Sea ice modelling I

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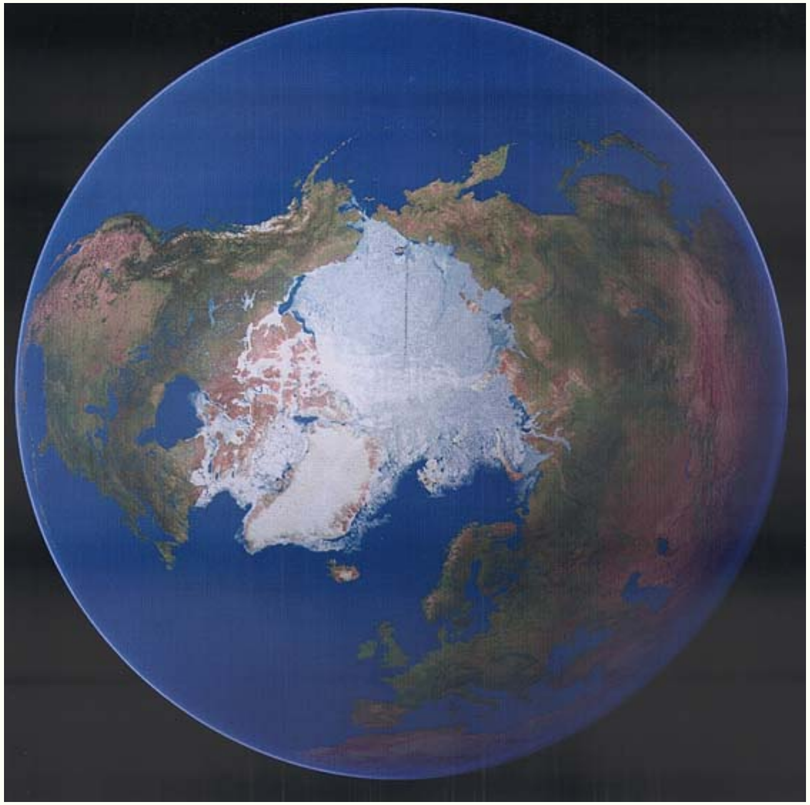
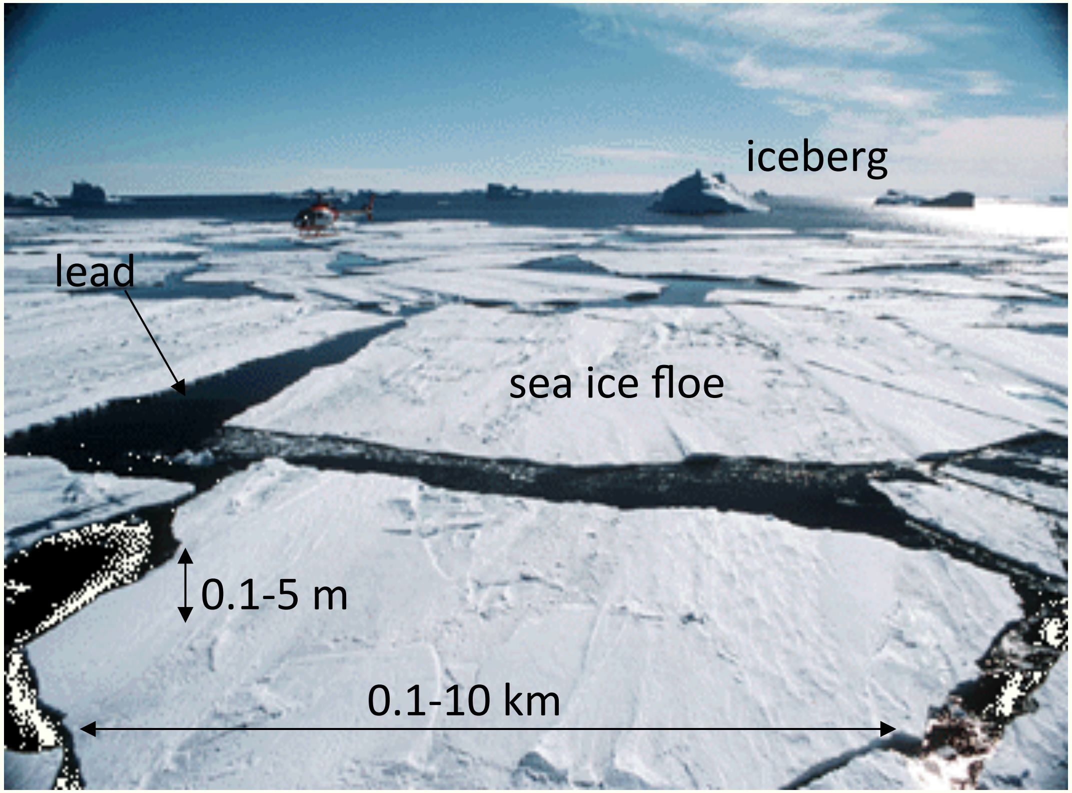
Thanks to David Schroeder, Michel Tsamados, Daniela Flocco

Talk structure

- What is sea ice and why is it important?
- Modelling sea ice
- Recent changes in the sea ice cover and model performance

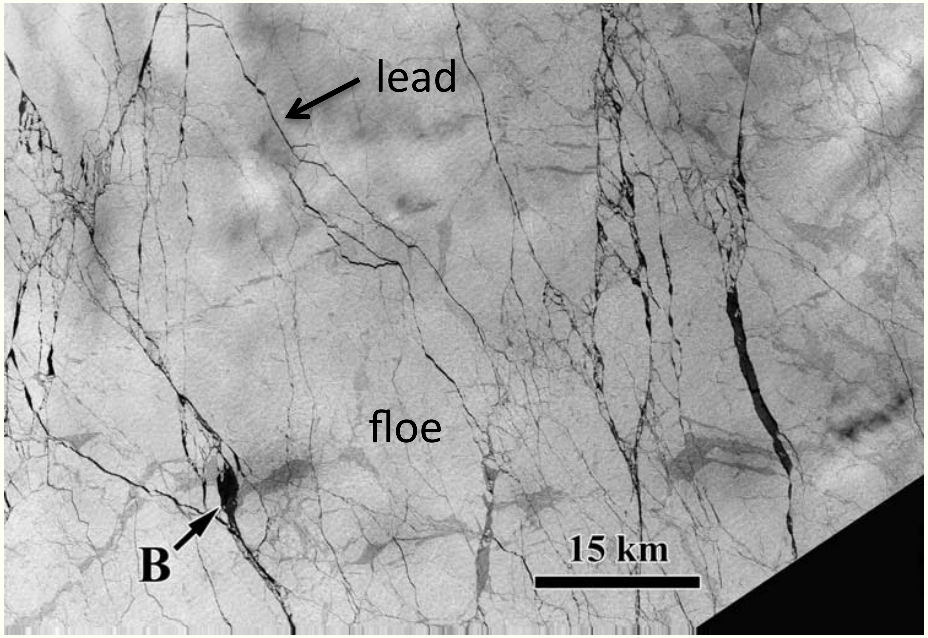
What is sea ice?

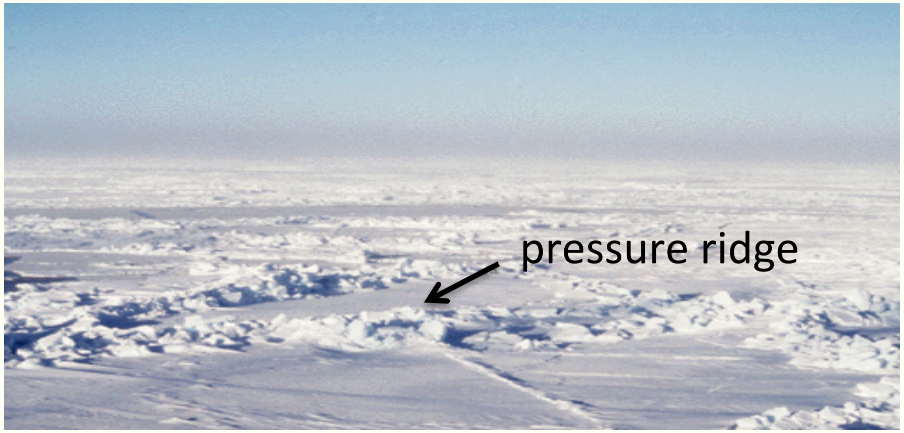


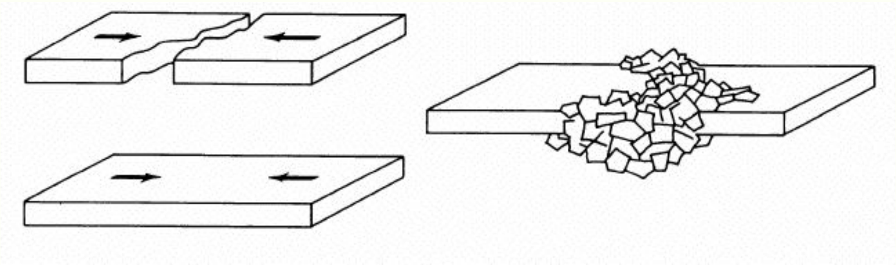


- **Sea ice is frozen seawater**. It floats as a layer of ice a few metres thick on the polar oceans.
- Sea ice is a partial barrier to transports of heat, moisture and momentum between the air and ocean.
- Sea ice covers about 8% of the Earth's oceans at maximum extent.
- The sea ice covers of the Arctic and Antarctic wax and wane seasonally.

