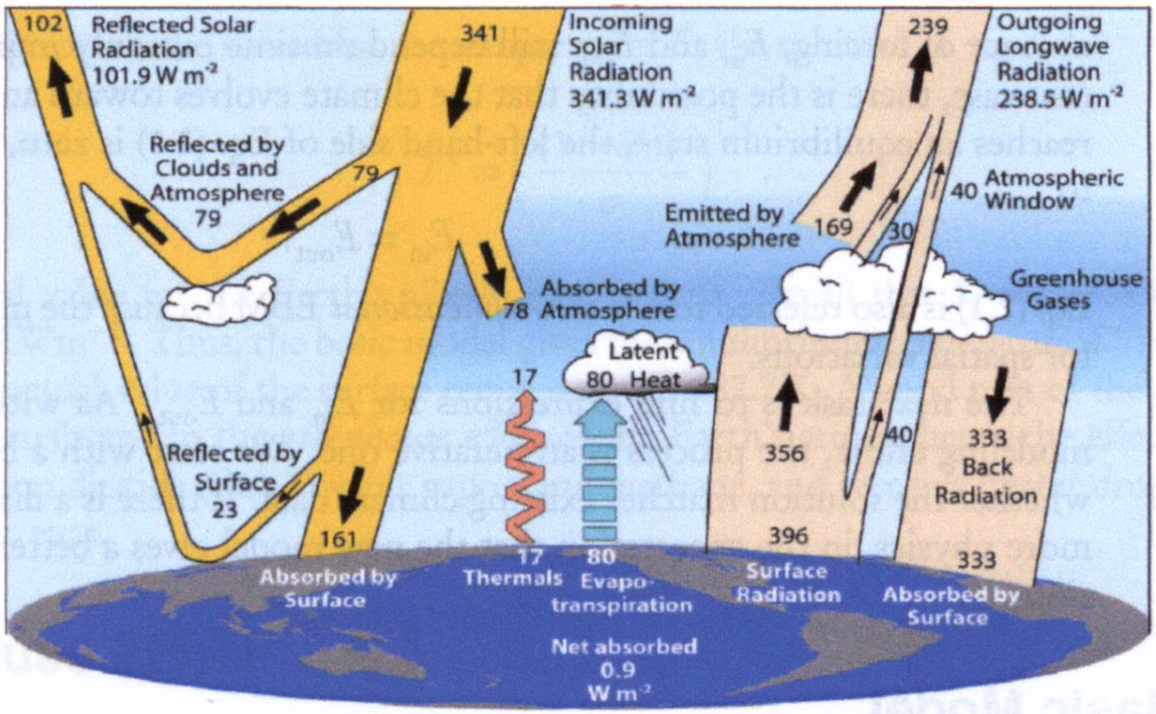


Figure 2.3. *Detailed radiative energy balance* [112].

