

Linking Scales in Earth's Sea Ice System

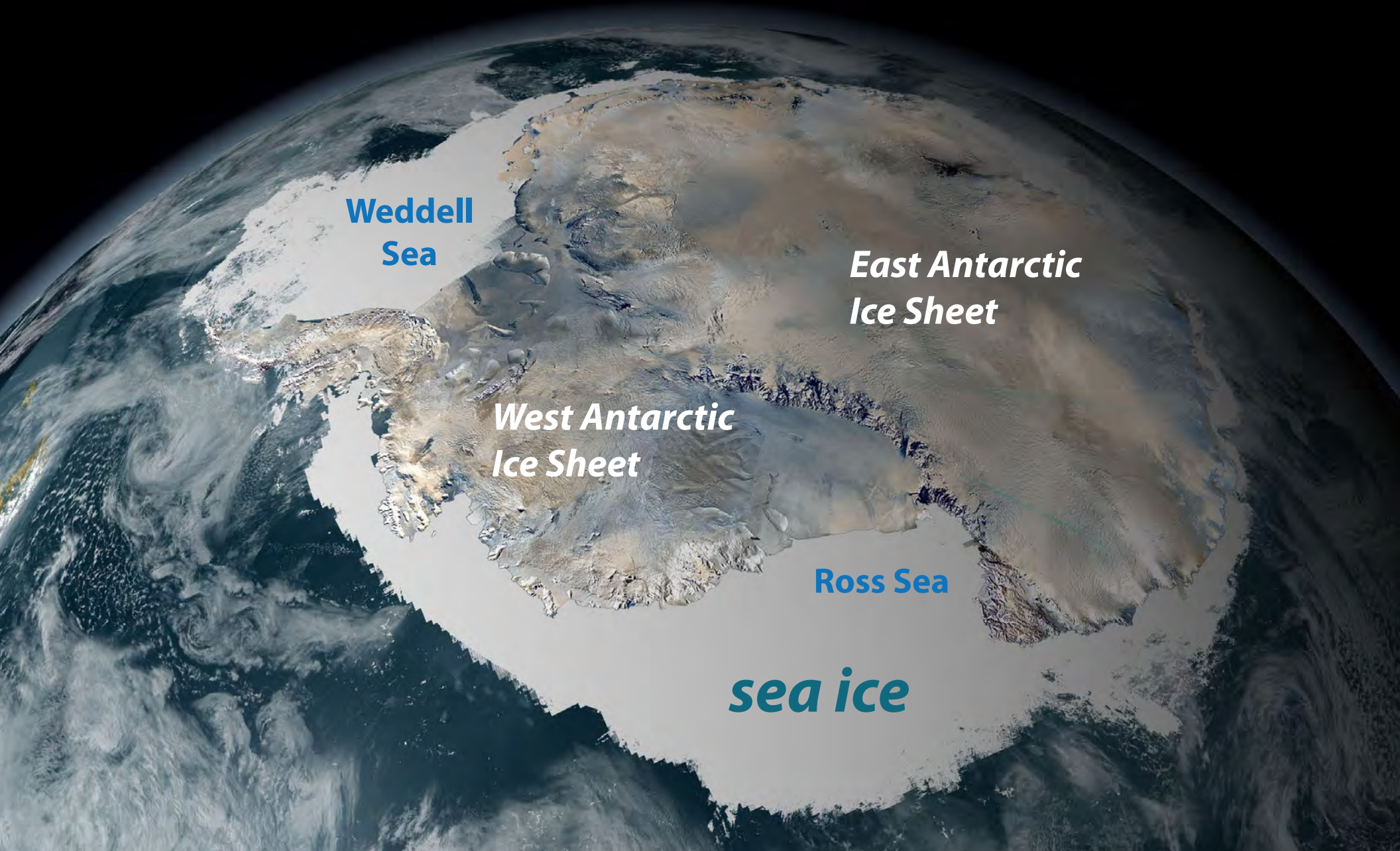
Kenneth M. Golden
Department of Mathematics
University of Utah



Alison Kohout 2012

ANTARCTICA

southern cryosphere



**Weddell
Sea**

***East Antarctic
Ice Sheet***

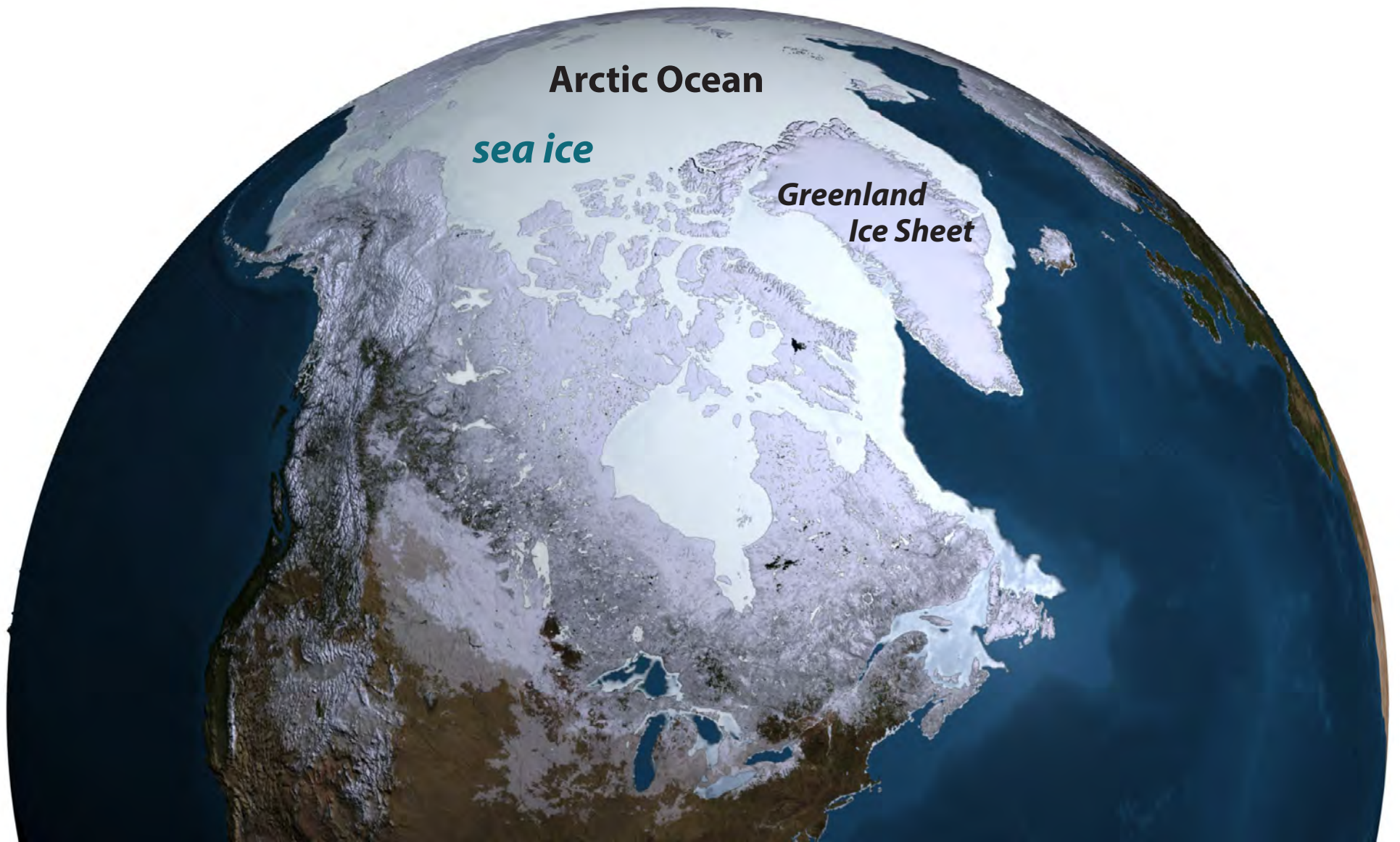
***West Antarctic
Ice Sheet***

Ross Sea

sea ice

THE ARCTIC

northern cryosphere



SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of **climate change**



polar ice caps critical to global climate in reflecting incoming solar radiation



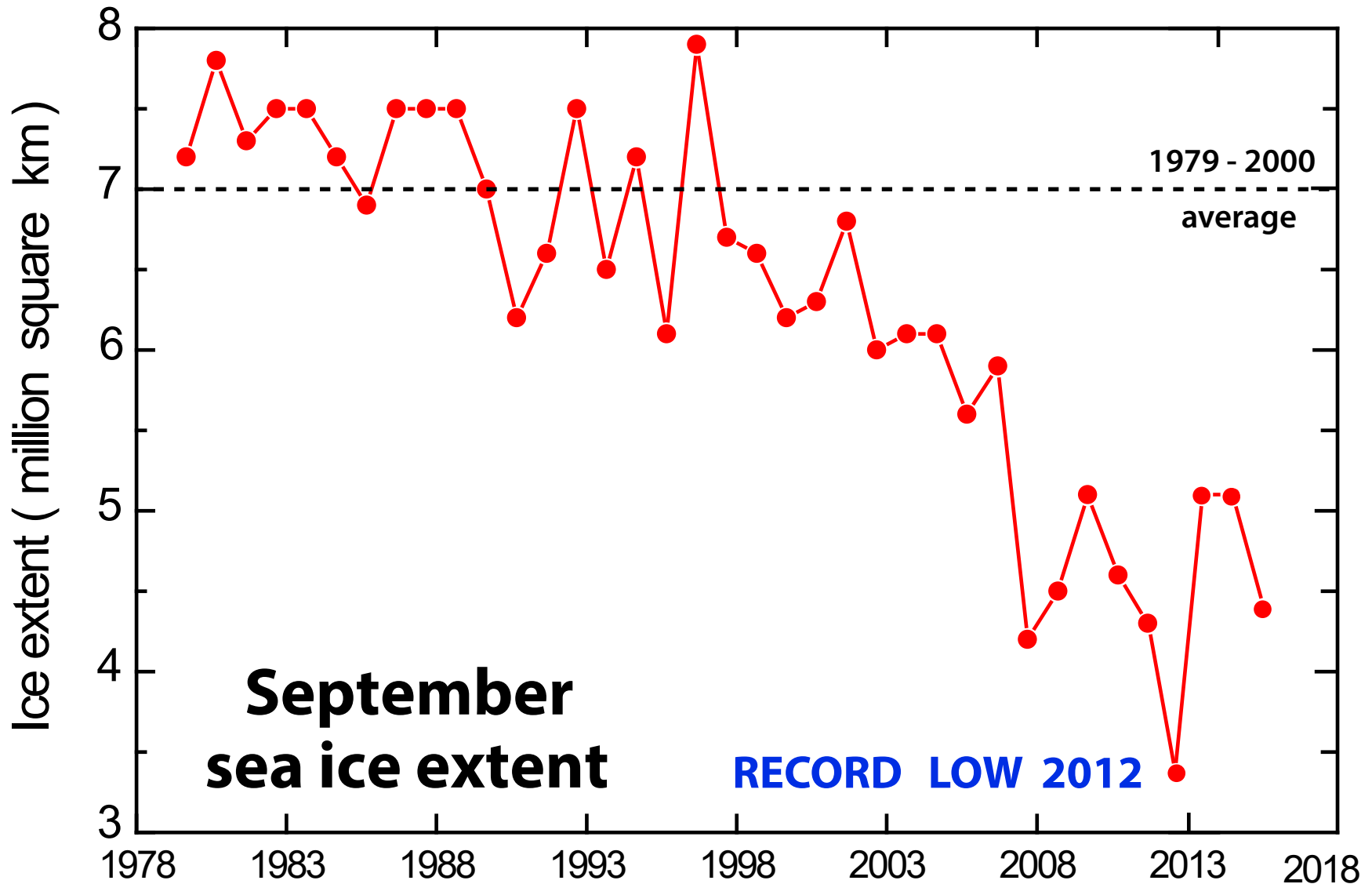
white snow and ice
reflect



dark water and land
absorb

$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

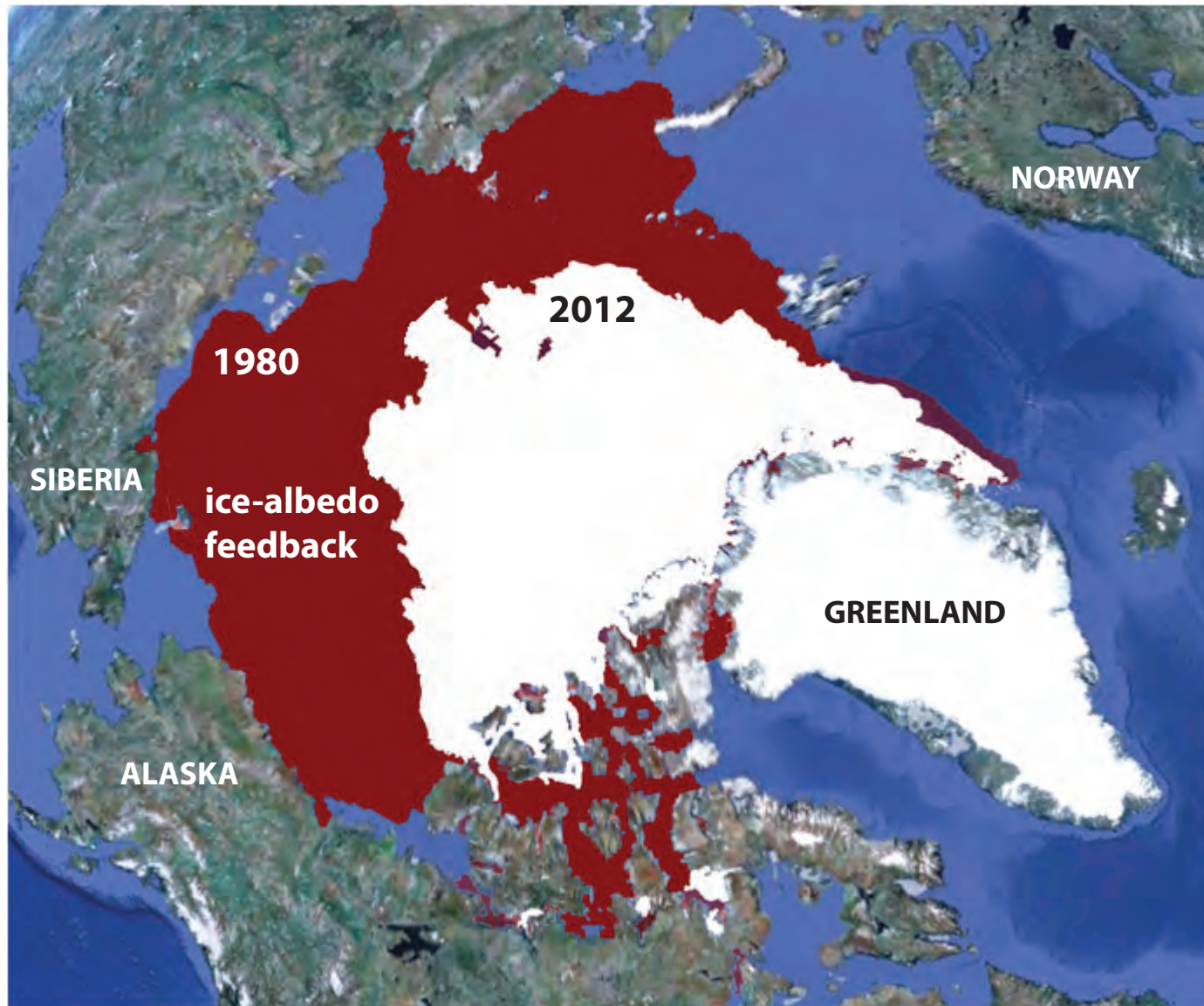
the summer Arctic sea ice pack is melting



Change in Arctic Sea Ice Extent

September 1980 -- **7.8** million square kilometers

September 2012 -- **3.4** million square kilometers





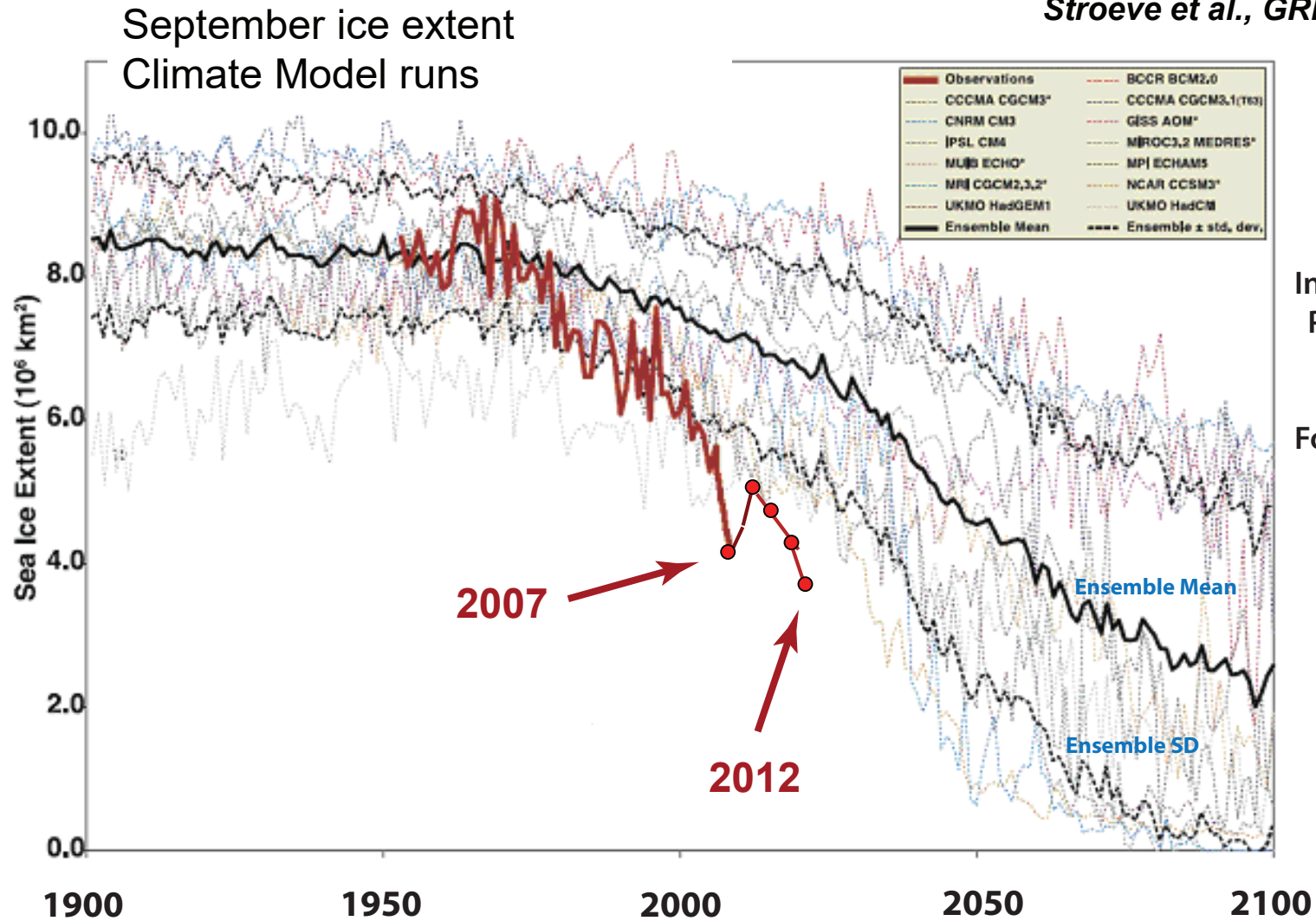
***recent losses
in comparison to
the United States***



Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007

Stroeve et al., GRL, 2012



**IPCC AR4
Models**

Intergovernmental
Panel on Climate
Change (IPCC)

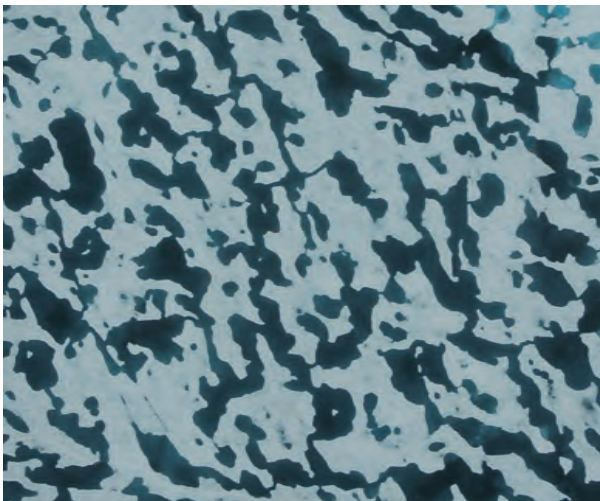
Fourth Assessment
AR4, 2007

challenge

represent sea ice more rigorously in climate models

account for key processes

such as melt pond evolution



Impact of melt ponds on Arctic sea ice
simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