Sea ice floes, leads, and ridges

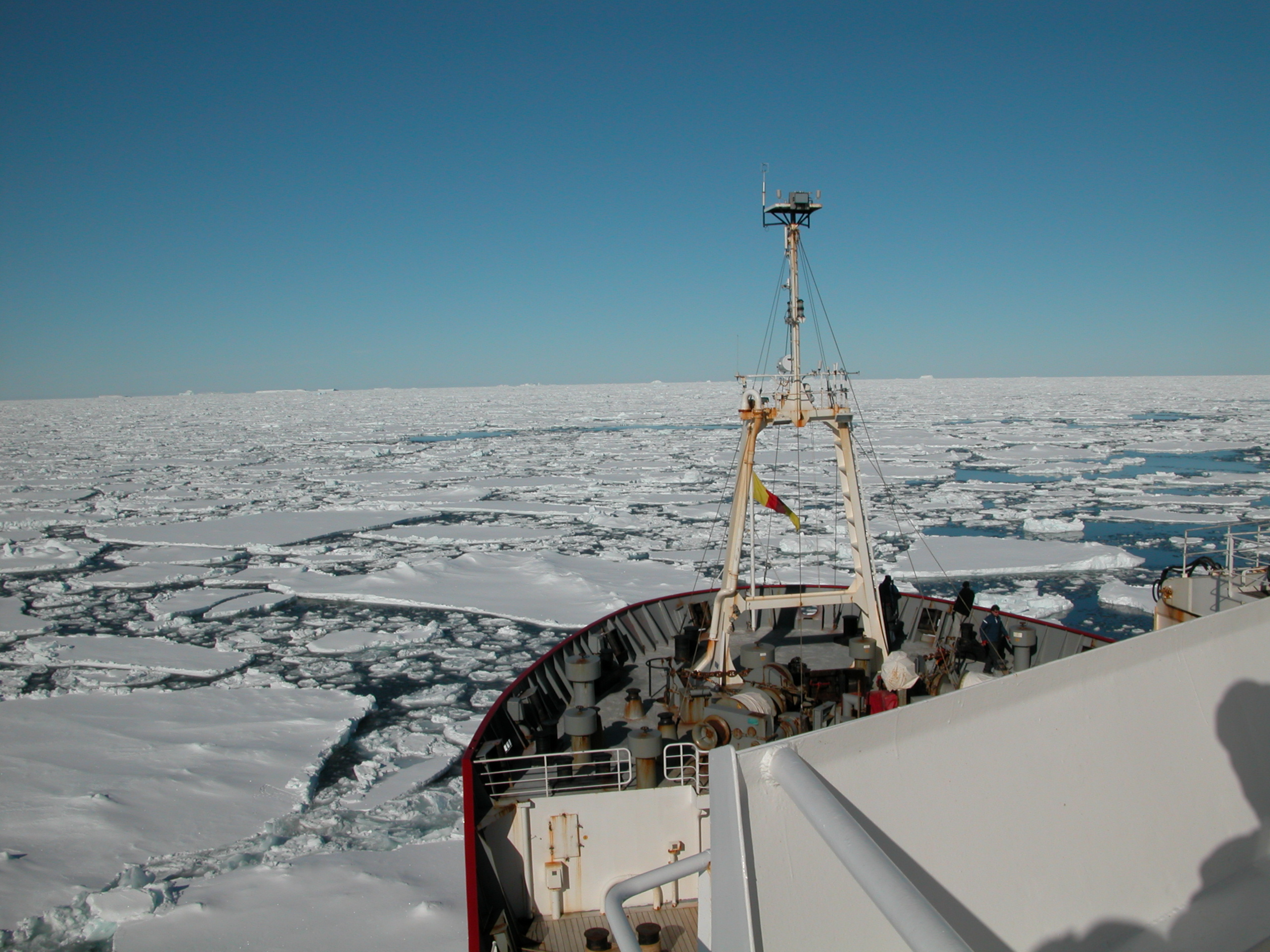


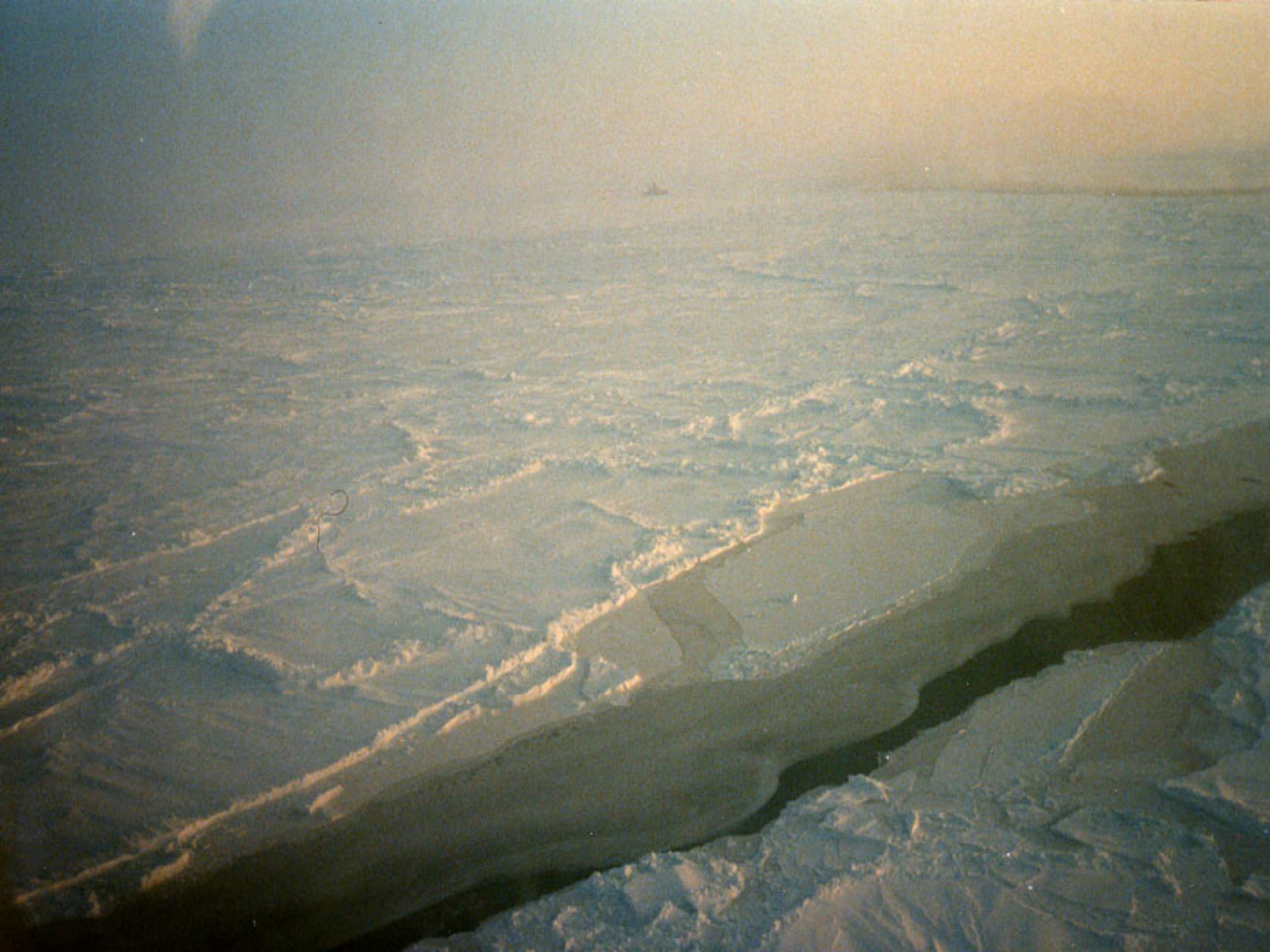




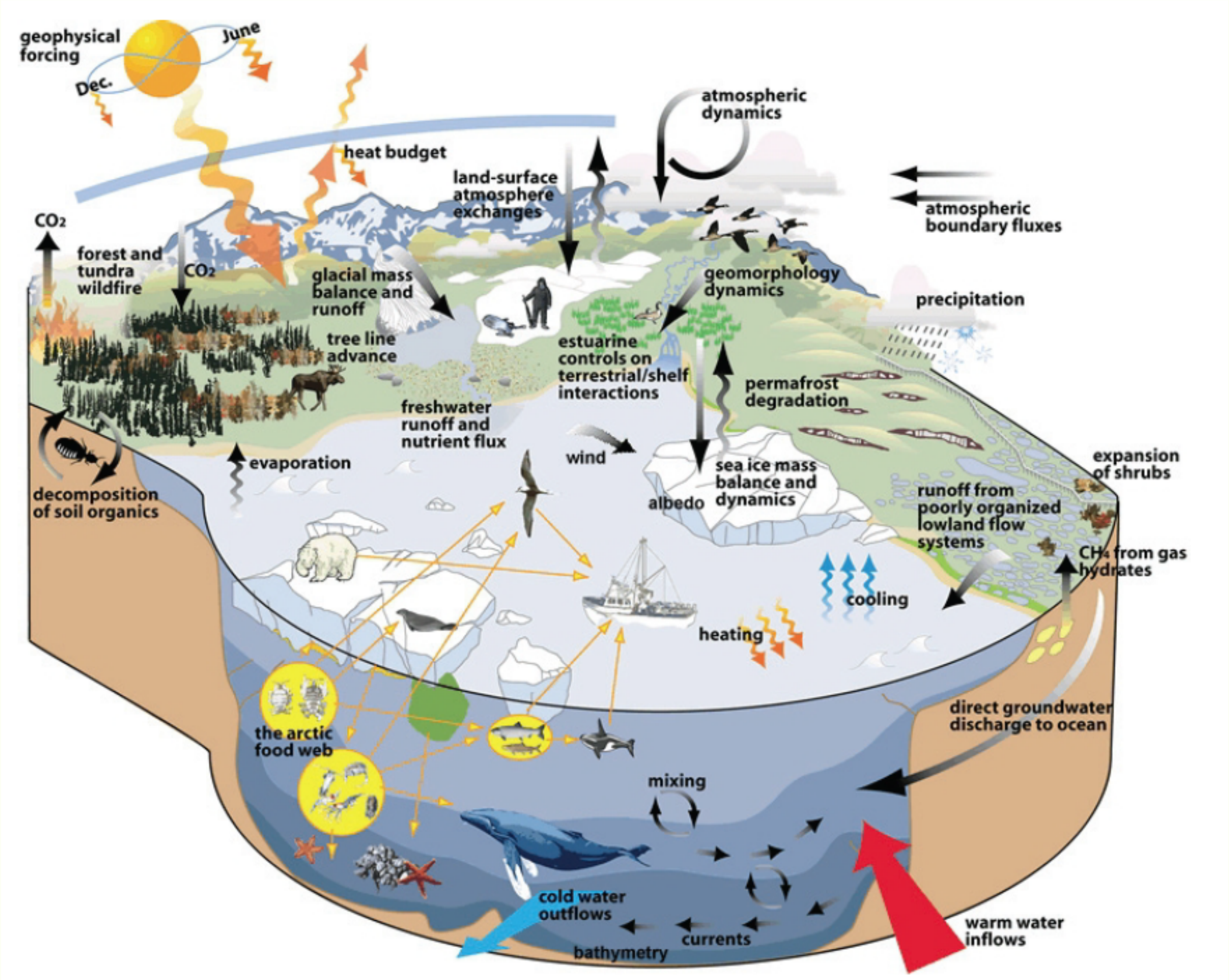
The sea ice cover consists of **floes** that are 0.1-10 km wide and 0.1-5 m thick; they are separated by cracks or **leads**. The floes may be frozen together to form **floe aggregates** or be separate. Ice area concentration typically 0.90-0.99.

Pressure ridges form when floes collide with, and over ride, each other. The ice sheet breaks up into blocks that are pushed into **sails** and **keels**. Pressure ridges can be many kilometres in length; the sails and keels are approximately triangular.





Sea ice is a major component of the cryosphere

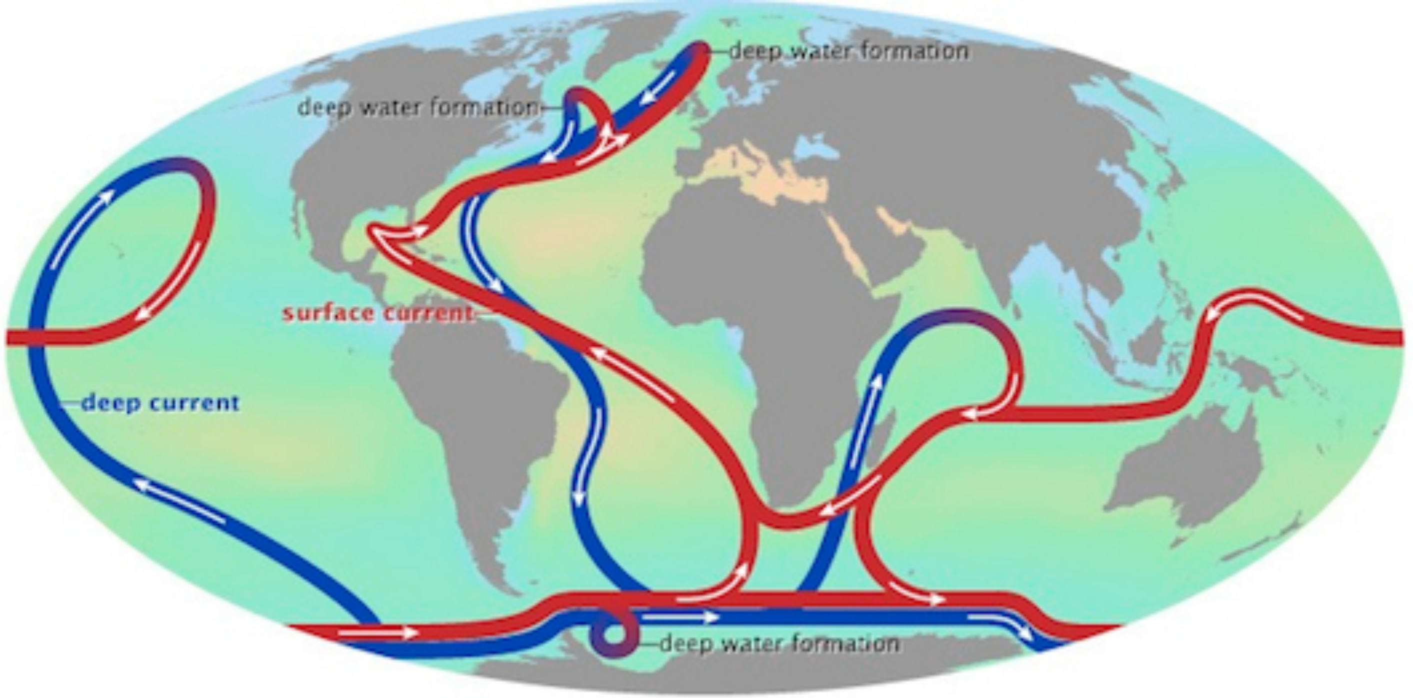


Sea ice has a high albedo (reflectivity). The difference in ocean and sea ice albedo drives an <u>albedo feedback mechanism</u>:

the albedo of ice and snow is much higher than seawater so a reduction in ice/snow cover results in greater absorption of solar radiation;

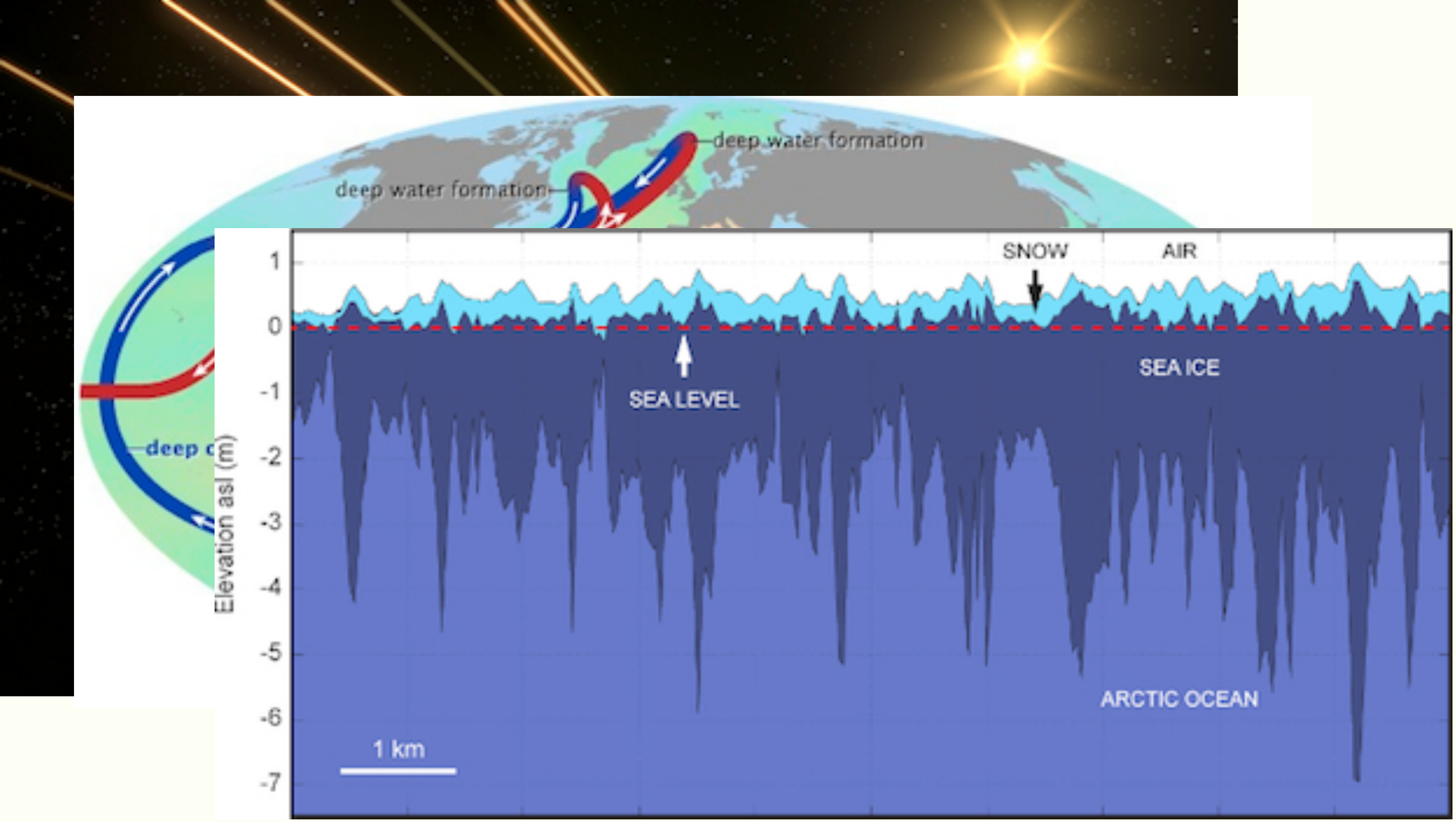
the absorbed heat can melt ice and snow, reducing the albedo further; and so on…

Surface temperature increases under global warming are predicted to be up to 5 times greater in the polar regions than the global average due to the albedo feedback mechanism.



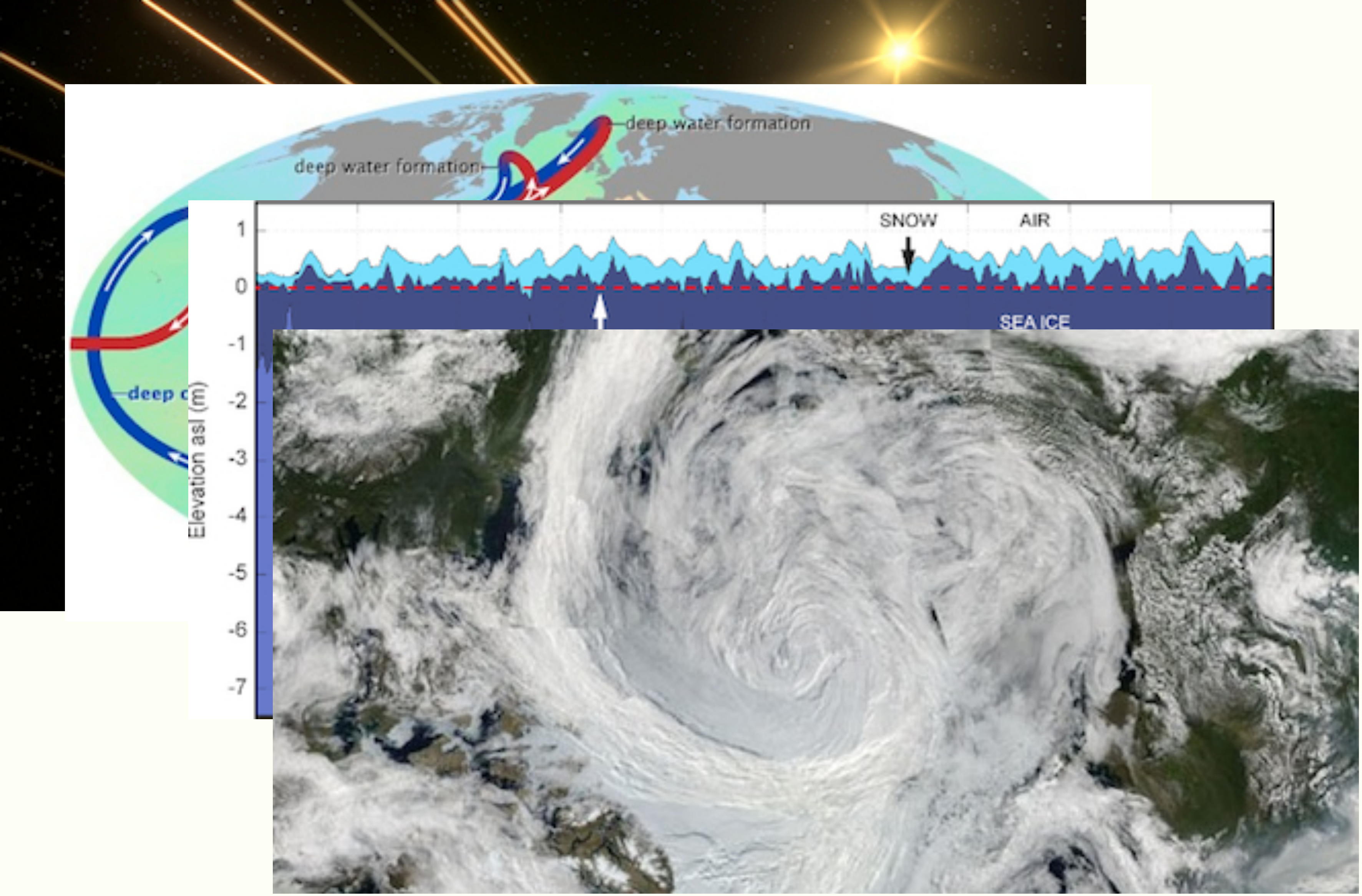
Sea ice formation releases salt, and melting releases freshwater; sea ice plays a fundamental role in the global thermohaline circulation.

As surface waters reach the poles it cools and sinks to form bottom water, which then returns to the equatorial regions at depth. Sea ice formation, by releasing salt and densifying water plays an important role in bottom water formation, especially in the Southern Ocean.

