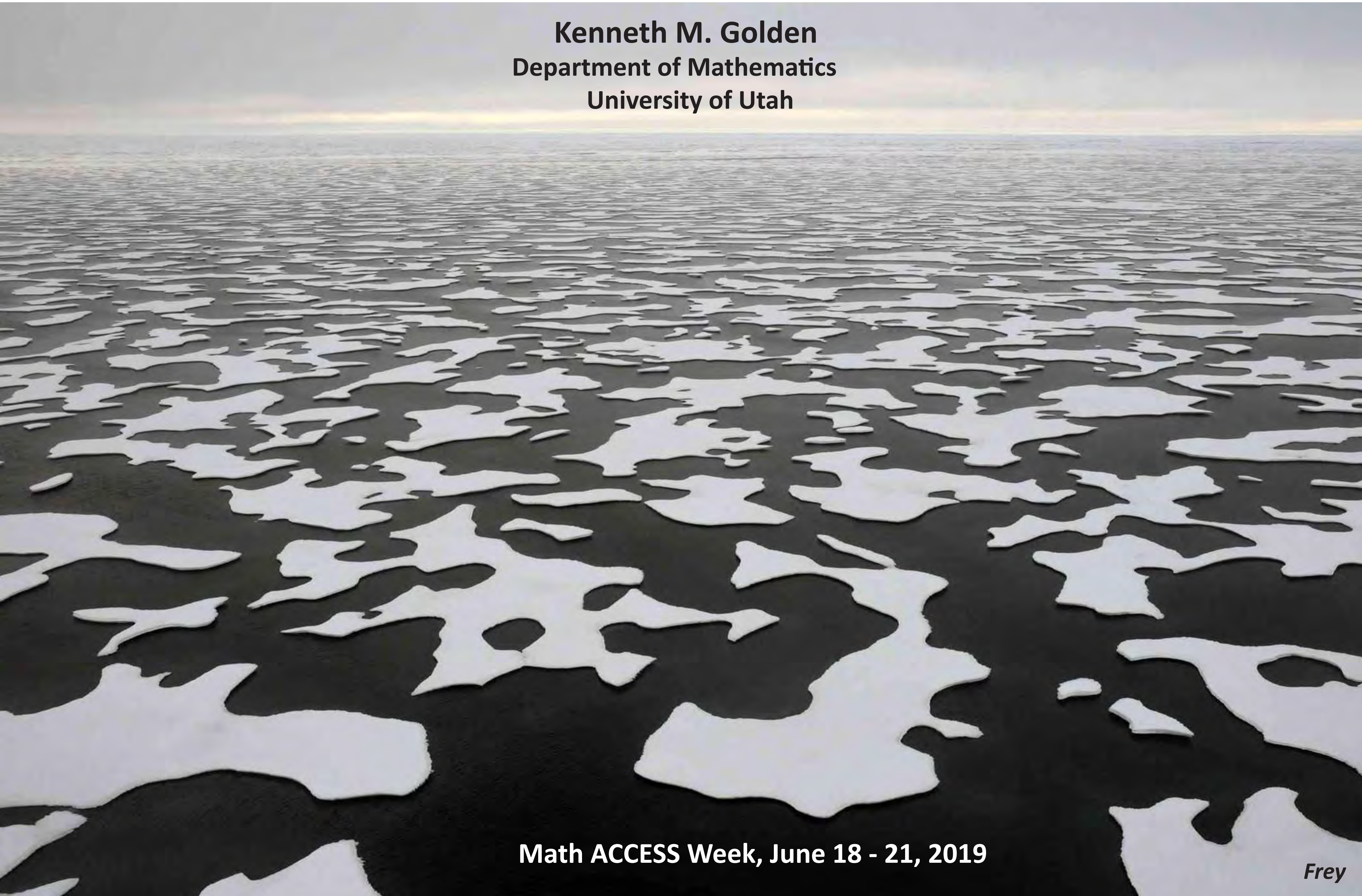
MODELING the MELT:
what math tells us about disappearing polar sea ice



Kenneth M. Golden
Department of Mathematics
University of Utah

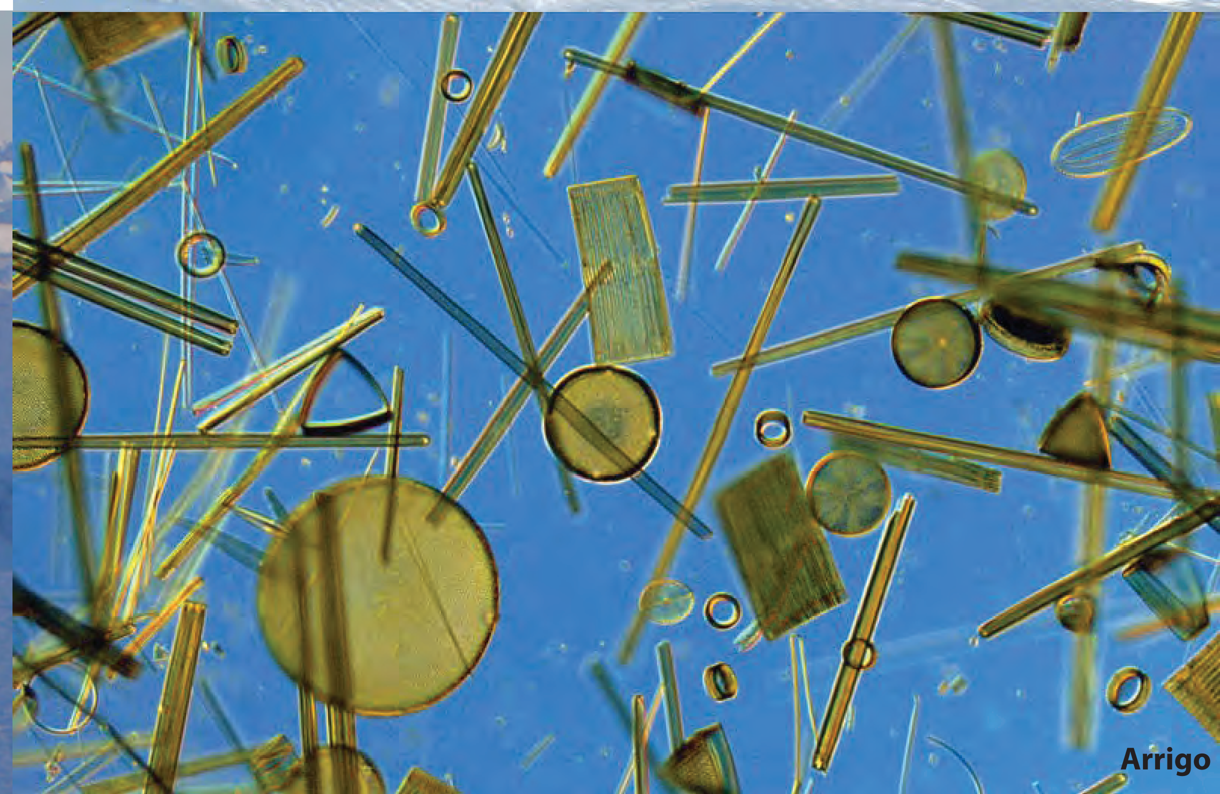
Math ACCESS Week, June 18 - 21, 2019

Frey

Microbial Ecology and the Physics of Sea Ice

Kenneth M. Golden
Department of Mathematics
University of Utah

Golden

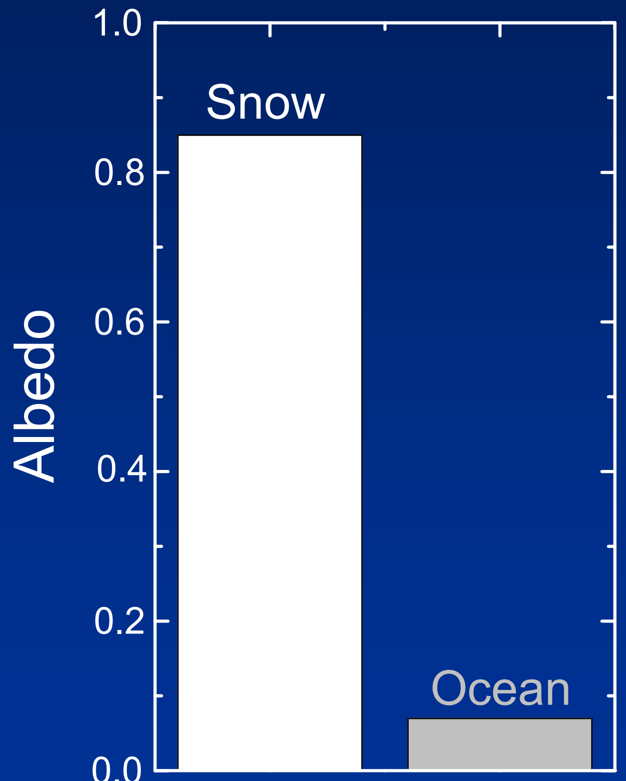


Arrigo

SEA ICE covers ~12% of Earth's ocean surface

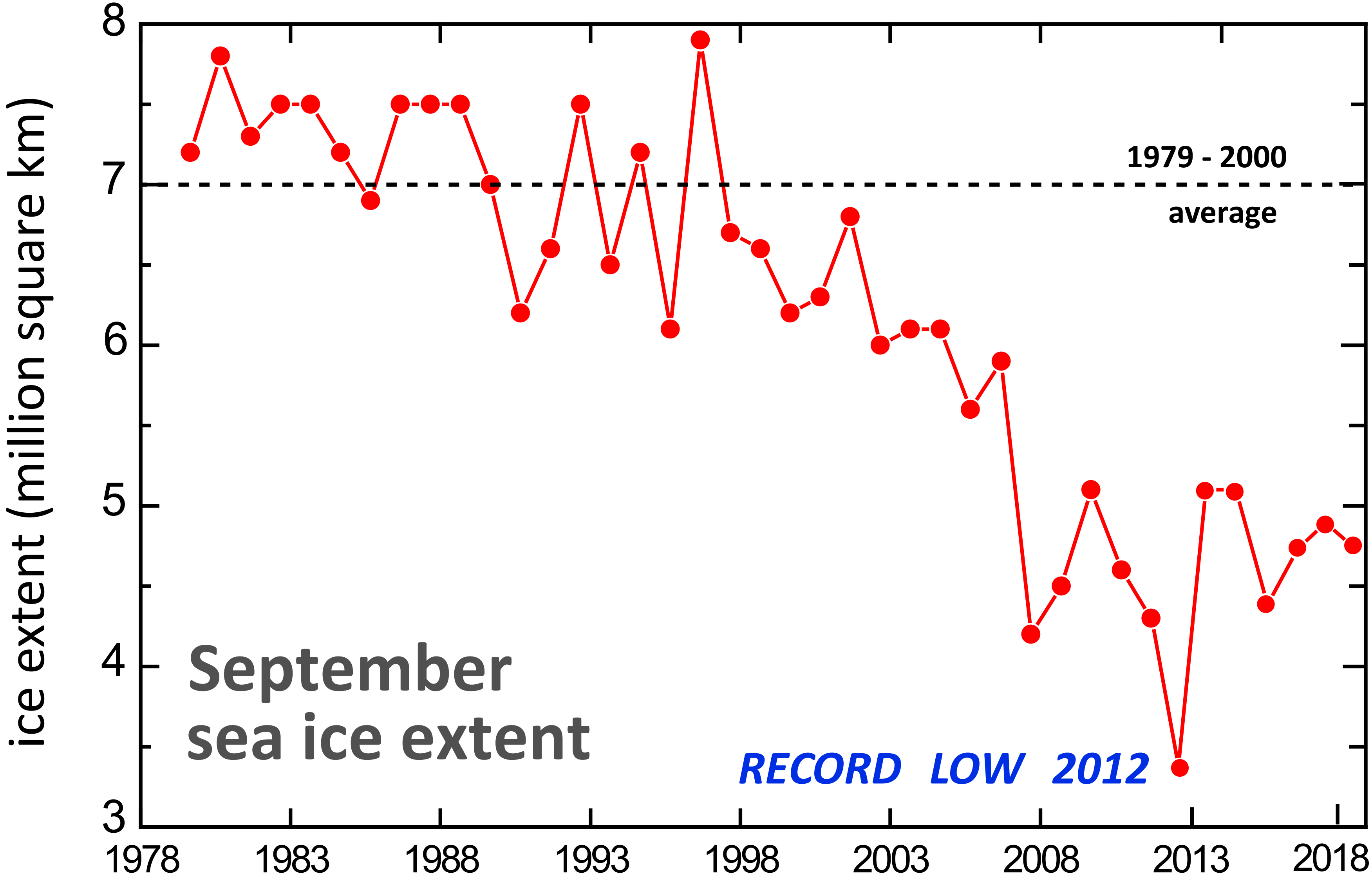
- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of climate change

$$\textit{Albedo} = \frac{\textit{reflected sunlight}}{\textit{incident sunlight}}$$



Albedo = fraction of sunlight reflected (Perovich)

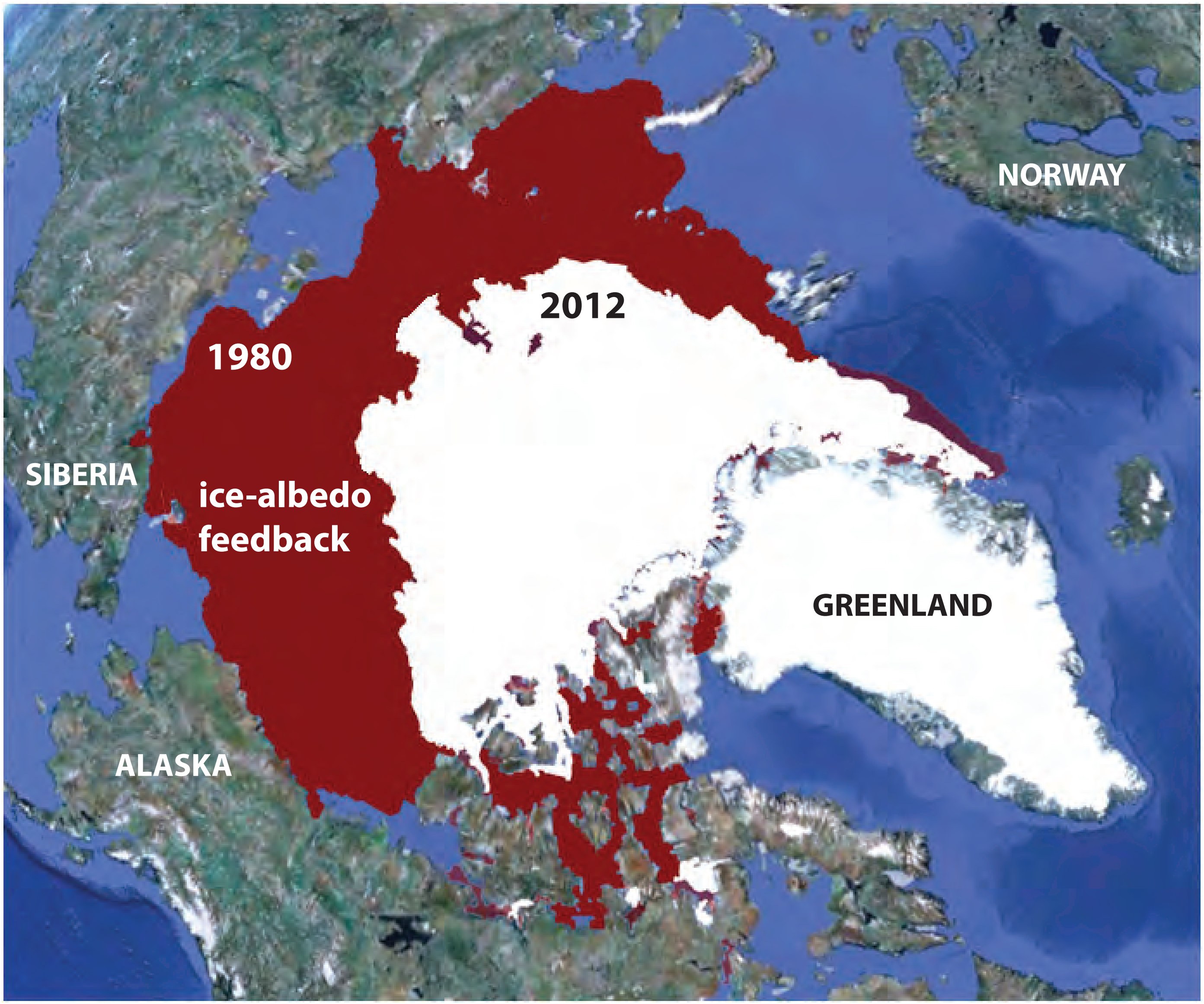
the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