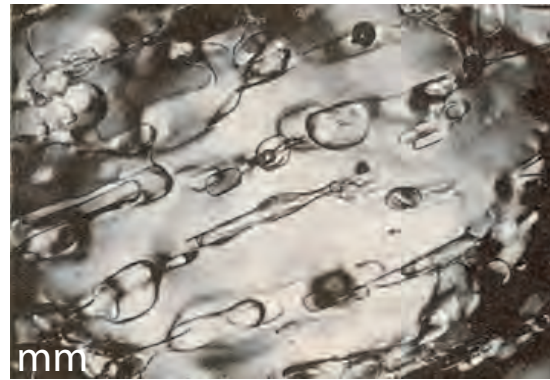
**For simulations with ponds
September ice volume is nearly 40% lower.**

... and other sub-grid scale structures and processes

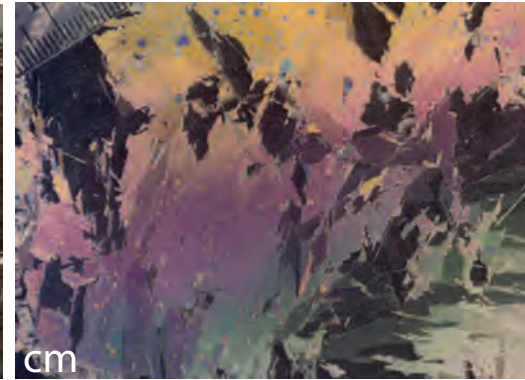
linkage of scales

sea ice is a multiscale composite
displaying structure over 10 orders of magnitude

0.1 millimeter



brine inclusions



polycrystals



horizontal

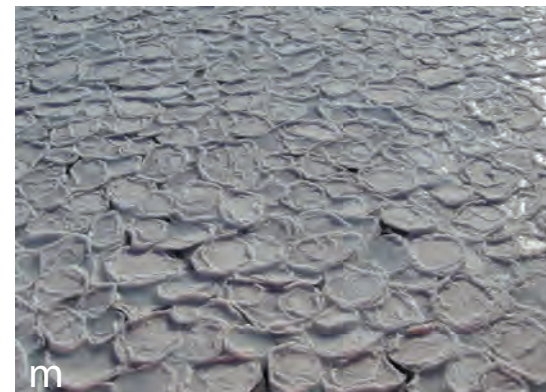


brine channels



vertical

1 meter

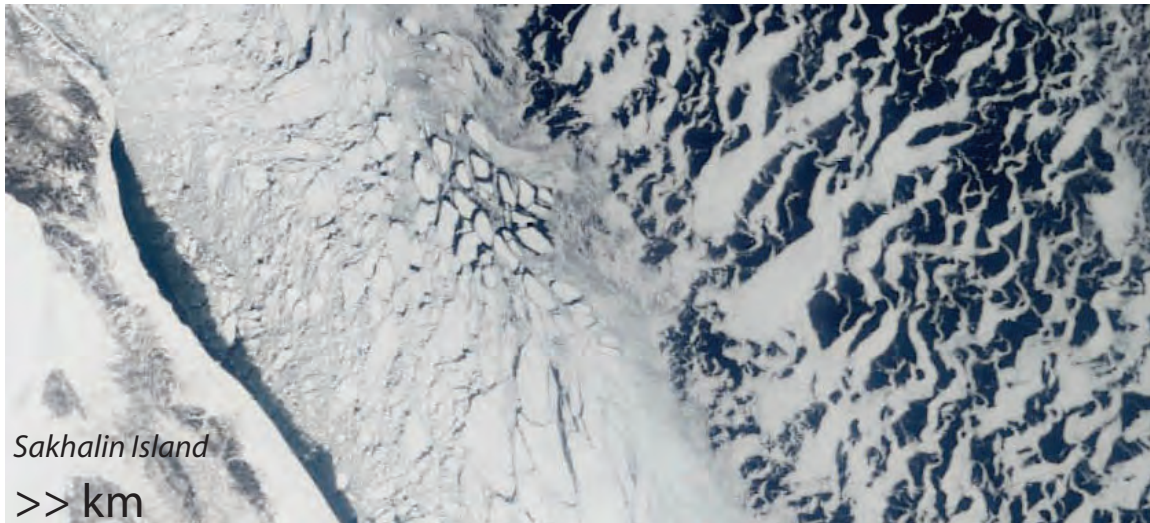


pancake ice

1 meter



100 kilometers



What is this talk about?

HOMOGENIZATION

*Using mathematics of composite materials and statistical physics
to LINK SCALES in the sea ice system and
rigorously compute effective behavior ... to improve climate projections.*

- 1. Climate models, sea ice components, homogenization*
- 2. Sea ice microphysics and porous media*
- 3. EM monitoring of sea ice, remote sensing*
- 4. Advection diffusion; polycrystals; waves in the MIZ*
- 5. Low order models and basin scale homogenization*
- 6. Evolution of Arctic melt ponds, fractal geometry*

critical behavior

cross - pollination

Global Climate Models

Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

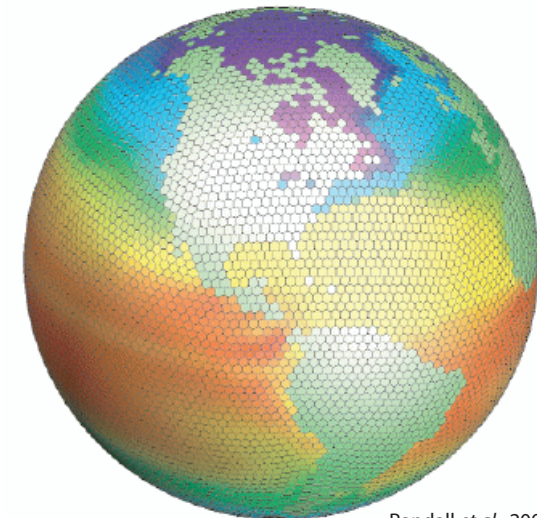
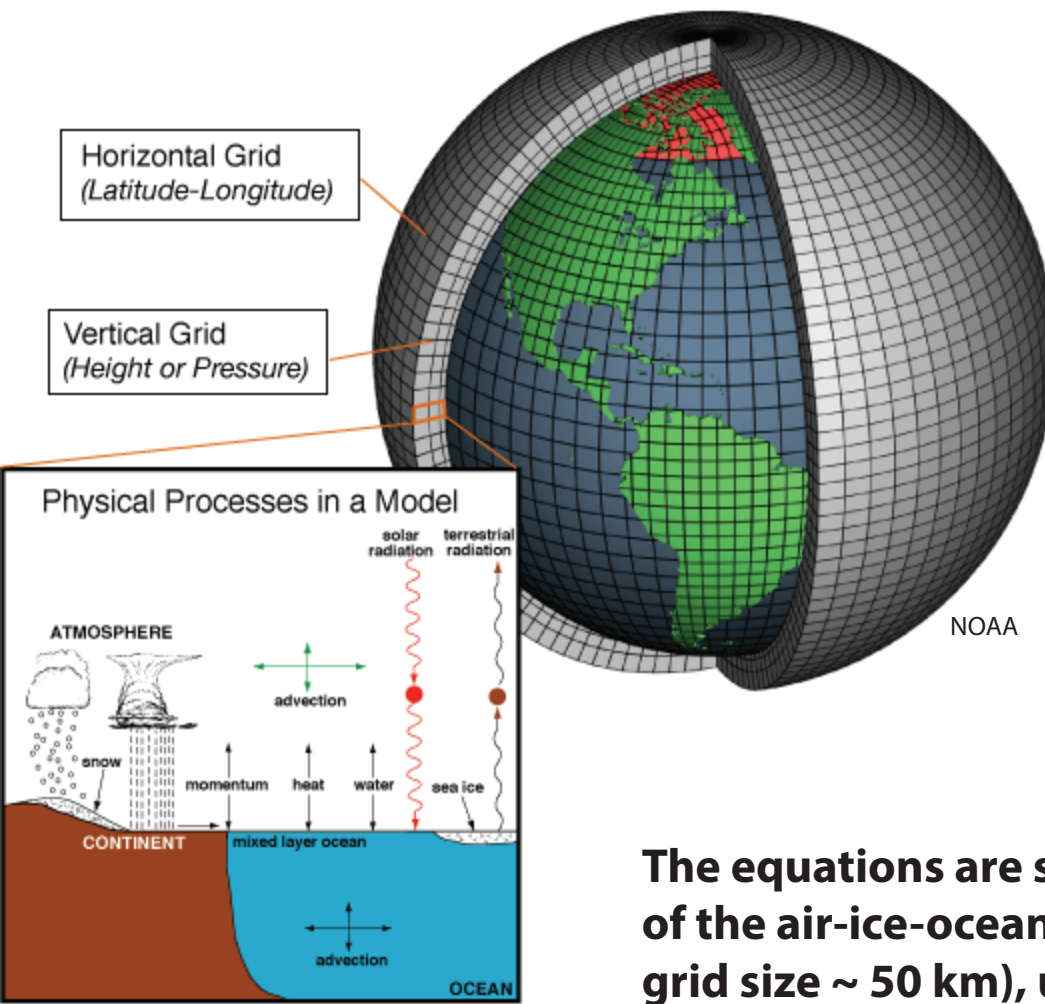
They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 50 km), using very powerful computers.

key challenge :

incorporating sub - grid scale processes

linkage of scales



sea ice components of GCM's

What are the key ingredients -- or **governing equations** that need to be solved on grids using powerful computers?

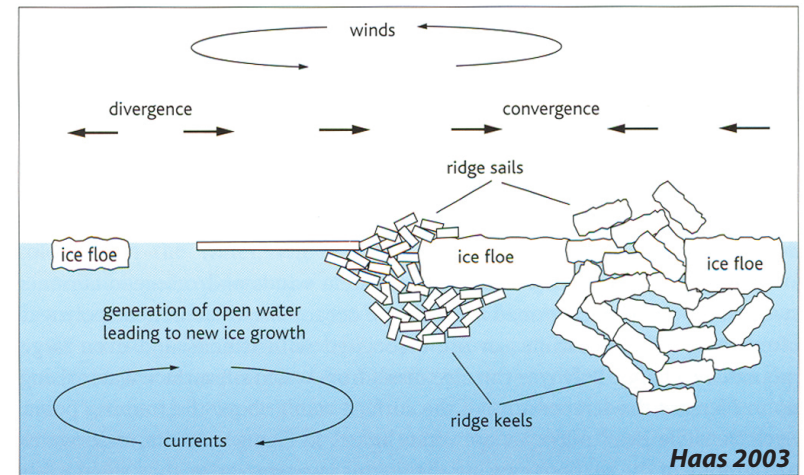
1. Ice thickness distribution $g(x, y, h, t)$ evolution equation **dynamics** + **thermodynamics**
(Thorndike et al. 1975)

$$\frac{Dg}{Dt} = -g \nabla \cdot \mathbf{u} + \Psi(g) - \frac{\partial}{\partial h} (\tau g) + \mathcal{L}$$

**nonlinear PDE with
ice velocity field**

**ice growth
ice melting**

**mechanical redistribution
- ridging and opening**



2. Conservation of momentum, stress vs. strain relation (Hibler 1979)

$$m \frac{D\mathbf{u}}{Dt} = -m f \mathbf{k} \times \mathbf{u} + \boldsymbol{\tau}_a + \boldsymbol{\tau}_o - mg \nabla H + \mathbf{F}_{int} \quad \mathbf{F} = m\mathbf{a} \text{ for sea ice dynamics}$$

3. Heat equation of sea ice and snow

(Maykut and Untersteiner 1971)

$$\frac{\partial T}{\partial t} + \mathbf{u}_{br} \cdot \nabla T = \nabla \cdot k(T) \nabla T$$

thermodynamics

**+ balance of radiative and
thermal fluxes on interfaces**

transform ice thickness distribution equation to Fokker-Planck type equation; Boltzmann framework

Toppaladoddi and Wettlaufer, *PRL*, 2015

thickness h is a diffusion process with probability density $g(h,t)$

“microscopic” mechanical processes that influence ice thickness distribution—rafting, ridging, and open water formation occur over very rapid time scales relative to geophysical-scale changes of $g(h)$

$$\Psi = \int_0^\infty [g(h+h')w(h+h',h') - g(h)w(h,h')]dh' \quad w = \text{transition probability}$$

moments k_1, k_2

Fokker-Planck

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial h} \left[\left(\frac{\epsilon}{h} - k_1 \right) g \right] + \frac{\partial^2}{\partial h^2} (k_2 g)$$

Langevin

$$\frac{dh}{dt} = \left(\frac{\epsilon}{h} - k_1 \right) + \sqrt{2k_2} \xi(t) \quad \xi(t) = \text{Gaussian white noise}$$

How do scales interact in the sea ice system?



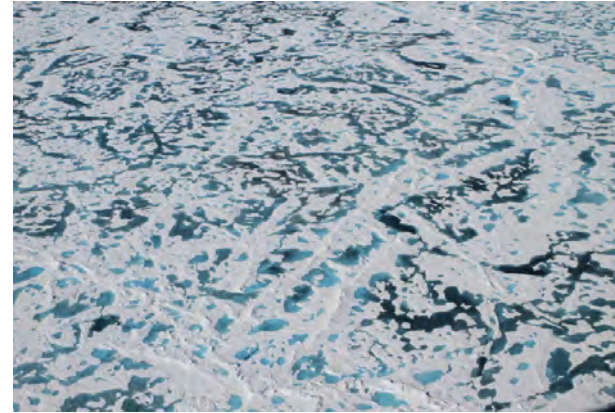
basin scale -
grid scale
albedo

Linking Scales

km
scale
melt
ponds



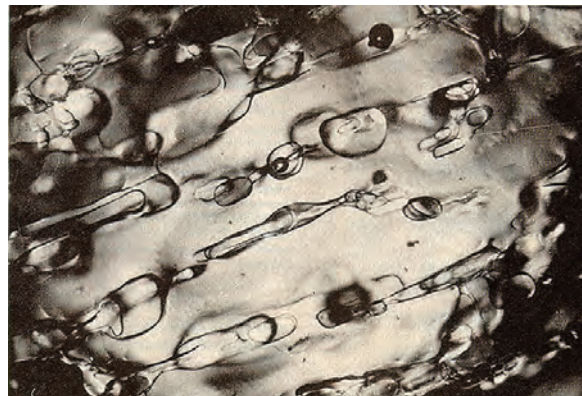
km
scale
melt
ponds



Linking

Scales

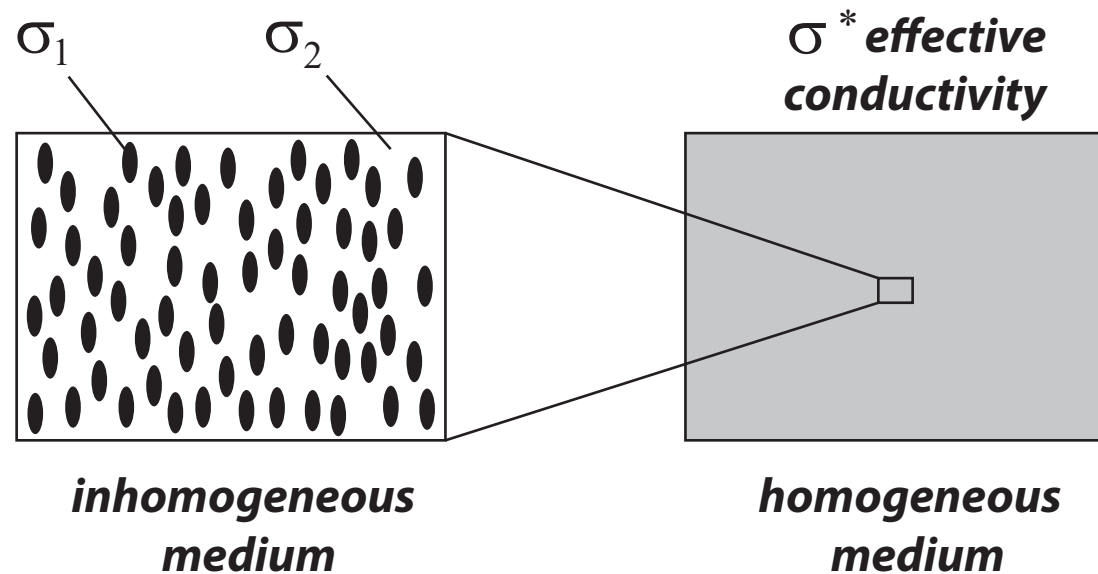
mm
scale
brine
inclusions



meter
scale
snow
topography



HOMOGENIZATION - Linking Scales in Composites



find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres

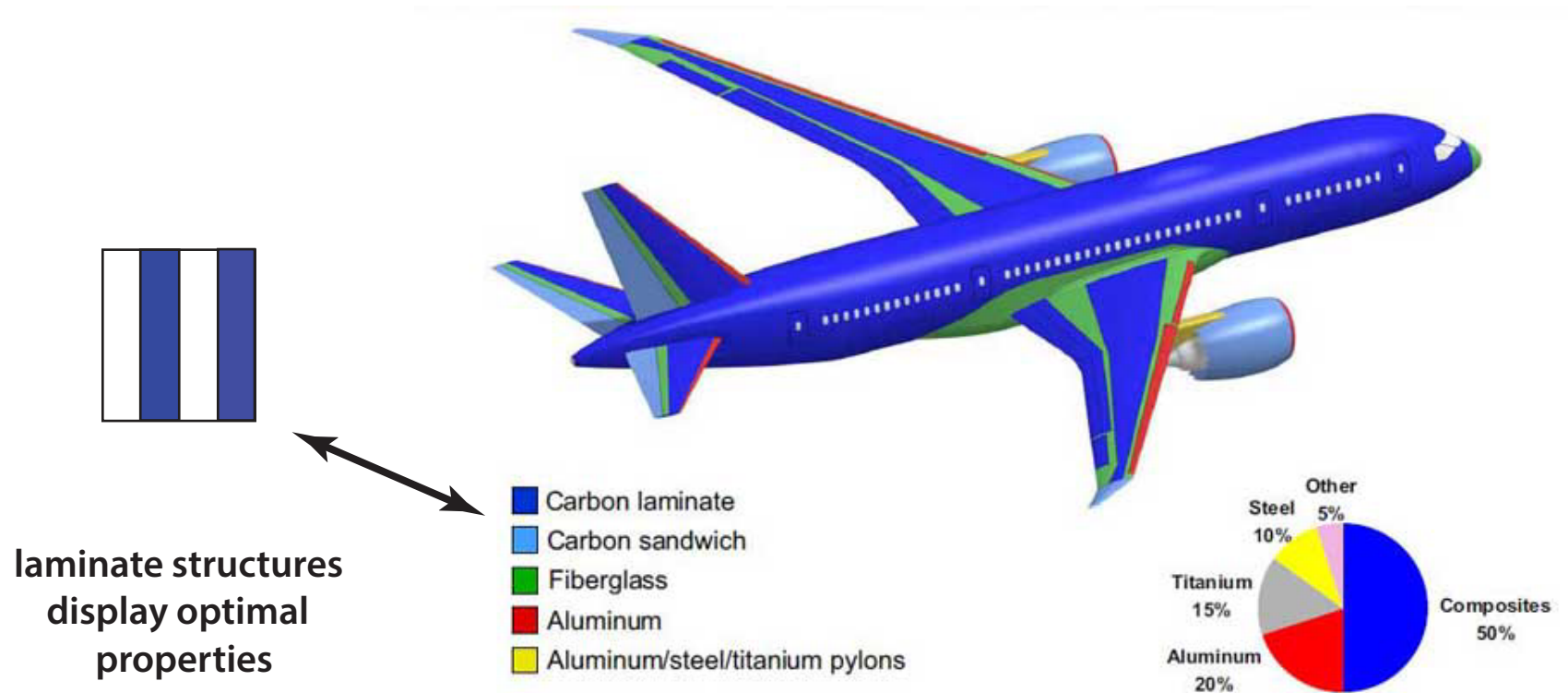
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*