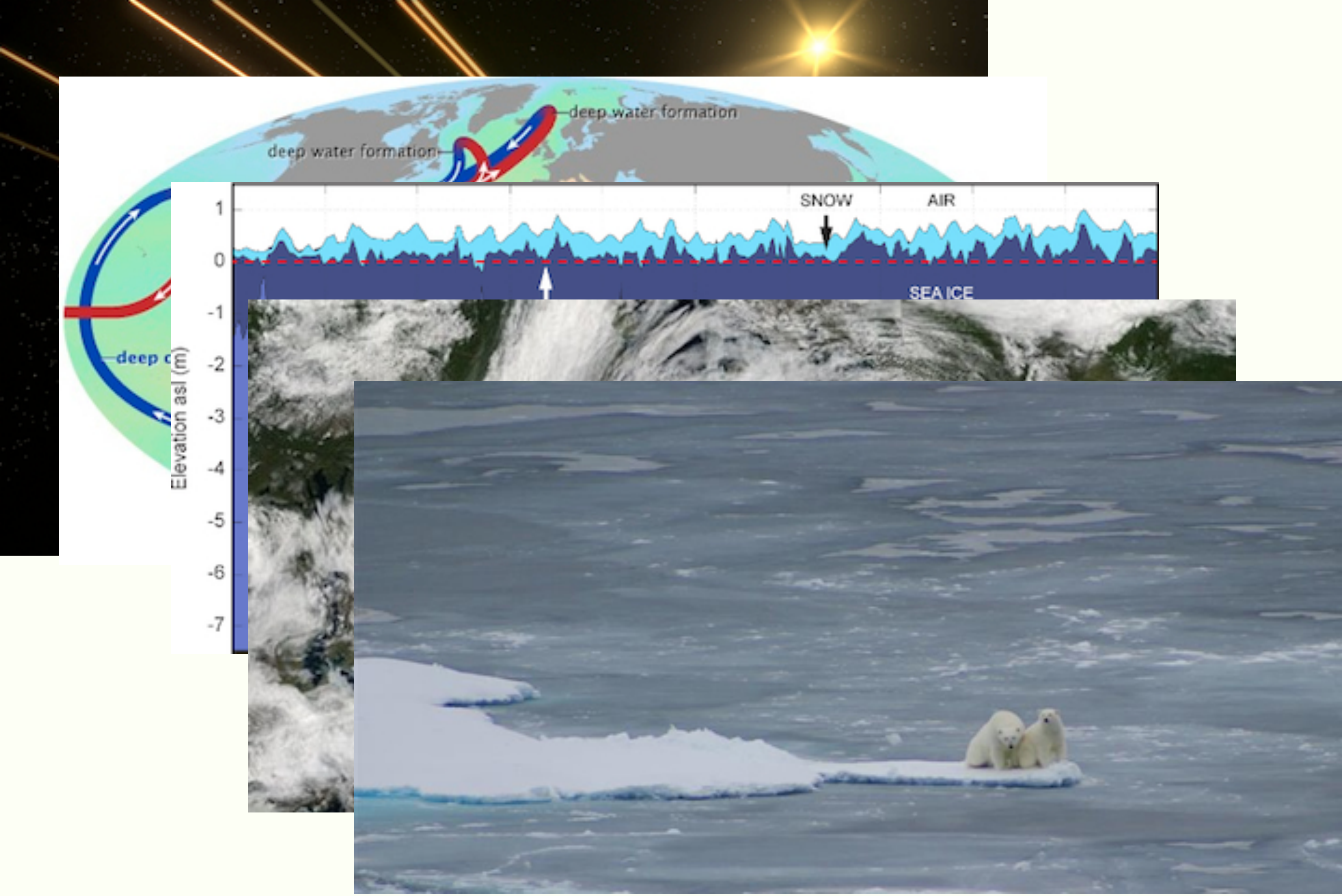


Sea ice insulates the (relatively) warm ocean at about -2^{o}C from the cold air (e.g. -20^{o}C).

Cracks in the sea ice cover (leads) although only occupying about 1-2% of the ice cover in winter account for half the ocean-air heat flux,
e.g. heat flux through sea ice is about 2-5 Wm^{-2} compared to 300-500 Wm^{-2} through leads.



The ocean to air heat flux has an impact on storms and is controlled by the ice cover.



Sea ice is an essential habitat for iconic wildlife and a hunting platform for indigenous people.

Modelling sea ice in climate models

Sea ice models are formulated as **continuum** expressions of **local balances of momentum, mass, and heat,** which are mediated through various **processes**. In practise, sea ice processes are divided into:

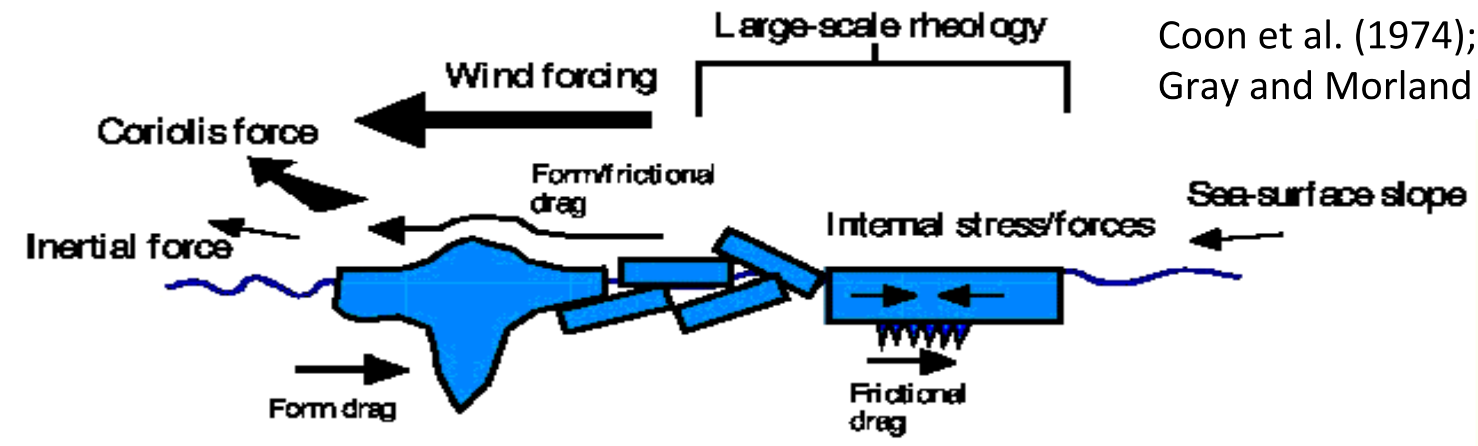
- **Dynamic** processes, which control the motion of ice cover, deformation, and redistribution of thickness. Example processes are air and ocean drag, ridging, sliding, and rupture (rheology).
- **Thermodynamic** processes, which control melting, freezing, and dissolving. Example processes are thermal conduction, brine convection, and solar radiation absorption.
- Both dynamic and thermodynamic processes involve **coupled** interactions with the atmosphere and ocean.
- The amount (extent, concentration, volume) of sea ice is determined by an **intimate mixture** of dynamic and thermodynamic processes.

A sea ice lead, formed by divergence results in rapid new ice growth.

Modelling sea ice

- Local momentum balance
- Local mass balance
- Local heat/salt balance
- Typical model output

Local momentum balance of sea ice



Coon et al. (1974);
Gray and Morland (1994)

Vertically-integrated (i.e. horizontal) momentum balance is:

$$\underbrace{m\frac{D\mathbf{u}}{Dt}}_{\sim 0.001} = \underbrace{\boldsymbol{\tau}_{\mathbf{a}}}_{\sim 0.1} + \underbrace{\boldsymbol{\tau}_{\mathbf{w}}}_{\sim 0.1} + \underbrace{\nabla\cdot\boldsymbol{\sigma}}_{\sim 0.1} - \underbrace{mf_C\mathbf{k}\times\mathbf{u}}_{\sim 0.01} - \underbrace{mg\nabla H}_{\sim 0.01} \qquad \mathrm{Nm}^{-2}$$

mass X acceleration = air drag + ocean drag + ice-ice force + Coriolis "force" + gravity force from sea surface tilt

$$\boldsymbol{\sigma} \equiv \int \boldsymbol{\sigma}' dz - \tfrac{1}{2}\rho_{\text{ice}}\mathbf{g}h^2$$ is the stress caused by frictional sliding between floes, pressure ridging, and collisions.

The dominant forces are air drag, ocean drag, and the ice-ice force.

Air-ice and ocean-ice drag laws

- Air-ice and ice-ocean drag laws are typically given by bulk formulae of the form

$$\boldsymbol{\tau}_{\mathbf{a}} = \rho_a C_a \left|\mathbf{U}_g^a\right| \left(\mathbf{U}_g^a \cos\varphi + \mathbf{k}\times\mathbf{U}_g^a \sin\varphi\right), \qquad \left|\mathbf{U}_g^a\right| >> \mathbf{u}$$

$$\boldsymbol{\tau}_{\mathbf{w}} = \rho_w C_w \left|\mathbf{U}_g^w\right| \left(\left(\mathbf{U}_g^w - \mathbf{u}\right)\cos\theta + \mathbf{k}\times\left(\mathbf{U}_g^w - \mathbf{u}\right)\sin\theta\right)$$

Geostrophic ocean current $\mathbf{U}_g^w = \mathbf{g} f_C^{-1} \mathbf{k}\times\nabla H$

- Current models treat the neutral drag coefficients C_a and C_w at the air-ice and ice-ocean interfaces as **constants**.
- The reality is that the topography of the ice cover creates spatially and temporally variable **form drag**.
- Atmospheric boundary layer stability is separately accounted for.

Sea ice internal stresses $\nabla \cdot \sigma$

Sea ice rheology describes the forces (stresses) necessary to create a deformation of the sea ice cover.

Early sea ice modellers supposed that ice stress depends only upon rate of deformation and scalars:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}\left(\frac{\partial u_i}{\partial x_j}, \text{scalars}\right)$$

Material frame indifference then leads to the Reiner-Rivlin form:

$$\sigma_{ij} = 2\eta\dot{\varepsilon}_{ij} + [\varsigma - \eta]\dot{\varepsilon}_{kk}\delta_{ij} - \frac{P}{2}\delta_{ij} + \gamma\dot{\varepsilon}_{is}\dot{\varepsilon}_{sj}$$

which depends on the symmetric part of the deformation rate, i.e. the strain rate

$$\dot{\varepsilon}_{ij} \equiv \tfrac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

and η, ς, γ, P are functions of the scalars and the invariants $\dot{\varepsilon}_I$, $\dot{\varepsilon}_{II}$

The RR equation is quite general; certain choices of the scalar functions lead to Newtonian fluid flow or plastic flow. Typically set $\gamma = 0$.

The Reiner-Rivlin description treats sea ice as an isotropic continuum.

Aside: Strain rate invariants

$$\dot{\varepsilon}_I \equiv \dot{\varepsilon}_{kk} = |\dot{\varepsilon}|\cos\theta = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 = \nabla\cdot\mathbf{u},$$ divergence

$$\dot{\varepsilon}_{II} \equiv 2\sqrt{-\det\left(\dot{\boldsymbol{\varepsilon}} - \tfrac{1}{2}\boldsymbol{\delta}\dot{\varepsilon}_I\right)} = |\dot{\varepsilon}|\sin\theta = \dot{\varepsilon}_1 - \dot{\varepsilon}_2$$ maximum shear rate

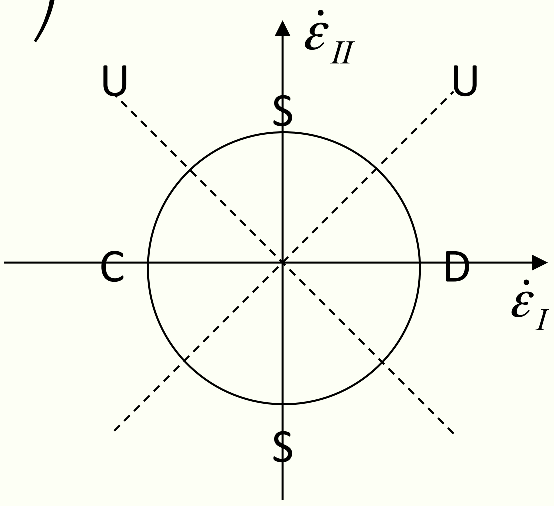
$\dot{\varepsilon}_1, \quad \dot{\varepsilon}_2$, the principal components of strain rate, are also invariants

and

$$|\dot{\varepsilon}| = \sqrt{\dot{\varepsilon}_I^2 + \dot{\varepsilon}_{II}^2}, \quad \theta = \tan^{-1}\left(\frac{\dot{\varepsilon}_{II}}{\dot{\varepsilon}_I}\right)$$

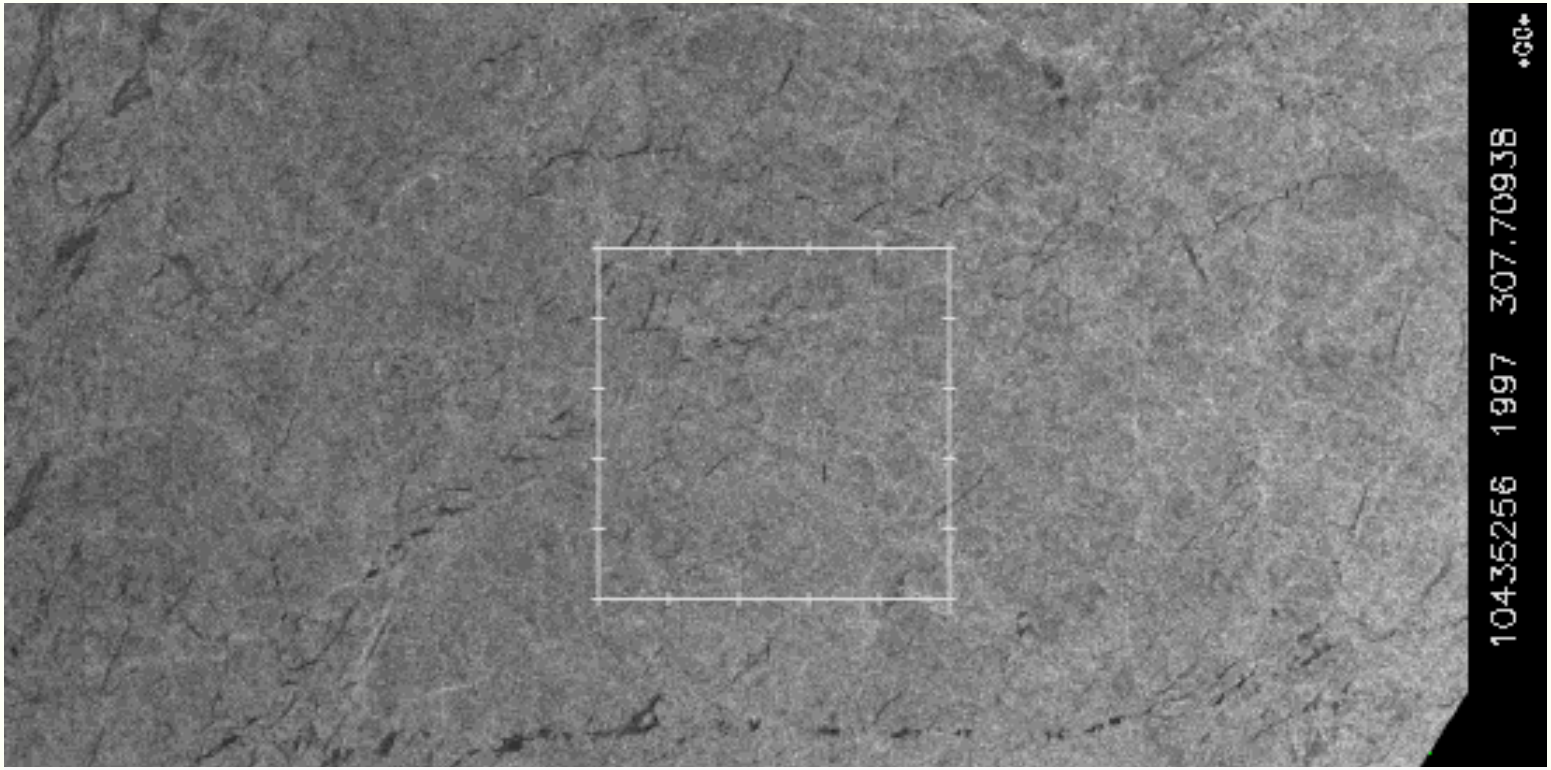
Note that:

$\theta = 0,$	pure divergence,
$\tfrac{1}{4}\pi,$	uniaxial extension,
$\tfrac{1}{2}\pi,$	pure shear,
$\tfrac{3}{4}\pi,$	uniaxial extension,
π	pure convergence



Sea ice deformation

[Kwok, 2001]



Initial square is 50 km x 50 km.
Deformation from SAR imagery.