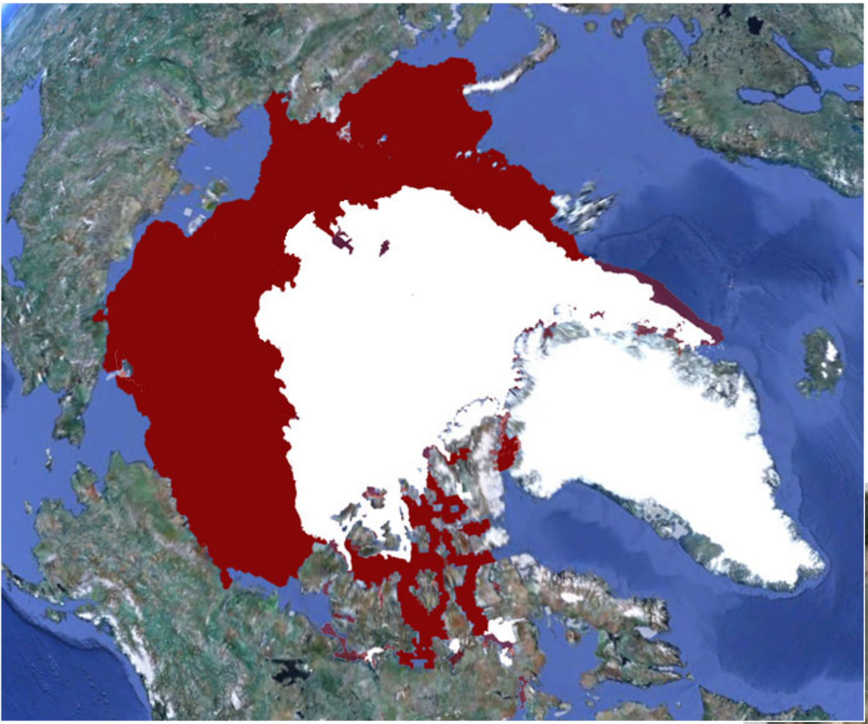
Change in Arctic Sea Ice Extent

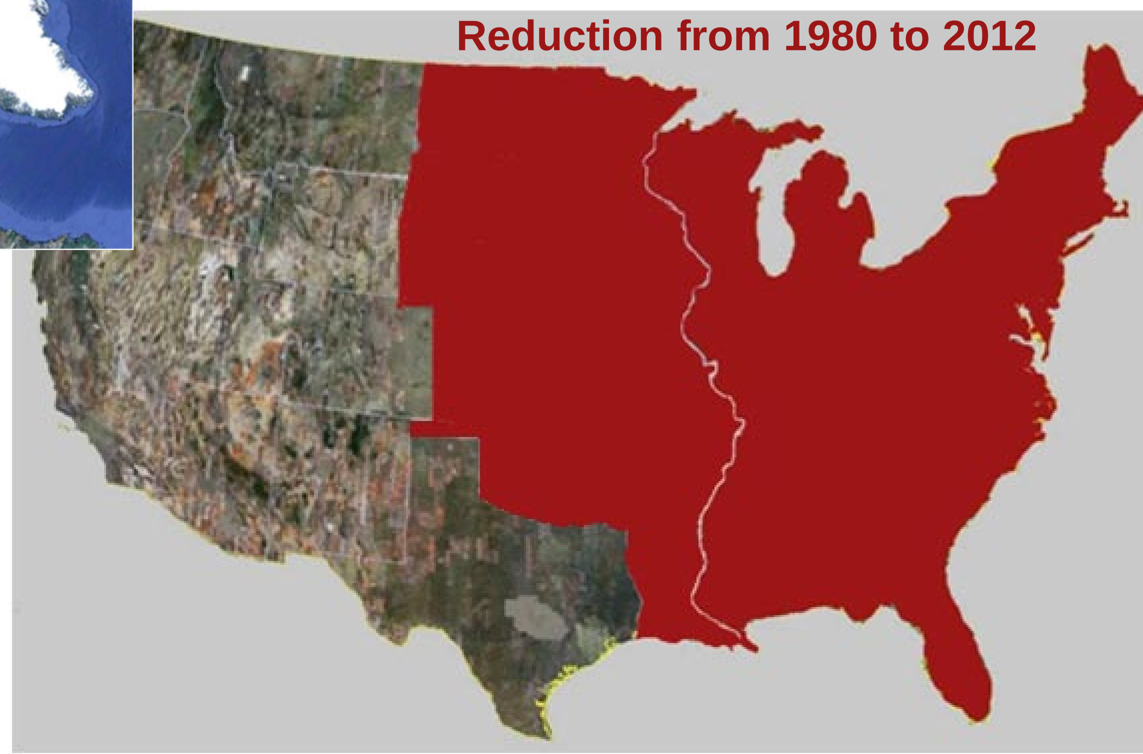
September 1980 -- 7.8 million square kilometers
September 2012 -- 3.4 million square kilometers



Perovich

Changes in sea ice extent



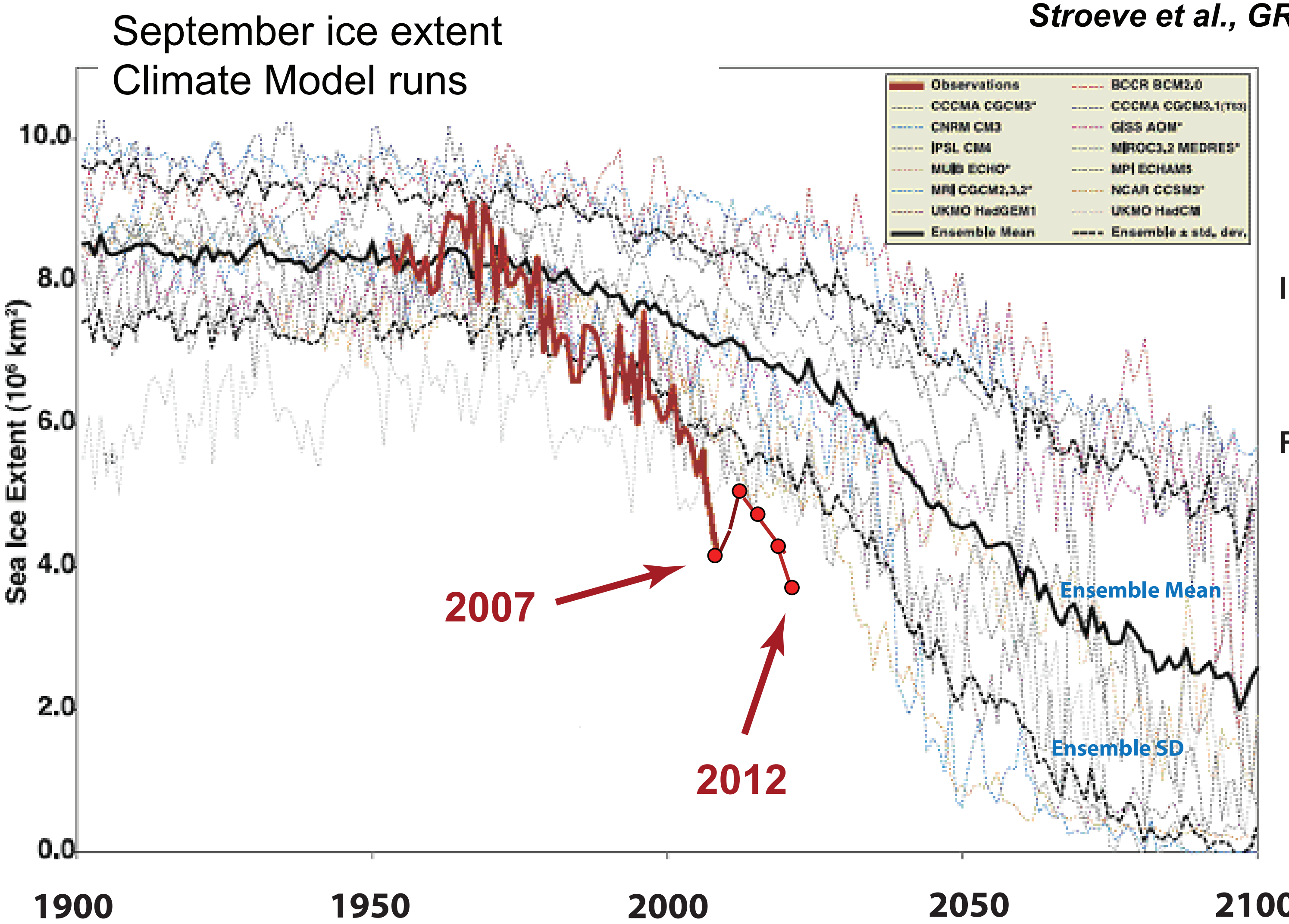


Huge decrease in ice extent

Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007

Stroeve et al., GRL, 2012



IPCC AR4 Models

Intergovernmental Panel on Climate Change (IPCC)

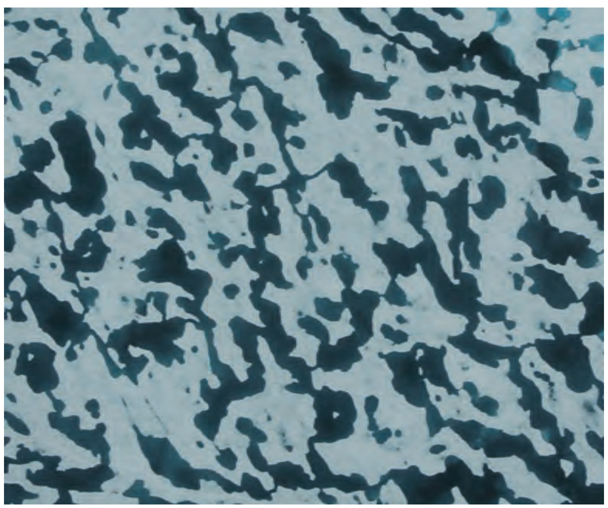
Fourth Assessment AR4, 2007

challenge

represent sea ice more rigorously in climate models

account for key processes

such as melt pond evolution



Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

For simulations with ponds September ice volume is nearly 40% lower.

... and other sub-grid scale structures and processes

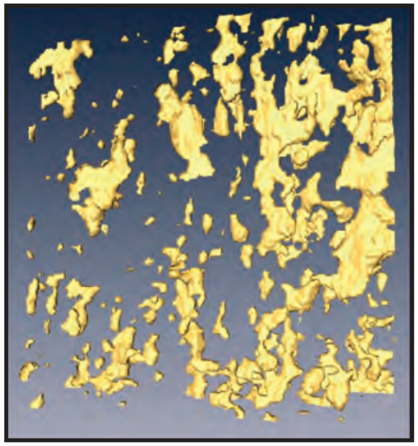
linkage of scales

Sea Ice is a Multiscale Composite Material

sea ice microstructure

brine inclusions

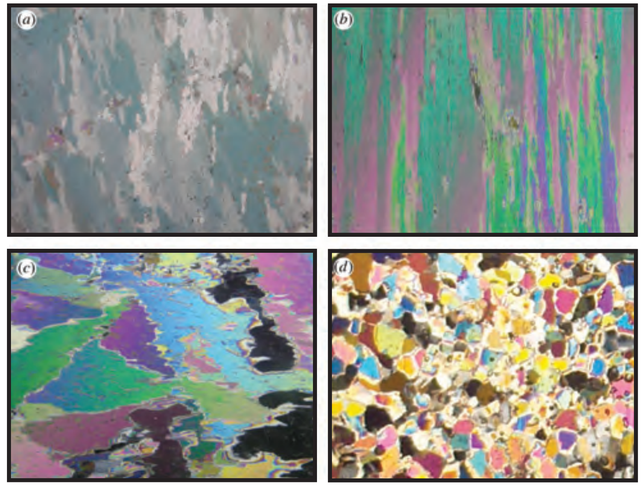
Weeks & Assur 1969



H. Eicken

Golden *et al. GRL* 2007

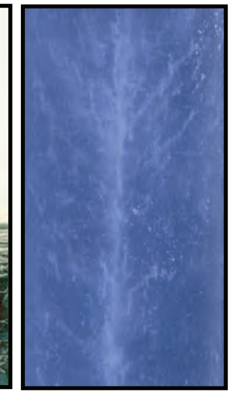
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

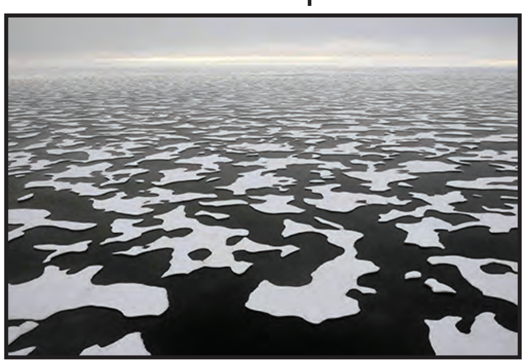
millimeters

centimeters

---

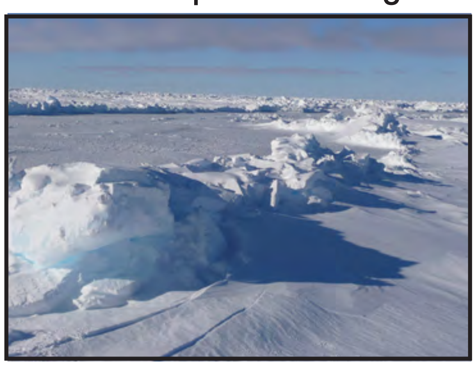
sea ice mesostructure

Arctic melt ponds



K. Frey

Antarctic pressure ridges



K. Golden

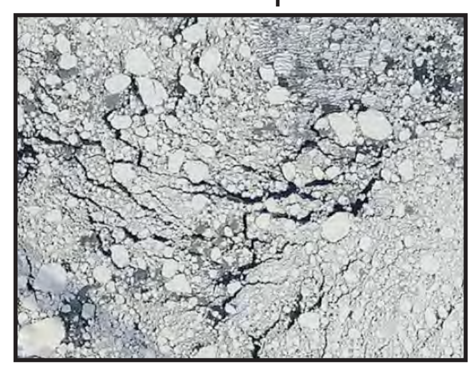
meters

sea ice macrostructure

sea ice floes

J. Weller

sea ice pack



NASA

kilometers

What is this talk about?