*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

Composite materials in the Boeing 787 Dreamliner



sea ice microphysics

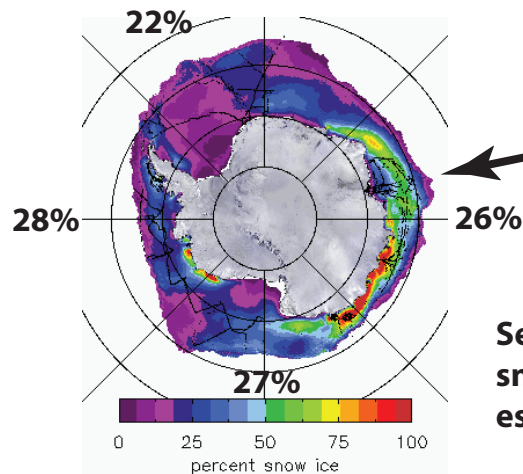
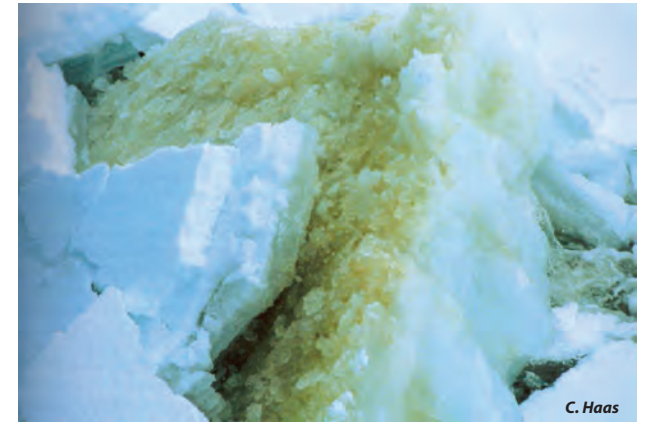
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities



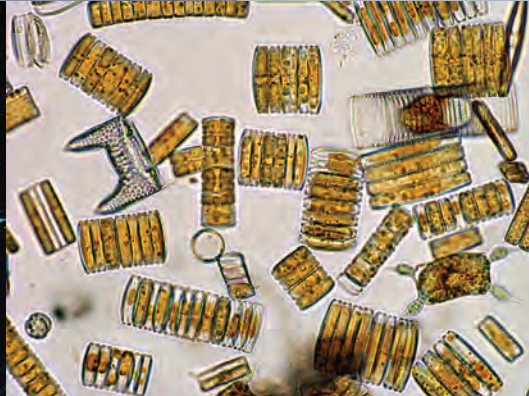
T. Maksym and T. Markus, 2008

*Antarctic surface flooding
and snow-ice formation*

September
snow-ice
estimates

- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO₂*

sea ice ecosystem



sea ice algae
support life in the polar oceans

fluid permeability k of a porous medium

porous
concrete



how much water
gets through the
sample per unit
time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

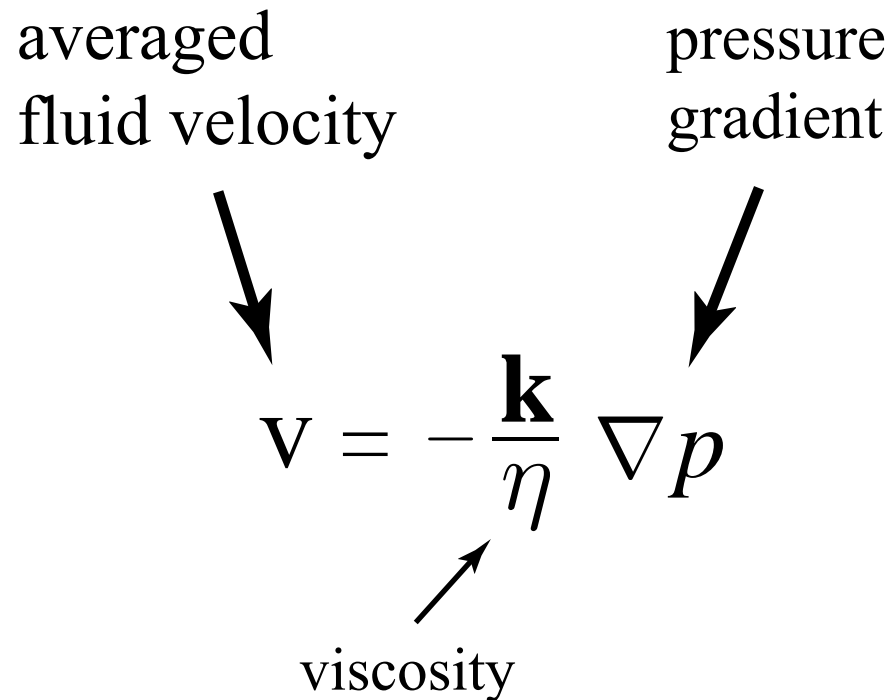
Darcy's Law for slow viscous flow in a porous medium

averaged
fluid velocity

pressure
gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

viscosity

The diagram shows the equation $\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$ centered on the slide. Three labels with arrows point to parts of the equation: 'averaged fluid velocity' points to \mathbf{v} , 'pressure gradient' points to ∇p , and 'viscosity' points to η .

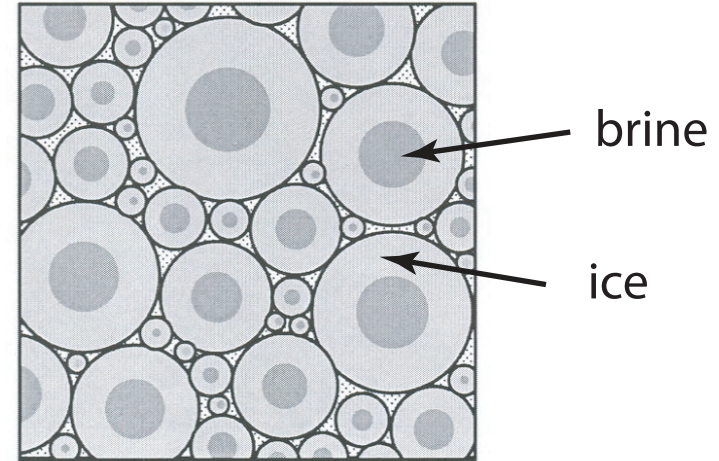
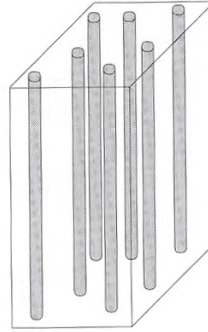
\mathbf{k} = fluid permeability tensor

PIPE BOUNDS on vertical fluid permeability k

Golden, Heaton, Eicken, Lytle, Mech. Materials 2006

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007

vertical pipes
with appropriate radii
maximize k



optimal coated
cylinder geometry

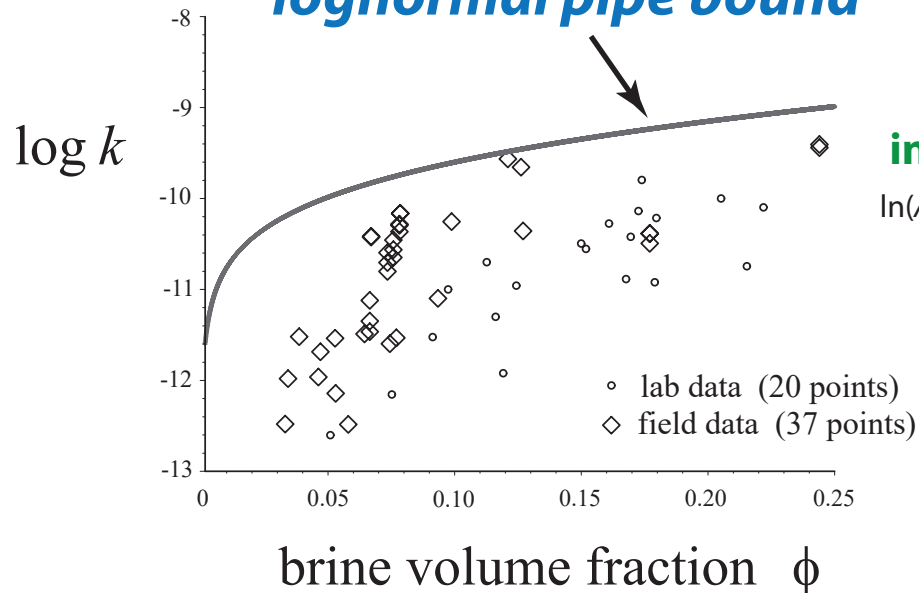
fluid analog of arithmetic mean upper bound for effective conductivity of composites (Wiener 1912)

$$k \leq \frac{\phi \langle R^4 \rangle}{8 \langle R^2 \rangle} = \frac{\phi}{8} \langle R^2 \rangle e^{\sigma^2}$$

inclusion cross sectional areas A lognormally distributed

$\ln(A)$ normally distributed, mean μ (increases with T) variance σ^2 (Gow and Perovich 96)

lognormal pipe bound



Golden et al., Geophys. Res. Lett. 2007

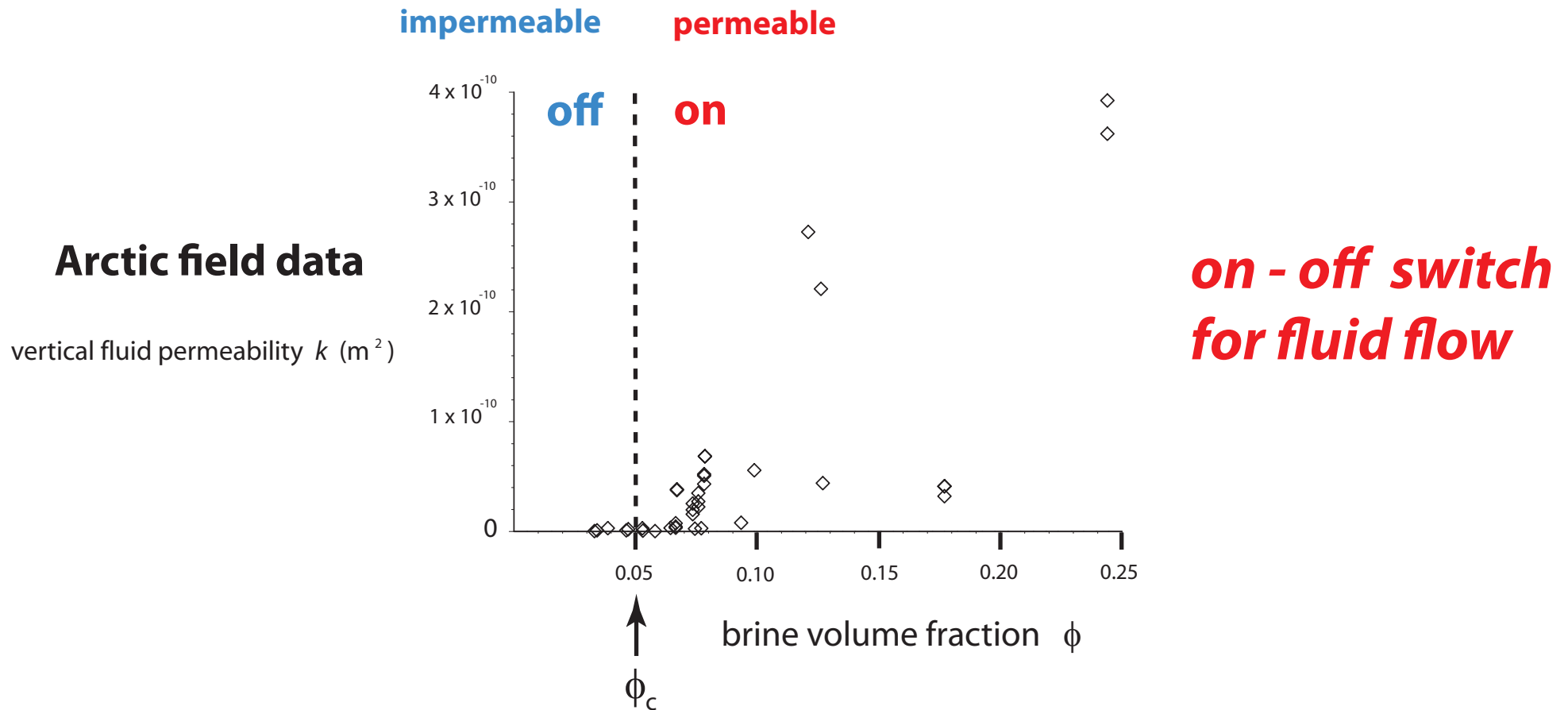
get bounds through variational analysis of
trapping constant γ for diffusion process
in pore space with absorbing BC

Torquato and Pham, PRL 2004

$$\mathbf{k} \leq \gamma^{-1} \mathbf{I}$$

for any ergodic porous medium
(Torquato 2002, 2004)

Critical behavior of fluid transport in sea ice



critical brine volume fraction $\phi_c \approx 5\%$ \longleftrightarrow $T_c \approx -5^\circ \text{C}$, $S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009



sea ice algal communities

D. Thomas 2004

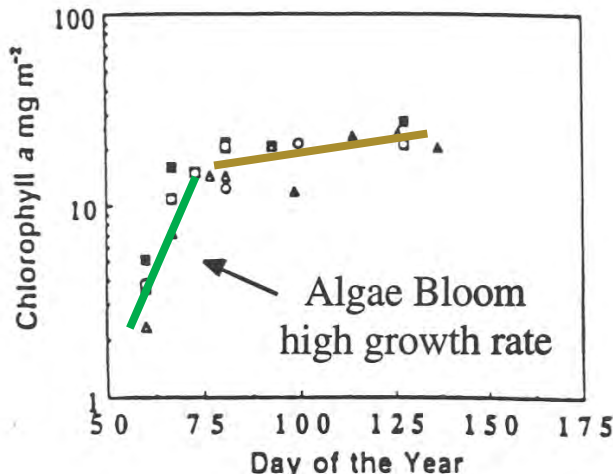
nutrient replenishment
controlled by ice permeability

biological activity turns on
or off according to
rule of fives

Golden, Ackley, Lytle *Science* 1998

Fritsen, Lytle, Ackley, Sullivan *Science* 1994

critical behavior of microbial activity

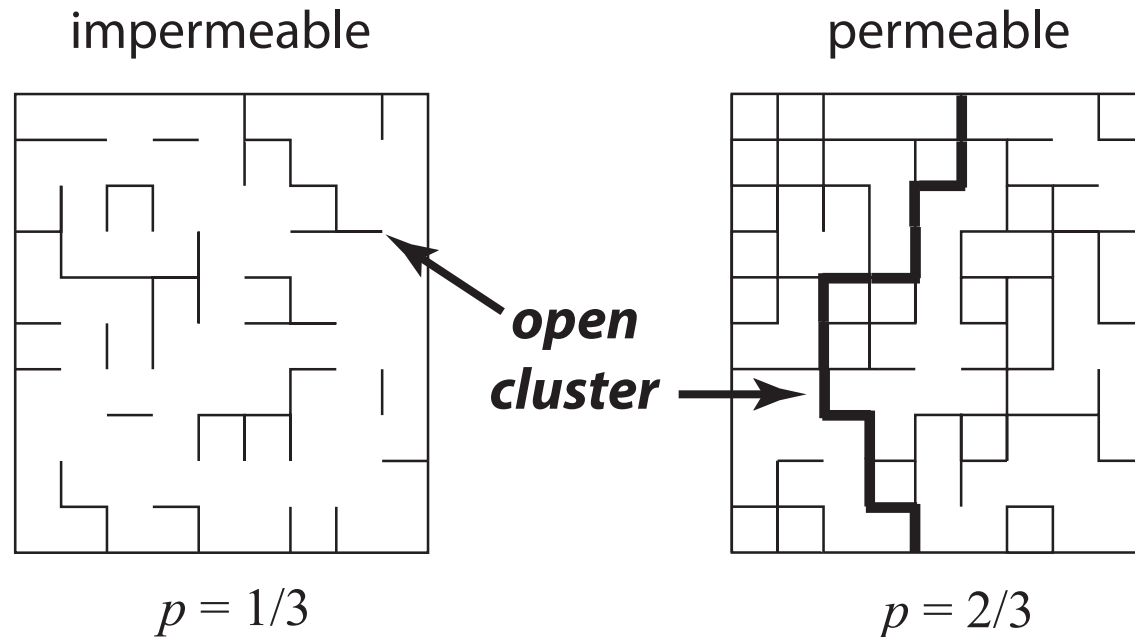


Convection-fueled algae bloom
Ice Station Weddell

Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond \longrightarrow *open* with probability p
closed with probability $1-p$

percolation threshold

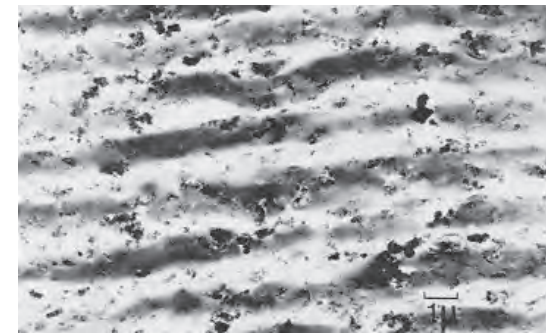
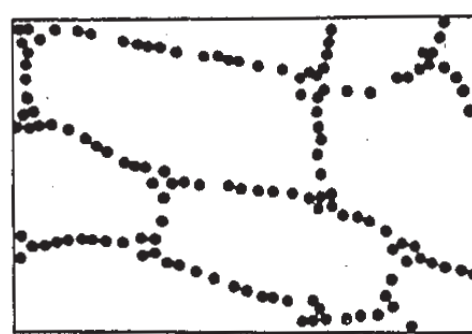
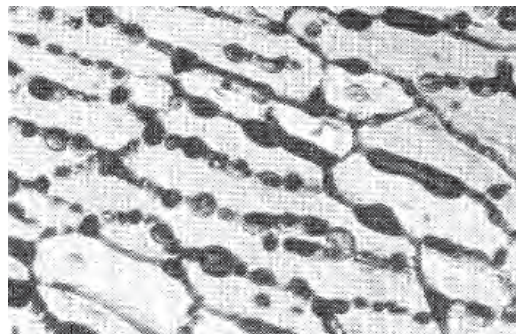
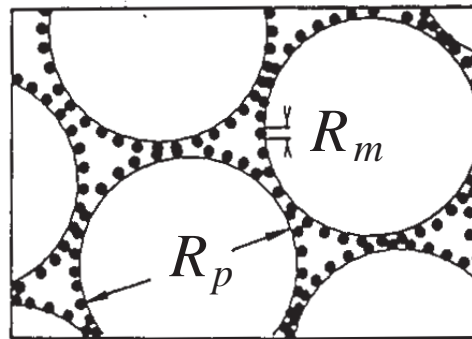
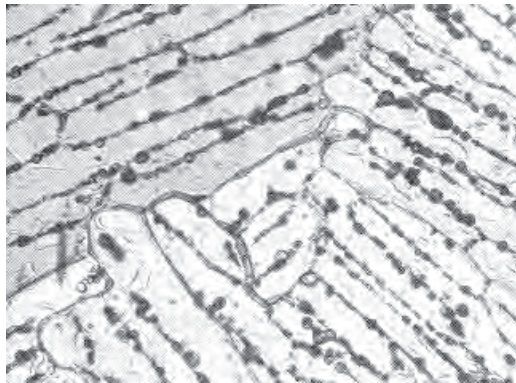
$$p_c = 1/2 \quad \text{for } d = 2$$

smallest p for which there is an infinite open cluster

Continuum percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5 \%$$

Golden, Ackley, Lytle, *Science*, 1998



sea ice

compressed
powder

radar absorbing
composite

sea ice is radar absorbing



***rigorous bounds
percolation theory
hierarchical model
network model***

field data

**X-ray tomography for
brine inclusions**

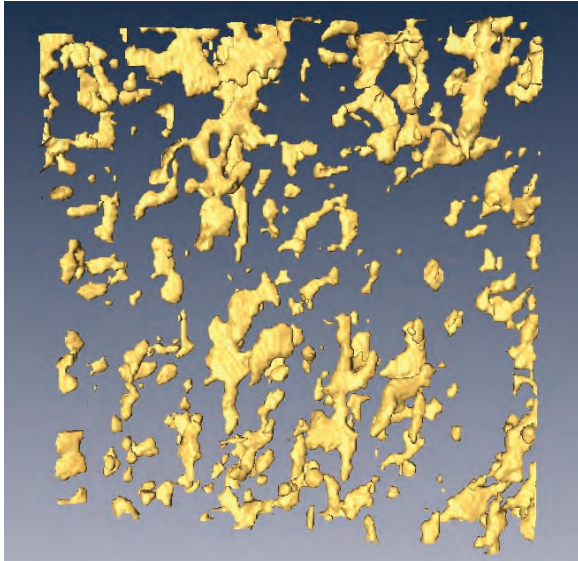
***unprecedented look
at thermal evolution
of brine phase and
its connectivity***

micro-scale
controls
macro-scale
processes

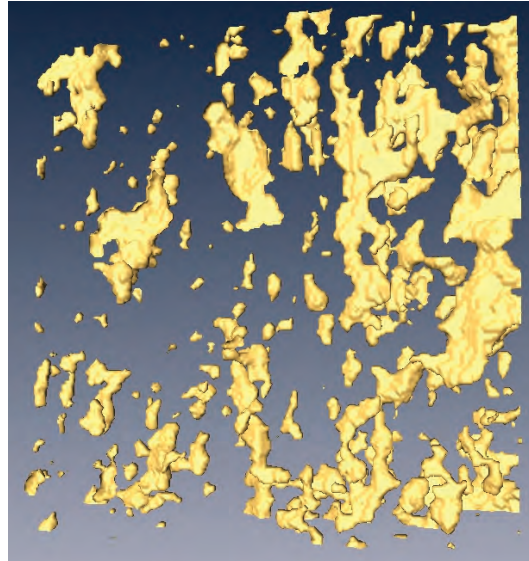
A unified approach to understanding permeability in sea ice • Solving the mystery of
booming sand dunes • Entering into the "greenhouse century": A case study from Switzerland

brine connectivity (over cm scale)