Some features of sea ice deformation

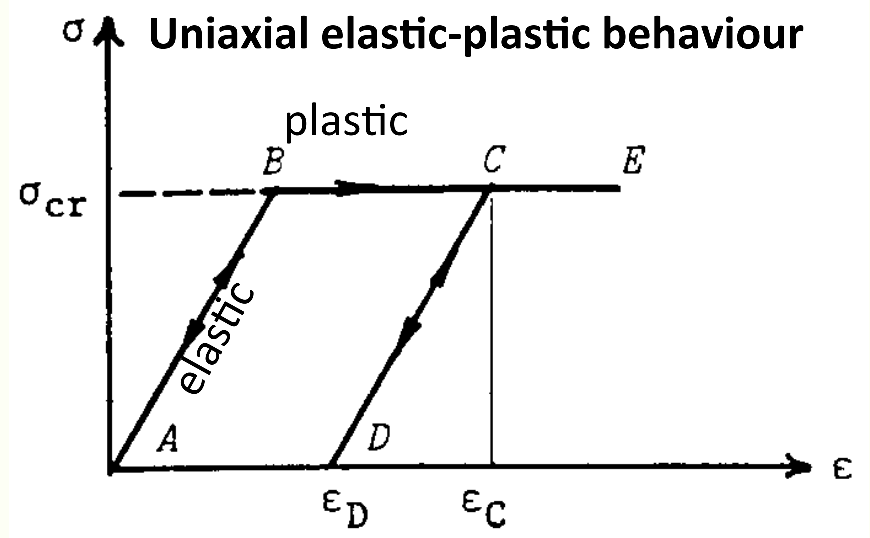
- Sea ice cover is permeated with **cracks**, e.g. uneven isostatic loading, long-period ocean waves, thermal expansion.
- Deformation occurs at cracks but ice between cracks is **rigid**.
- A crack may open to form a lead or close to form a pressure ridge with or without shear.
- Deformation occurs sporadically *as though* a **critical stress state** had been reached. (Air and ocean tractions are **smooth**.)

These features suggest sea ice is a PLASTIC material.

Other plastic materials are glass, metal, wood, and.. plastic.

Plasticity

The response of sea ice is elastic and/or viscous until the stress is great enough to fulfil a **yield criterion**.



Once this criterion is met, further deformation is plastic
i.e. non-recoverable and independent of the magnitude of strain rate.

Due to isotropy, the yield criterion may be expressed as a function of
the principal invariants of stress and scalars:

$$F\left(\sigma_I,\ \sigma_{II},\ \text{scalars}\right) = 0$$

$$\sigma_I \equiv \tfrac{1}{2}\sigma_{ii} = \tfrac{1}{2}\left(\sigma_1 + \sigma_2\right),$$ Negative pressure

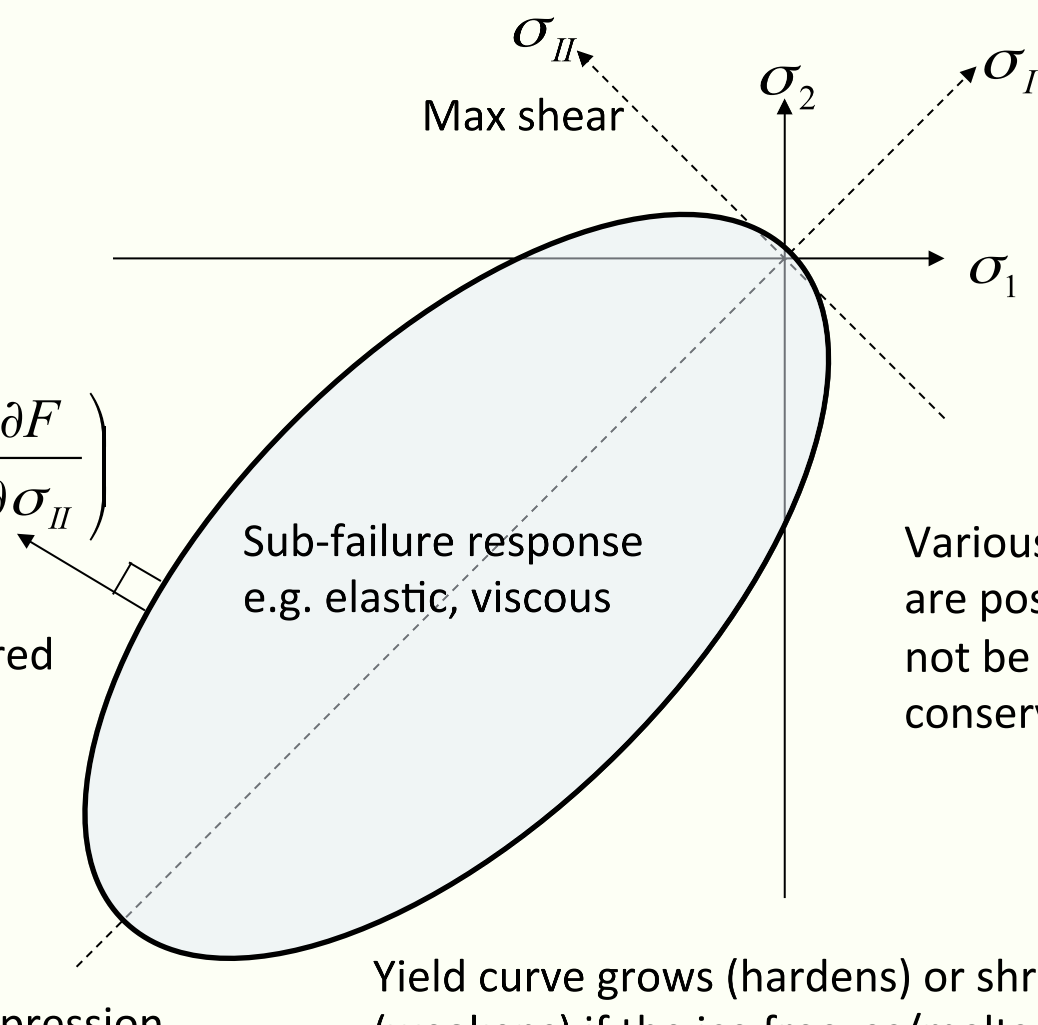
$$\sigma_{II} \equiv \sqrt{-\det\left(\sigma_{ij} - \delta_{ij}\sigma_I\right)} = \tfrac{1}{2}\left(\sigma_2 - \sigma_1\right)$$ Max. shear stress

Plastic yield curve $F\left(\sigma_I,\ \sigma_{II},\ \text{scalars}\right)=0$ for sea ice

Plastic flow occurs **on** the yield curve:

$$\left(\dot{\varepsilon}_I,\ \dot{\varepsilon}_{II}\right)=\lambda\left(\frac{\partial F}{\partial \sigma_I},\ \frac{\partial F}{\partial \sigma_{II}}\right)$$

Plastic flow is any deformation not recovered when the stresses are removed, e.g. pressure ridging, sliding of floes past each other.



Various yield curves are possible but must not be concave to satisfy conservation of energy.

Yield curve grows (hardens) or shrinks (weakens) if the ice freezes/melts or increases/decreases concentration.

Modelling sea ice

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Local mass balance for sea ice

The local sea ice mass balance alters through melting/freezing and advection.

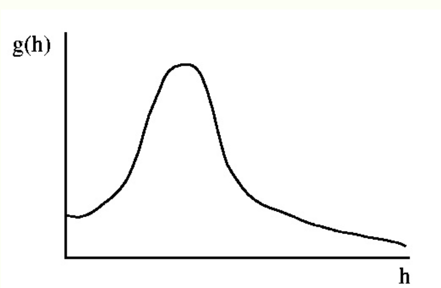
Sea ice is also redistributed in thickness space through deformation (ridging).

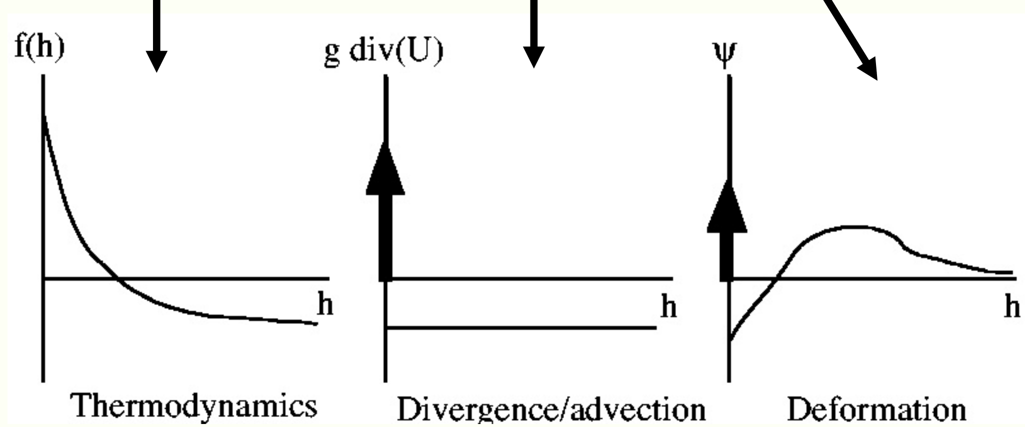
The normalised thickness distribution function *g(h)* is defined such that *g(h)dh* is the area of ice with thickness *h* to *h+dh*.

The local mass balance equation is expressed as

$$\frac{\partial g}{\partial t} = -\frac{\partial (fg)}{\partial h} - \nabla \cdot (Ug) + \psi$$

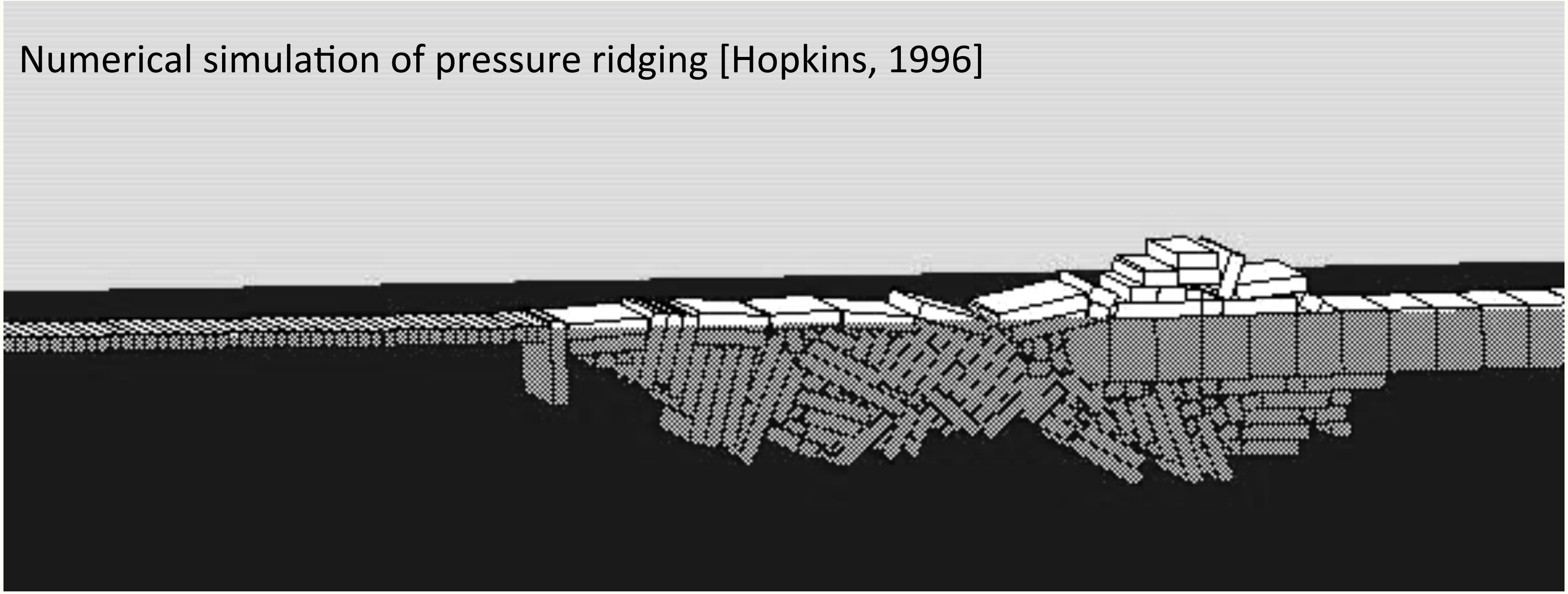
Thorndike and Maykut [1973]





Note that in modern climate models, *g(h)* is typically discretised into 5 thickness classes

Thickness redistribution function $\psi(h)$



Simulations, laboratory experiments, and field observations have been used to develop mathematical expressions for the thickness redistribution function $\psi(h)$

While there are various formulations, all describe the redistribution of thin ice to thick ice during the pressure ridging process.

(Older sea ice models) two level thickness distribution

Computational constraints and observational ignorance of thickness led to a simple, two level thickness distribution still used in many climate models.

Area concentration *A* of ice with average thickness *h* greater than a cut-off thickness h_0 = 0.5m :

$$\frac{\partial h}{\partial t} = -\nabla.(\mathbf{u}h) + S_h + \zeta_1 \nabla^2 h + \zeta_2 \nabla^4 h,$$

$$\frac{\partial A}{\partial t} = -\nabla.(\mathbf{u}A) + S_A + \zeta_3 \nabla^2 A + \zeta_4 \nabla^4 A$$

$S_h, \; S_A$ are complicated thermodynamic source/sink terms of thickness and concentration

$\zeta_1 - \zeta_4$ are small diffusion coefficients, used to regularise the solution

Modelling sea ice

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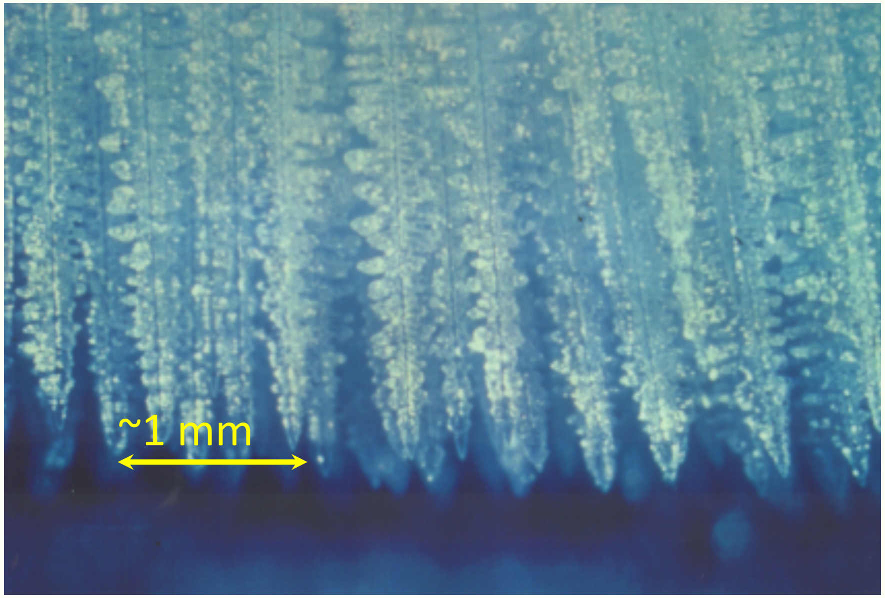
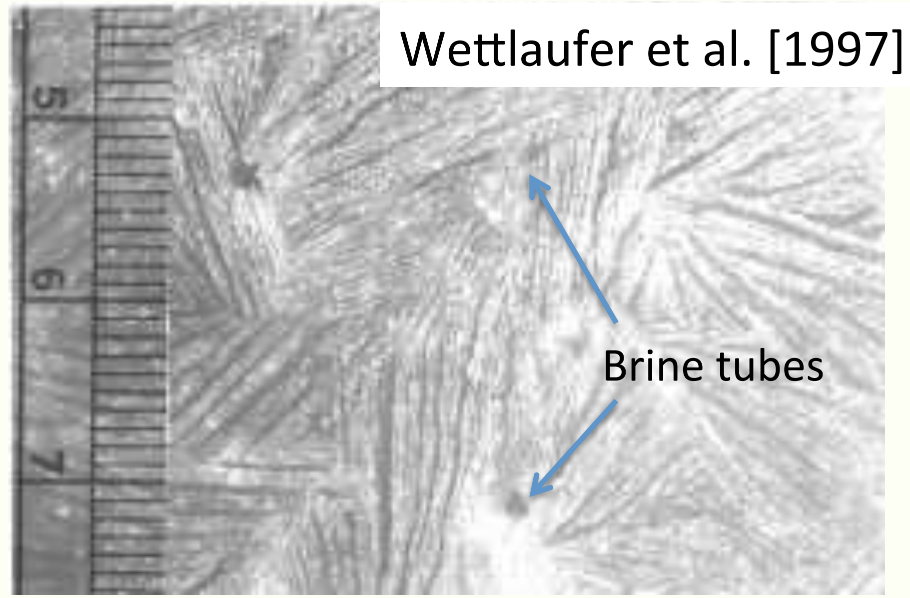
Sea ice is not just a slab of ice, it is a **mushy layer**





A mushy layer consists of a (typically) porous matrix of (almost) pure solid bathed in (highly concentrated) interstitial liquid.

The convoluted geometry enhances expulsion of solute and heat.

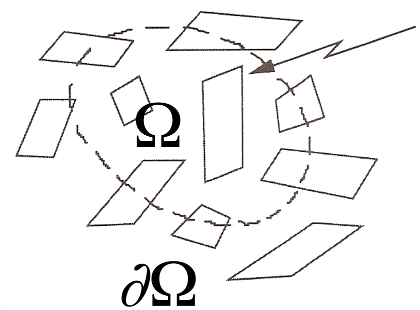
In sea ice, the solid matrix is pure ice and the liquid phase is brine. The brine can escape from the sea ice in brine tubes.