1. LIFE IN THE ICE

   sea ice microphysics and fluid transport

2. LIFE UNDER THE ICE

   melt ponds, under-ice light field, algal blooms

3. Species competition - resources depend on climate

Solving problems in physics and biology of sea ice drives advances in theory of composite materials and ecological systems.

Microbial life IN sea ice

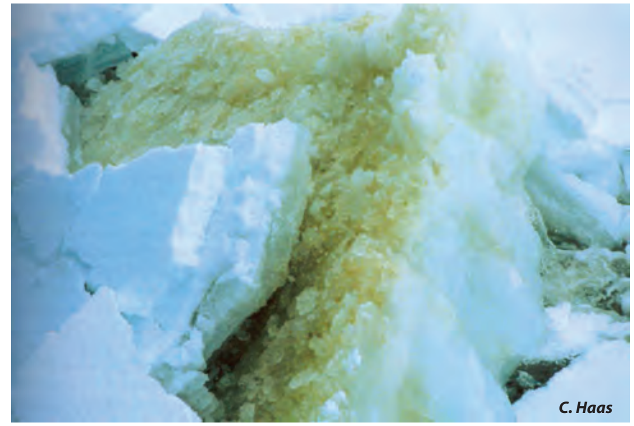
sea ice microphysics

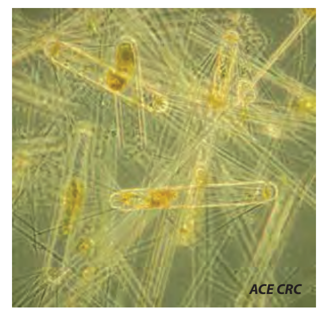
fluid transport

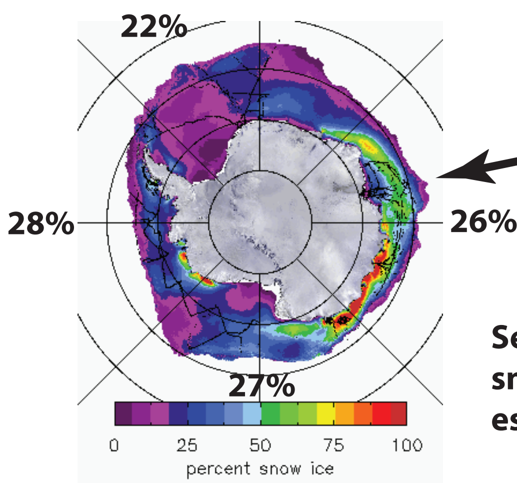
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

nutrient flux for algal communities







T. Maksym and T. Markus, 2008

September snow-ice estimates

Antarctic surface flooding and snow-ice formation

- evolution of salinity profiles

- ocean-ice-air exchanges of heat, CO_2

sea ice ecosystem

sea ice algae
support life in the polar oceans

fluid permeability *k* of a porous medium

porous concrete

how much water gets through the sample per unit time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Darcy's Law for slow viscous flow in a porous medium

averaged fluid velocity

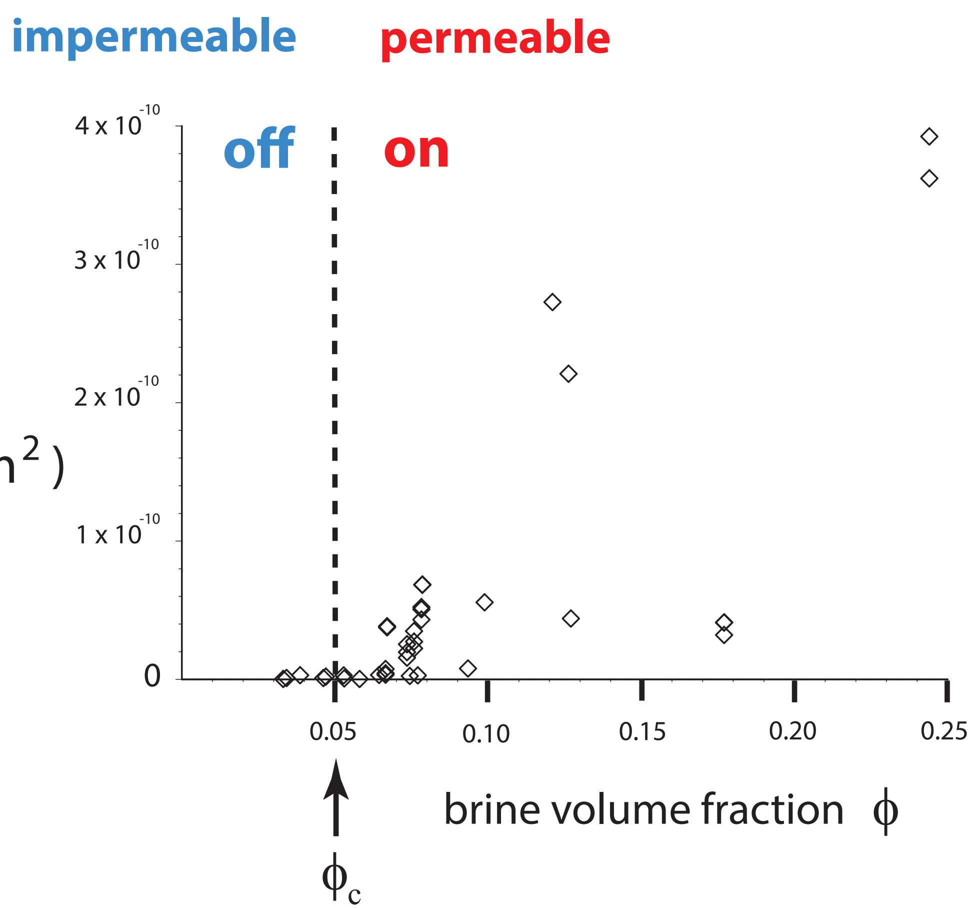
pressure gradient

$$\mathrm{v} = -\frac{\mathbf{k}}{\eta}\,\nabla p$$

viscosity

$\mathbf{k}$ = fluid permeability tensor

Critical behavior of fluid transport in sea ice



Arctic field data

vertical fluid permeability k (m^2)

"on - off" switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ$C, $S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

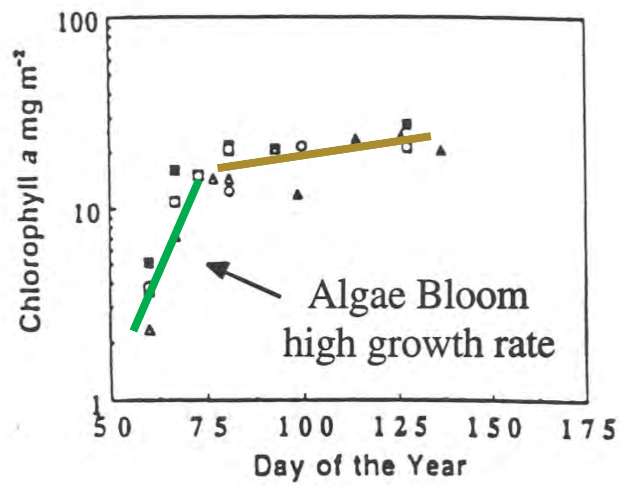
sea ice algal communities

D. Thomas 2004

nutrient replenishment controlled by ice permeability

biological activity turns on or off according to ***rule of fives***

critical behavior of microbial activity



Convection-fueled algae bloom
Ice Station Weddell

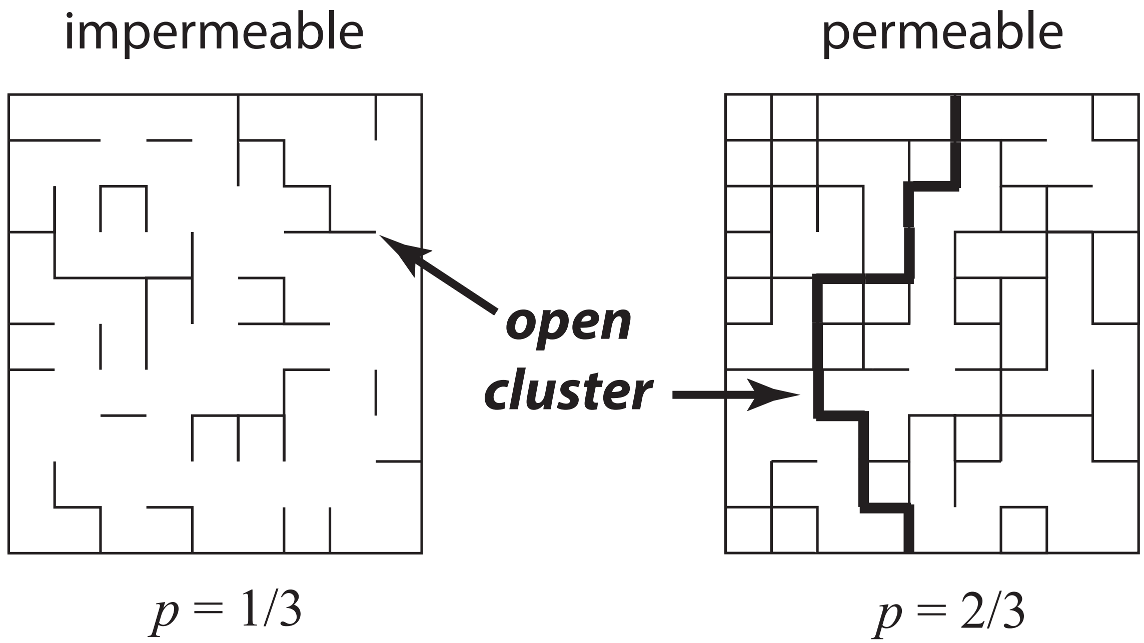
Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond → **open** *with probability* p
closed *with probability* $1-p$

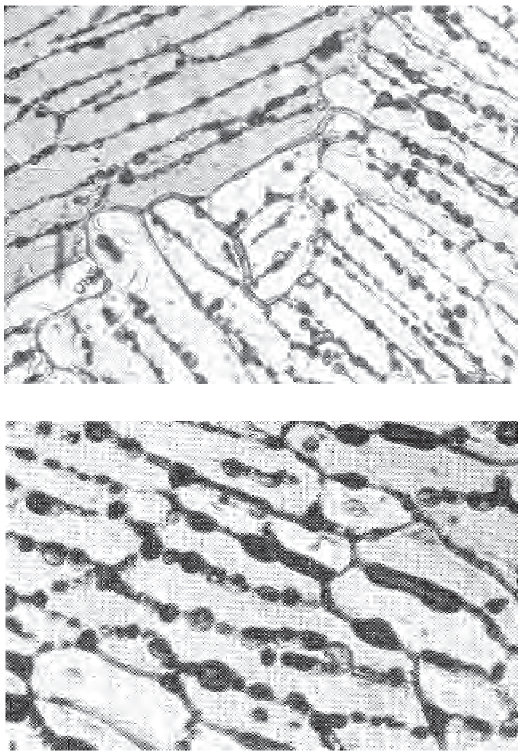
percolation threshold

$$p_c = 1/2 \quad \text{for } d = 2$$

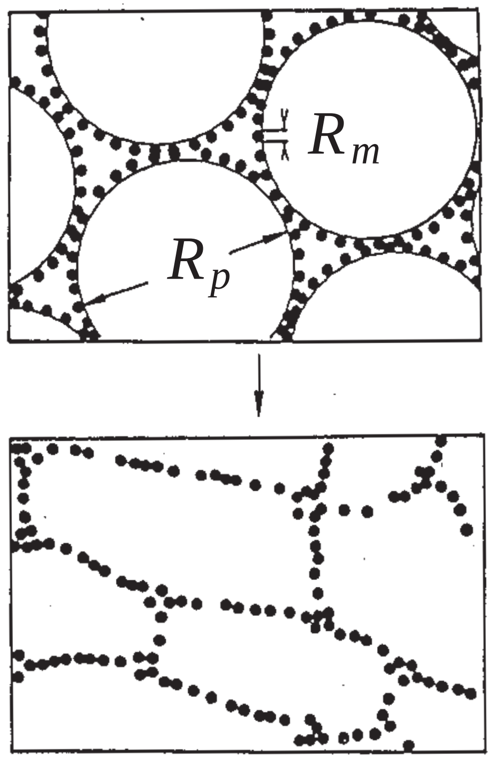
smallest *p* for which there is an infinite open cluster

Continuum percolation model for *stealthy* materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