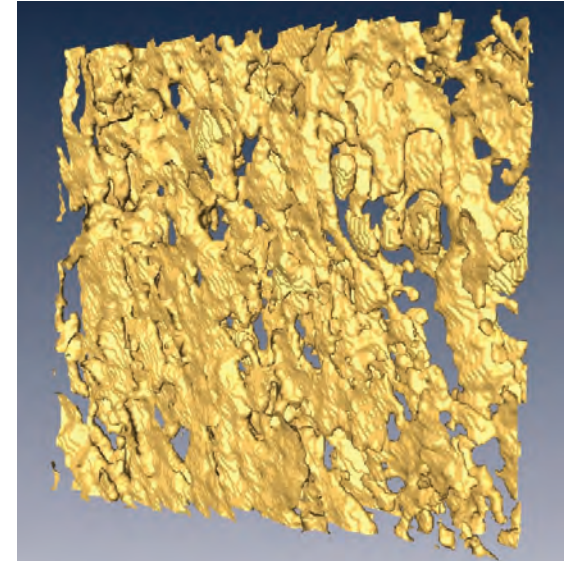
8 x 8 x 2 mm



-15 °C, $\phi = 0.033$



-6 °C, $\phi = 0.075$



-3 °C, $\phi = 0.143$

X-ray tomography confirms percolation threshold

3-D images
pores and throats



3-D graph
nodes and edges

analyze graph connectivity as function of temperature and sample size

- ***use finite size scaling techniques to confirm rule of fives***
- ***order parameter data from a natural material***

lattice and continuum percolation theories yield:

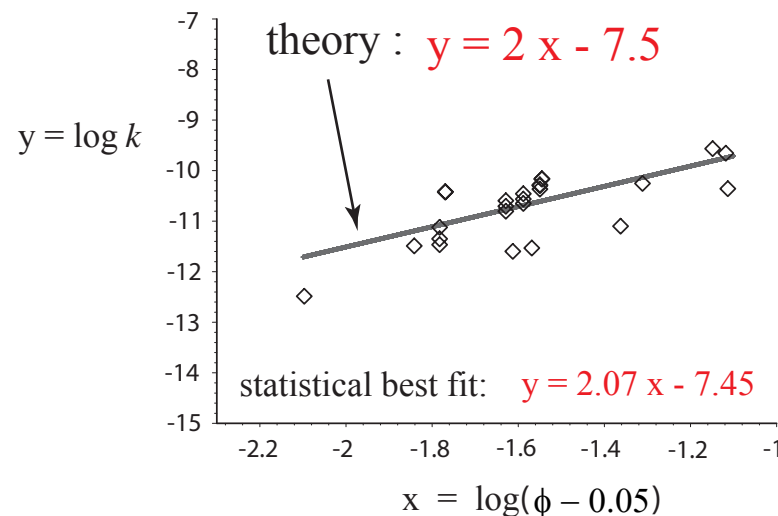
$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical
exponent

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

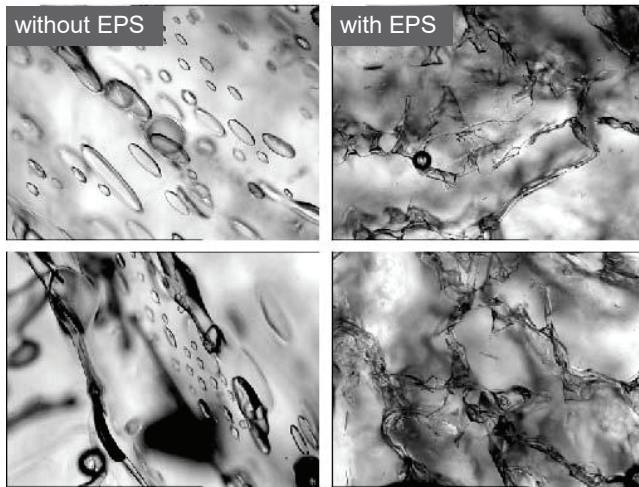
t

- exponent is **UNIVERSAL** lattice value $t \approx 2.0$
- **sedimentary rocks** like sandstones also exhibit universality
- **critical path analysis** -- developed for electronic hopping conduction -- yields scaling factor k_0

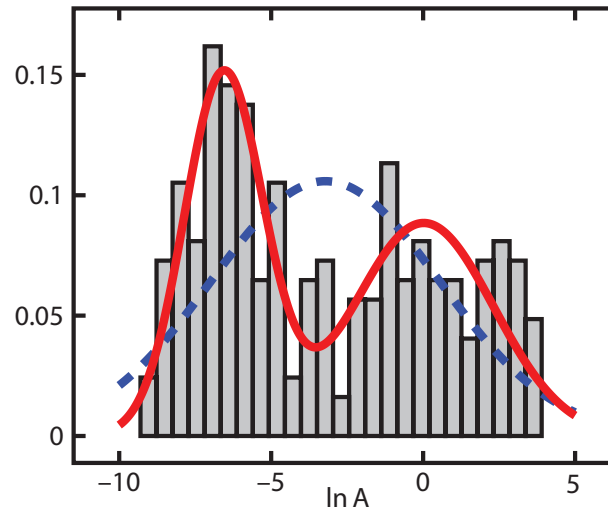


Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

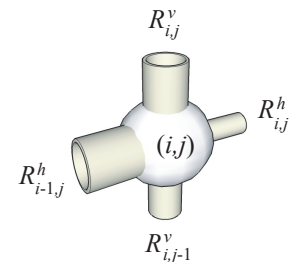
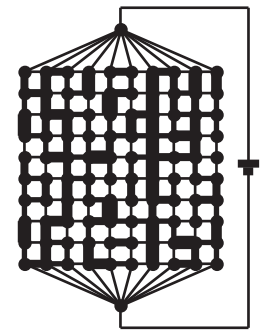
How does EPS affect fluid transport?



Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



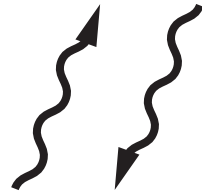
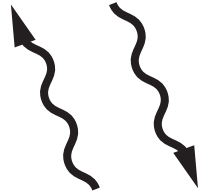
- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution; Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability k .
- Rigorous bound on k for bimodal distribution of pore sizes

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden,
Eicken, Morris
Ann. Glac. 2006

How does the biology affect the physics?

Remote sensing of sea ice



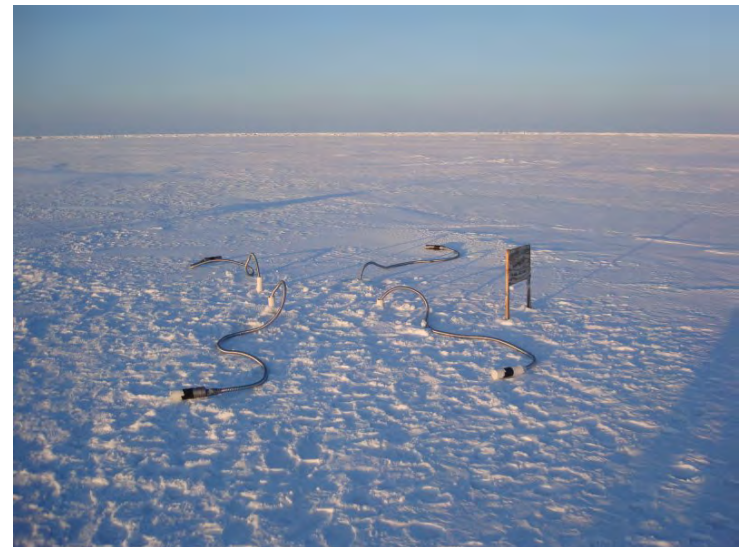
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice
properties from
electromagnetic
(EM) data

$$\epsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)

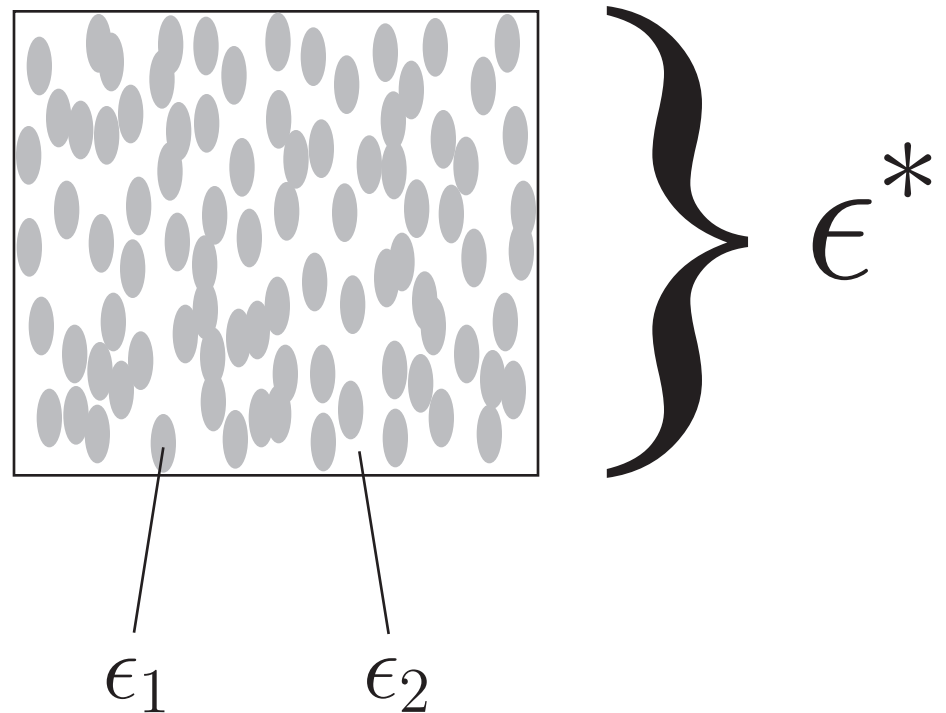


brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite
in the quasistatic (long wavelength) limit

**What are the effective propagation characteristics
of an EM wave (radar, microwaves) in the medium?**

HOMOGENIZATION



p_1, p_2 = volume fractions of brine and ice

Theory of Effective Electromagnetic Behavior of Composites

analytic continuation method

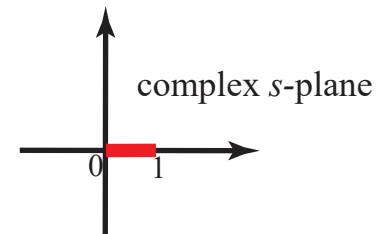
Forward Homogenization Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)
Theory of Composites, Milton (2002)

composite geometry
 (spectral measure μ) $\longrightarrow \epsilon^*$

Stieltjes integral representation

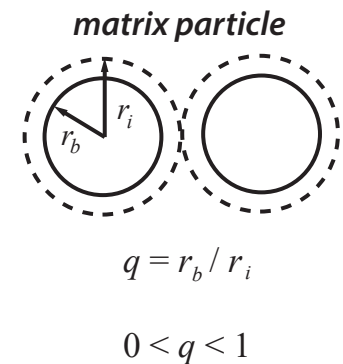
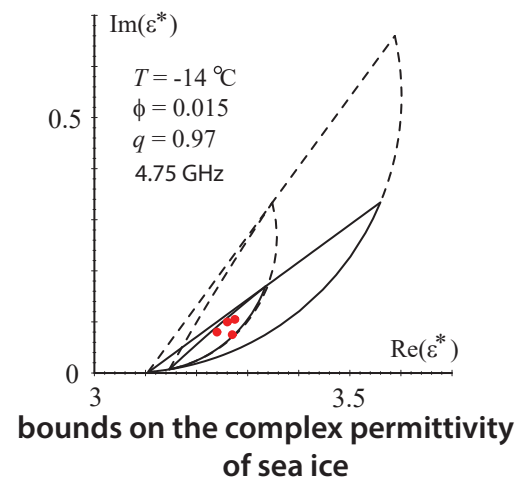
$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$



- μ /
- spectral measure of self adjoint operator $\chi \Gamma \chi$
 - mass = p_1
 - higher moments depend on n -point correlations

Golden and Papanicolaou 1983



Golden 1995, 1997

Stieltjes integral representation

separates geometry from parameters

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

geometry ←

← *material parameters*

- μ {
- spectral measure of self adjoint operator $\Gamma\chi$
 - mass = p_1
 - higher moments depend on n -point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla.$$

χ = characteristic function of the brine phase

$$E = (s + \Gamma\chi)^{-1}e_k$$

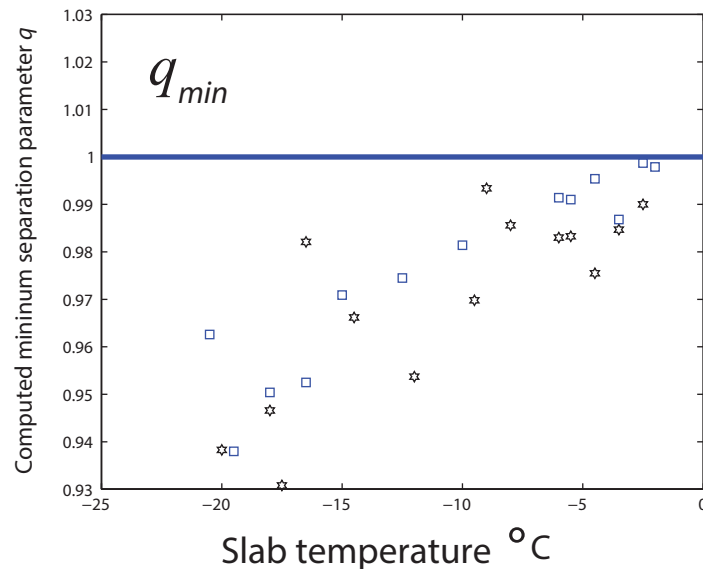
$\Gamma\chi$: microscale \rightarrow macroscale

$\Gamma\chi$ *links scales*

Inverse Homogenization Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001) McPhedran, McKenzie, Milton (1982), *Theory of Composites*, Milton (2002)



recover brine volume fraction, brine connectivity, etc.



Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

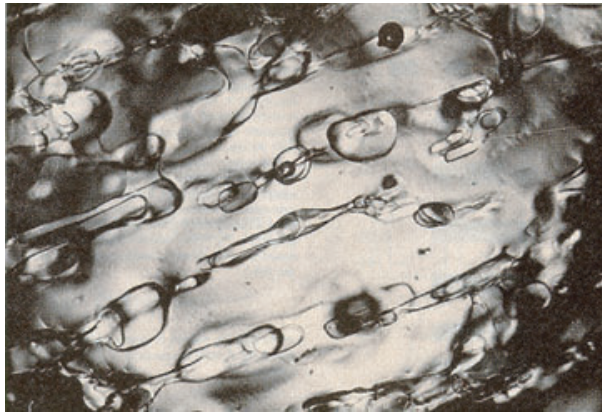
**inversion for brine inclusion
separations in sea ice from
measurements of effective
complex permittivity ε^***

**rigorous inverse bound
on spectral gap**

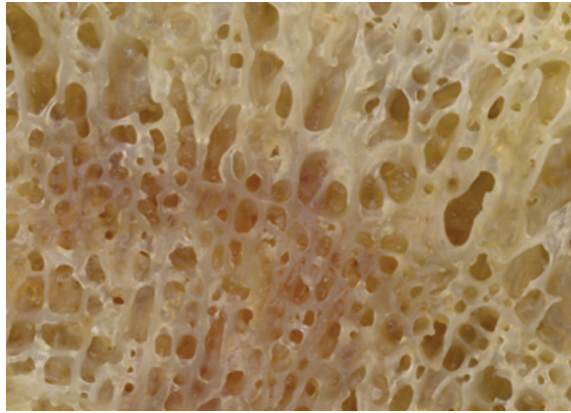
**inverse bounds and
recovery of brine porosity**

Gully, Backstrom, Eicken, Golden
Physica B, 2007

SEA ICE

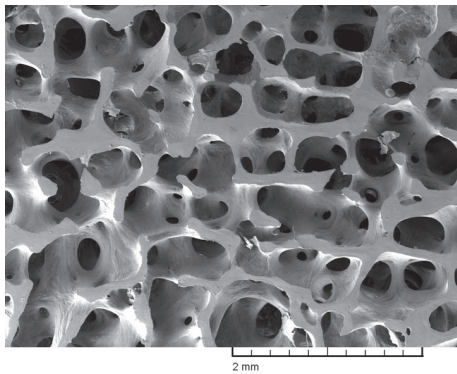


HUMAN BONE

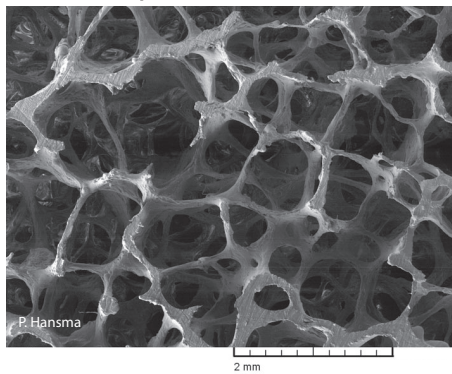


*spectral characterization
of porous microstructures
in human bone*

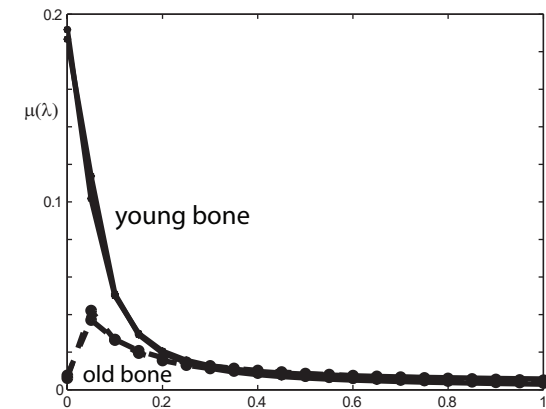
young healthy trabecular bone



old osteoporotic trabecular bone



reconstruct spectral measures
from complex permittivity data



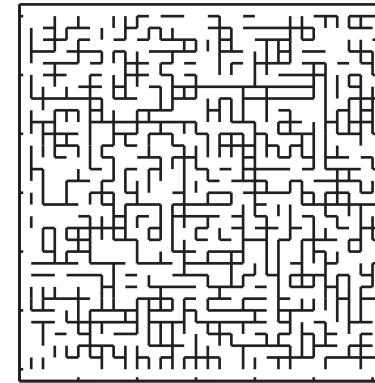
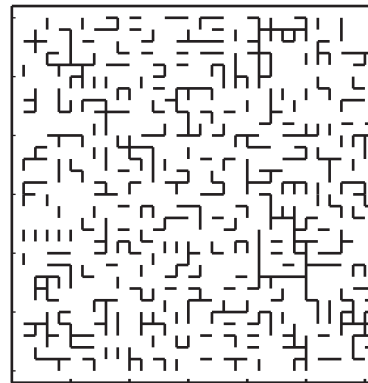
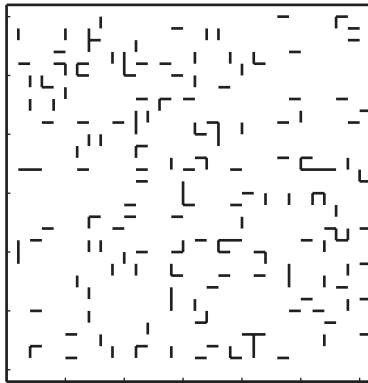
use regularized inversion scheme

*apply spectral measure analysis of brine connectivity and
spectral inversion to electromagnetic monitoring of osteoporosis*

Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

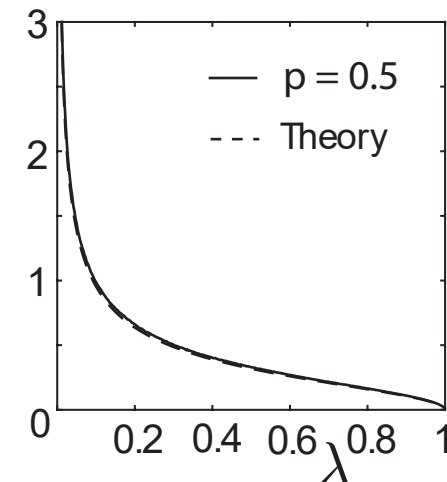
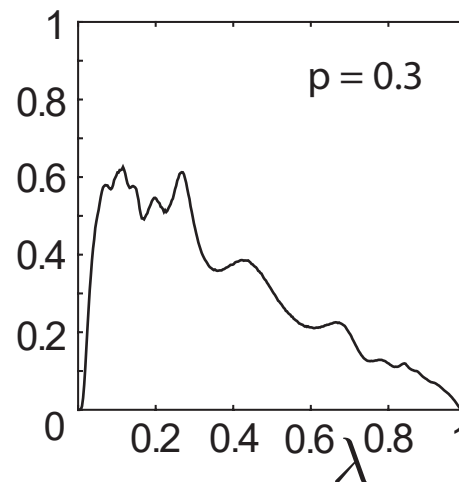
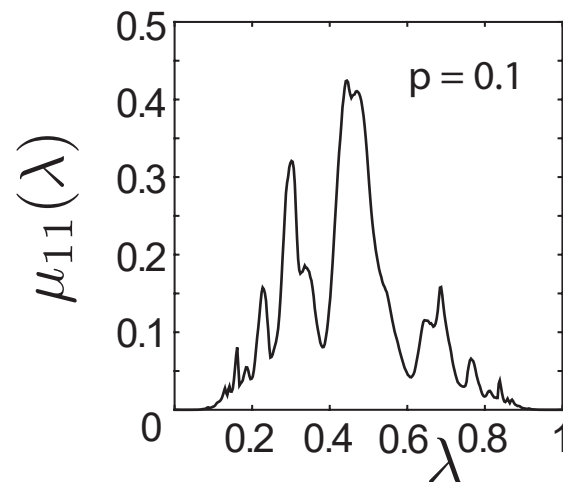
Spectral statistics for 2D random resistor network



Spectral Measures

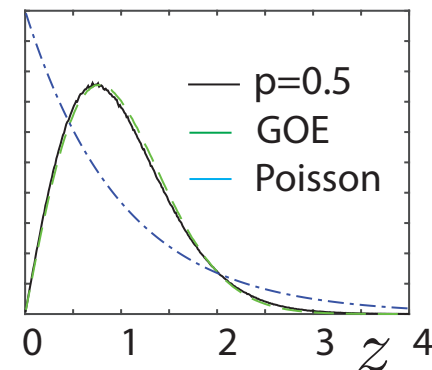
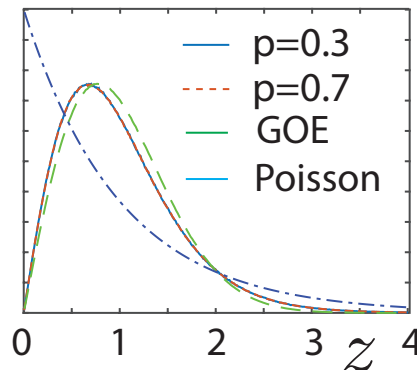
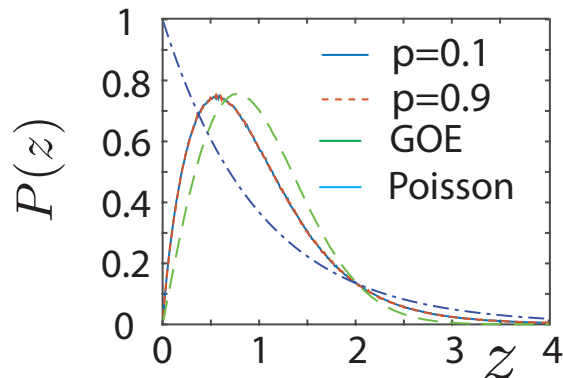
Murphy and Golden, *J. Math. Phys.*, 2012

Murphy et al. *Comm. Math. Sci.*, 2015



$p_c = 0.5$

Eigenvalue Spacing Distributions



Murphy,
Cherkaev,
Golden,
PRL, 2017

Eigenvalue Statistics of Random Matrix Theory

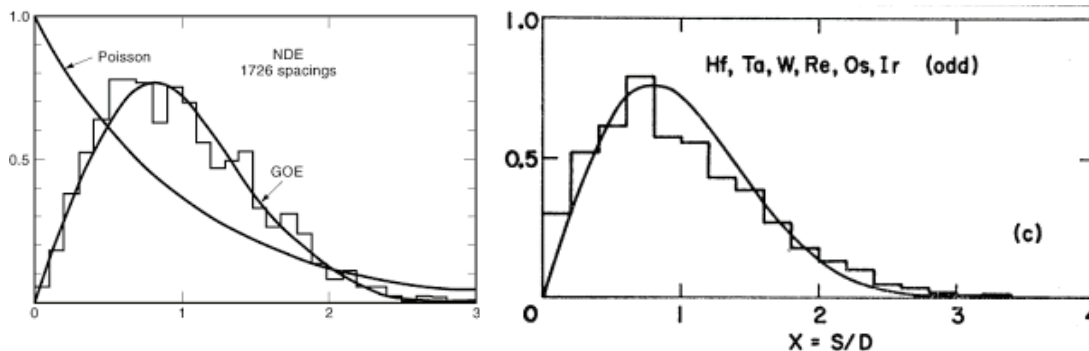
Wigner (1951) and Dyson (1953) first used random matrix theory (RMT) to describe quantized energy levels of heavy atomic nuclei.

$[N]_{ij} \sim N(0,1), \quad A = (N + N^T)/2 \quad \text{Gaussian orthogonal ensemble (GOE)}$

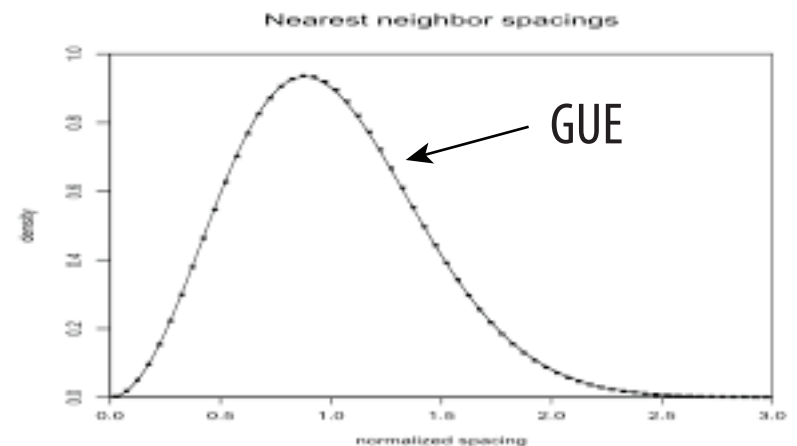
$[N]_{ij} \sim N(0,1) + iN(0,1), \quad A = (N + N^\dagger)/2 \quad \text{Gaussian unitary ensemble (GUE)}$

Short range and long range correlations of eigenvalues are measured by various eigenvalue statistics.

Spacing distributions of energy levels for heavy atomic nuclei



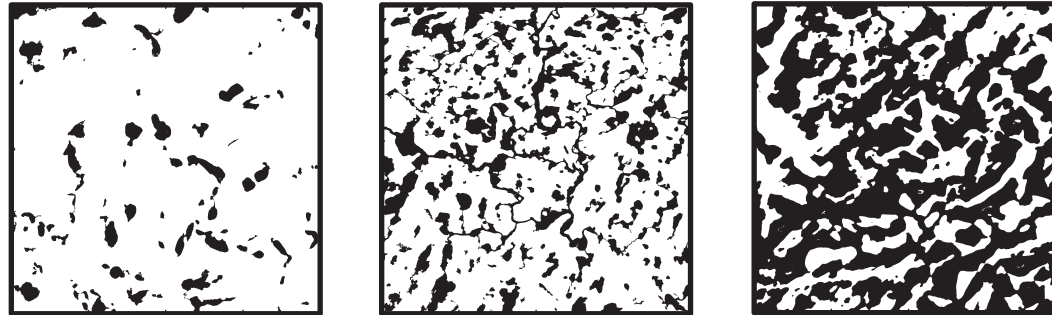
Spacing distributions of the first billion zeros of the Riemann zeta function



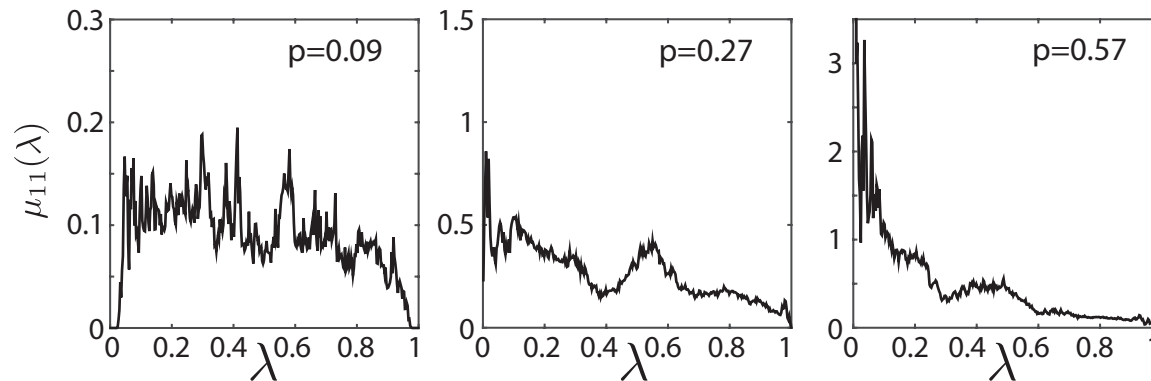
RMT used to characterize **disorder-driven transitions** in mesoscopic conductors, neural networks, random graph theory, etc.

Universal eigenvalue statistics arise in a broad range of “unrelated” problems!

Spectral computations for Arctic melt ponds

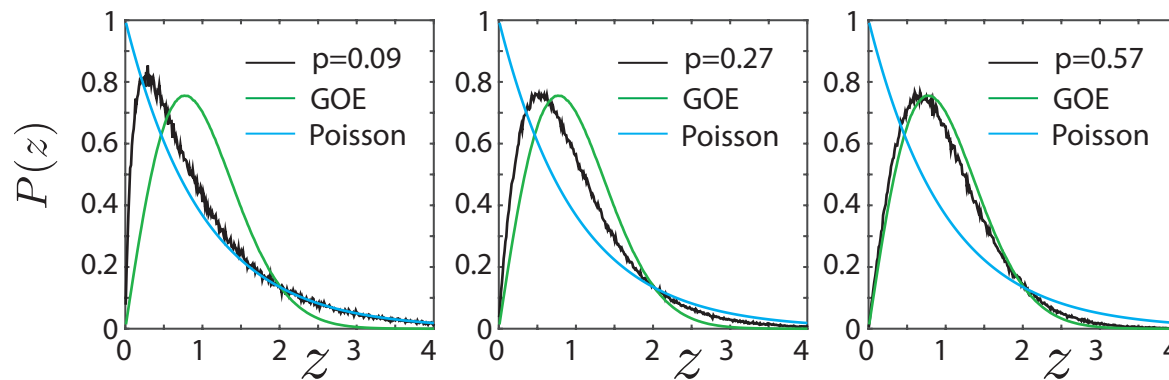


spectral
measures



Ben Murphy
Elena Cherkaev
Ken Golden
2017

eigenvalue
spacing
distributions



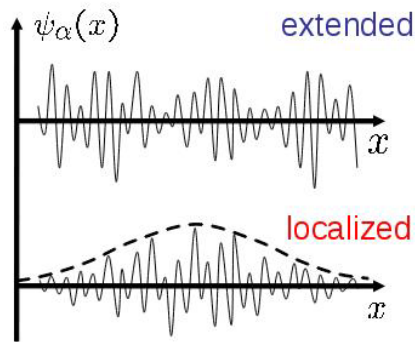
uncorrelated



level repulsion

TRANSITION

*eigenvalue statistics
for transport tend
toward the
UNIVERSAL
Wigner-Dyson
distribution
as the “conducting”
phase percolates*



metal / insulator transition **localization**

Anderson 1958
Mott 1949
Shklovshii et al 1993
Evangelou 1992

**Anderson transition in wave physics:
quantum, optics, acoustics, water waves, ...**

we find a surprising analog

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017

**PERCOLATION
TRANSITION**

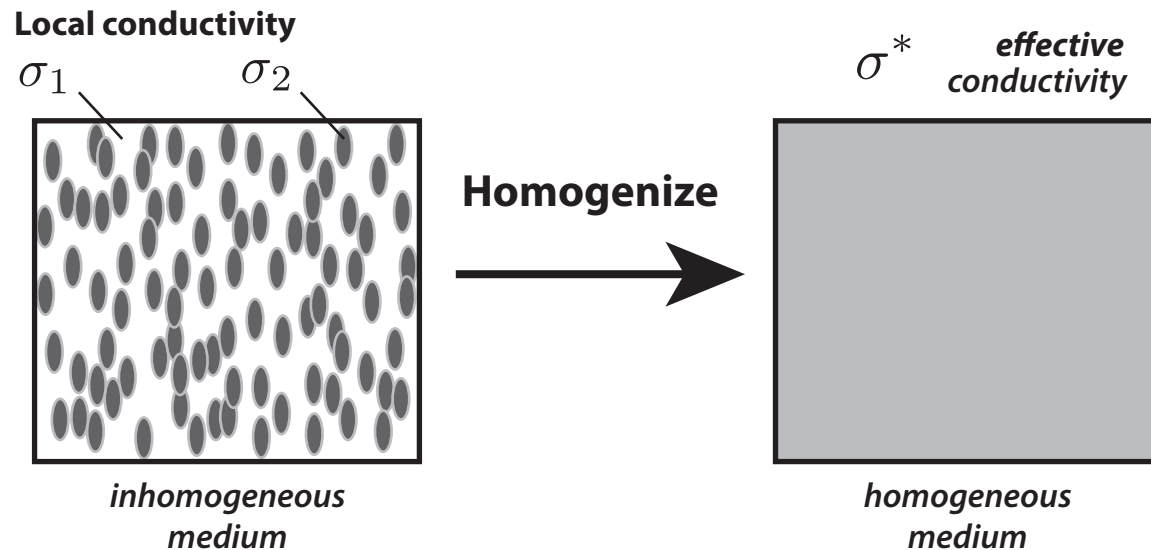


**transition to universal
eigenvalue statistics (GOE)
extended states, mobility edges**

-- but without wave interference or scattering effects ! --

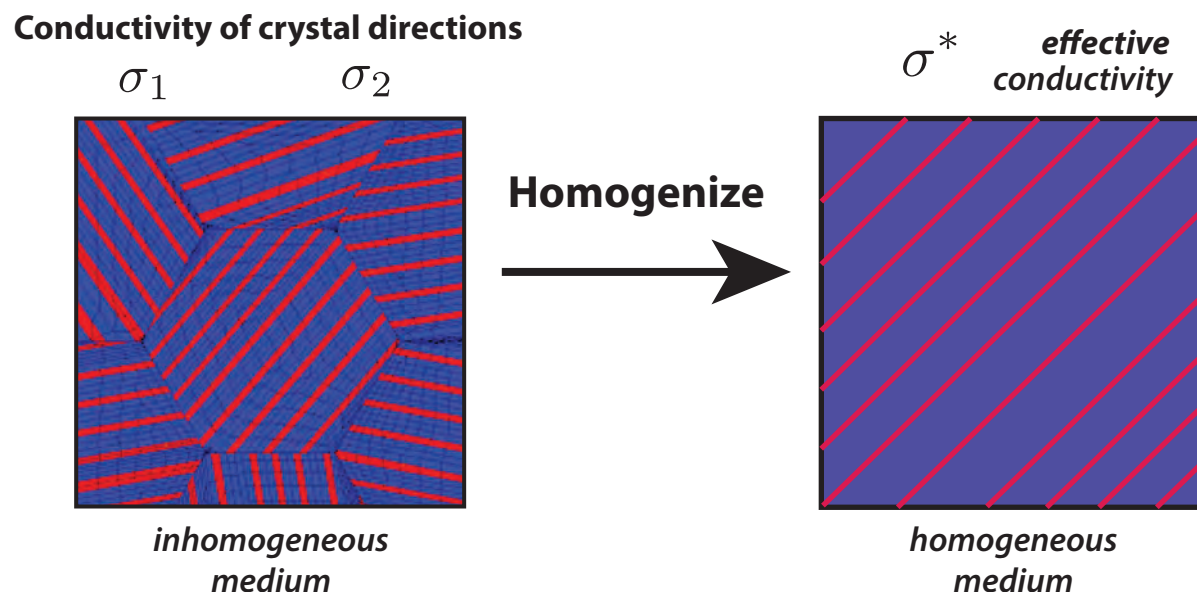
Homogenization for composite materials

**Two-component
composites**



Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

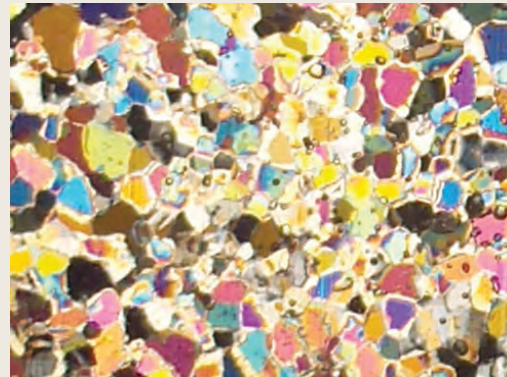
**Polycrystalline
media**



Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**
Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



PROCEEDINGS A

350 YEARS
OF SCIENTIFIC
PUBLISHING

An invited review
commemorating 350 years
of scientific publishing at the
Royal Society

A method to distinguish
between different types
of sea ice using remote
sensing techniques