Sea ice (all of it) is an example of a mushy layer, and the mushy layer equations are used for the heat and salt balances [Feltham et al, 2006].

Developing a continuum mushy layer model from conservation equations

Local conservation of an arbitrary scalar Φ, e.g. heat, salt etc, is

$$\frac{\partial}{\partial t}\int_{\Omega}\Phi dv = -\int_{\partial\Omega}\mathbf{n}\cdot\mathbf{J}ds + \int_{\Omega} S dv$$



"Infinitesimal" control volume used for defining averages and derivatives. Contains representative samples of both phases.

where S is a source/sink per unit volume and $\mathbf{J}$ is a flux of Φ

$$\mathbf{J} = \mathbf{U}\Phi - D_{\Phi}\nabla\Phi \quad \text{(+ radiative flux for heat)}$$

Using the divergence theorem $\int_{\partial\Omega}\mathbf{n}\cdot\mathbf{J}ds = \int_{\Omega}\nabla\cdot\mathbf{J}dv$

we can write, since the control volume is arbitrary and fixed, that

$$\frac{\partial\Phi}{\partial t} + (\nabla\cdot\mathbf{U})\Phi + \mathbf{U}\cdot\nabla\Phi = \nabla\cdot(D_{\Phi}\nabla\Phi) + S$$

Mathematical description of a mushy layer

$$(1-\varphi)\frac{\partial C}{\partial t}+U\cdot\nabla C=\nabla\cdot\left(\bar{D}\nabla C\right)+\frac{\rho_s}{\rho_l}\left(C-C_s\right)\frac{\partial\varphi}{\partial t}$$

Salt balance

φ is solid fraction

$$\bar{\rho}\bar{C}_p\frac{\partial T}{\partial t}+\rho_l C_{pl}U\cdot\nabla T=\nabla\cdot\left(\bar{k}\nabla T\right)+\rho_s L\frac{\partial\varphi}{\partial t}$$

Heat balance

$$\nabla\cdot U=\left(1-\frac{\rho_s}{\rho_l}\right)\frac{\partial\varphi}{\partial t}$$

Mass balance

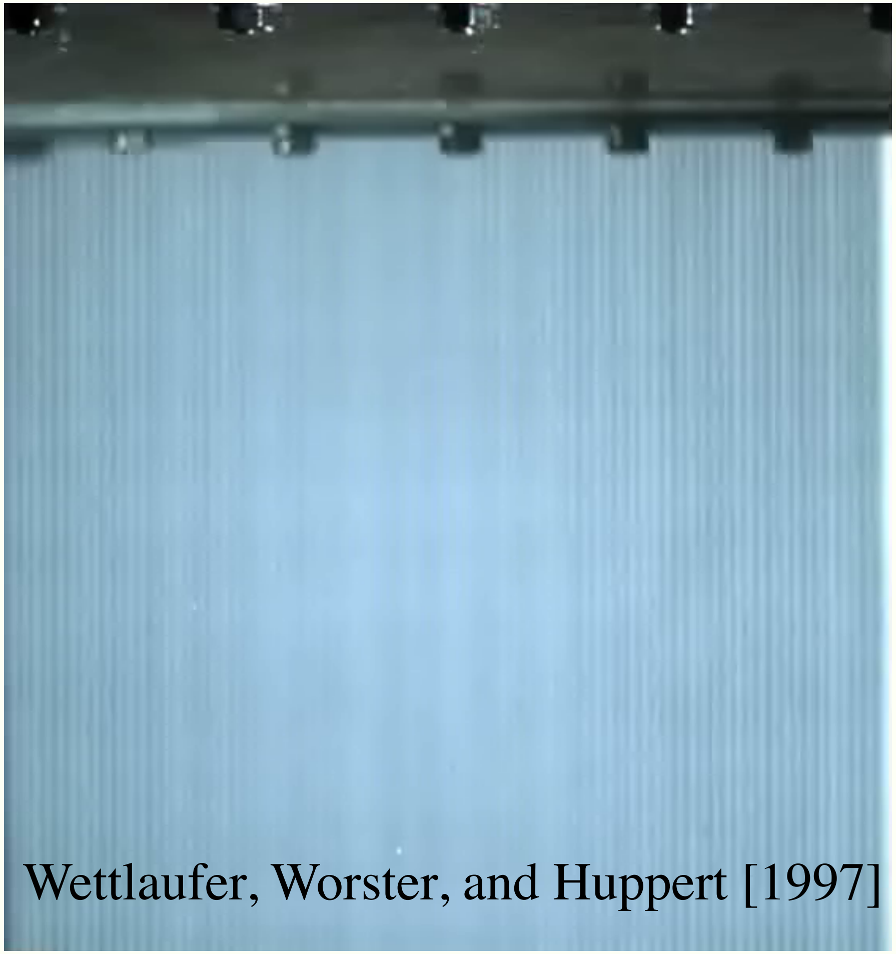
$$T=T_L\left(C\right)$$

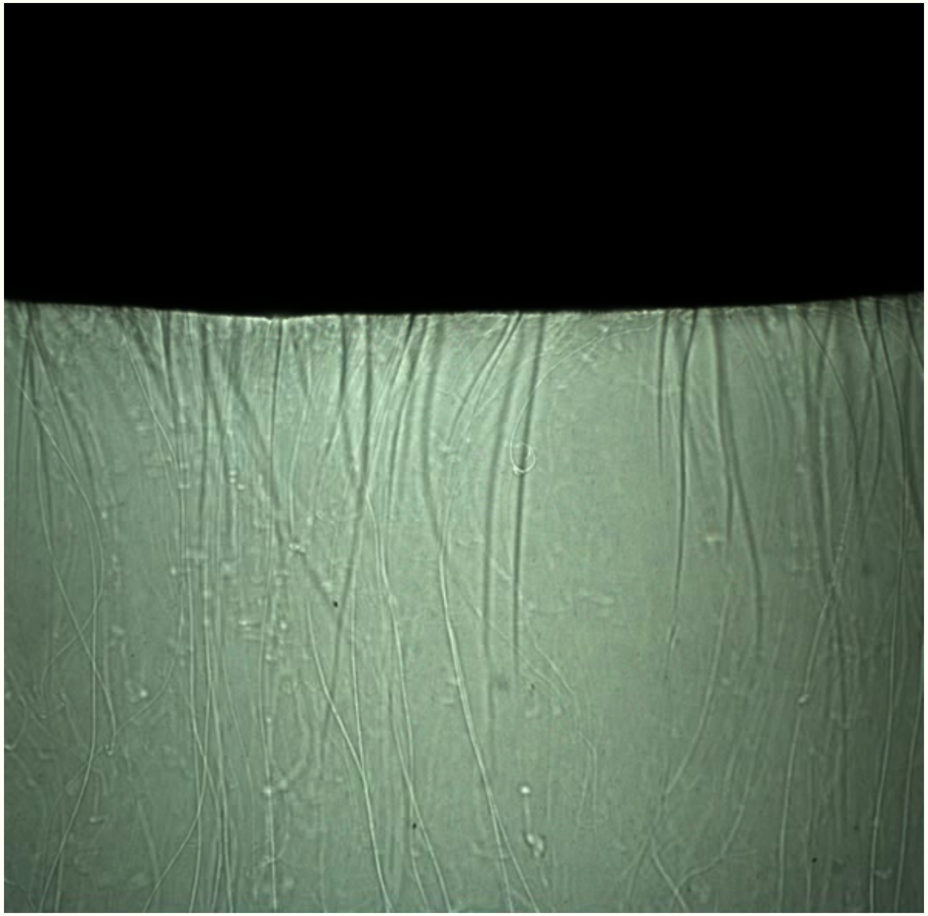
Thermodynamic equilibrium

Note: classical mushy layer theory does not include the momentum balance. Flow of brine in sea ice is typically well approximated by Darcy's Law

$$\mathbf{U}=-\frac{\Pi}{\mu}\nabla p'$$

Mushy layer theory allows us to describe brine drainage





Unstable density profile of brine salinity in sea ice can drive gravity drainage.
Accurate modelling of brine drainage is currently quite a "hot" topic in sea ice research.

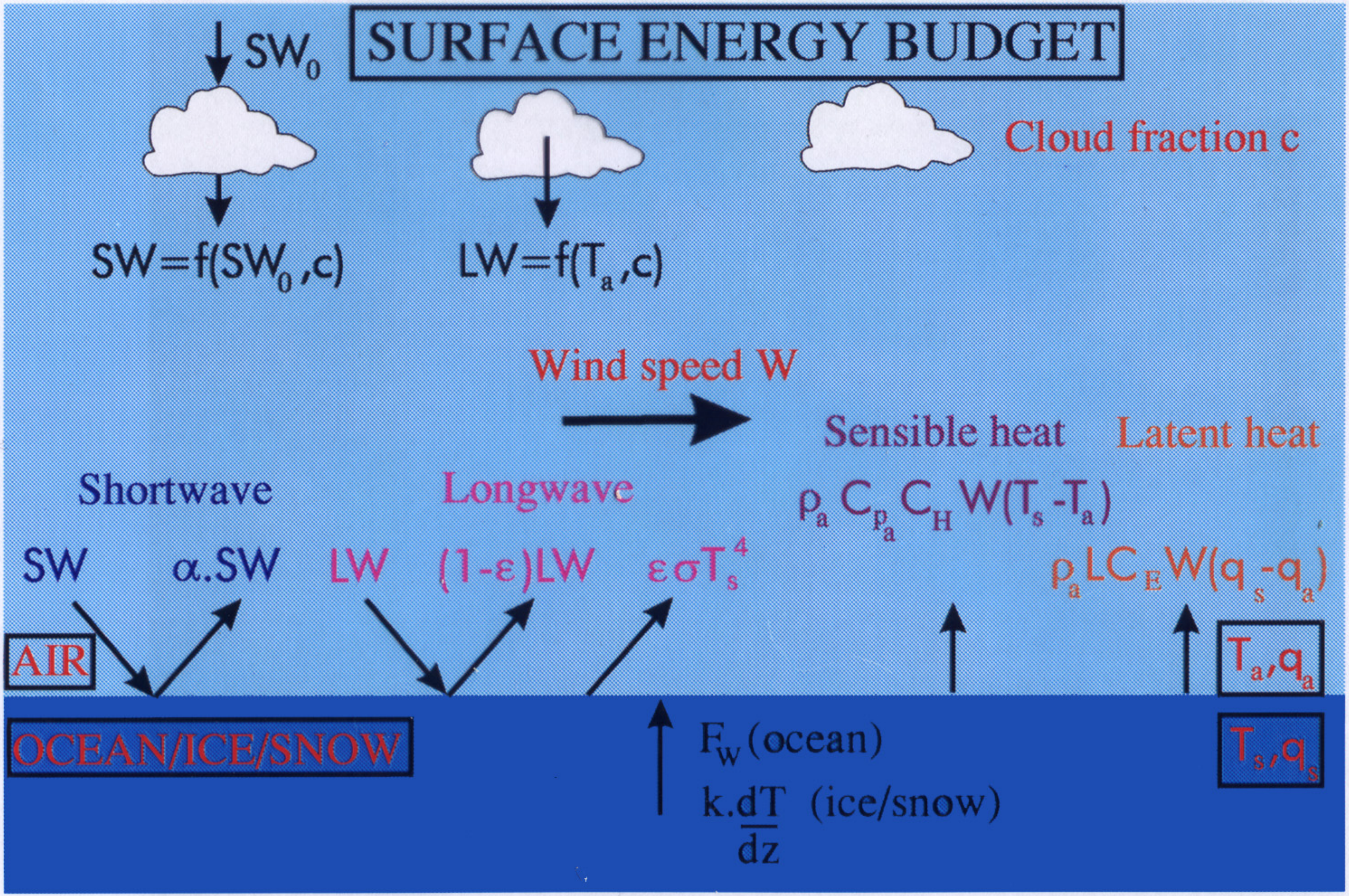
Brinicle

Brinicle

… also known as the ICE FINGER OF DEATH!

Modelling sea ice

- Local momentum balance
- Local mass balance
- Local heat balance
- Typical model output

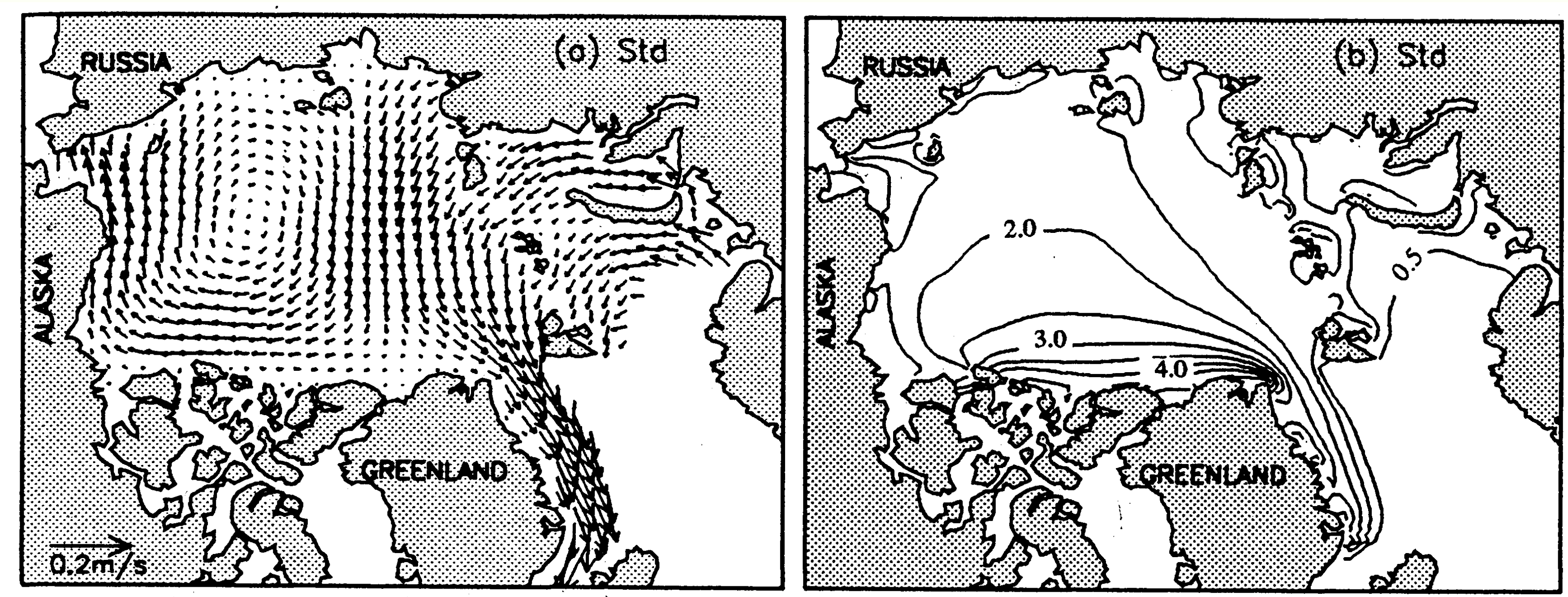


SURFACE ENERGY BUDGET
SW_0
Cloud fraction c
$SW=f(SW_0,c)$
$LW=f(T_a,c)$
Wind speed W
Sensible heat
Latent heat
Shortwave
Longwave
$\rho_a C_{p_a} C_H W(T_s - T_a)$
SW
$\alpha.SW$
LW
$(1-\varepsilon)LW$
$\varepsilon\sigma T_s^4$
$\rho_a L C_E W(q_s - q_a)$
AIR
T_a, q_a
OCEAN/ICE/SNOW
F_W (ocean)
$k.\frac{dT}{dz}$ (ice/snow)
T_s, q_s

Sea ice drift patterns and ice thickness using typical sea ice model

(a) Mean winter ice velocity; (b) thickness.