$\phi_c \approx 5\ \%$ Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed powder

radar absorbing composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

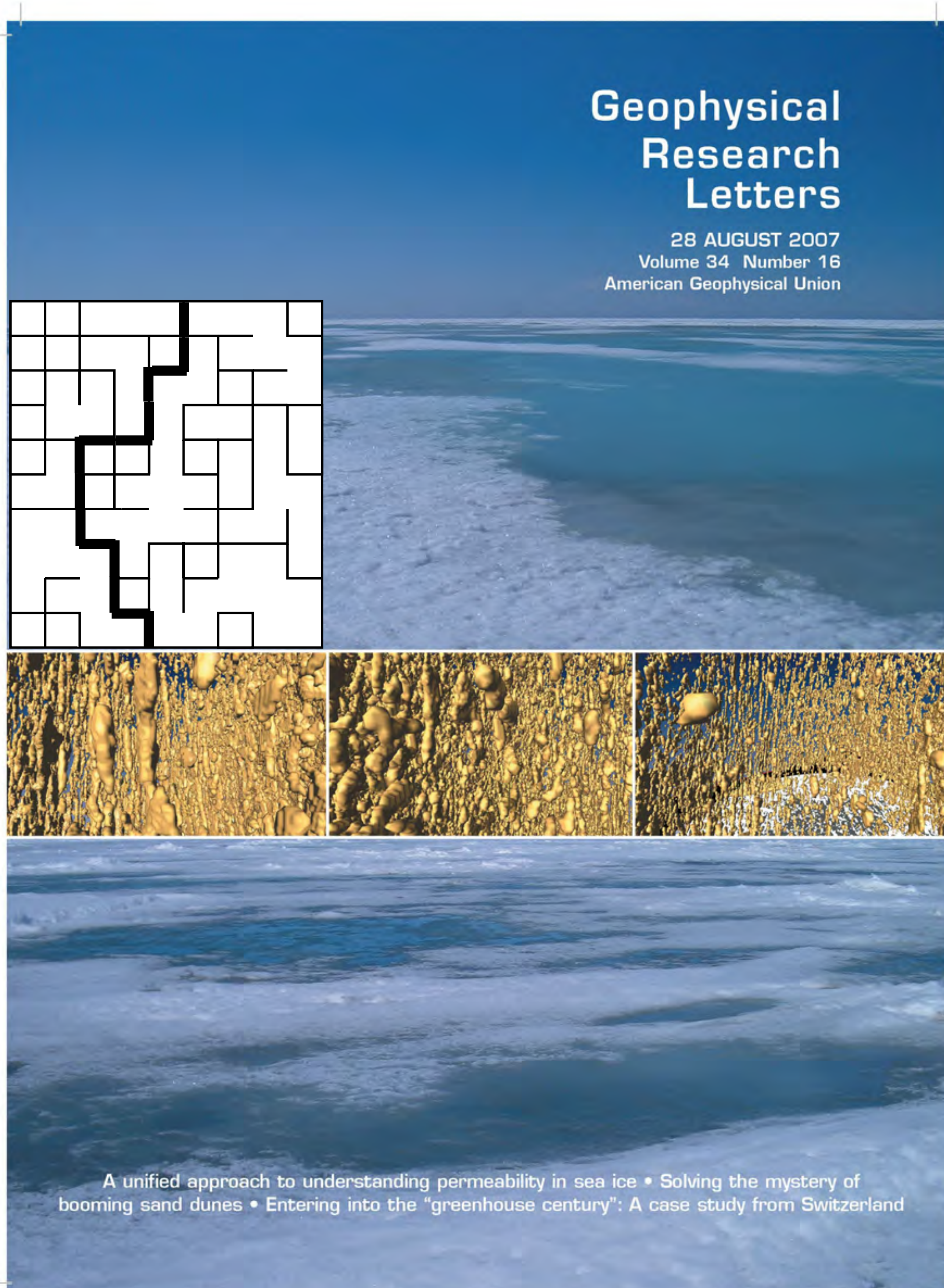
Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007

micro-scale

controls

macro-scale

processes



percolation theory

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8} \ \mathrm{m}^2$$

hierarchical model
network model
rigorous bounds

agree closely
with field data

X-ray tomography for brine inclusions

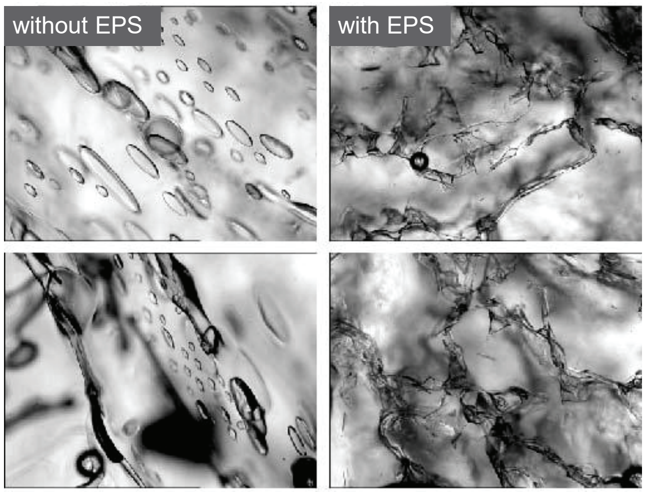
unprecedented look at thermal evolution of brine phase and its connectivity

confirms rule of fives

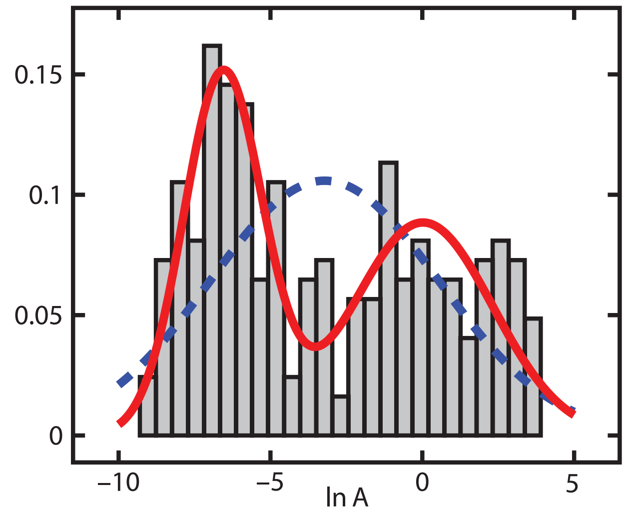
Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

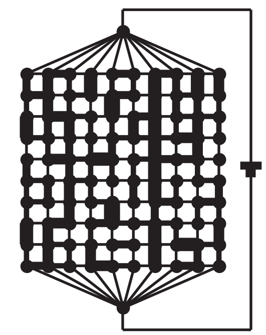
How does EPS affect fluid transport?

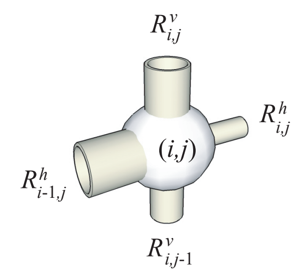


Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL





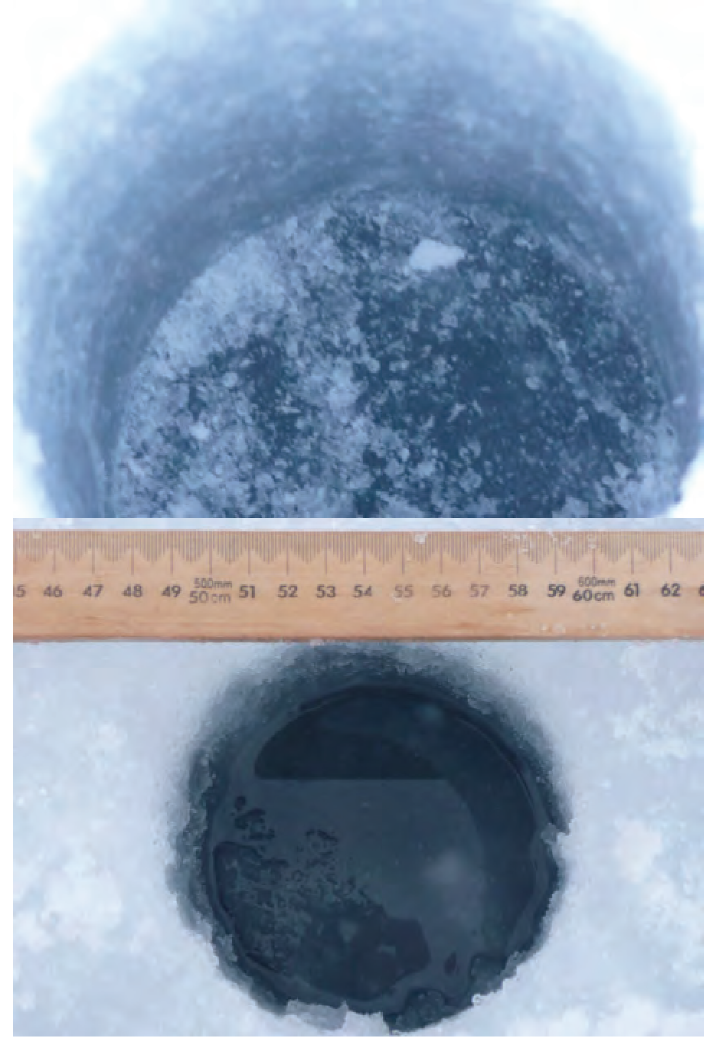
Zhu, Jabini, Golden, Eicken, Morris
Ann. Glac. 2006

- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution; Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability *k*.
- **Rigorous bound on *k* for bimodal distribution of pore sizes**

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

How does the biology affect the physics?

photo by Jan Lieser



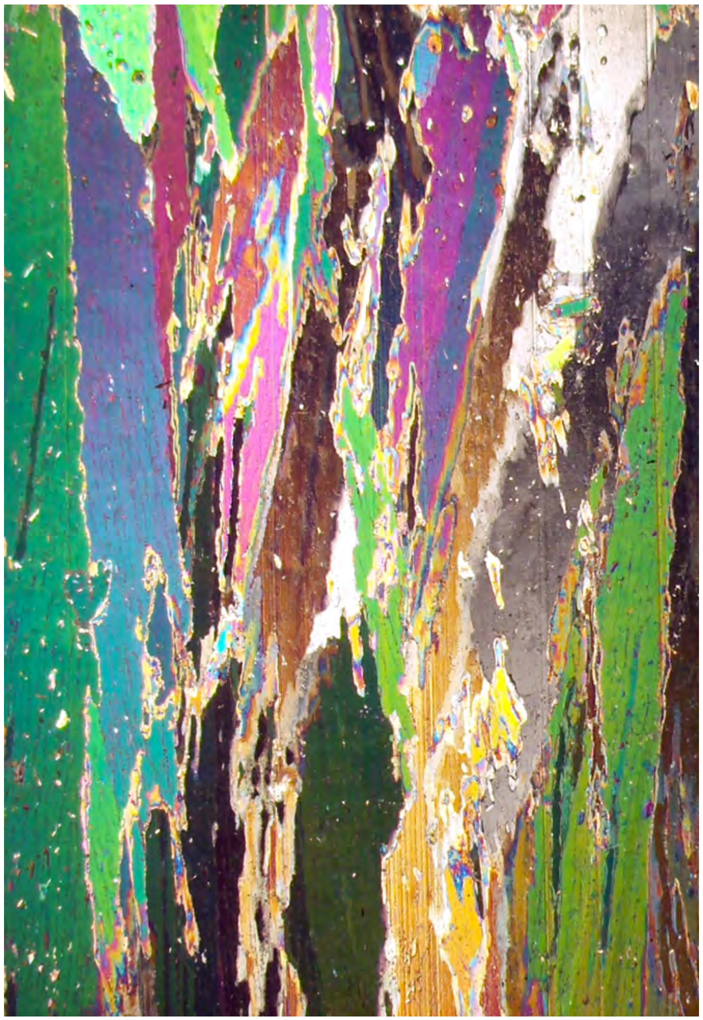
measuring fluid permeability of Antarctic sea ice

SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

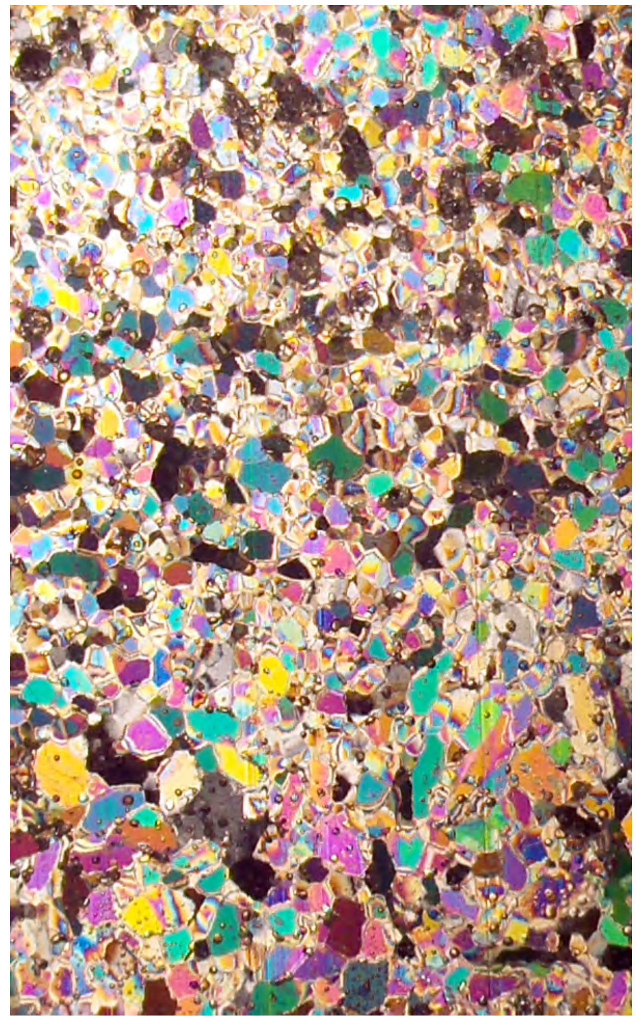
columnar

5%



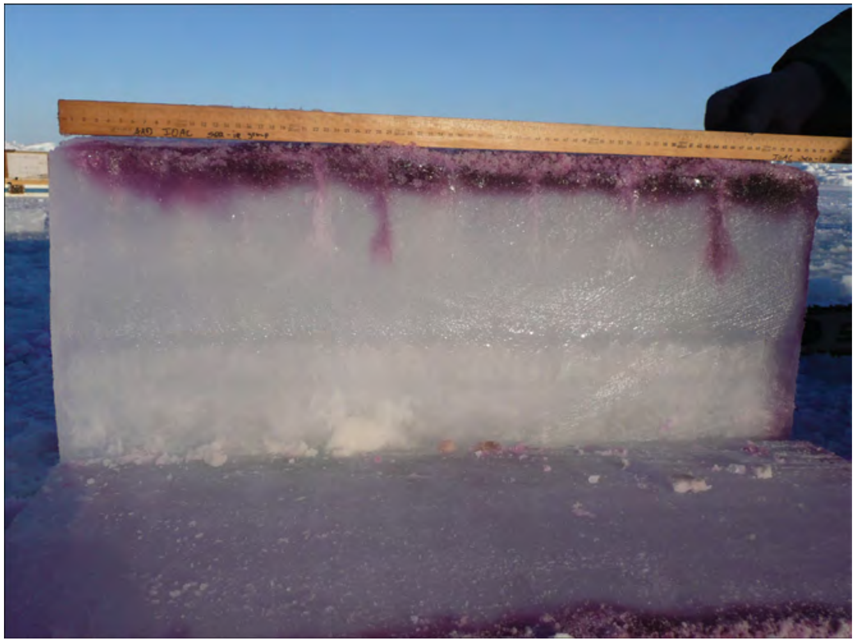
granular

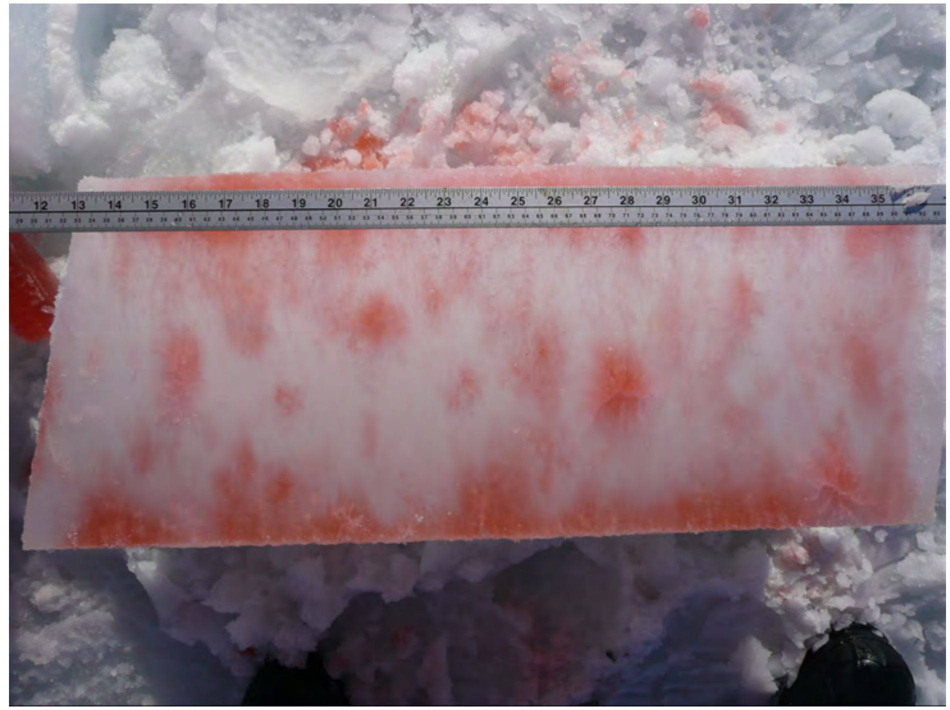
10%

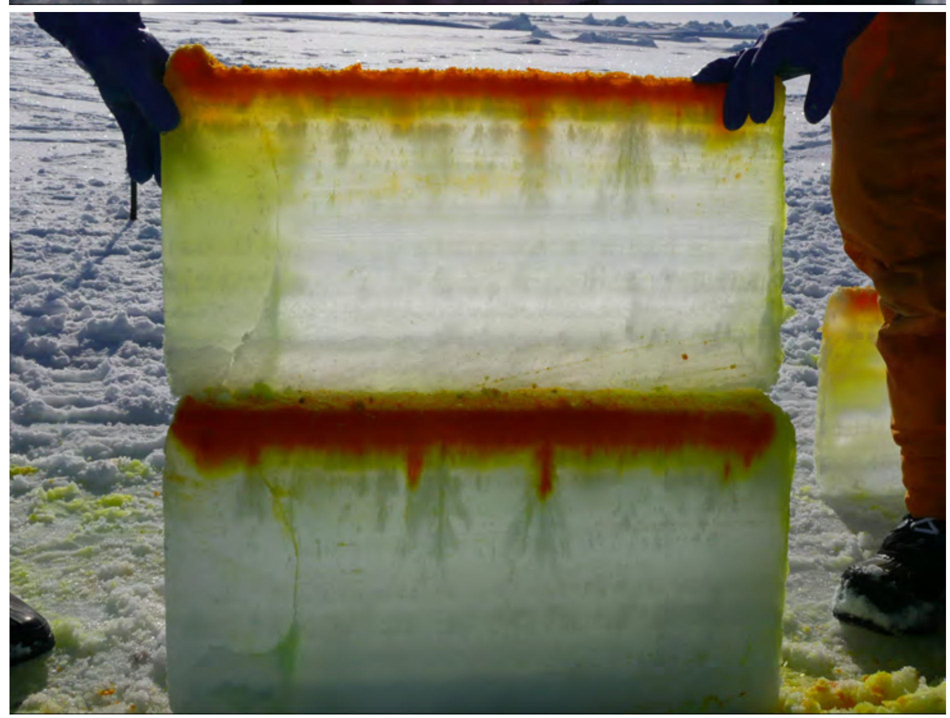


Golden, Sampson, Gully, Lubbers, Tison 2019

tracers flowing through inverted sea ice blocks

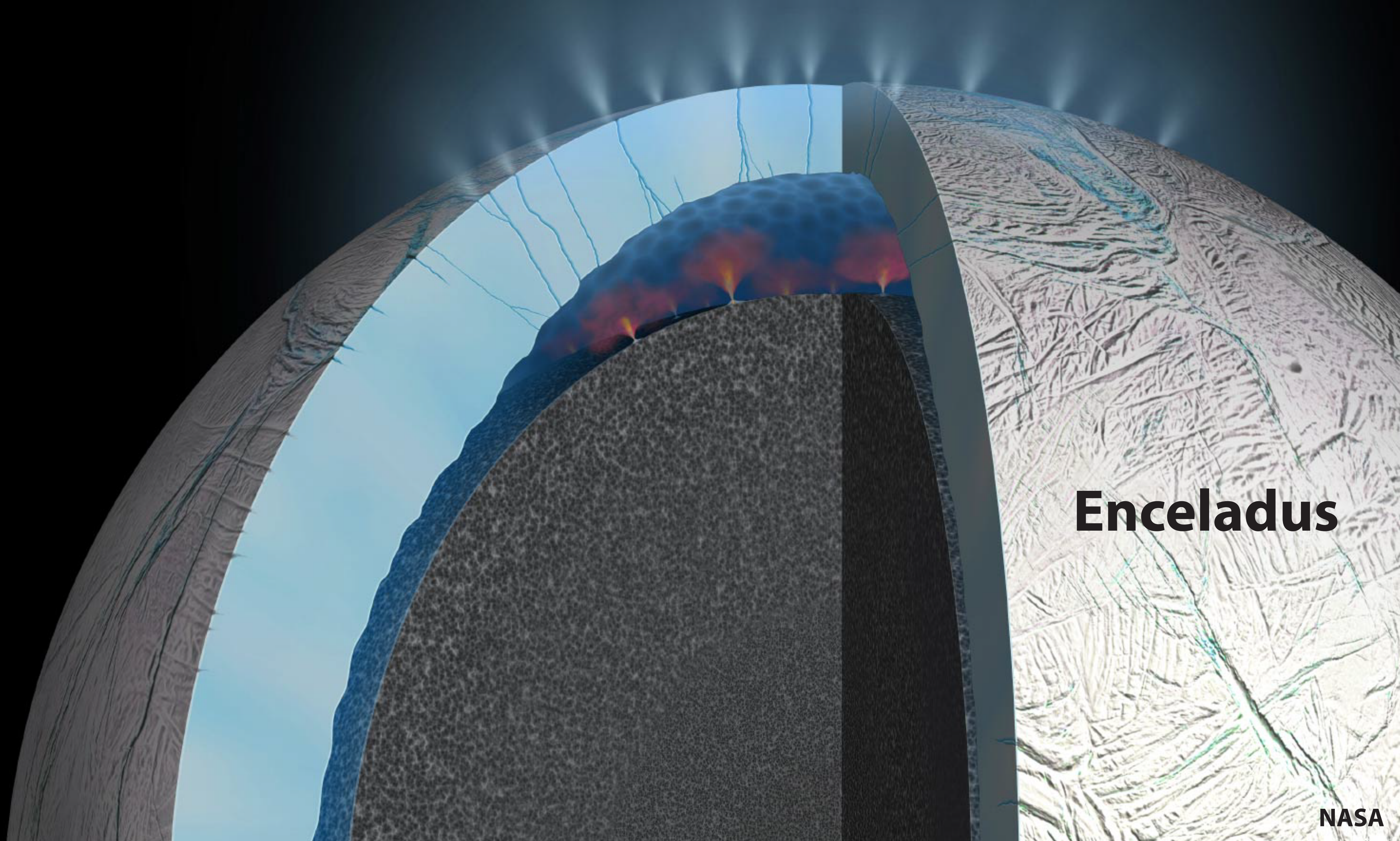






Microbial Habitability in the Icy Moons of Jupiter and Saturn

Ruby Bowers
David Morison
Ken Golden



Remote sensing of sea ice

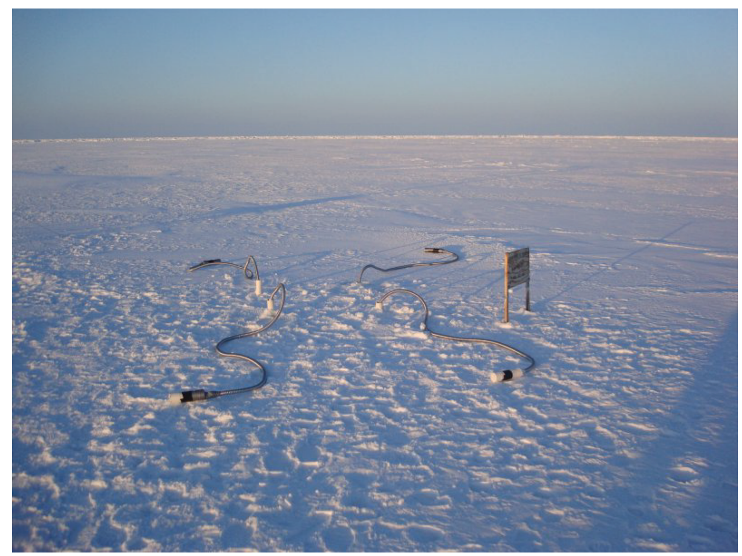
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

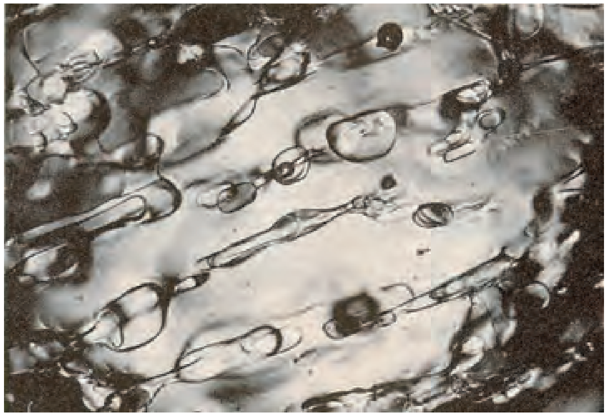
ε^*

effective complex permittivity
(dielectric constant, conductivity)

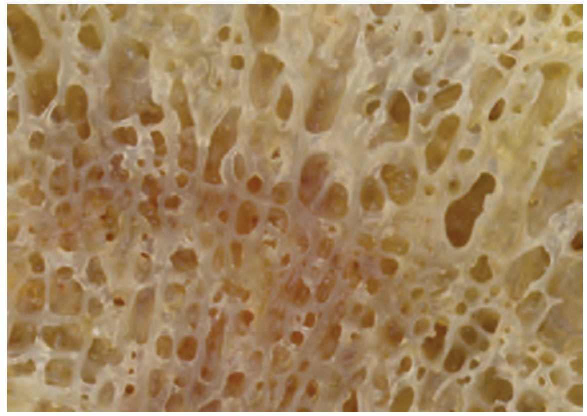


brine volume fraction
brine inclusion connectivity

SEA ICE

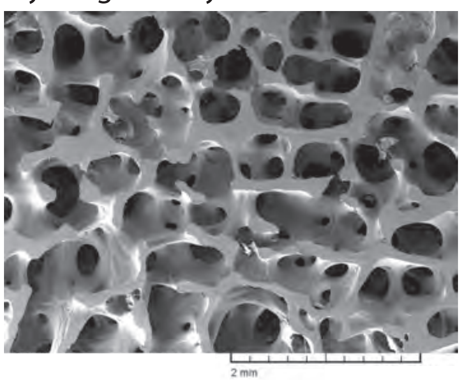


HUMAN BONE

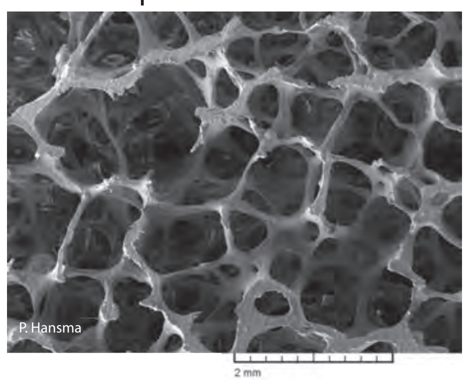


spectral characterization of porous microstructures in human bone

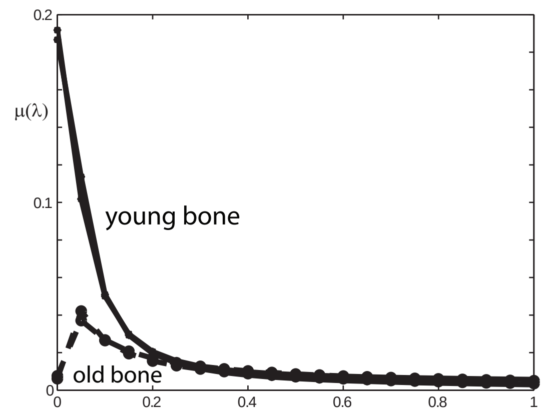
young healthy trabecular bone



old osteoporotic trabecular bone



reconstruct spectral measures from complex permittivity data



use regularized inversion scheme

apply spectral measure analysis of brine connectivity and spectral inversion to electromagnetic monitoring of osteoporosis

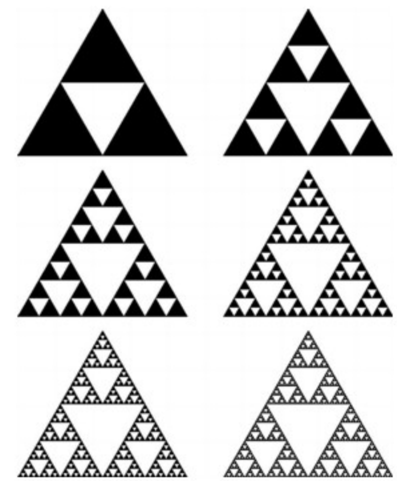
Golden, Murphy, Cherkaev, J. Biomechanics 2011

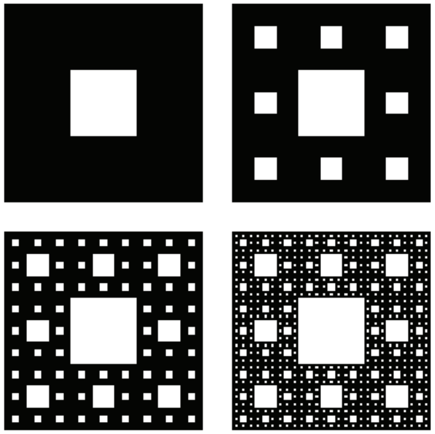
the math doesn't care if it's sea ice or bone!