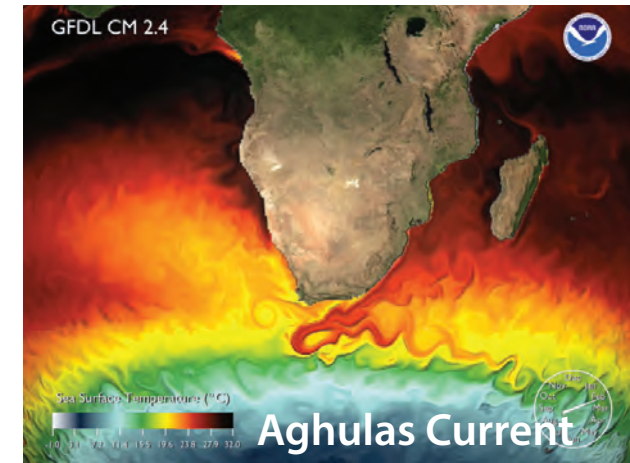
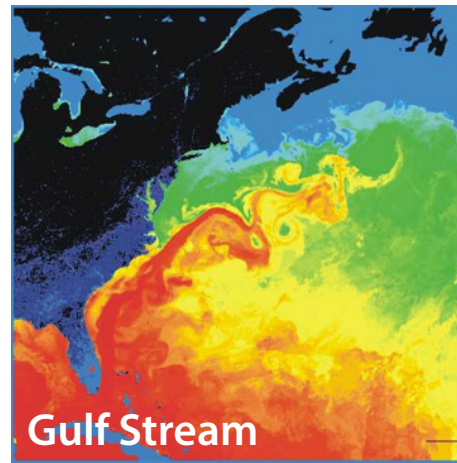
A computer model to
determine how a human
should walk so as to expend
the least energy



advection enhanced diffusion

effective diffusivity

sea ice floes diffusing in ocean currents
diffusion of pollutants in atmosphere
salt and heat transport in ocean
heat transport in sea ice with convection



advection diffusion equation with a velocity field \vec{u}

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

homogenize

$$\frac{\partial \bar{T}}{\partial t} = \kappa^* \Delta \bar{T}$$

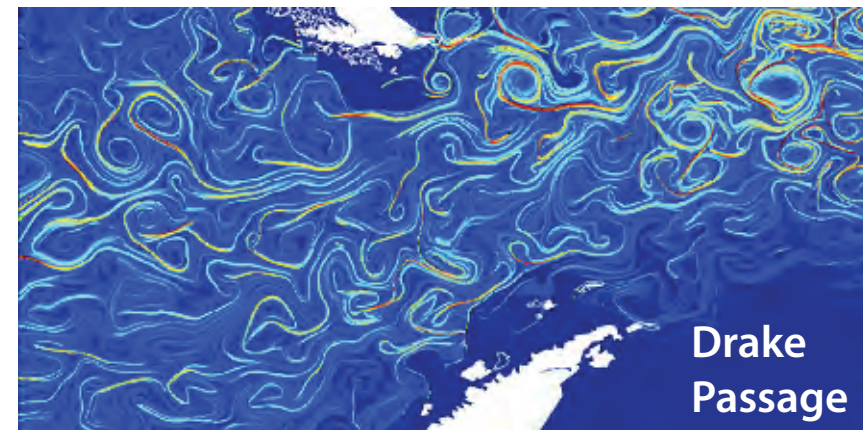
κ^* effective diffusivity

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

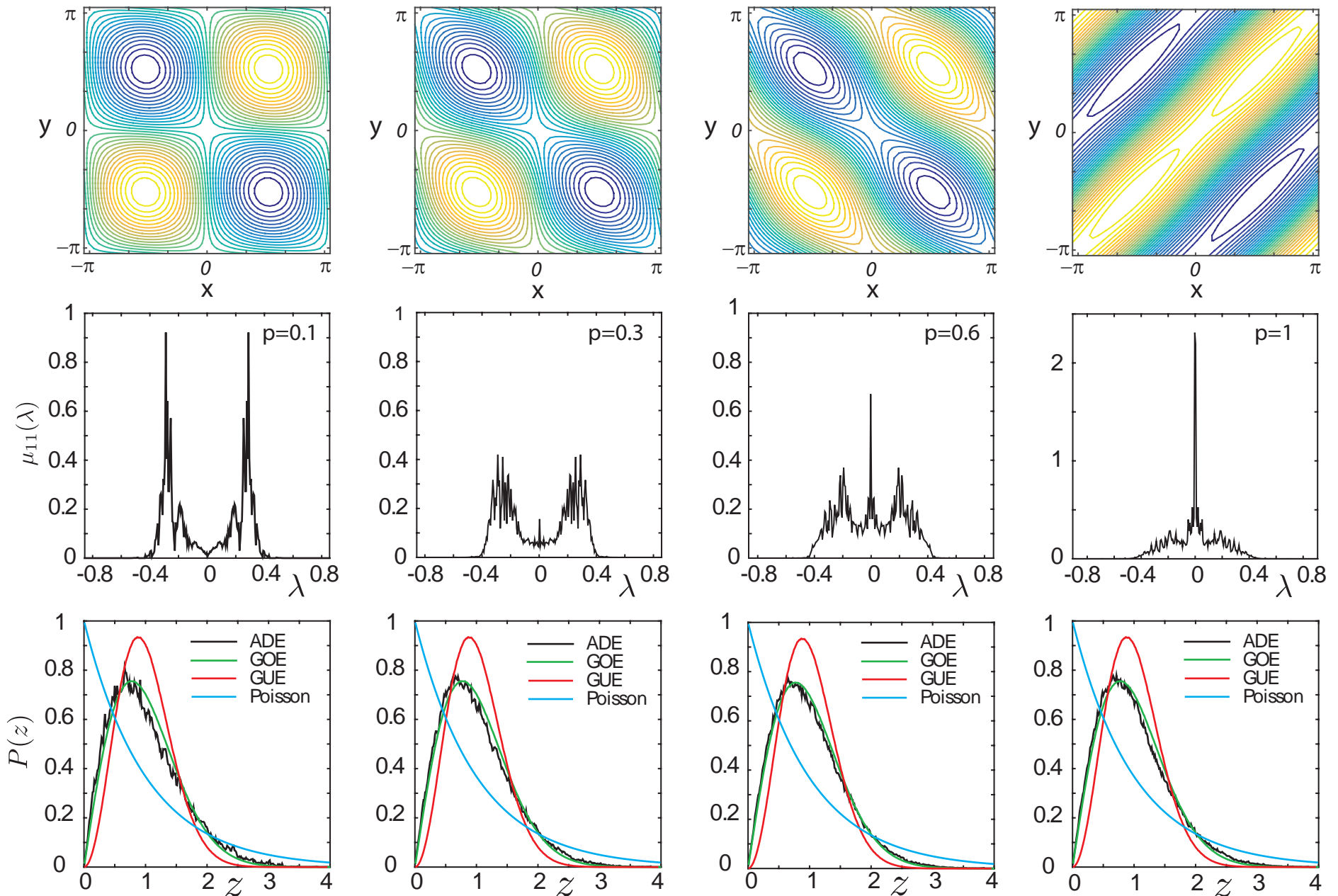
Murphy, Cherkaev, Xin, Zhu, Golden, Ann. Math. Sci. Appl. 2017

Murphy, Cherkaev, Zhu, Xin, Golden, 2018

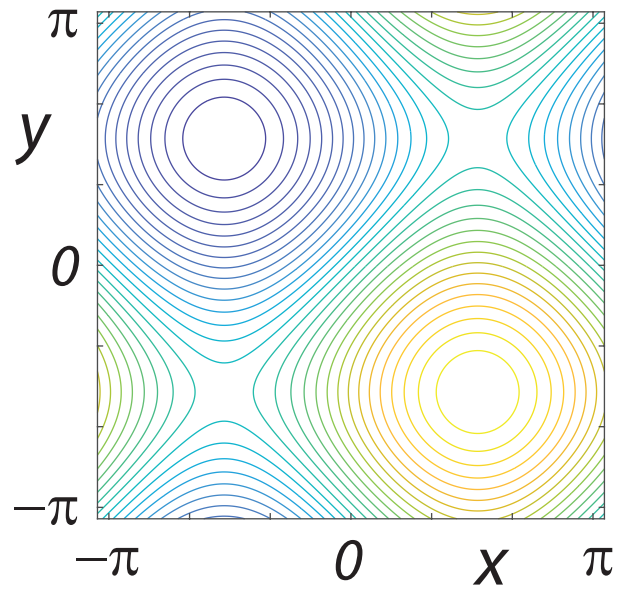


Spectral measures and eigenvalue spacings for cat's eye flow

$$H(x,y) = \sin(x) \sin(y) + A \cos(x) \cos(y), \quad A \sim U(-p,p)$$



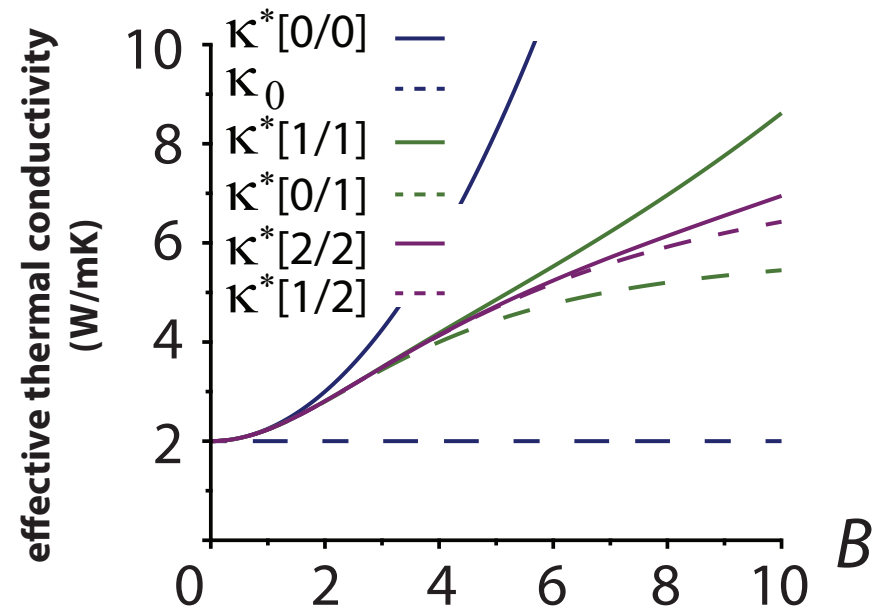
Convection - enhanced thermal conductivity of sea ice w/ BC - flow



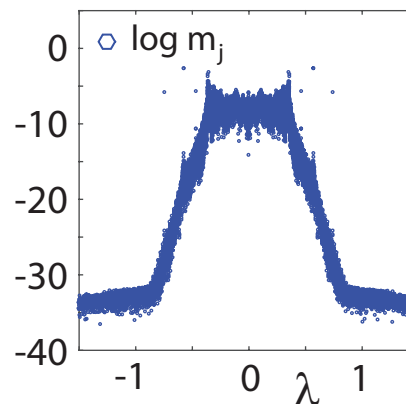
BC - flow streamlines

$$H = B \sin x - C \sin y \quad B = C$$

**Kraitzman, Hardenbrook,
Murphy, Zhu, Cherkaev,
Golden 2018**



**rigorous Padé bounds
from Stieltjes integral
+ analytical calculations
of moments of measure**



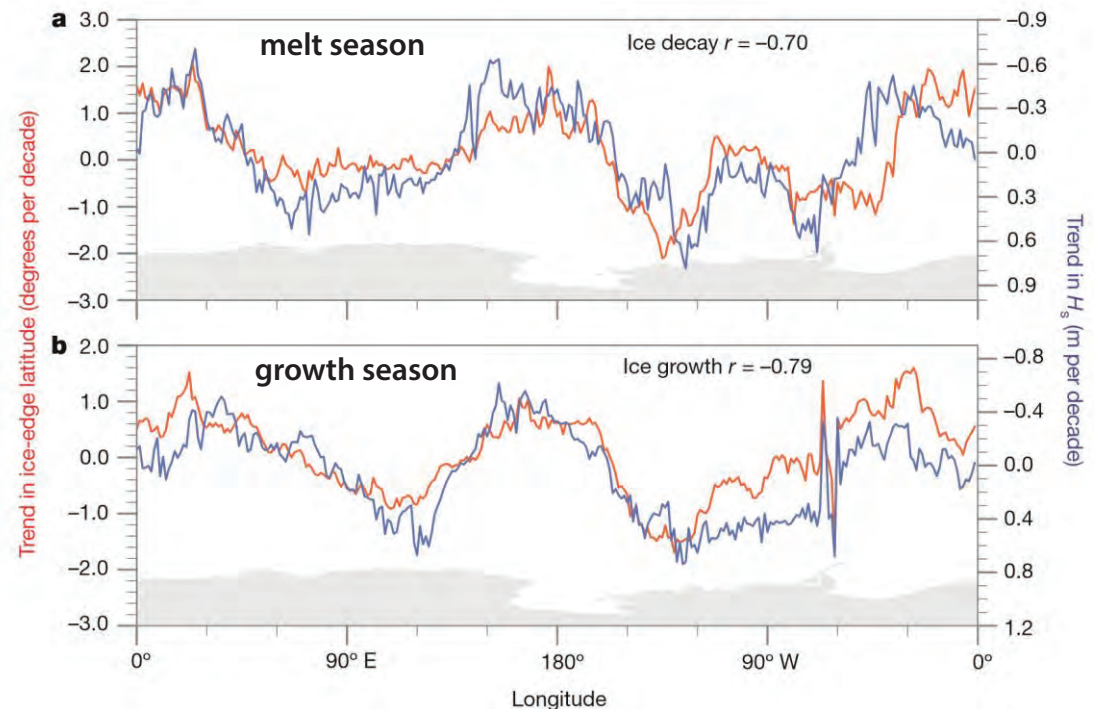
spectral masses

Murphy, Cherkaev, Zhu, Xin, Golden 2017

Storm-induced sea-ice breakup and the implications for ice extent

Kohout et al., *Nature* 2014

- during three large-wave events, significant wave heights did not decay exponentially, enabling large waves to persist deep into the pack ice.
- large waves break sea ice much farther from the ice edge than would be predicted by the commonly assumed exponential decay



ice extent compared with significant wave height

Waves have strong influence on both the floe size distribution and ice extent.

ocean swells propagating through a vast field of pancake ice

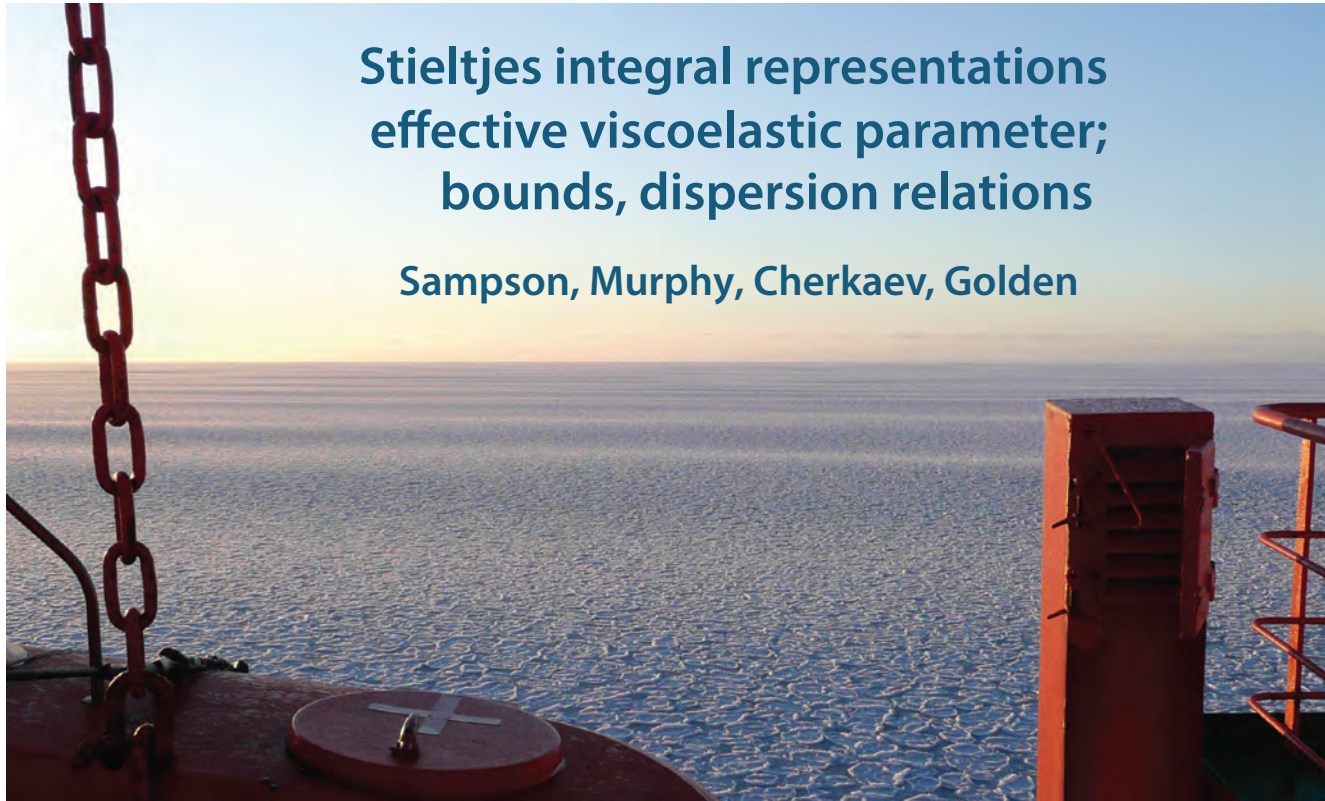
HOMOGENIZATION: long wave sees an effective medium, not individual floes



wave propagation in the marginal ice zone

Stieltjes integral representations
effective viscoelastic parameter;
bounds, dispersion relations

Sampson, Murphy, Cherkaev, Golden



bounds on the effective complex viscoelasticity

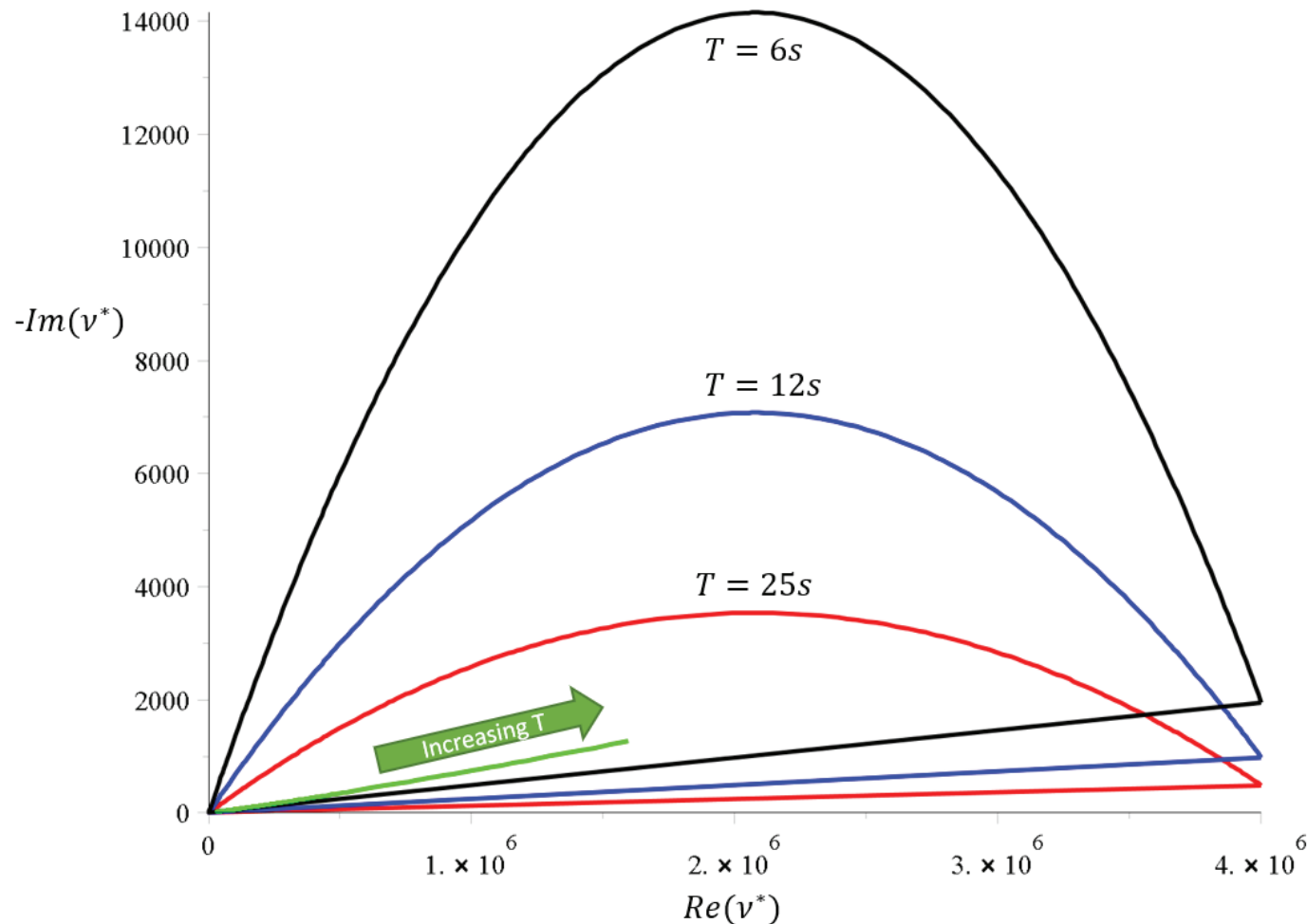
complex elementary bounds
(fixed area fraction of floes)

$$V_1 = 10^7 + i 4875$$

pancake ice

$$V_2 = 5 + i 0.0975$$

slush / frazil

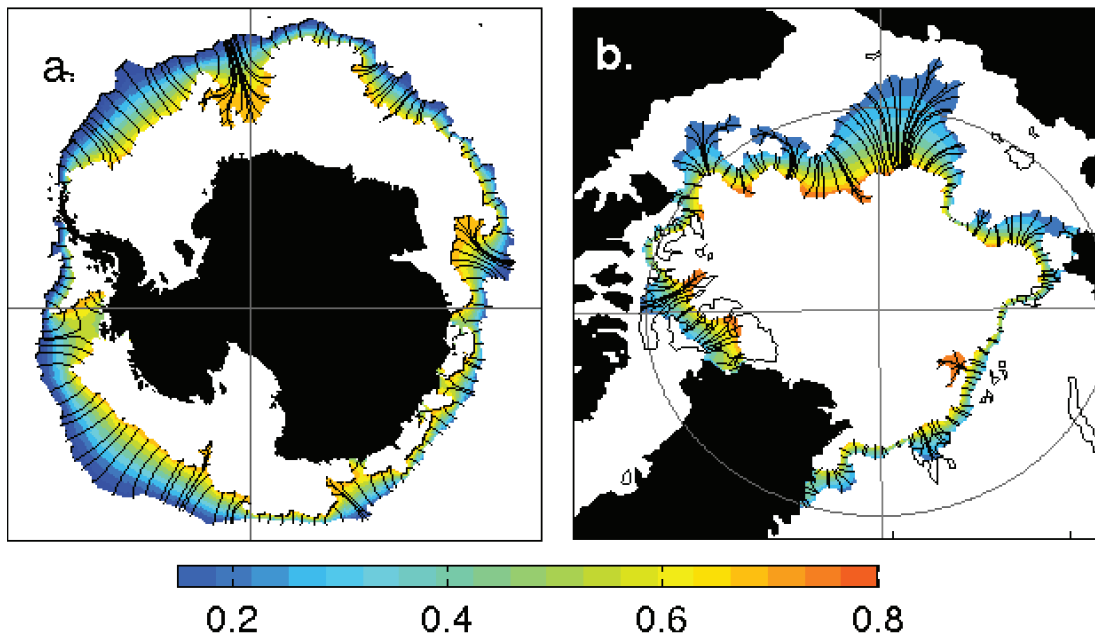


Sampson, Murphy, Cherkaev, Golden 2018

Marginal Ice Zone

MIZ

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



MIZ WIDTH

fundamental length scale of
ecological and climate dynamics

Strong, *Climate Dynamics* 2012

Strong and Rigor, *GRL* 2013

transitional region between
dense interior pack ($c > 80\%$)
sparse outer fringes ($c < 15\%$)