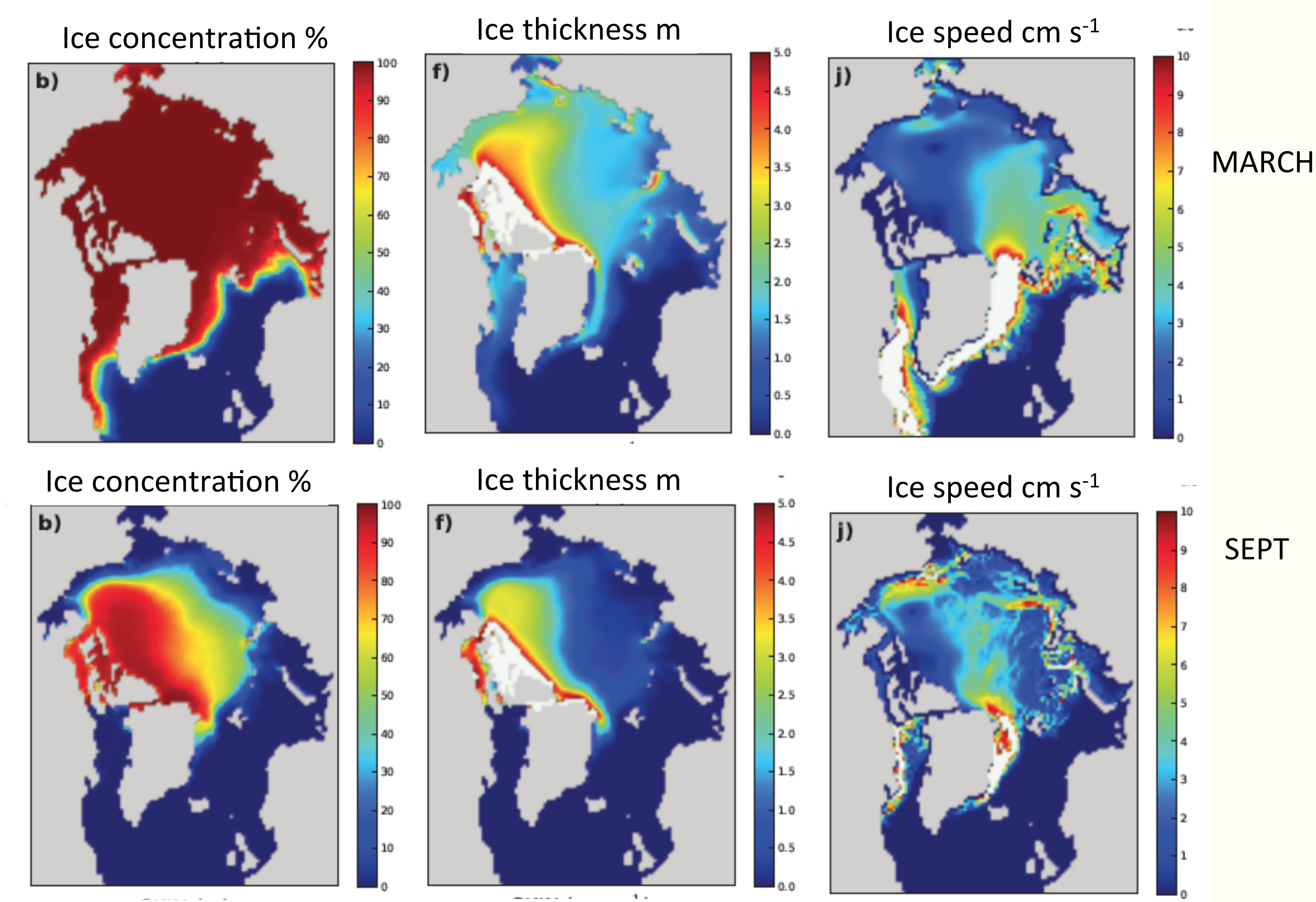
[Steele *et al*, 1997]



Dominant drift = Beaufort Gyre + Transpolar "stream"

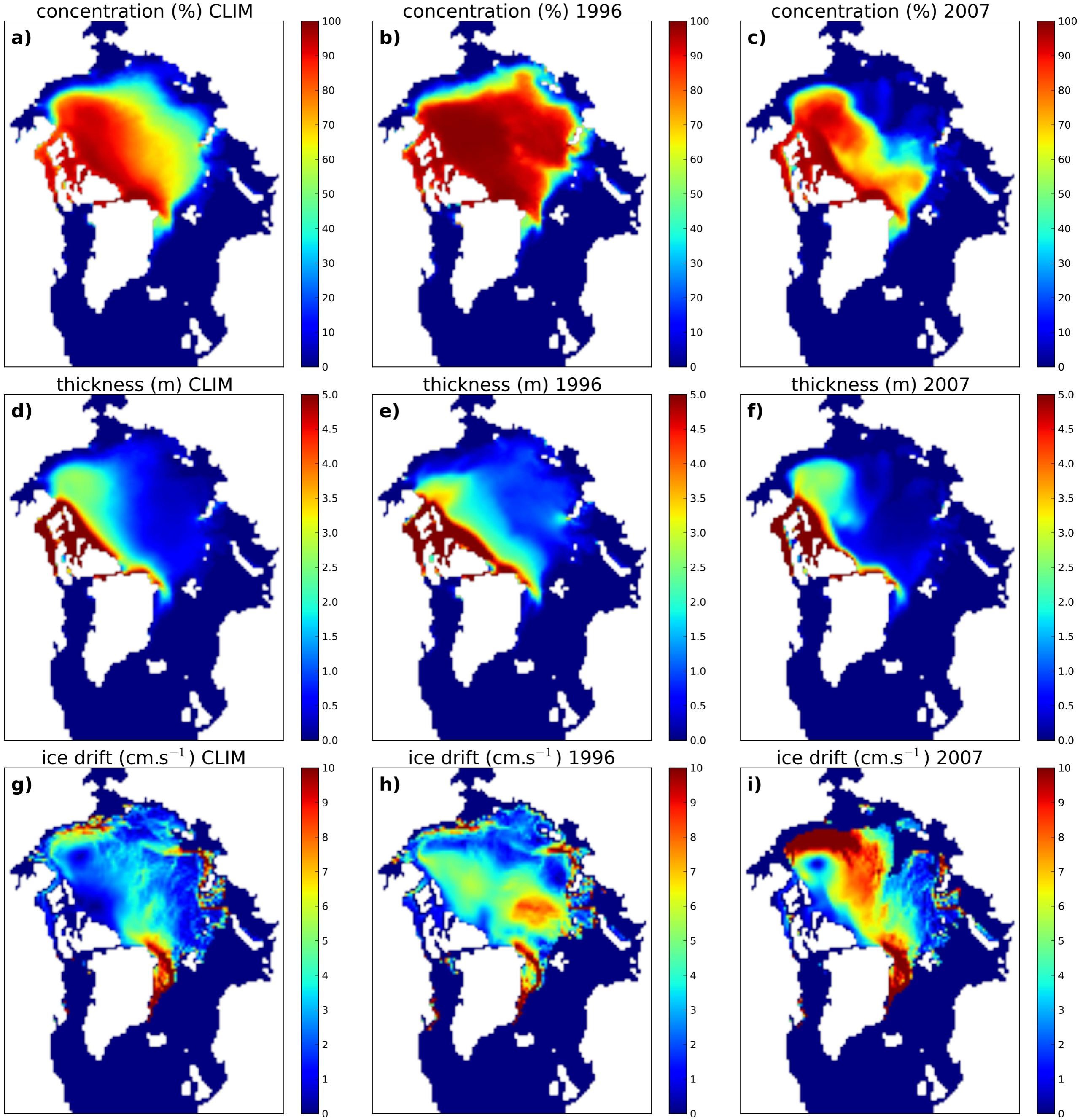
Note: ice piles up against Canadian archipelago and North Greenland.

Typical climatology from a highly calibrated, forced sea ice model (1979-2012)



Inter-annual variability in a forced sea ice model

September



Inter-annual variability in a forced sea ice model

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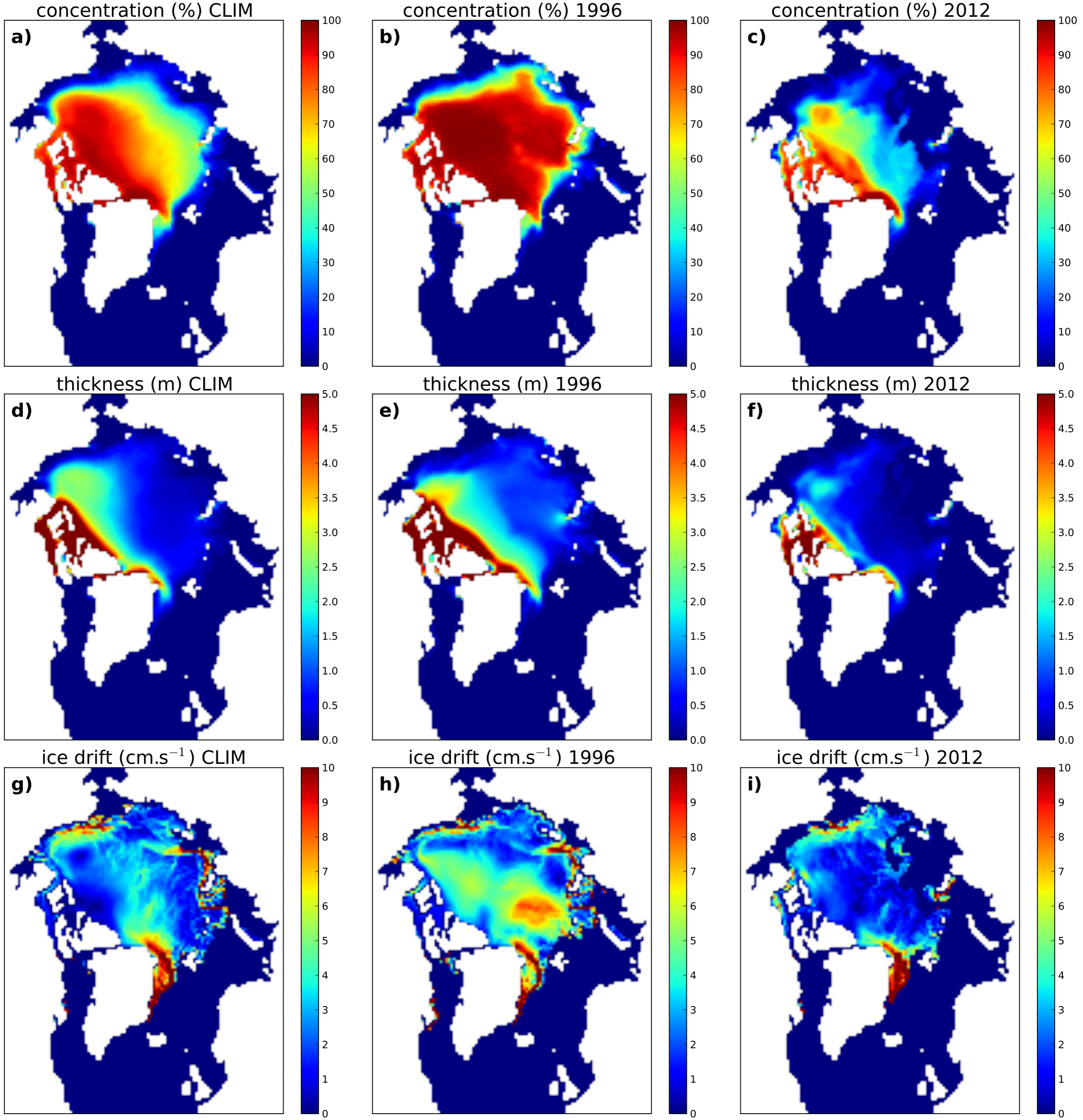
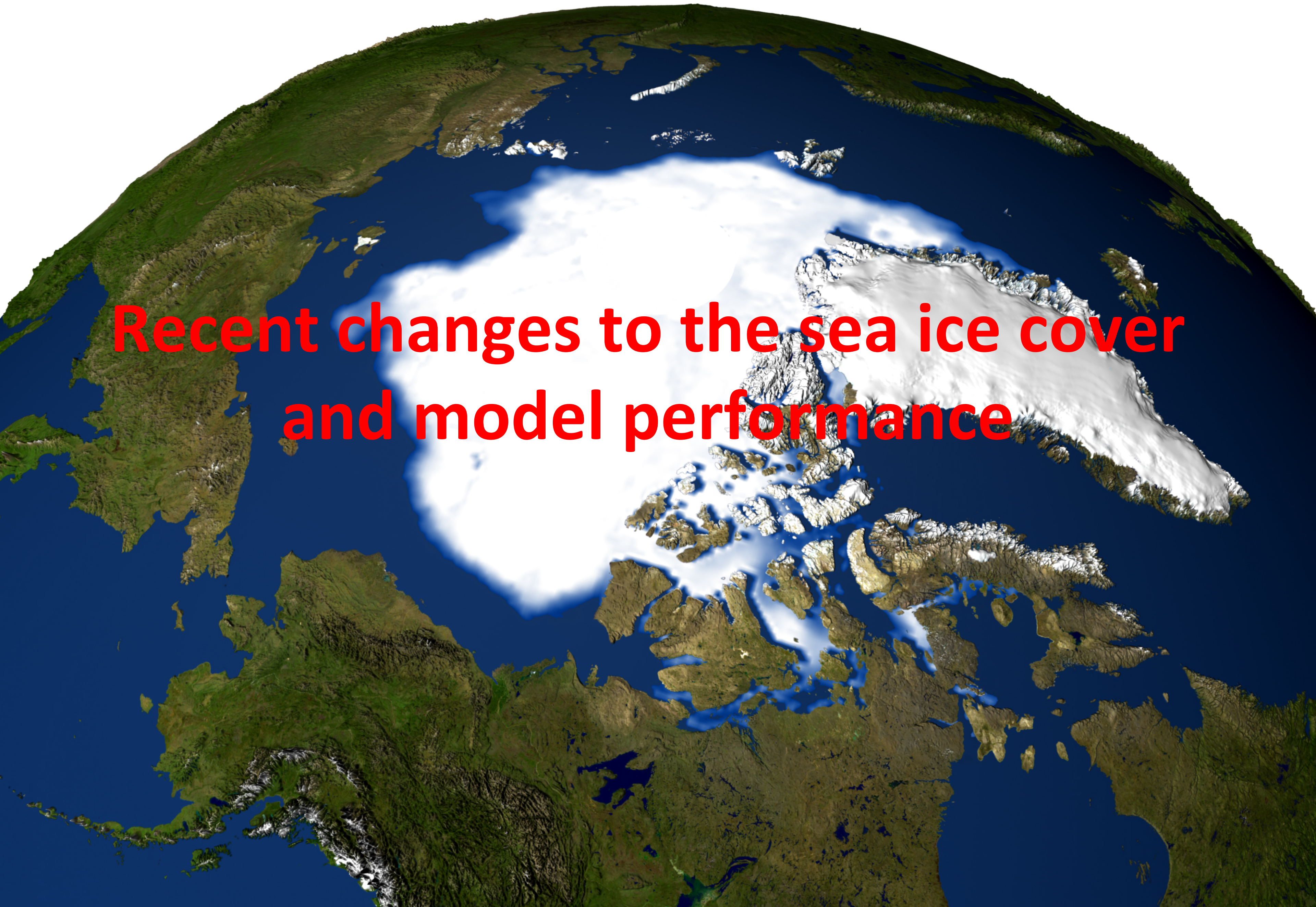


Recent changes to the sea ice cover and model performance

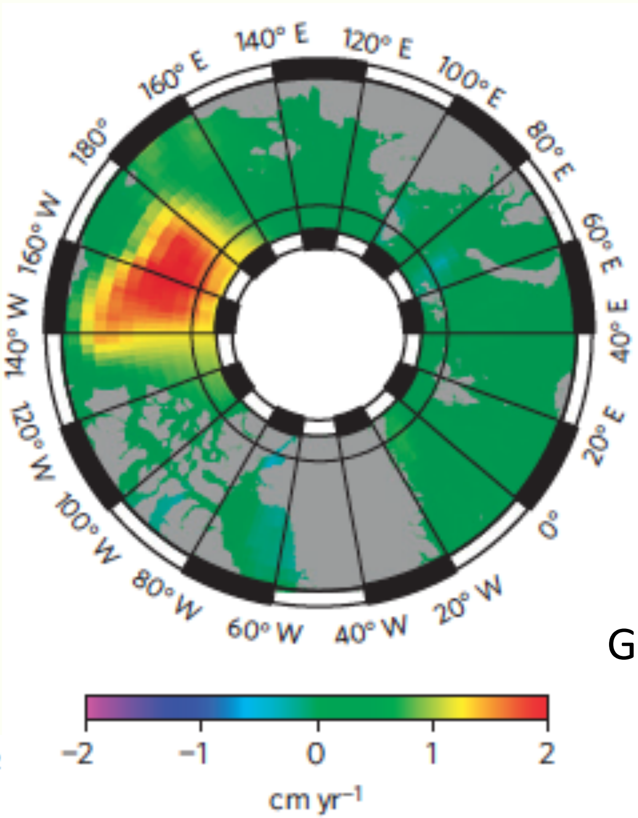


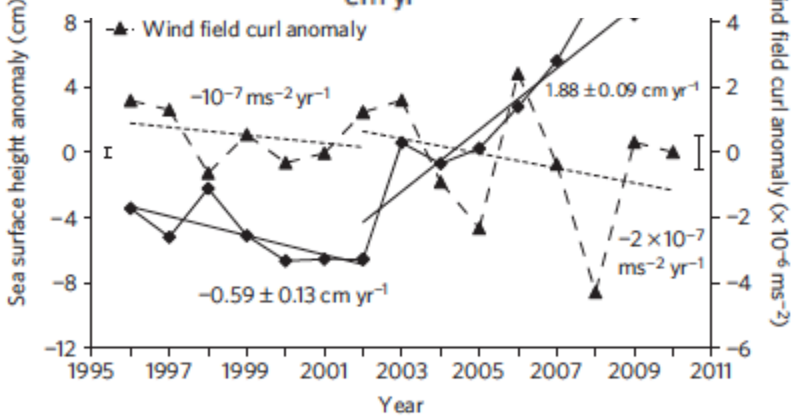
Sea ice concentration from satellites in the Arctic