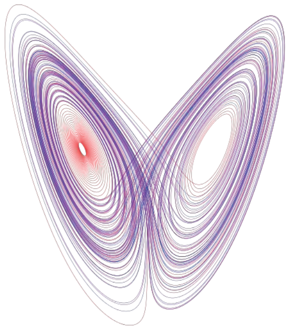
Microbial life UNDER sea ice

melt ponds, algal blooms

fractals and multiscale structure



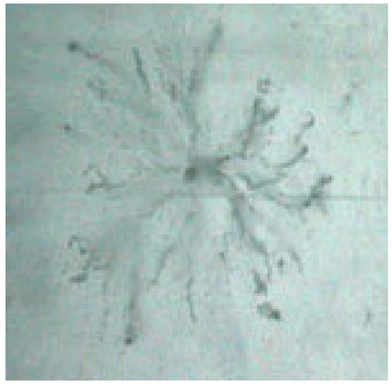




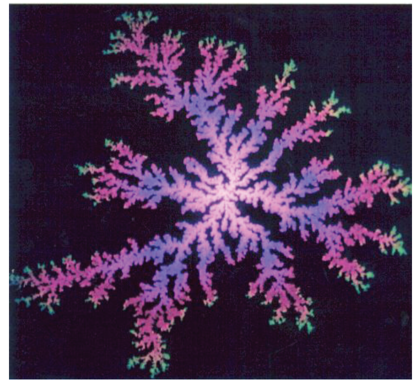
some
fractals

fractal microstructures

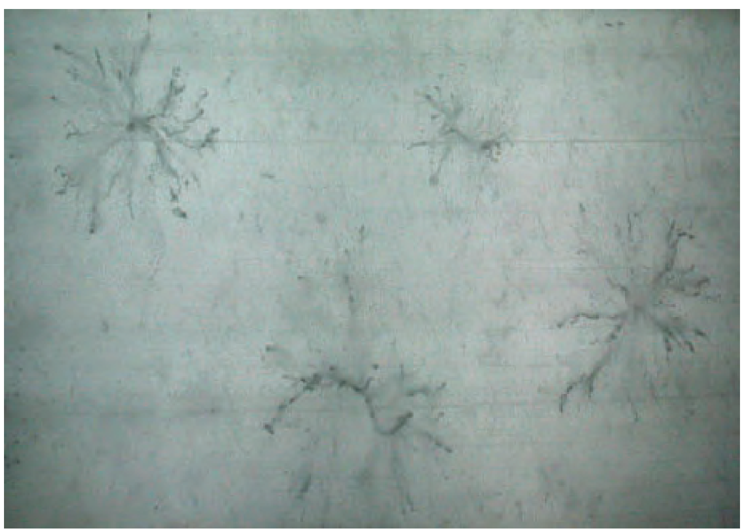
electrorheological fluid
with metal spheres

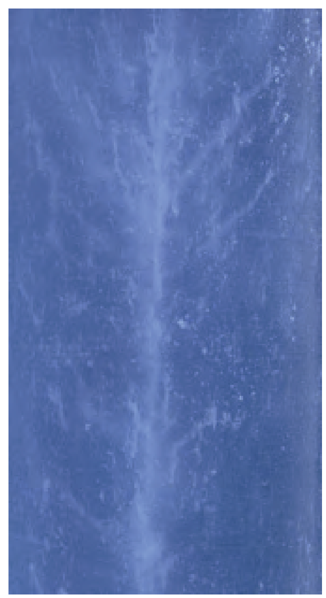


brine channel
in sea ice



diffusion limited
aggregation

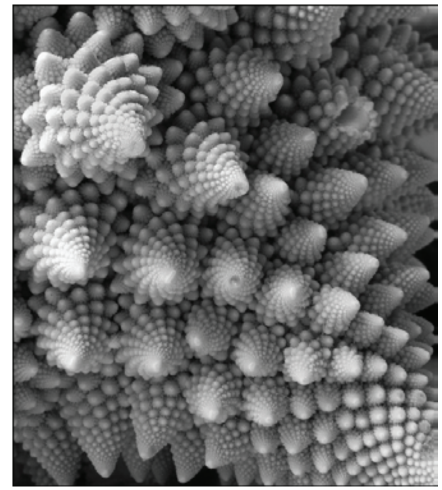


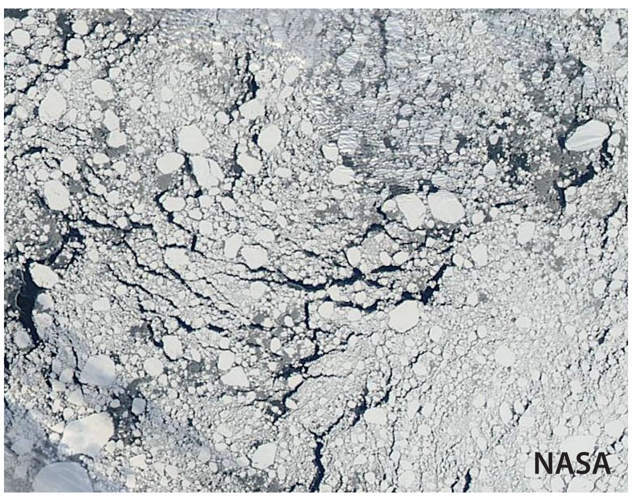


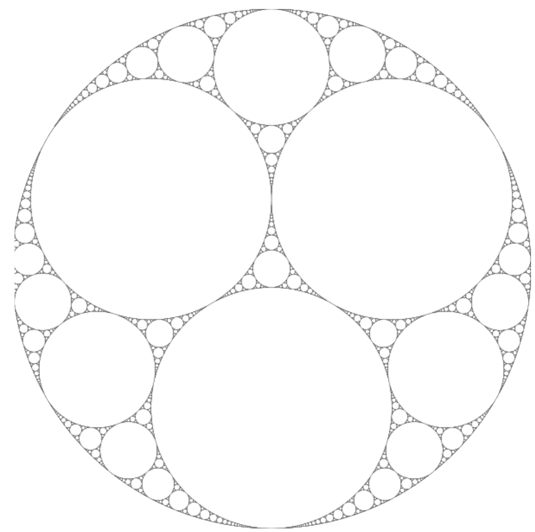
brine channels

the sea ice pack is a ***fractal***

displaying self-similar structure on many scales





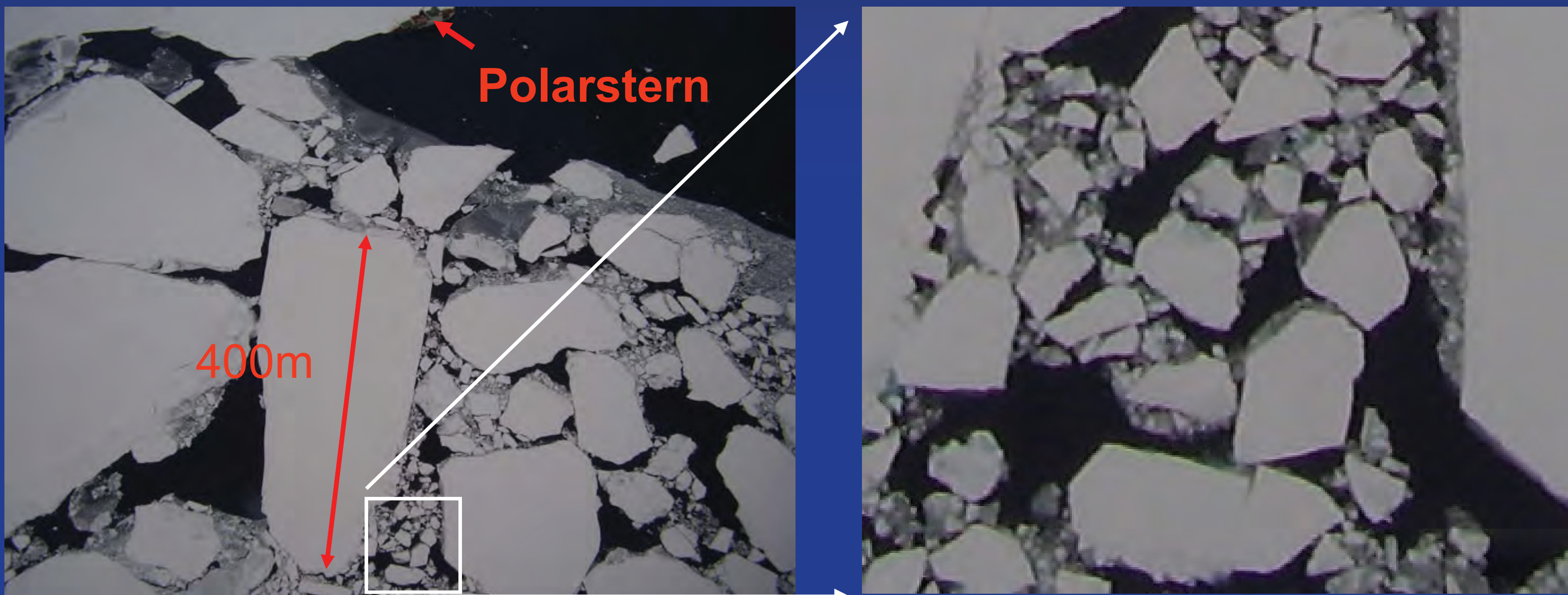


floe size distribution, area-perimeter relations, etc. important in dynamics (fracture), thermodynamics (melting)

Toyota, et al. Geophys. Res. Lett. 2006
Rothrock and Thorndike, J. Geophys. Res. 1984

Self-similarity of sea ice floes

Weddell Sea, Antarctica



fractal dimensions of Okhotsk Sea ice pack
smaller scales D~1.2, larger scales D~1.9

Toyota, *et al. Geophys. Res. Lett.* 2006
Rothrock and Thorndike, *J. Geophys. Res. 1984*

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

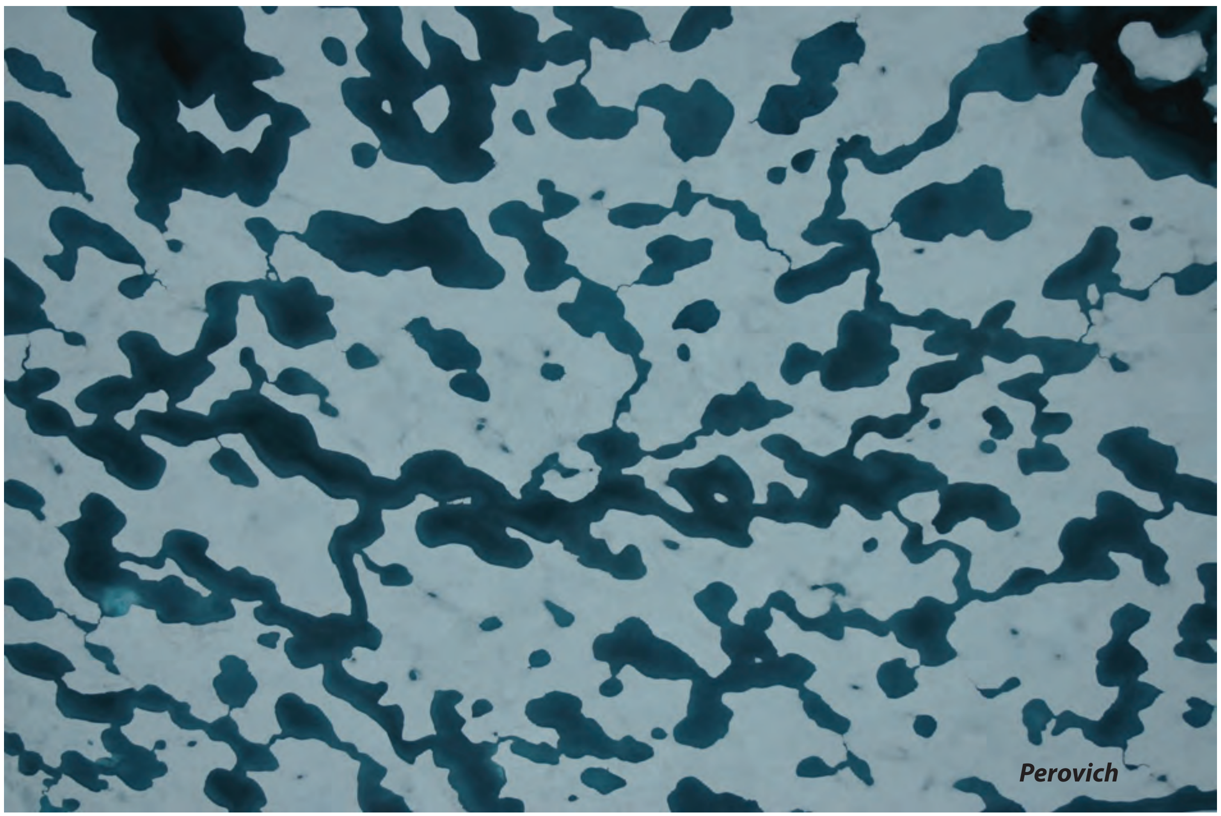
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

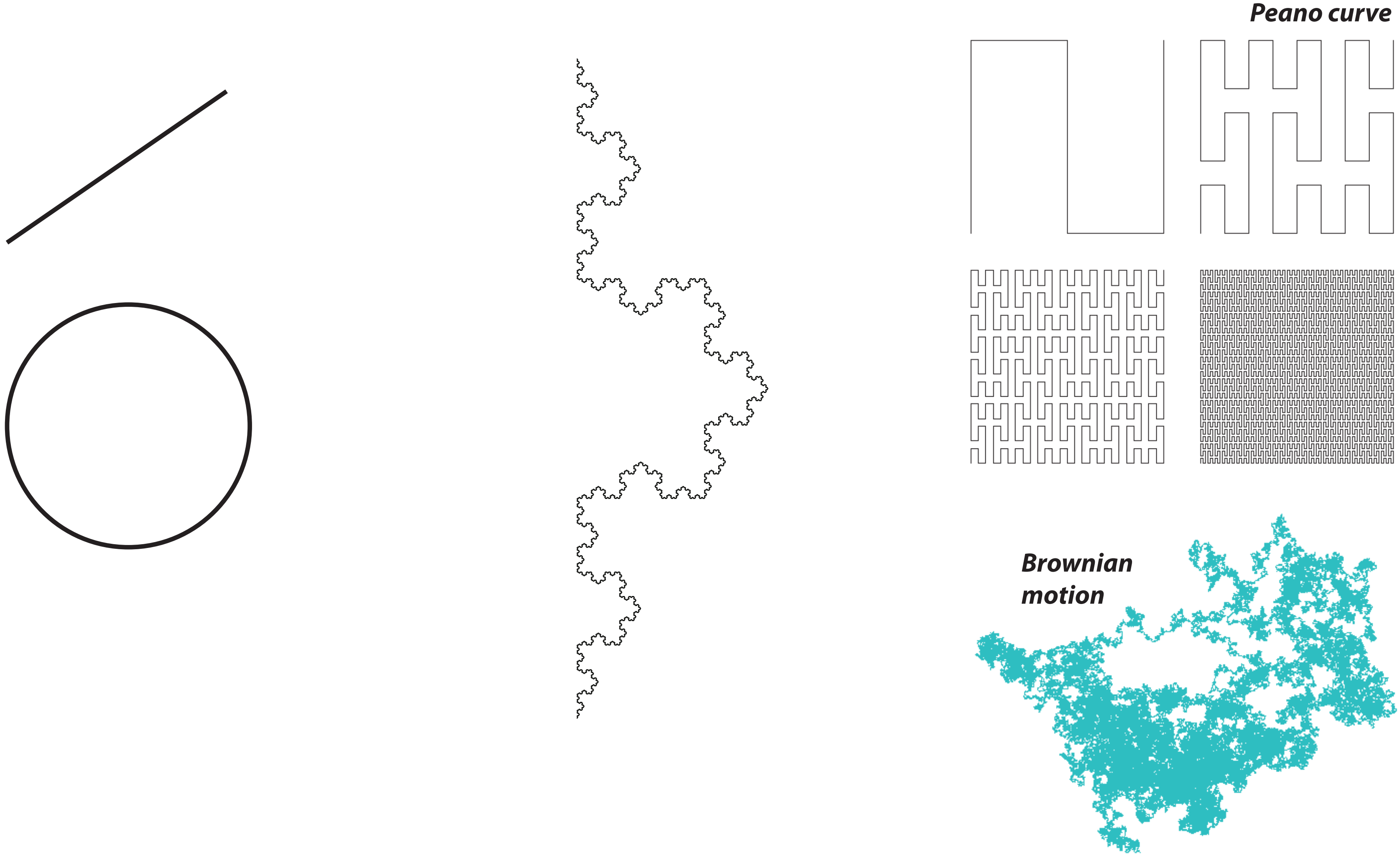
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

fractal curves in the plane

they wiggle so much that their dimension is >1



simple curves — $D = 1$

Koch snowflake — $D = 1.26$

space filling curves — $D = 2$

clouds exhibit fractal behavior from 1 to 1000 km

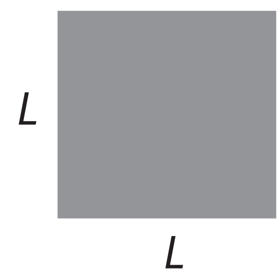
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