**How to objectively
measure the “width”
of this complex,
non-convex region?**

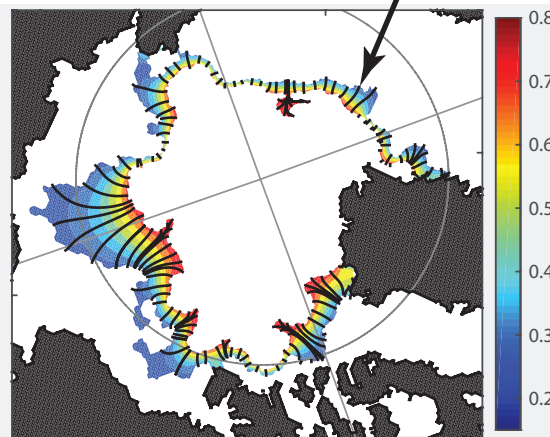
Objective method for measuring MIZ width motivated by medical imaging and diagnostics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

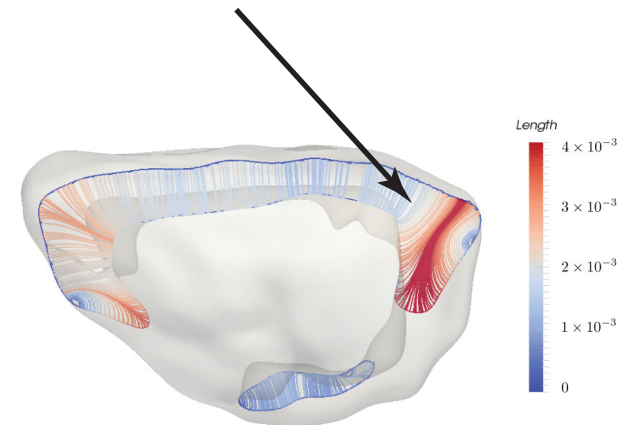
**39% widening
1979 - 2012**

“average” lengths of streamlines

streamlines of a solution
to Laplace’s equation



Arctic Marginal Ice Zone



**cross-section of the
cerebral cortex of a rodent brain**

analysis of different MIZ WIDTH definitions

Strong, Foster, Cherkaev, Eisenman, Golden
J. Atmos. Oceanic Tech. 2017

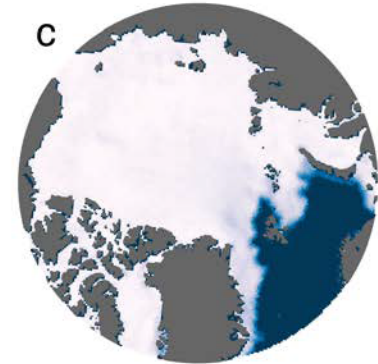
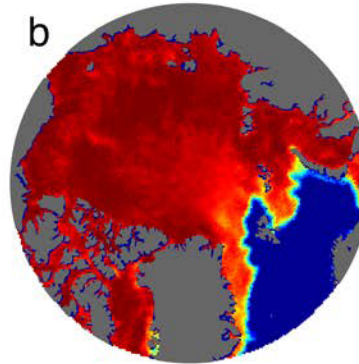
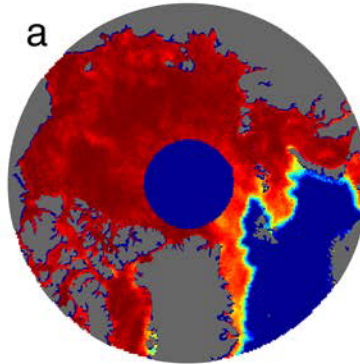
Strong and Golden
Society for Industrial and Applied Mathematics News, April 2017

Filling the polar data gap

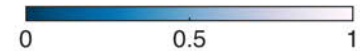
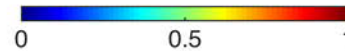
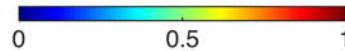
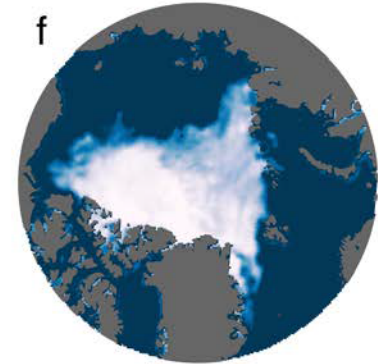
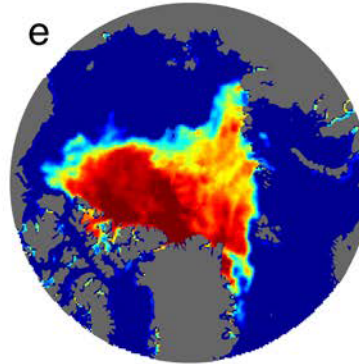
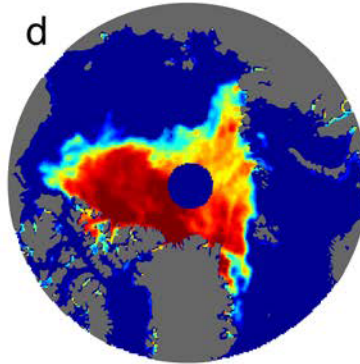
hole in satellite coverage
of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985



Gap radius: 311 km
30 August 2007



**fill with harmonic function satisfying
satellite BC's plus stochastic term**

Strong and Golden, *Remote Sensing* 2016

Strong and Golden, *SIAM News* 2017

Arctic and Antarctic field experiments

*develop electromagnetic methods
of monitoring fluid transport and
microstructural transitions*

extensive measurements of fluid and
electrical transport properties of sea ice:

2007 Antarctic SIPEX

2010 Antarctic McMurdo Sound

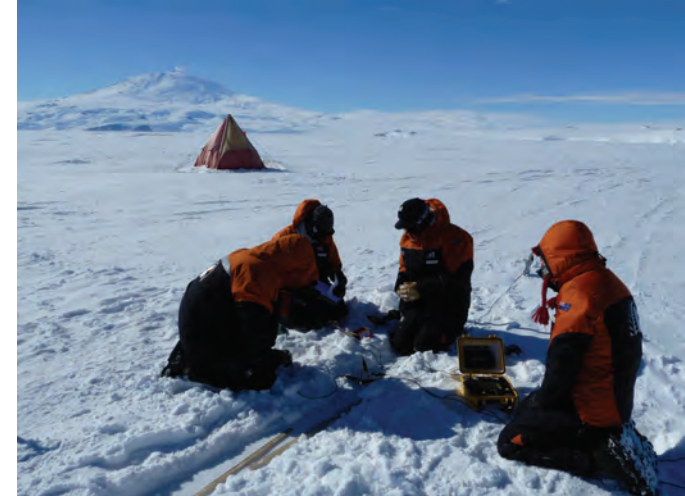
2011 Arctic Barrow AK

2012 Arctic Barrow AK

2012 Antarctic SIPEX II

2013 Arctic Barrow AK

2014 Arctic Chukchi Sea



Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and
the Mathematics of
Transport in Sea Ice

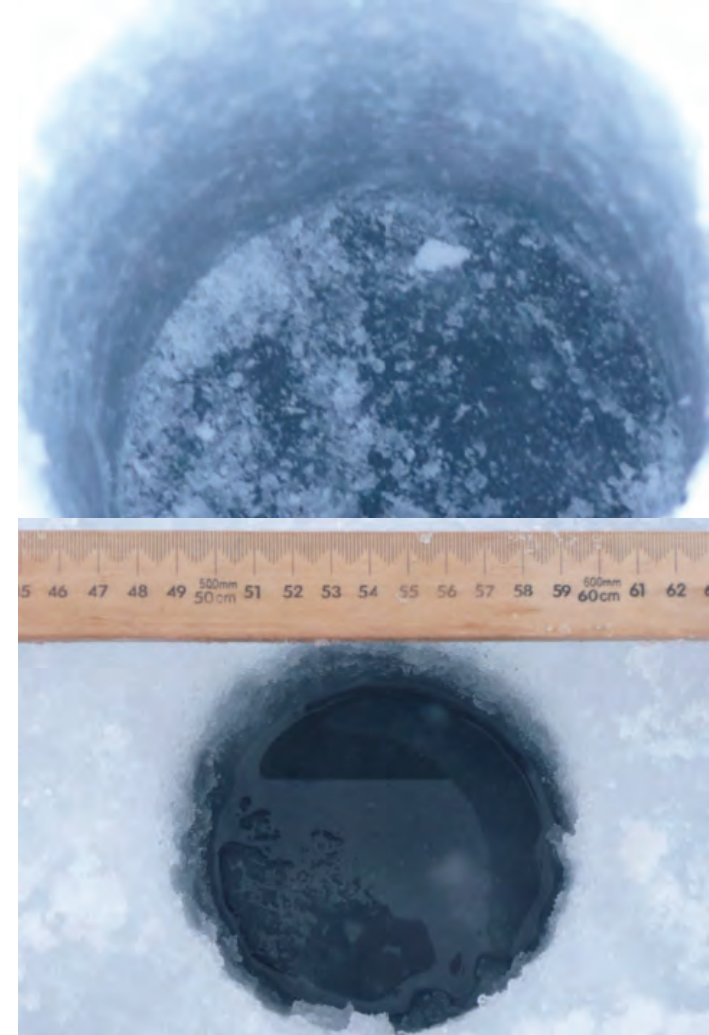
page 562

Mathematics and the
Internet: A Source of
Enormous Confusion
and Great Potential

page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



***measuring
fluid permeability
of Antarctic sea ice***

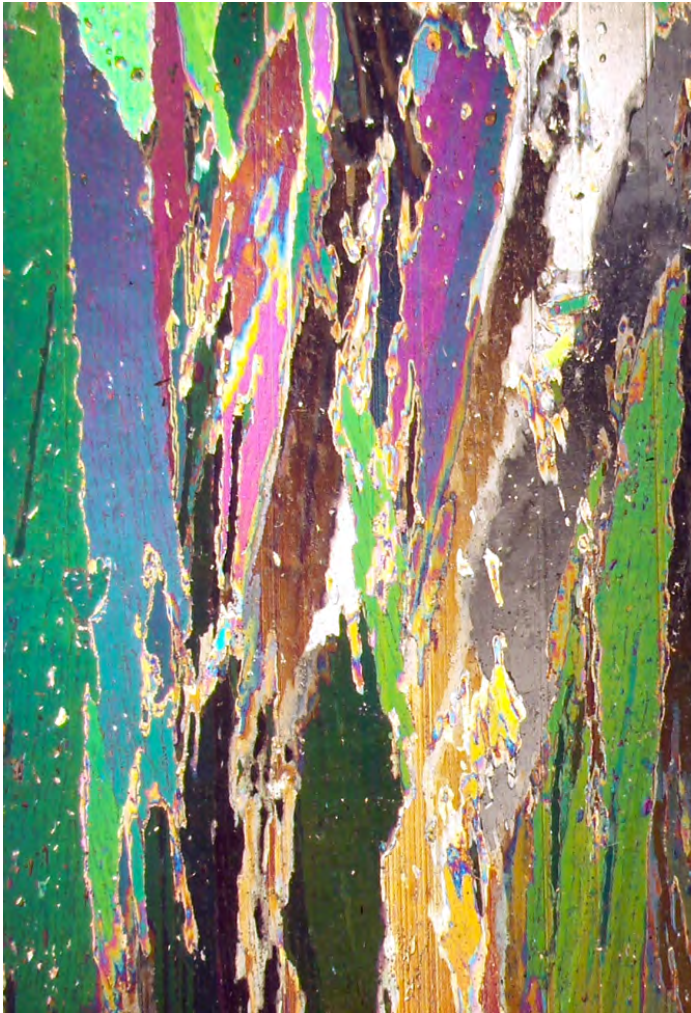
SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

columnar

granular

5%



10%

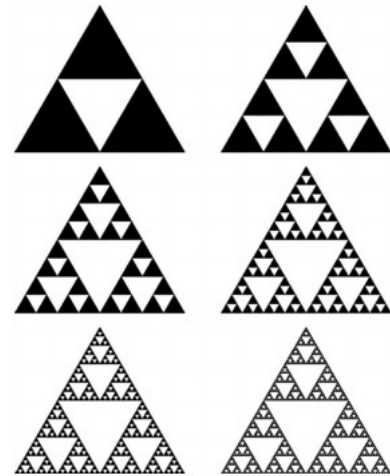


Golden, Sampson, Gully, Lubbers, Tison 2018

tracers flowing through inverted sea ice blocks



fractals and multiscale structure



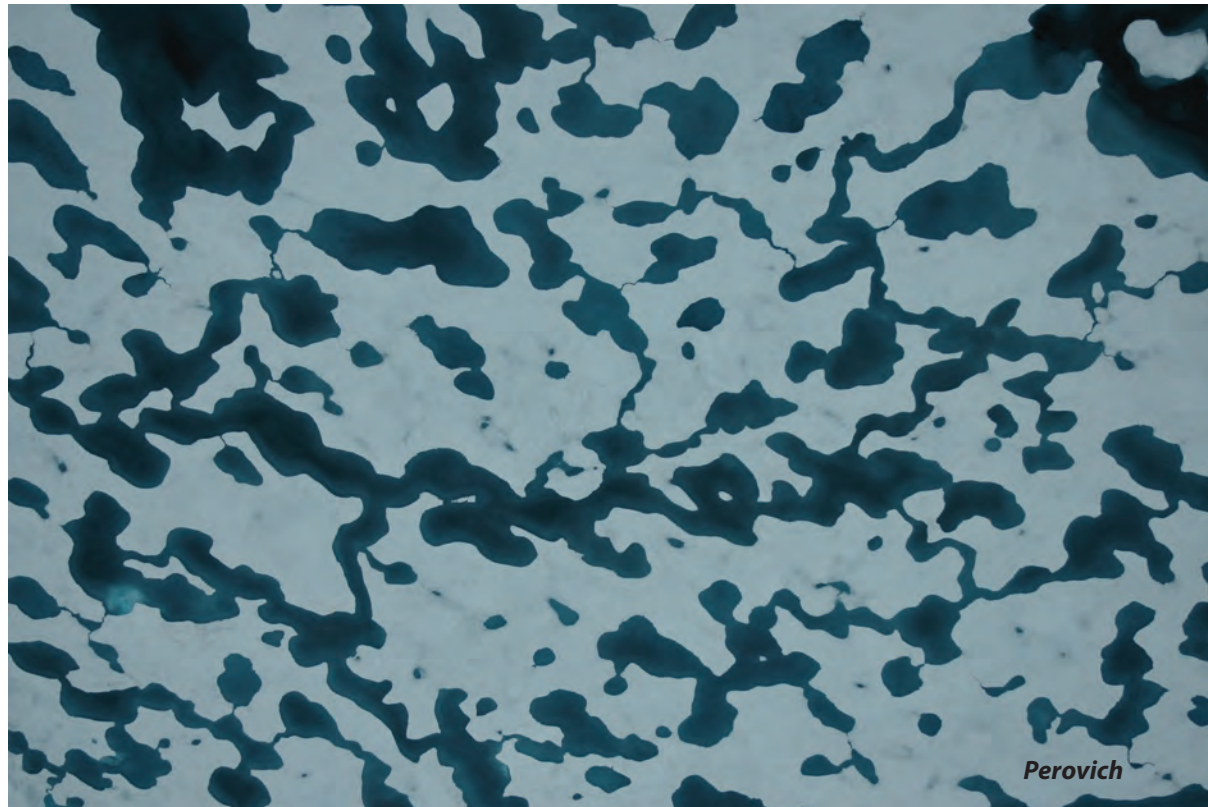
melt pond formation and albedo evolution:

- *major drivers in polar climate*
- *key challenge for global climate models*

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham,
Taylor, Worster 2006
Flocco, Feltham 2007

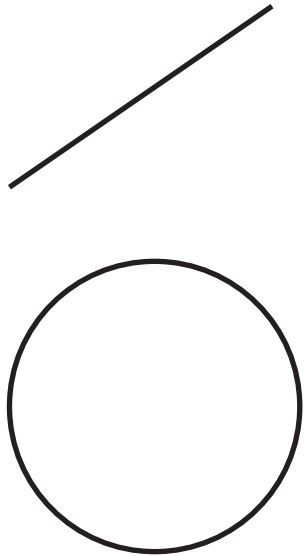
Skyllingstad, Paulson,
Perovich 2009
Flocco, Feltham,
Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

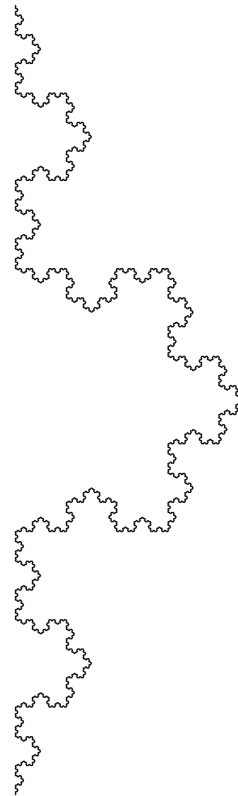
fractal curves in the plane

they wiggle so much that their dimension is >1



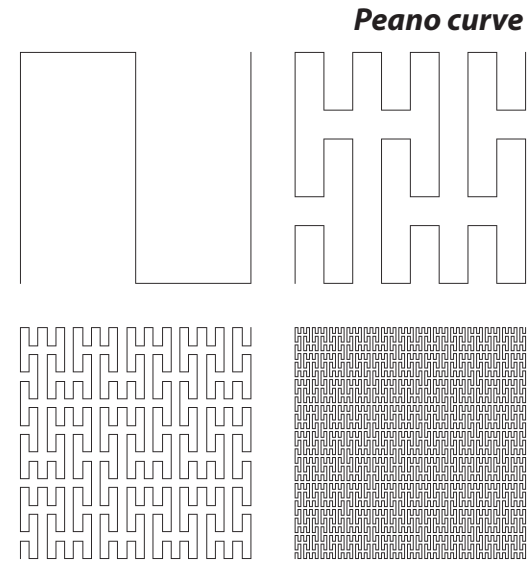
simple curves

$D = 1$



Koch snowflake

$D = 1.26$



Peano curve

Brownian motion

space filling curves

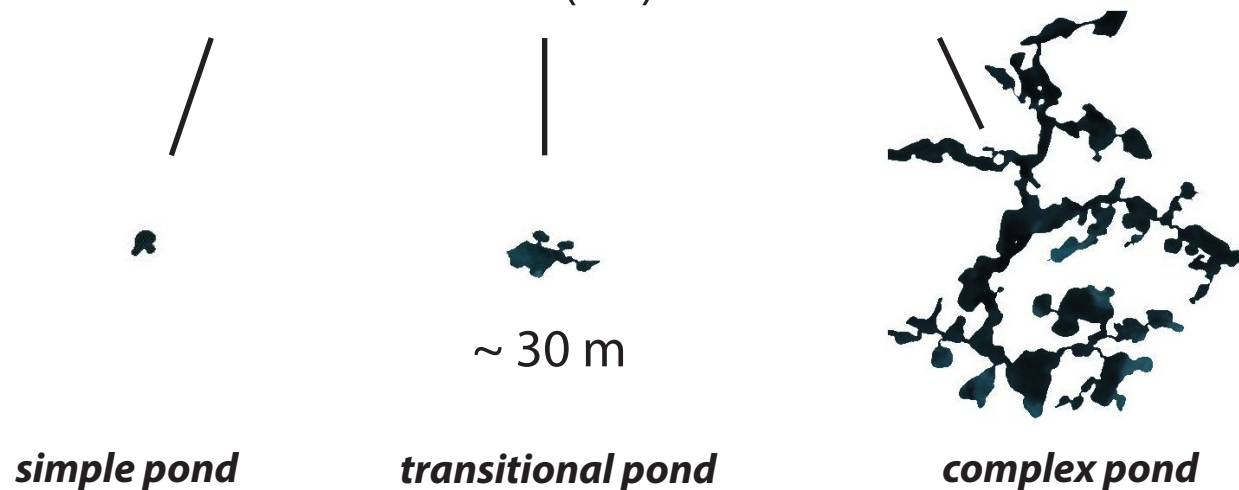
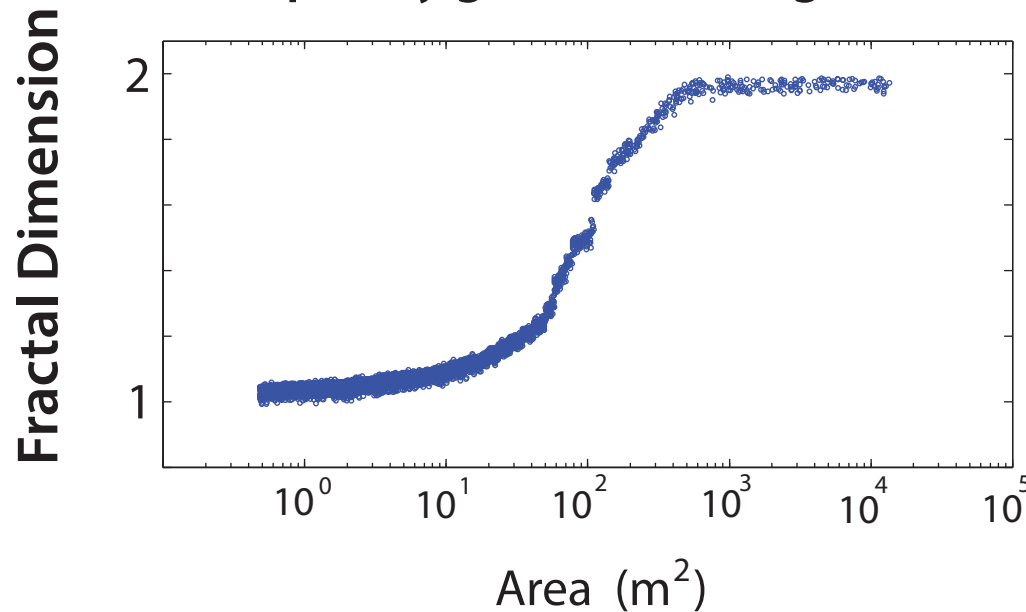
$D = 2$