Sea ice age

NOAA Climate.gov

Spin up of Arctic Ocean





Giles et al 2012

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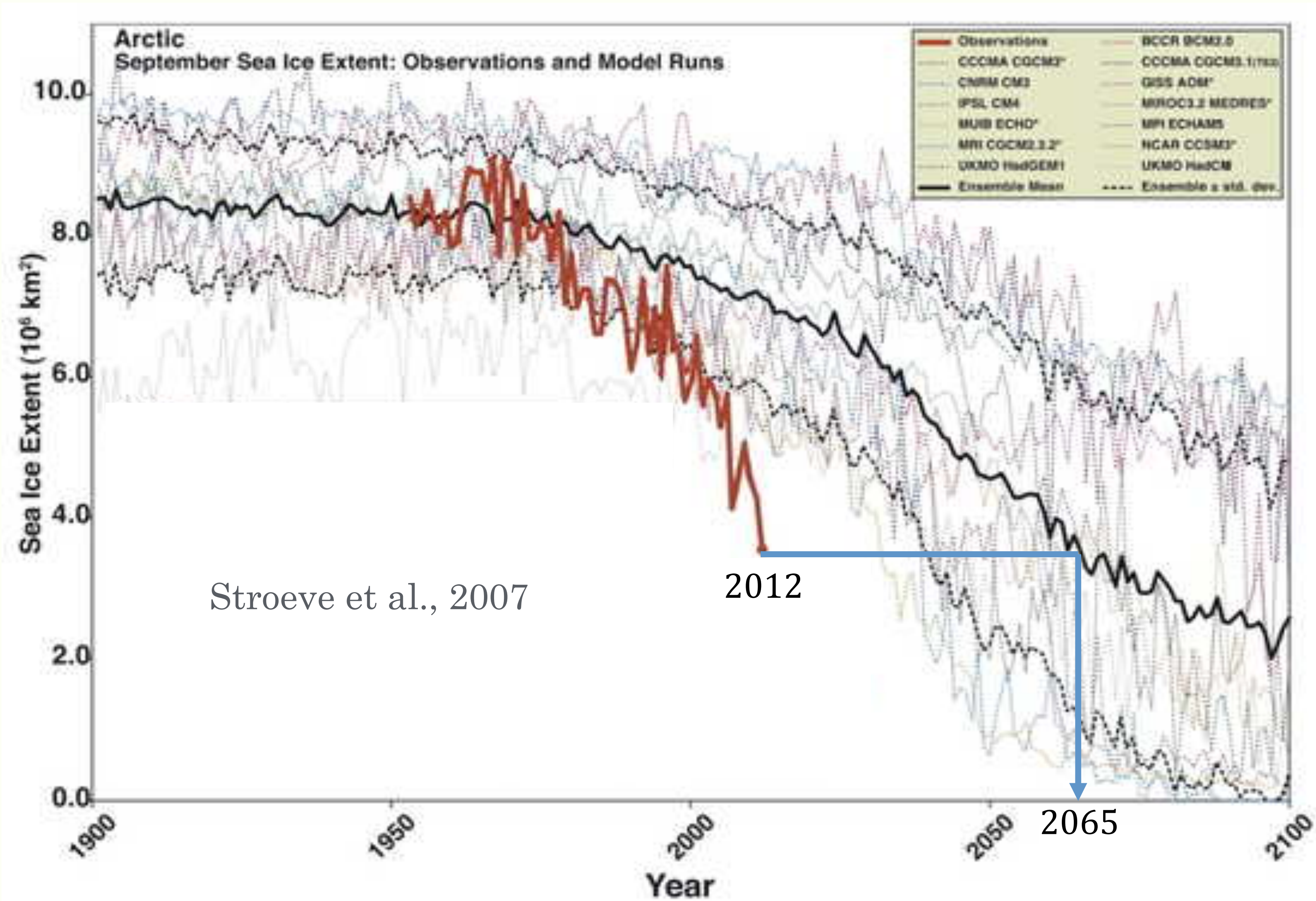
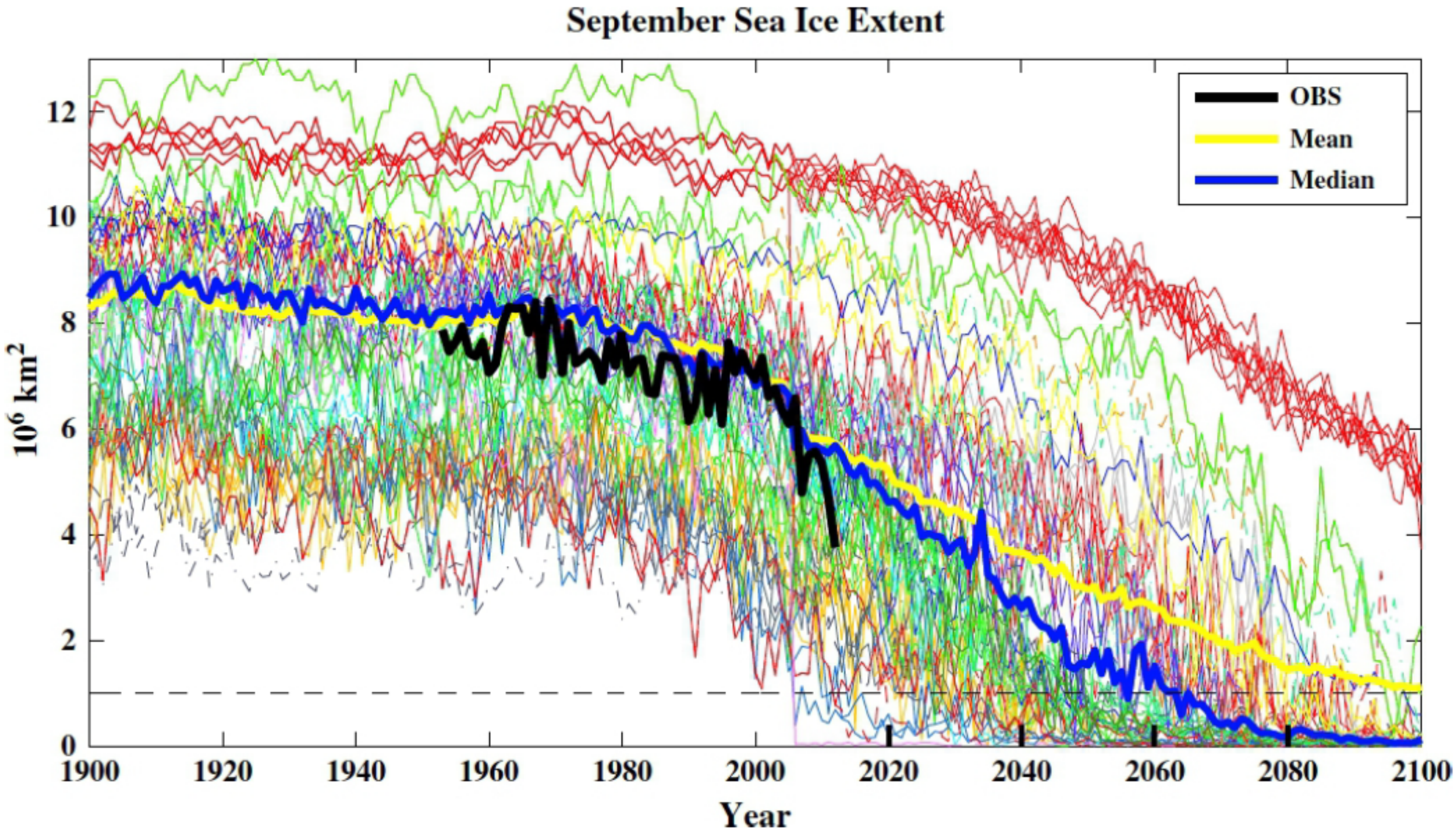
Rapid reduction of summer Arctic sea ice extent

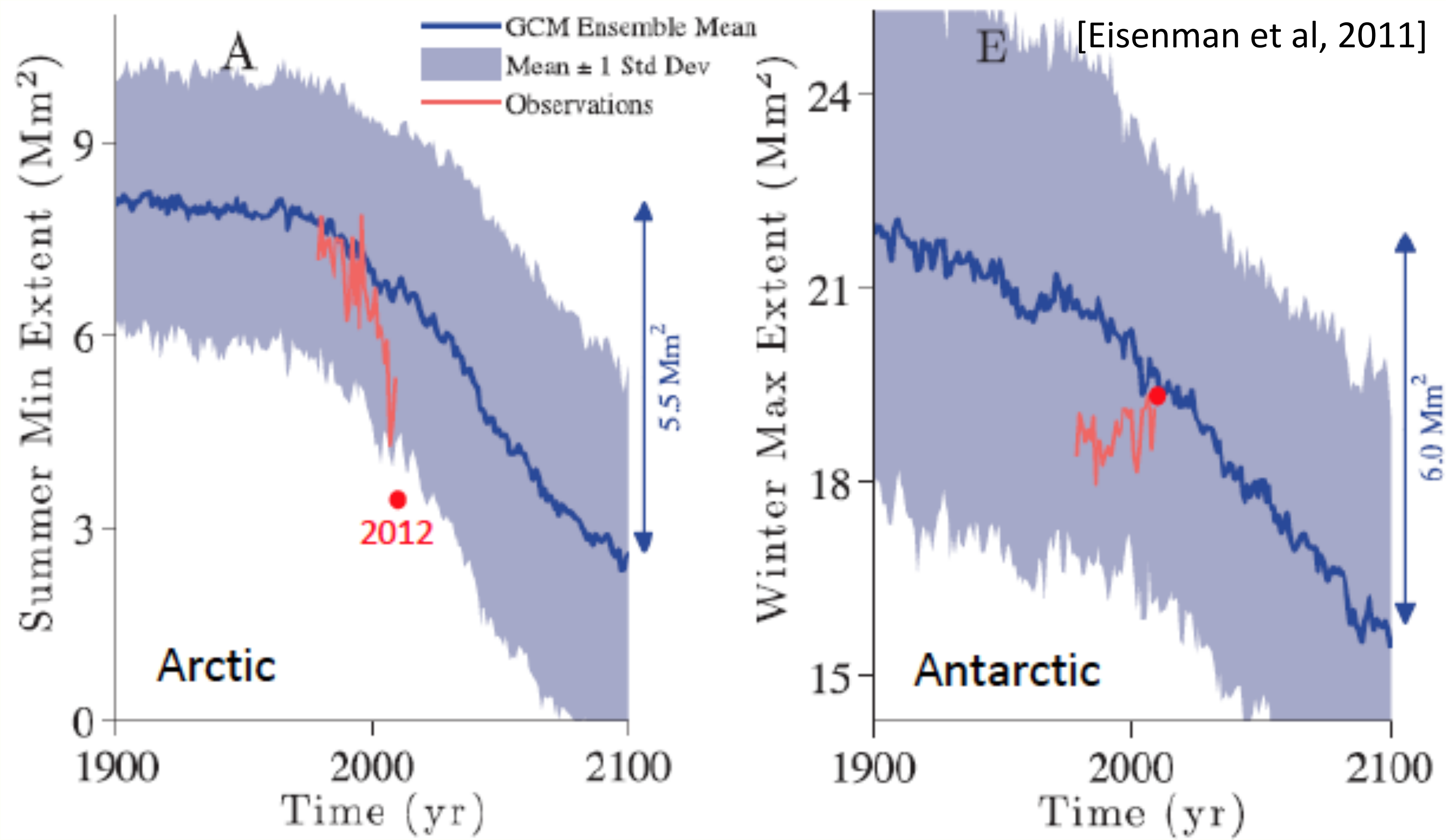


Model predictions CMIP5

Overland 2013



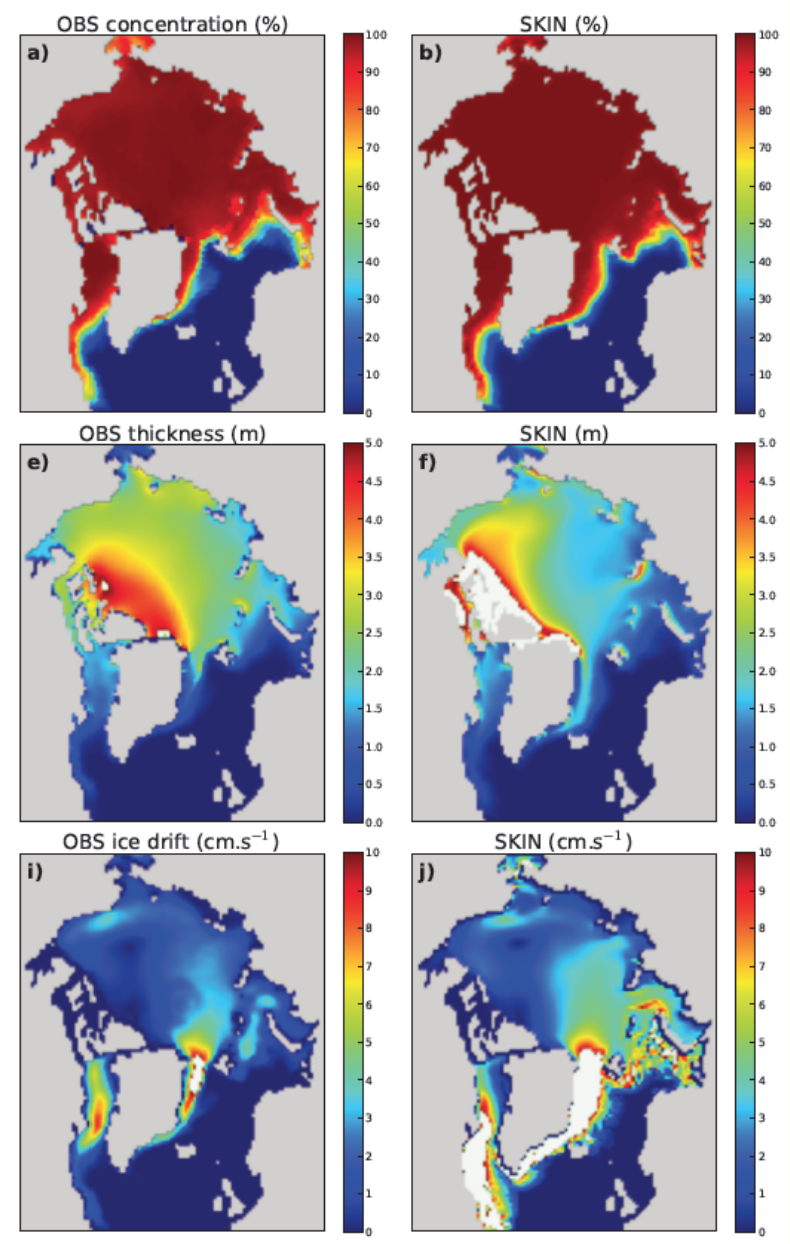
Sea ice extent: South versus North

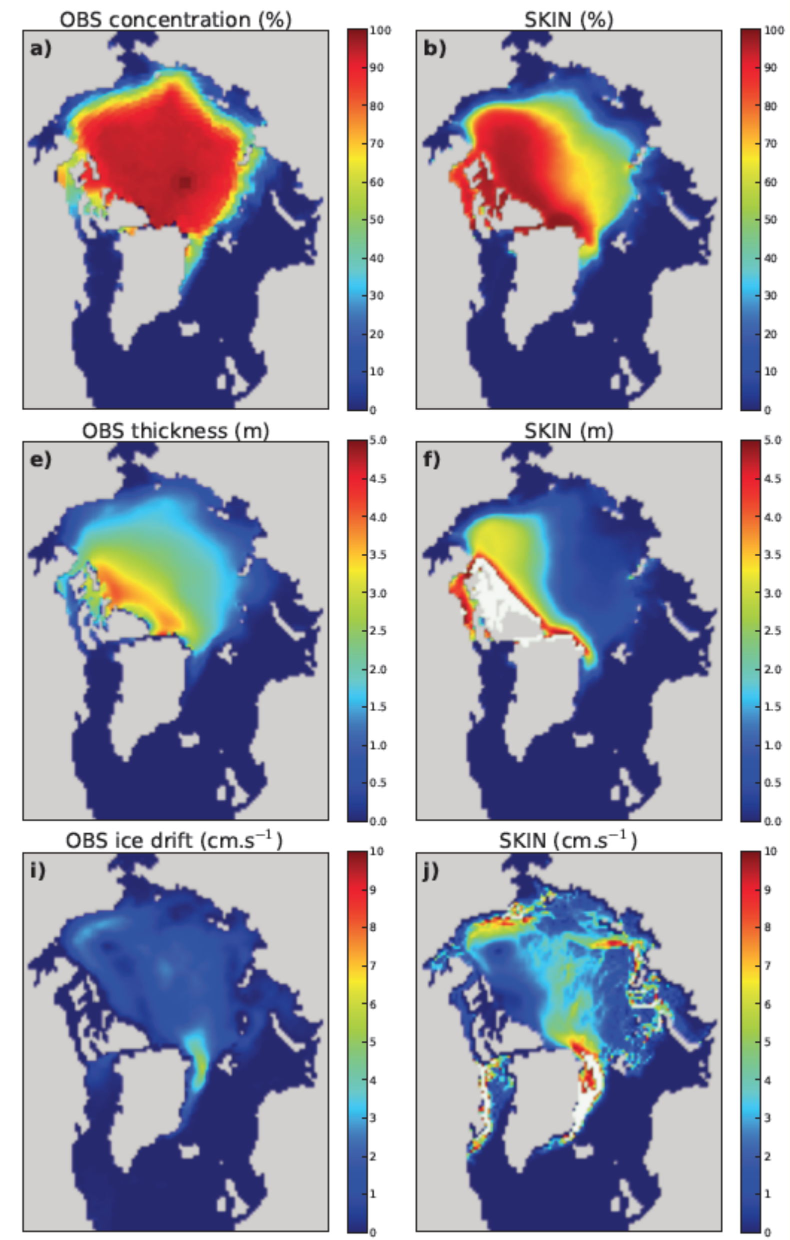


Small increasing trend of Antarctic sea ice extent not captured by models.
2012 saw a record in both poles.