Energy Balance Model

$$C\,\frac{dT}{dt} = E_{in} - E_{out}$$

C = heat capacity
= energy needed to raise temperature by one Kelvin

rate of change of mean global temperature

$$C\,\frac{dT}{dt} = (1-\alpha)Q - \sigma T^4$$

Find steady-state equilibrium temperature: $C\frac{dT}{dt} = E_{in} - E_{out}$

$$\frac{dT}{dt} = 0$$

constant temperature rate of change = 0

$$E_{in} = E_{out}$$

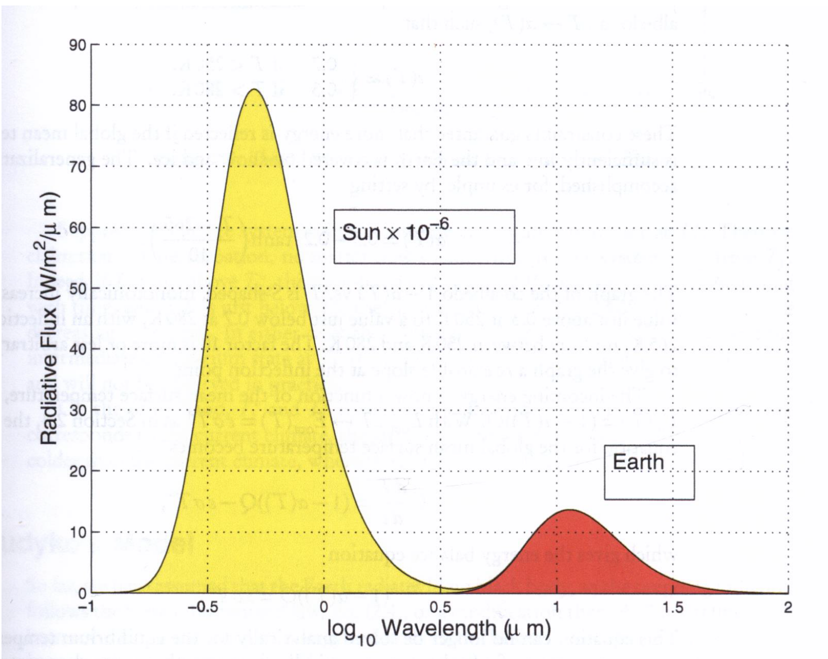
solve $$\sigma T^4 = (1-\alpha)Q$$

$$T_{eq} = \left(\frac{(1-\alpha)\,Q}{\sigma}\right)^{1/4}$$

$T_{eq} \approx 254.8\,\mathrm{K} = -1.03\,°\mathrm{F}$, which is quite cold!

The actual measured temperature of the surface of the Earth is $287.89\,\mathrm{K} = 58.5\,°\mathrm{F}$

Pretty good .... but what are we missing?



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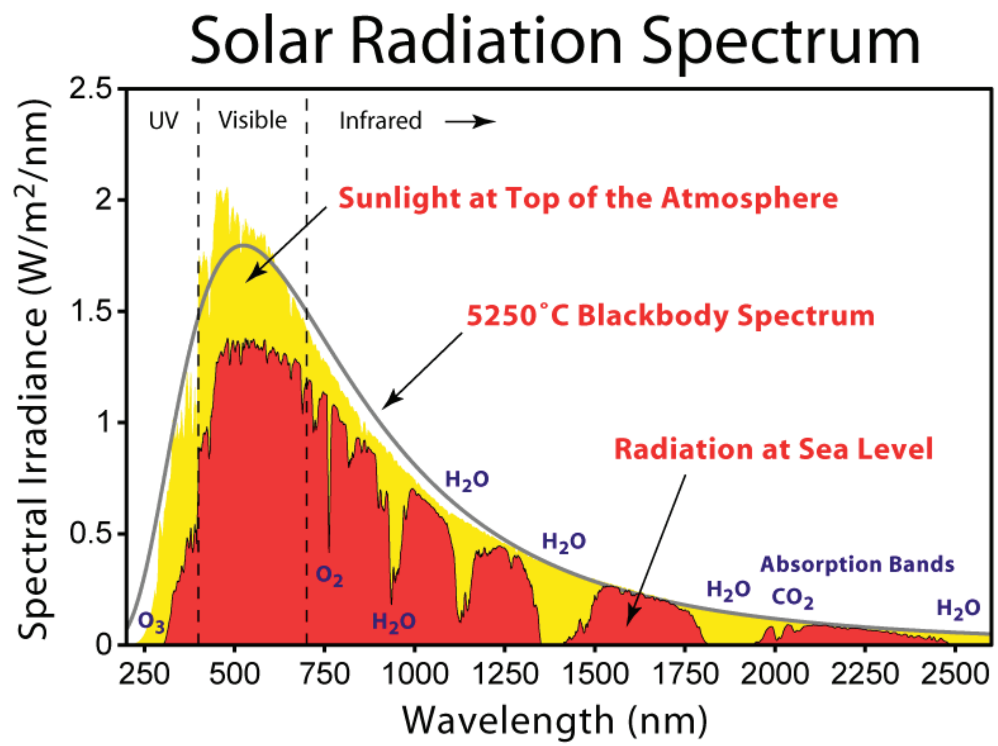
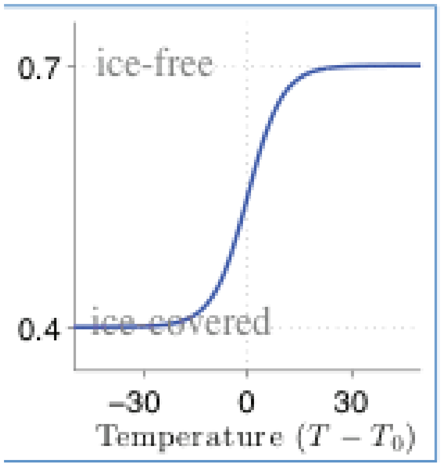