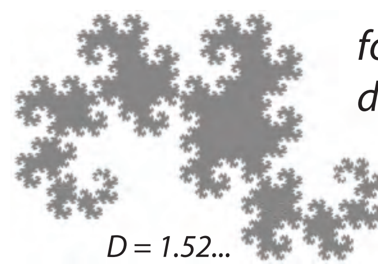
simple shapes



$A = L^2$

$P = 4L = 4\sqrt{A}$

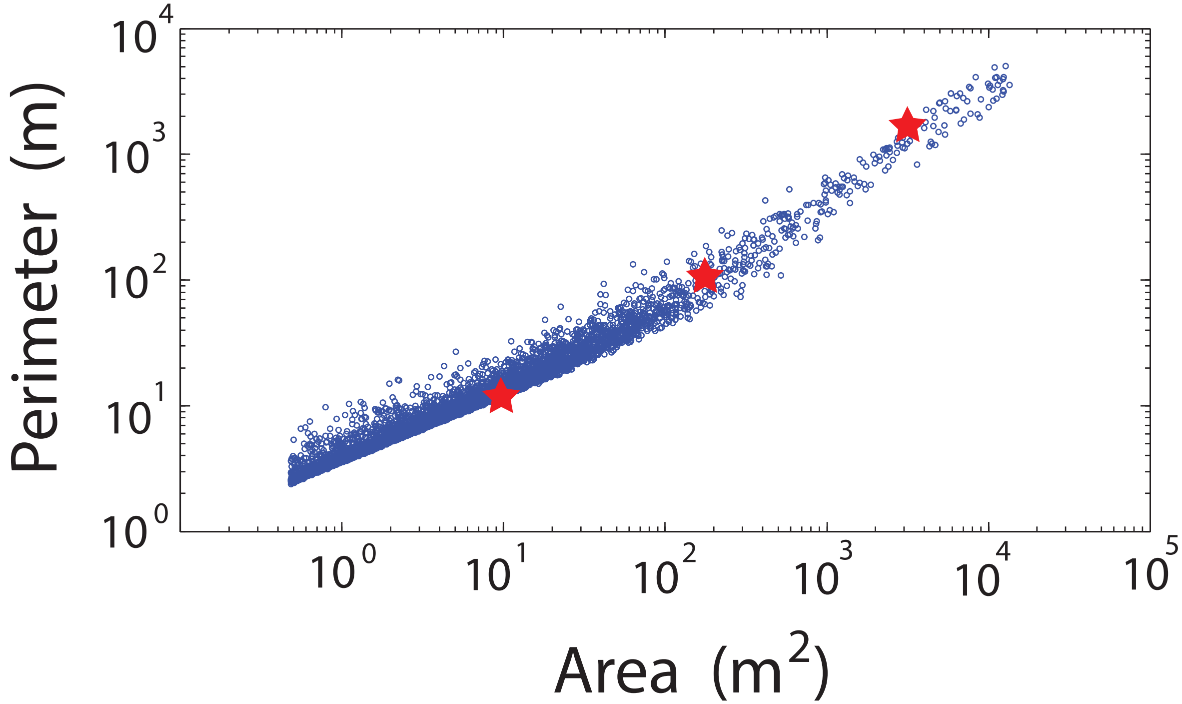
$$P \sim \sqrt{A}^{\,D}$$



for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds   *The Cryosphere, 2012*

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden



data from
5269 Arctic
melt ponds

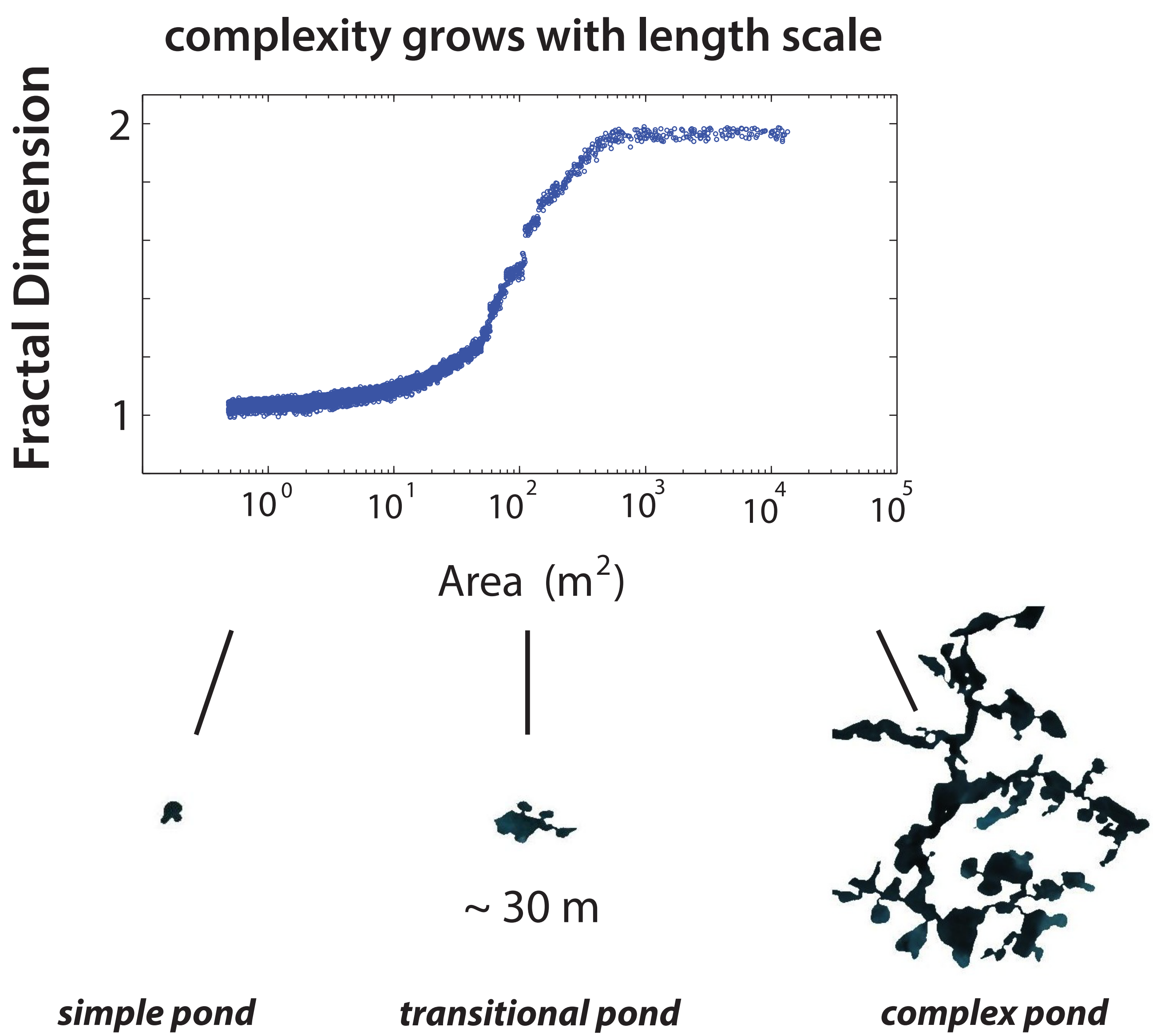
~ 30 m

simple pond   ***transitional pond***   ***complex pond***

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

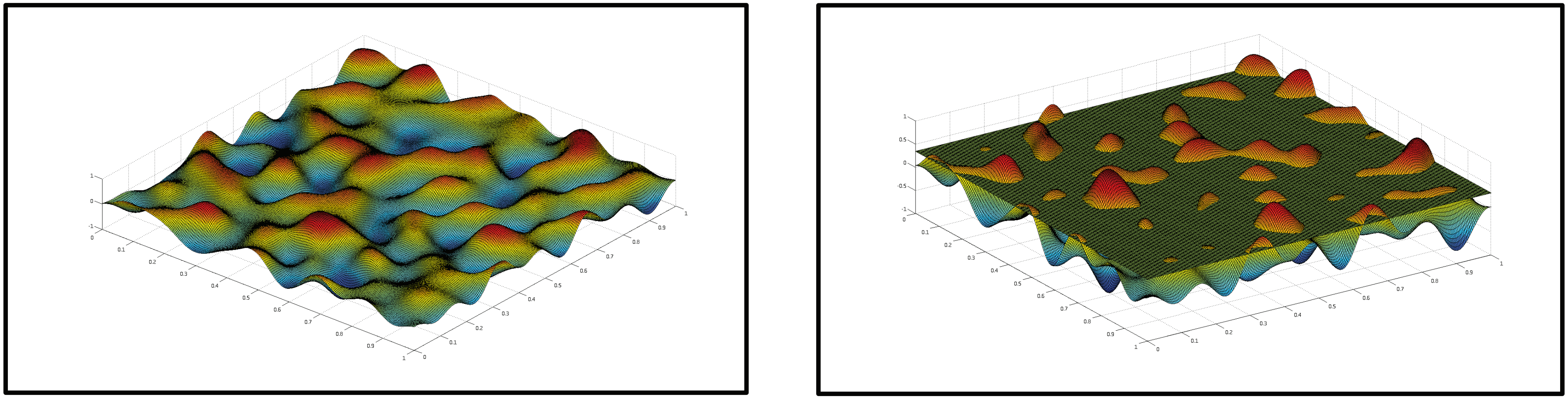
The Cryosphere, 2012



Continuum percolation model for melt pond evolution

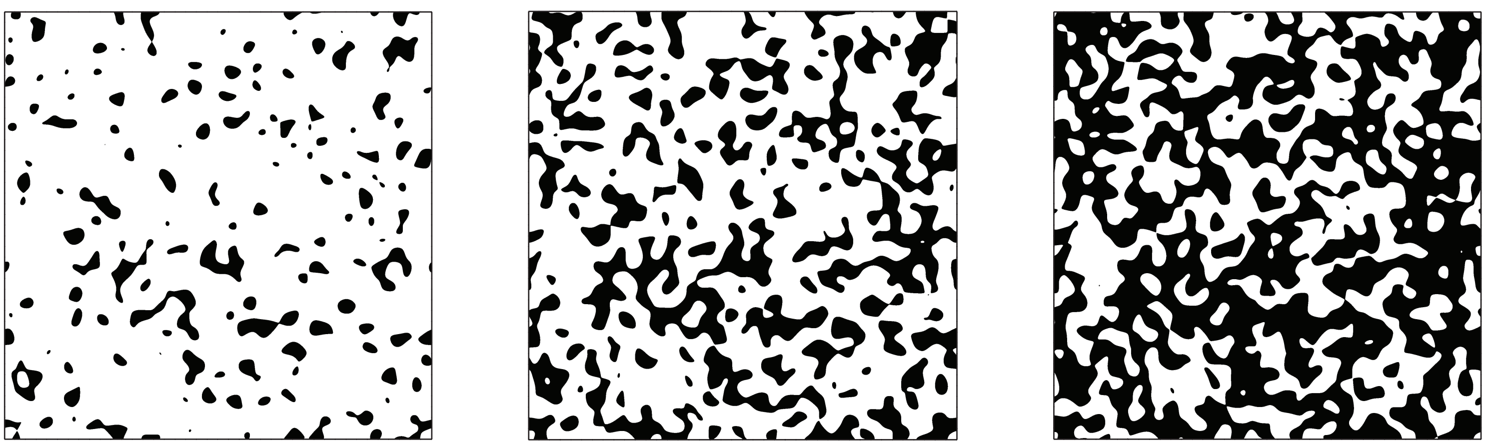
level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



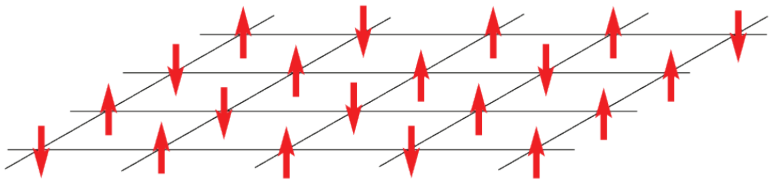
random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds

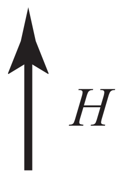


electronic transport in disordered media *diffusion in turbulent plasmas* *Isichenko, Rev. Mod. Phys., 1992*

Ising Model for a Ferromagnet



applied magnetic field



$$s_i = \begin{cases} +1 & \text{spin up} \\ -1 & \text{spin down} \end{cases}$$

blue

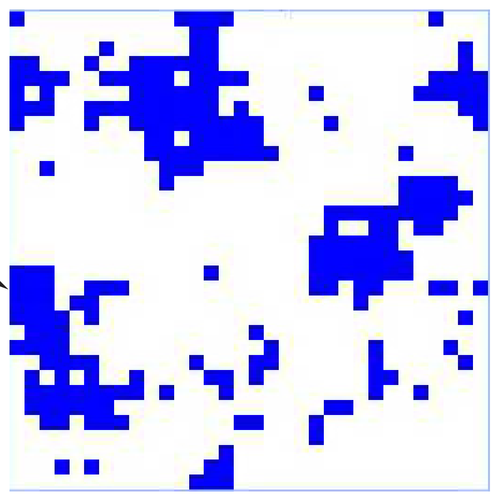
white

$$\mathcal{H} = -H \sum_i s_i - J \sum_{<i,j>} s_i s_j$$

nearest neighbor Ising Hamiltonian

ferromagnetic interaction $J \geq 0$

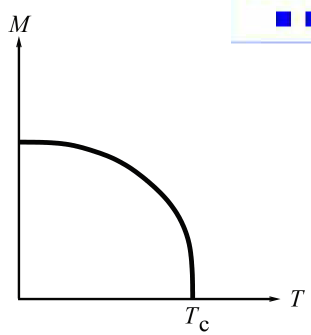
islands or ponds of like spins



magnetization

$$M(T,H) = \lim_{N \to \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

homogenized parameter like effective conductivity



Curie point critical temperature

Stieltjes integral representation for M

Baker, PRL 1968

Ising model for ferromagnets ⟶ Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

$$\mathcal{H} = -\sum_{i}^{N} H_i \, s_i - J \sum_{<i,j>}^{N} s_i s_j \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow & -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

random magnetic field represents snow topography

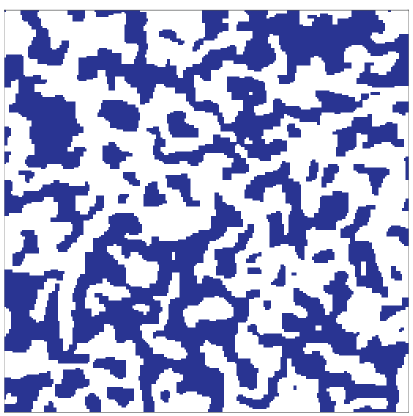
magnetization M **pond coverage** $\frac{(M+1)}{2}$

~ albedo

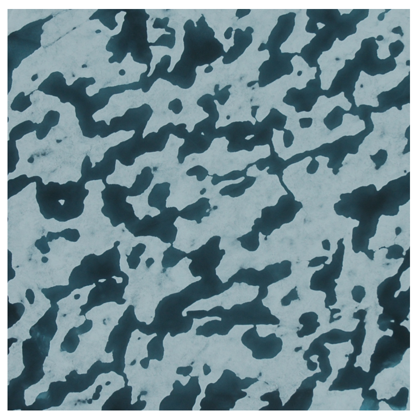
only nearest neighbor patches interact

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system "flows" toward metastable equilibria.