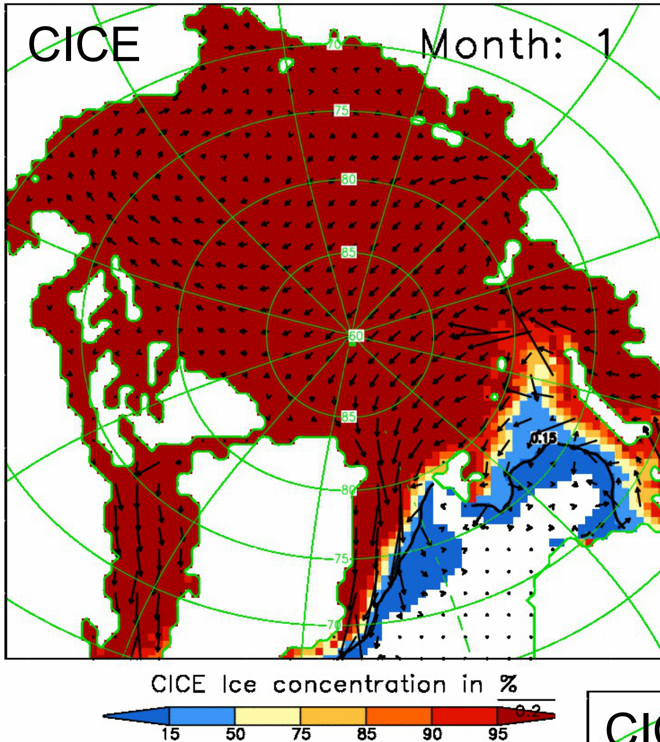
Typical climatology from a “good” forced sea ice model (1979-2012)

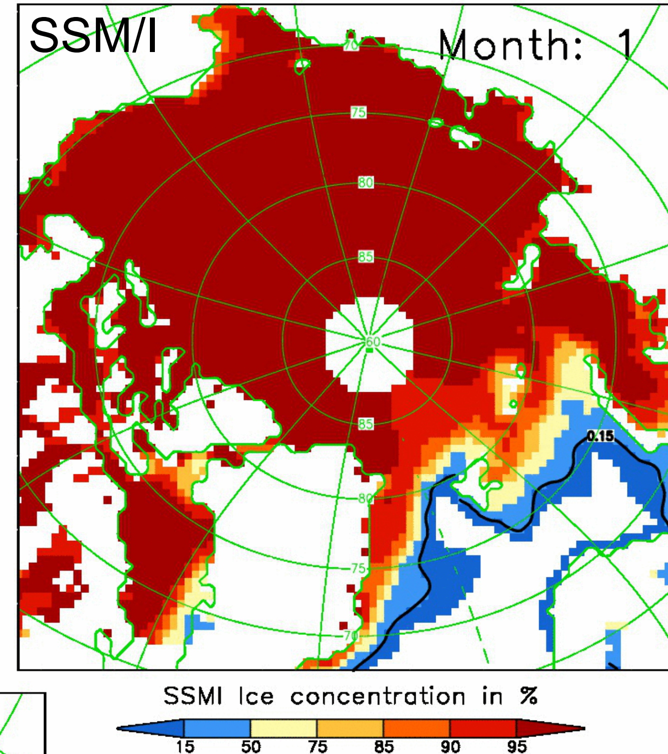
MARCH Tsamados et al, 2013 SEPTEMBER

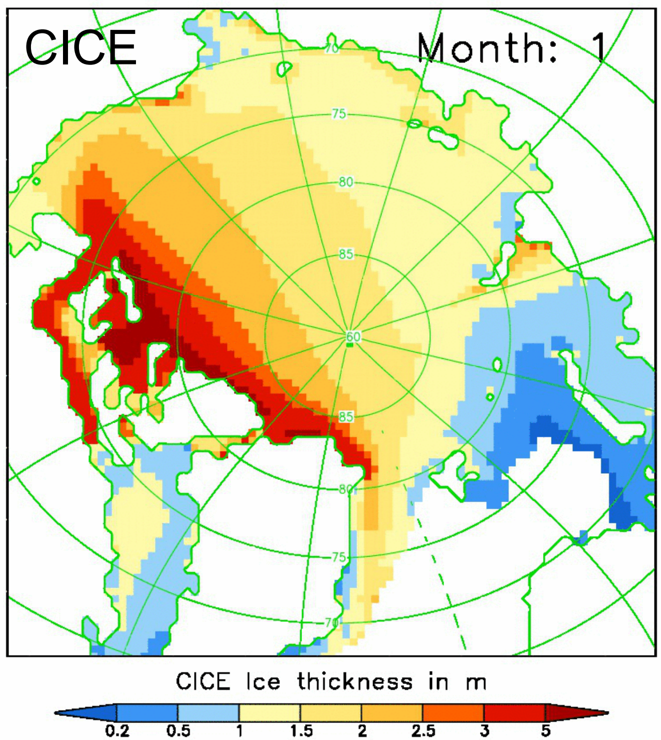




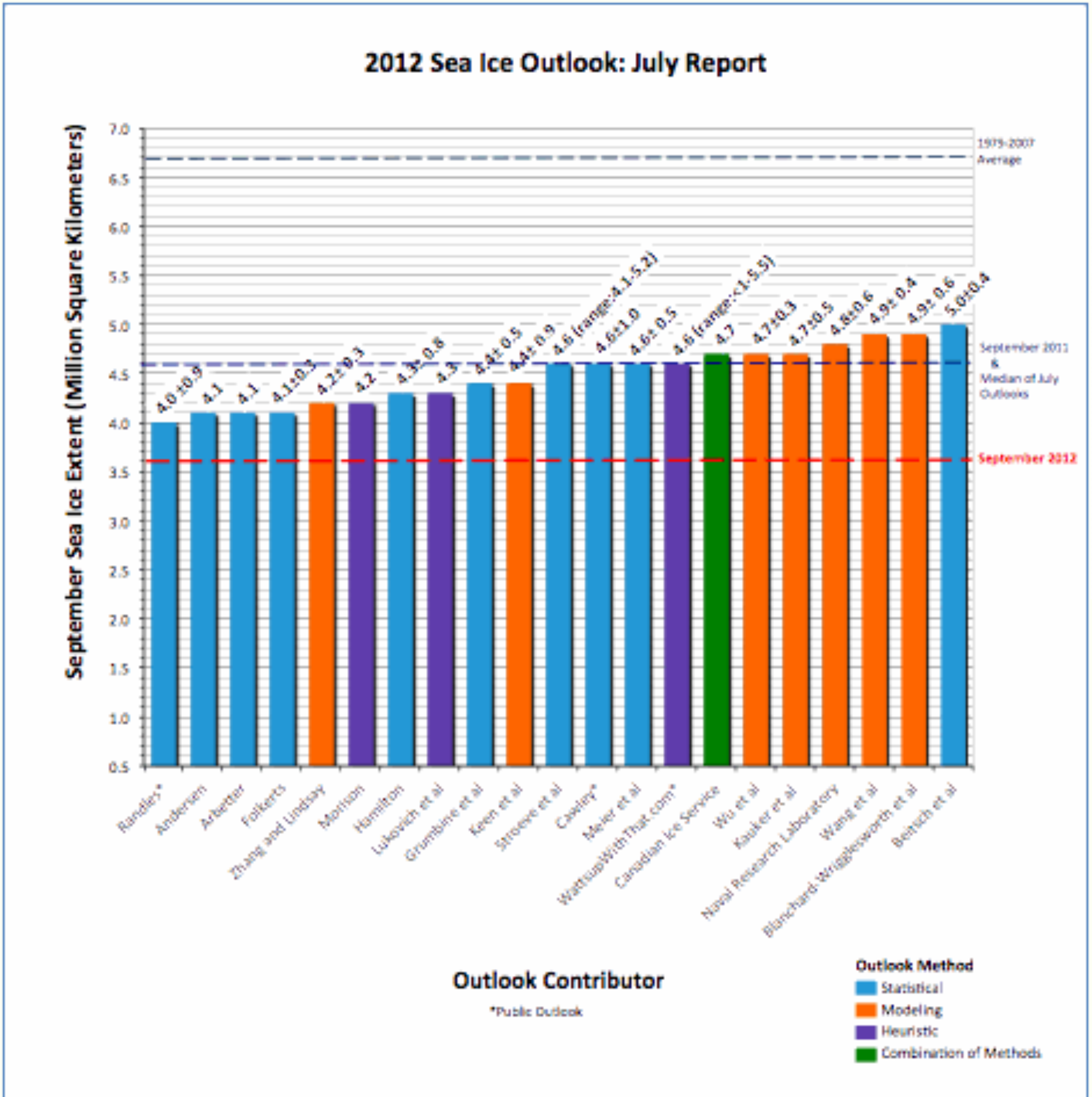
Climatology, 1990-2007

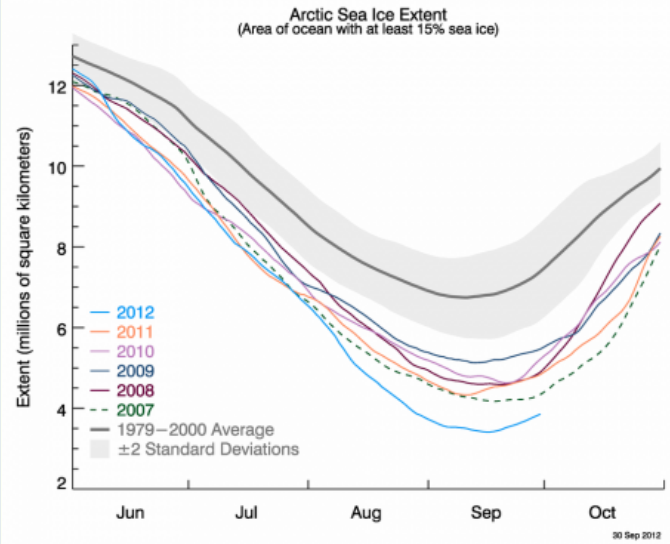






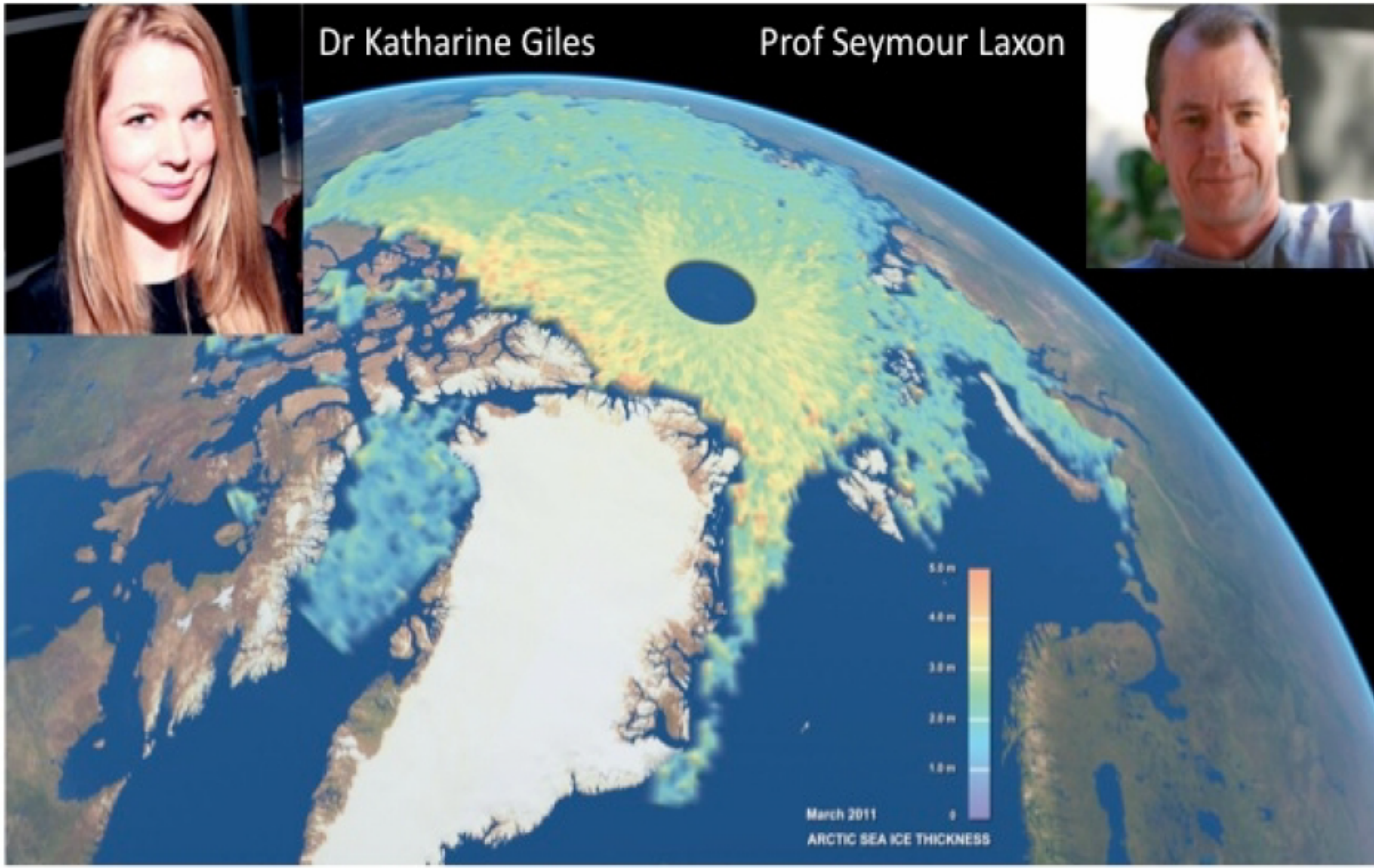
The difficulty of predicting a few months ahead





Study of Environmental Arctic Change (SEARCH)

Measuring the Arctic sea ice thickness - CryoSat-2



Sea ice model performance against observations

- **Coupled climate models** do not reproduce **actual** sea ice concentration, but do produce **realistic ice concentrations** and **levels of natural variability of concentration.**

Thickness and motion (and hence mass fluxes) are typically **wrong by a factor of 2 or more** [e.g. Holland, 2007; Stroeve, 2012]

- Climate models **do** predict a reduction of Arctic sea ice but **underestimate** the trend. Climate models typically predict an equal loss of sea ice in the Southern Ocean, which is **not observed**.
- **Highly-tuned, forced** (uncoupled) sea ice models can produce quite accurate hindcasts of ice concentration, but do less well for ice motion and thickness.
- **Poor performance** of sea ice models related to **atmospheric and oceanic** forcing but simulations also sensitive at leading order to **poorly constrained model physics** [e.g. Miller et al, 2006].

Summary

- Sea ice is an important **component** of the climate system and a **sensitive indicator of climate change**, with **rapid changes** seen in the Arctic
- Sea ice models are based on continuum local balances of **momentum, mass, and heat**
- Sea ice behaves as a **plastic** material and is a **mushy layer**
- Models of sea ice have **limited predictive** capability, due to uncertainty in **forcing (air/ocean)** and **model physics**
- More **realistic model physics** (e.g. melt ponds) **improves** the ability of models to accurately **reproduce** and **predict** the evolution of the sea ice cover