greenhouse effect

Assuming albedo independent of Earth's temperature too simplified....

planetary co-albedo

$$1-\alpha$$

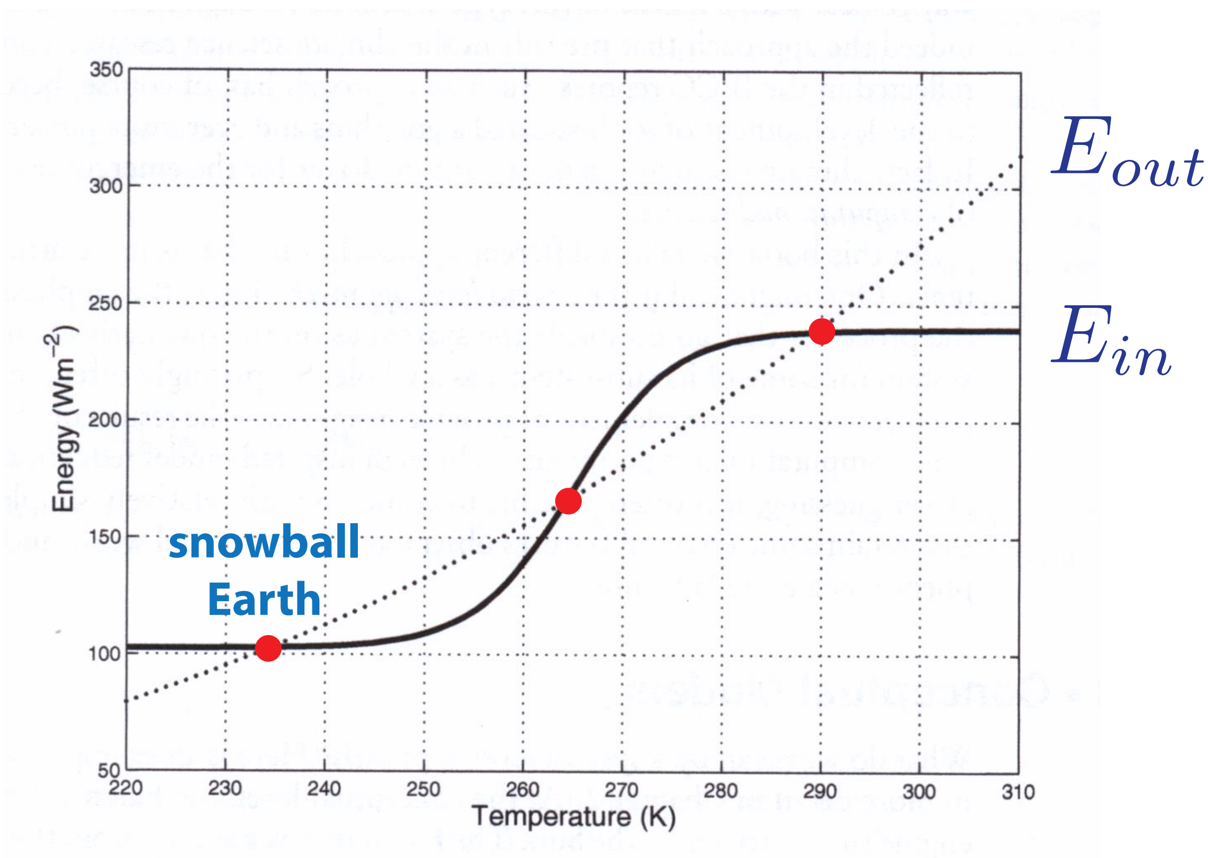


albedo nonlinear in temperature

Now solve for equilibrium temperature $E_{in} = E_{out}$