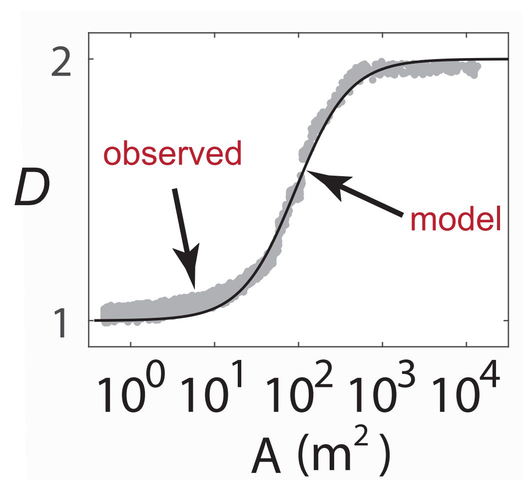
Order from Disorder



Ising model



melt pond photo (Perovich)



pond size distribution exponent

observed -1.5

(Perovich, *et al.* 2002)

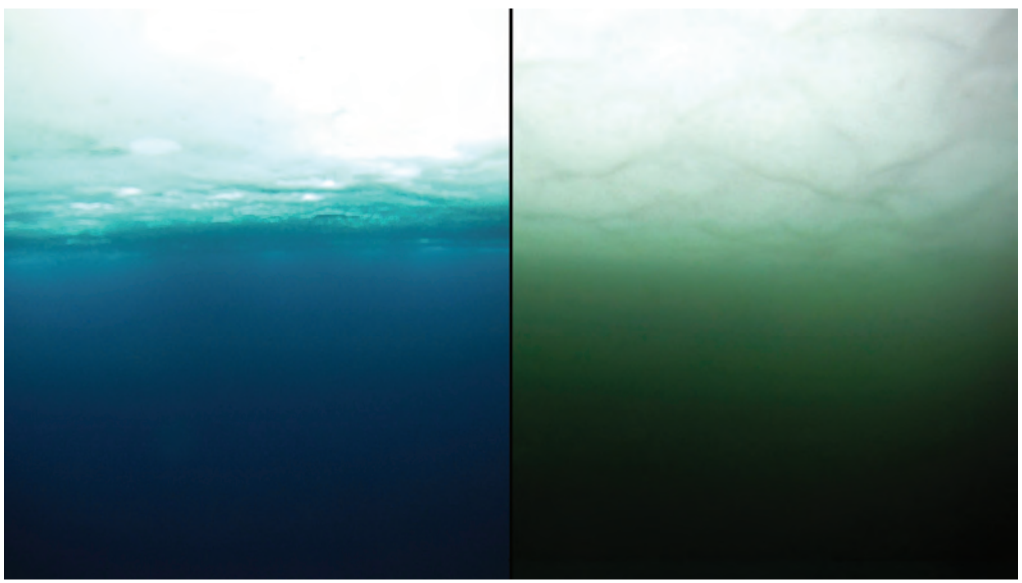
model -1.58

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data

2011 massive under-ice algal bloom

Arrigo et al., *Science* 2012

melt ponds act as *WINDOWS* allowing light through sea ice



no bloom   bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

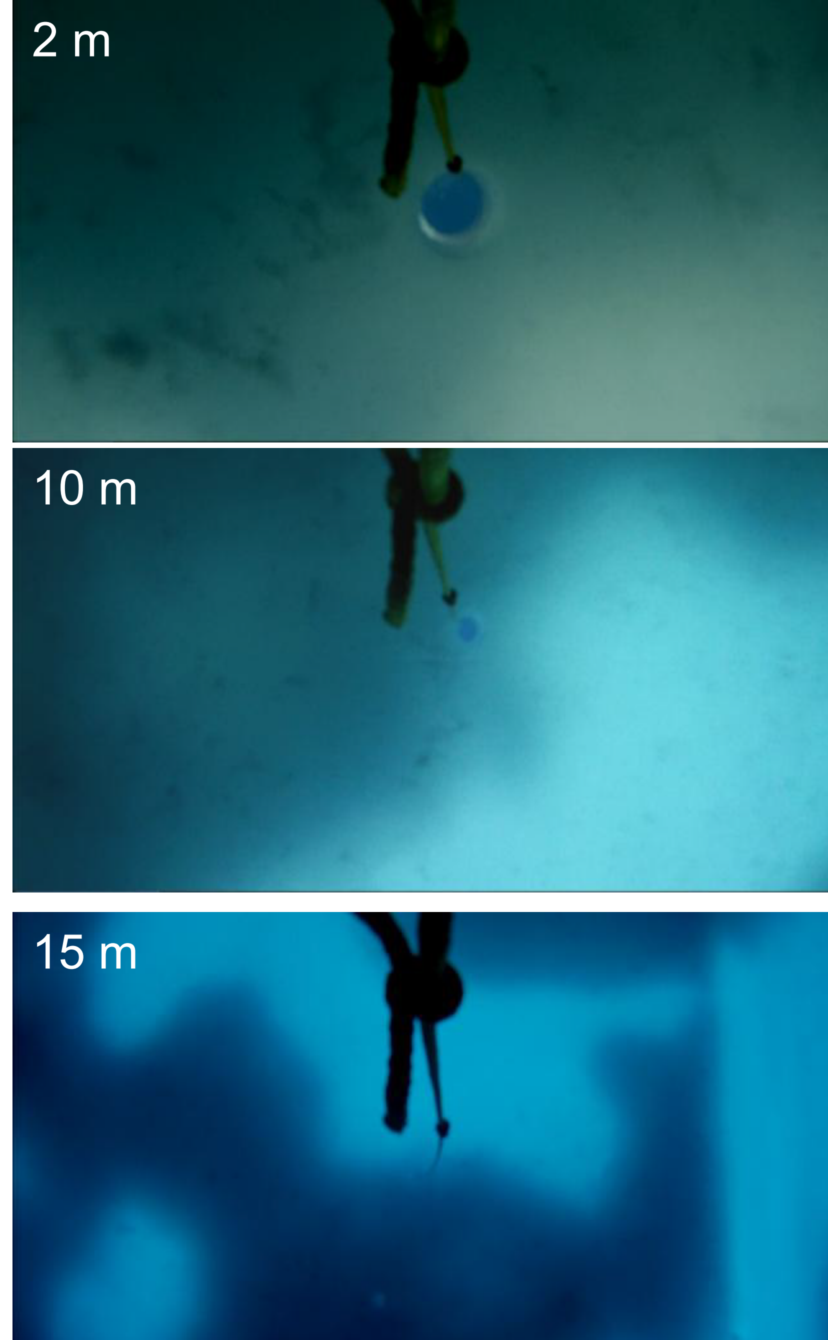
Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, ***Science Advances***, 2017

The distribution of solar energy under ponded sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, 2019

(2015 AMS MRC @ Snowbird)

the view from underneath →



melt ponds are WINDOWS

light reaches the upper ocean

Perovich

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, ***Science Advances***, 2017

- Model for initiation of light-limited phytoplankton bloom (depth of mixed layer, ice thickness, melt pond area fraction, ...)
- Thinner summertime Arctic sea ice is increasingly covered in melt ponds, which permit more light penetration.
- Marked increase in light conditions conducive to sub-ice blooms.
- As little as 20 years ago, conditions for sub-ice blooms may have been uncommon; frequency has increased so that nearly 30% of the ice-covered Arctic Ocean in July permits sub-ice blooms.

Recent climate change may have significantly altered the ecology of the Arctic Ocean.

The Melt Pond Conundrum:

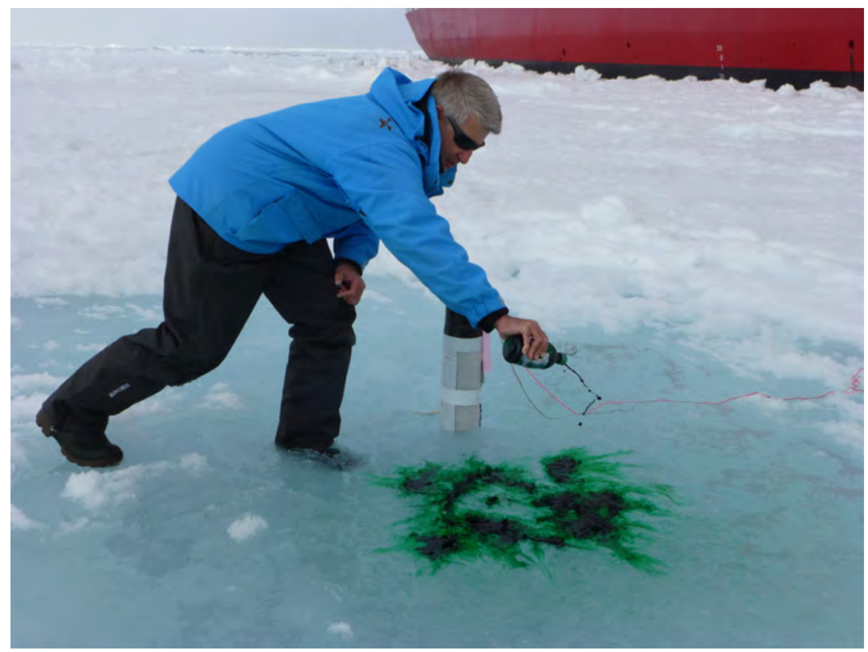
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE) aboard USCGC Healy



Large ecosystems in transition: bifurcations and mass extinction

I. Sudakov, S. Vakulenko, D. Kirievskaya, K. M. Golden, *Ecological Complexity,* 2017

model of multispecies populations competing for distributed resources

coupling climate and population dynamics via resources

feedback between species abundances and resources through temperature

resource competition model

Leon & Tumpson, *J. Theor. Biol.* 1975
Tilman, *Ecology*, 1977
Huisman & Weissing, *Nature*, 1999 (solved "plankton paradox")

$$\frac{dx_i}{dt} = x_i(-r_i + \phi_i(v) - \sum_{j=1}^{N} \gamma_{ij}\, x_j), \quad 1 \leq i \leq N$$

$$\frac{dv}{dt} = D(S - v) - \sum_{j=1}^{N} c_j\, x_j\, \phi_j(v)$$

$$\phi_j(v) = \frac{a_j v}{K_j + v}\,, \quad a_j,\ K_j > 0$$

- x_i species abundance
- ϕ_i species growth rate
- r_i species mortality rate
- γ_{ij} describe competition (e.g. toxic compounds) diagonals ~ self-regulation restricting abundances
- S supply concentration of resource
- v resource availability
- D resource turnover rate
- c_j determine how species share resource
- K_j saturation constants

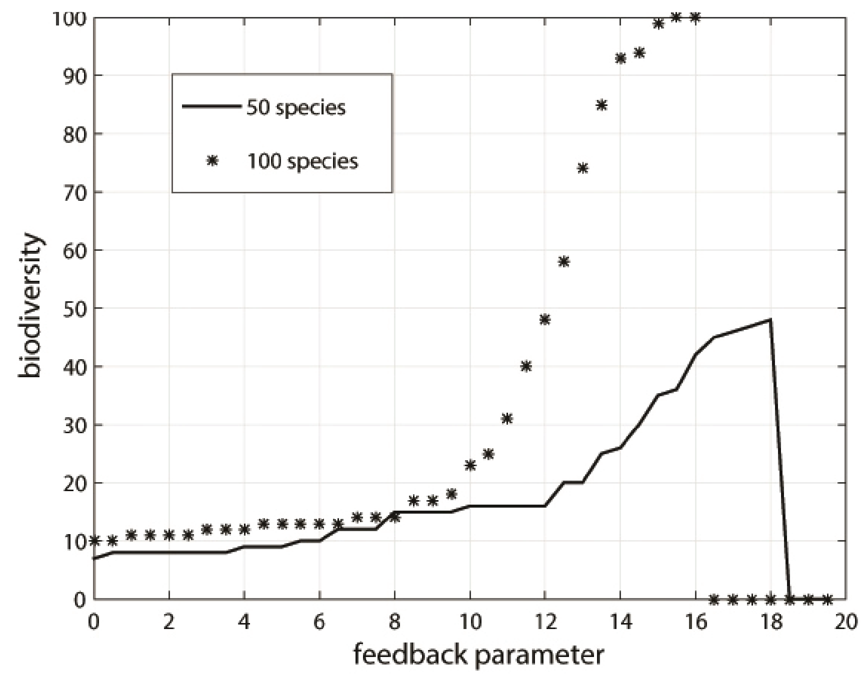
Extend model to M resources, whose supplies depend on temperature T, which depends on species abundances.

$$T = \bar{T} + \Delta T, \quad \Delta T = \sum_{k=1}^{N} \mu_{kj} x_j$$

$$S_k = \bar{S}_k(\bar{T}) + \Delta S_k + O(\Delta T^2)$$

$$\Delta S_k = \sum_{k=1}^{N} b_{kj}(\bar{T}) x_j, \quad k = 1, \ldots, M$$

$$b_{kj} = \frac{dS_k(\bar{T})}{d\bar{T}} \mu_{kj} \begin{cases} > 0 & \textit{positive feedback} \\ < 0 & \textit{negative feedback} \end{cases}$$



mass extinctions

reduction to Lotka-Volterra system

$|b_{ik}| \ll \gamma$ ***weak climate coupling***

close to 'competitive systems' - exhibit no stable periodic or chaotic regimes: almost all trajectories converge to equilibria

$|b_{ik}| \gg \gamma$ ***strong climate coupling***

$M = 1$, possible that all N species survive in equilibrium, or coexistence of many equilibria

$M = 2$, feedback coeffs have different signs, periodic sols

$M > 2$, system can produce time chaotic solutions

when the number of species increases, so does the likelihood of a sharp drop in species number as the climate changes and feedback processes grow stronger

The model exhibits coexistence of many species, yet also displays the possibility of catastrophic bifurcations, where all species become extinct under the influence of abiotic factors (strong climate coupling).

Conclusions

1. The physics of fluid transport in porous composites regulates microbial life inside sea ice.

2. The geometry of melt ponds controls light in the upper ocean, and initiation of under-ice algal blooms.

3. Resource competition models provide tools to study the complexity of these microbial communities.

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999