$$\sigma T^4 = (1 - \alpha(T))Q$$

3 solutions!
multiple equilibria



stable *vs.* unstable equilibria

Bifurcation Diagram

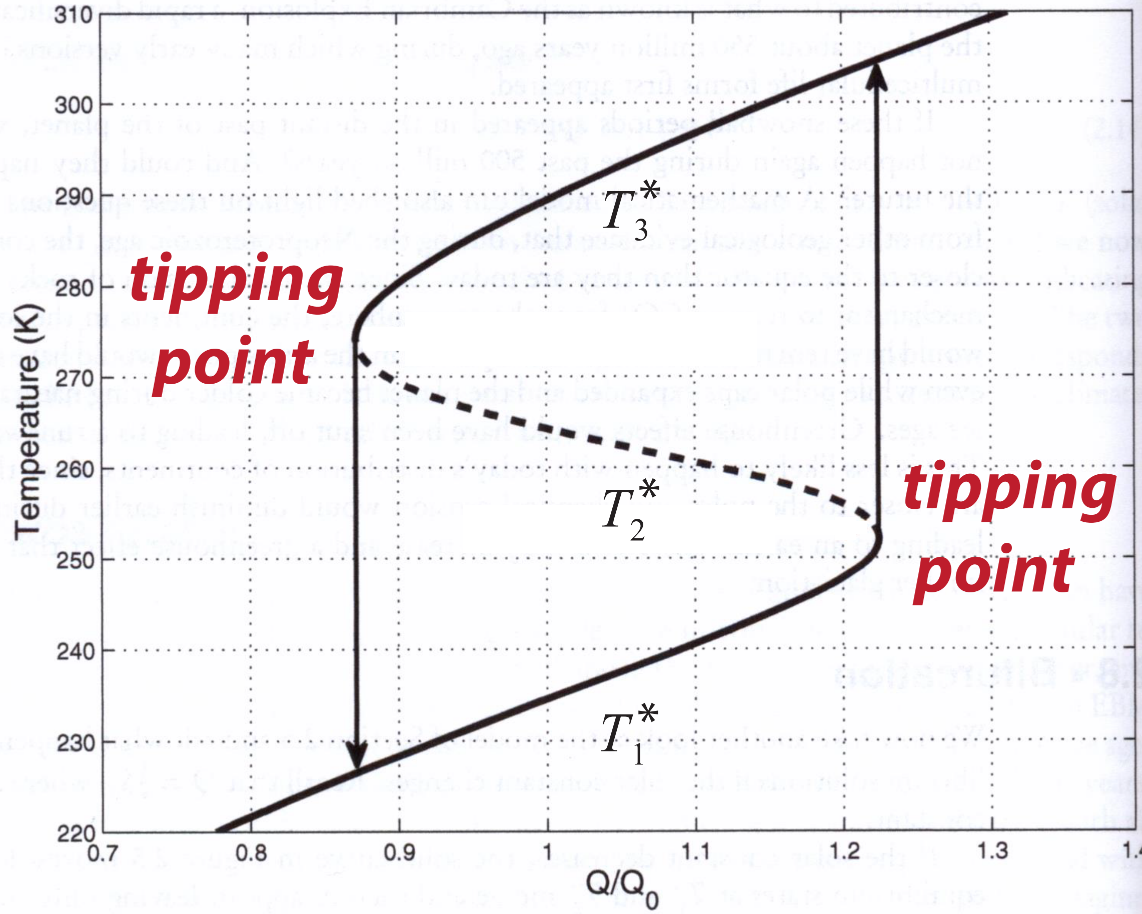


Figure 2.6. *Mean surface temperatures at equilibrium as a function of the solar constant (in units of its present value).*