Transition in the fractal geometry of Arctic melt ponds

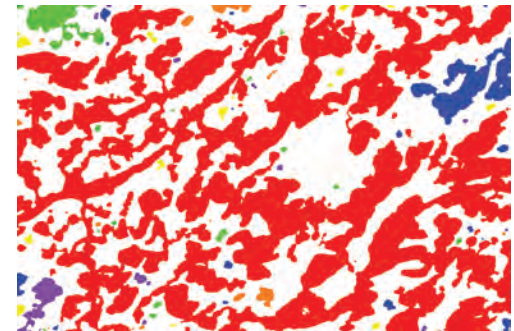
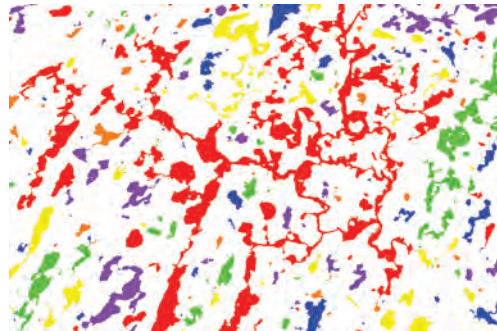
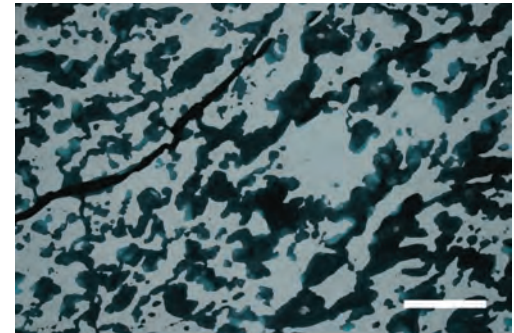
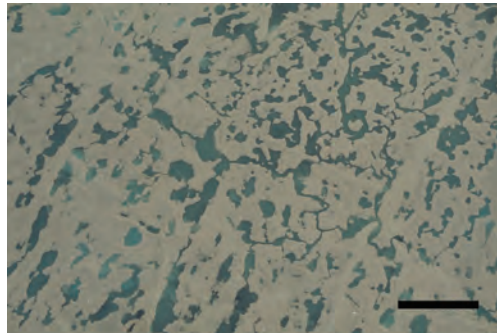
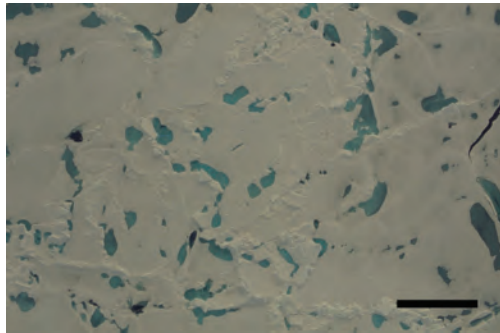
Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

complexity grows with length scale



***small simple ponds coalesce to form
large connected structures with complex boundaries***



melt pond percolation

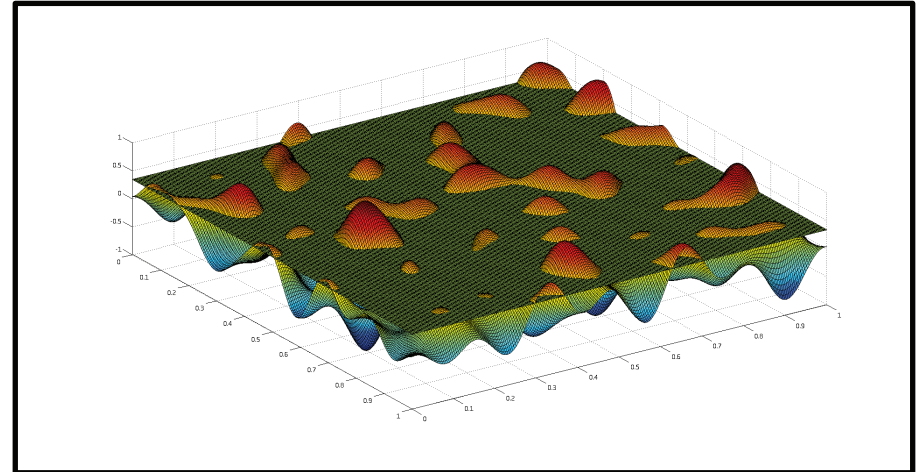
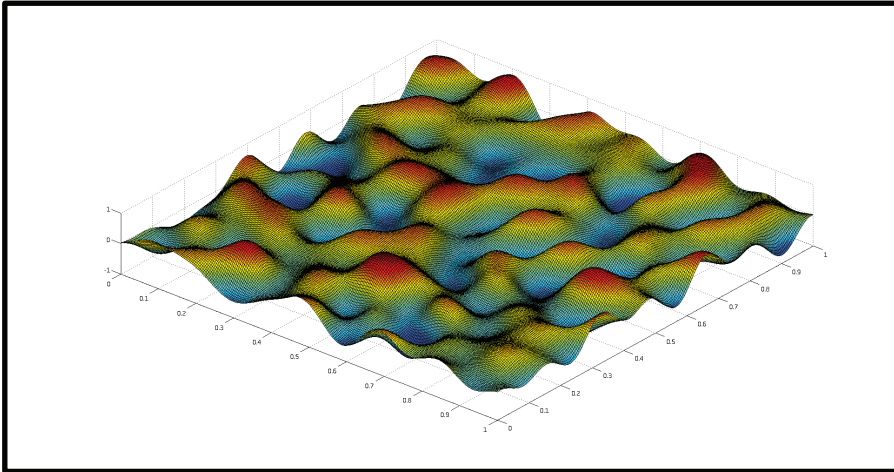
results on percolation threshold, correlation length, cluster behavior

Anthony Cheng (Hillcrest HS), Dylan Webb (Skyline HS), Court Strong, Ken Golden

Continuum percolation model for melt pond evolution

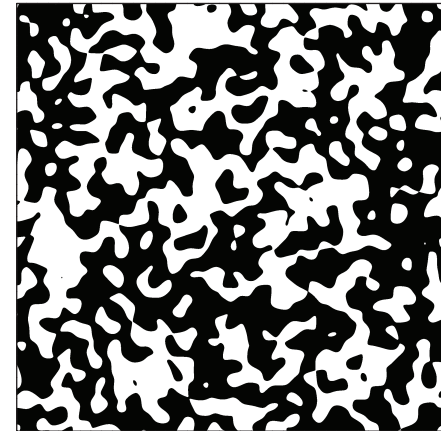
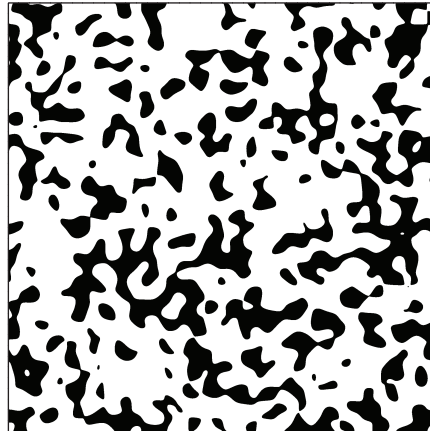
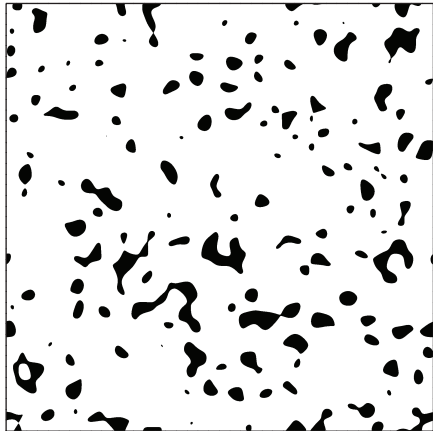
level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds



electronic transport in disordered media

diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

Ising model for ferromagnets

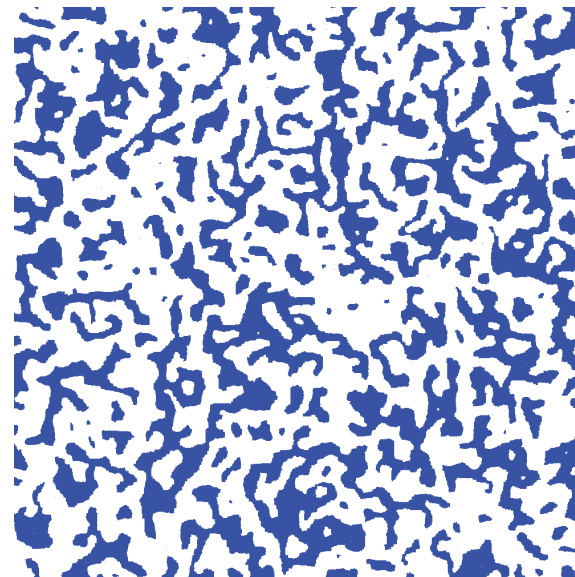
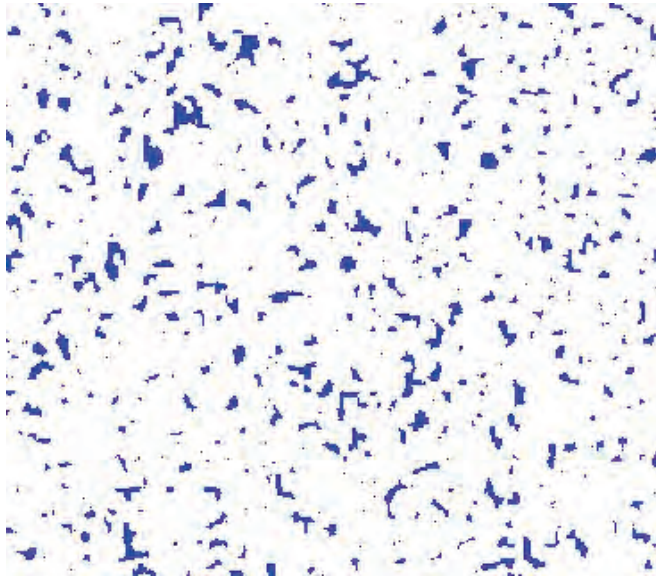


Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{\langle i,j \rangle}^N s_i s_j - H \sum_i^N s_i \quad s_i = \begin{cases} \uparrow & +1 & \text{water} & (\text{spin up}) \\ \downarrow & -1 & \text{ice} & (\text{spin down}) \end{cases}$$

magnetization $M = \lim_{N \rightarrow \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$

pond coverage $\frac{(M+1)}{2}$



“melt ponds” are clusters of magnetic spins that align with the applied field

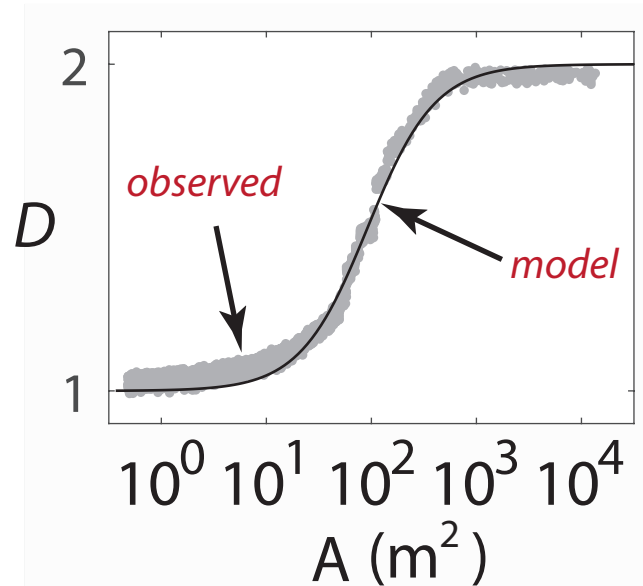
predictions of fractal transition, pond size exponent Ma, Sudakov, Strong, Golden 2018

Ising model results

Minimize Ising Hamiltonian energy

Random magnetic field represents snow topography; interaction term represents horizontal heat transfer.

*Melt ponds —
metastable islands of like spins
in our random field Ising model.*

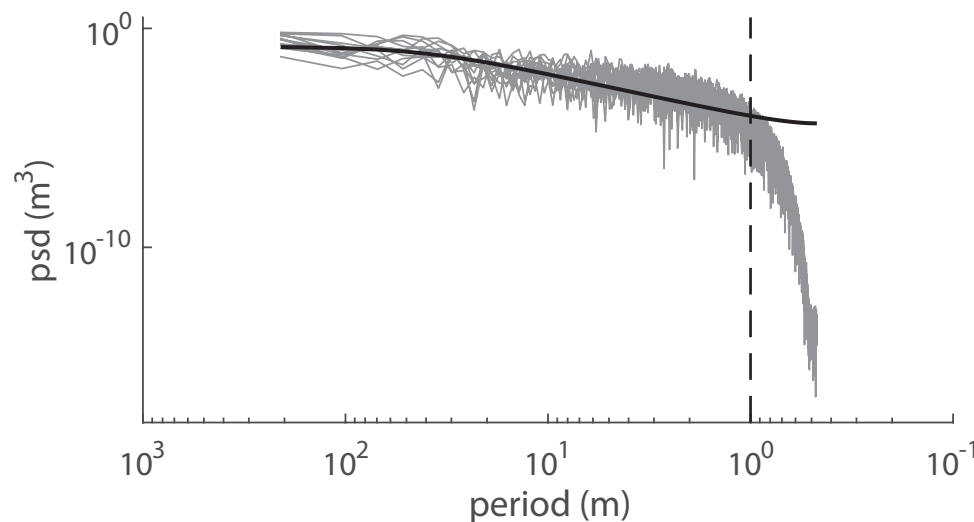


*pond size distribution
exponent*

*observed -1.5
(Perovich, et al 2002)*

model -1.58

order out of disorder



The lattice constant a must be small relative to the 10-20 m length scales prominent in sea ice and snow topography. We set $a=1$ m as the length scale above which important spatially correlated fluctuations occur in the power spectrum of snow topography.



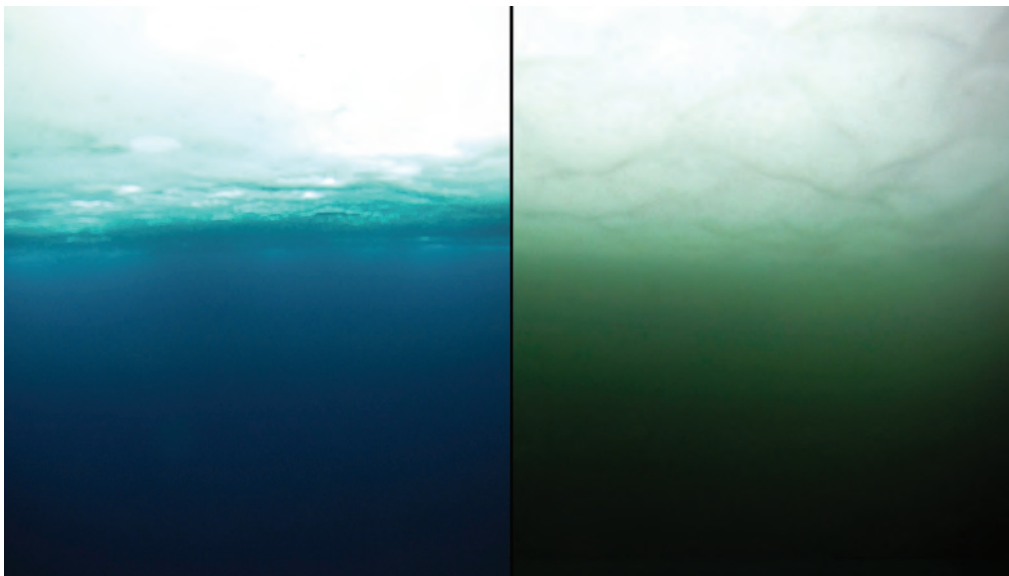
2011 massive under-ice **algal bloom**

Arrigo et al., *Science* 2012

melt ponds act as

WINDOWS

allowing light
through sea ice



no bloom

bloom

***Have we crossed into a
new ecological regime?***

The frequency and extent of sub-ice
phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder,
Flocco, Feltham, *Science Advances*, 2017

(2015 AMS MRC, Snowbird)

The Melt Pond Conundrum:

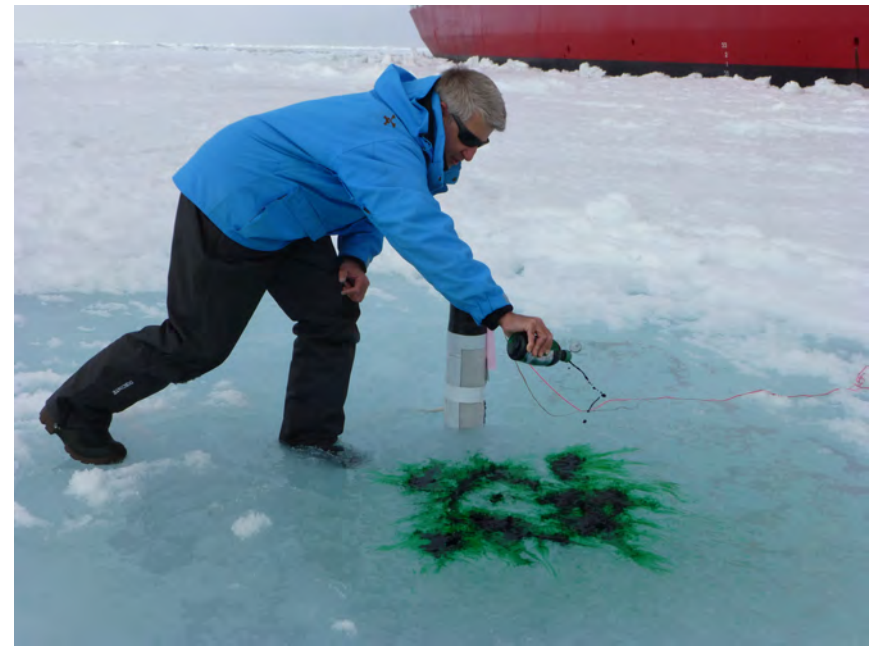
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

*2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE)
aboard USCGC Healy*



Conclusions

1. Summer Arctic sea ice is **melting rapidly**, and **melt ponds** and other processes must be accounted for in order to predict melting rates.
2. **Fluid flow** through sea ice mediates **melt pond evolution** and many processes important to climate change and polar ecosystems.
3. **Statistical physics and homogenization help link scales**, provide rigorous methods for finding effective behavior, and advance how sea ice is represented in climate models.
4. Critical behavior (in many forms) is inherent in the climate system.
5. Field experiments are essential to developing relevant mathematics.
6. Our research will help to **improve projections of climate change**, the fate of Earth's sea ice packs, and the ecosystems they support.

THANK YOU

National Science Foundation

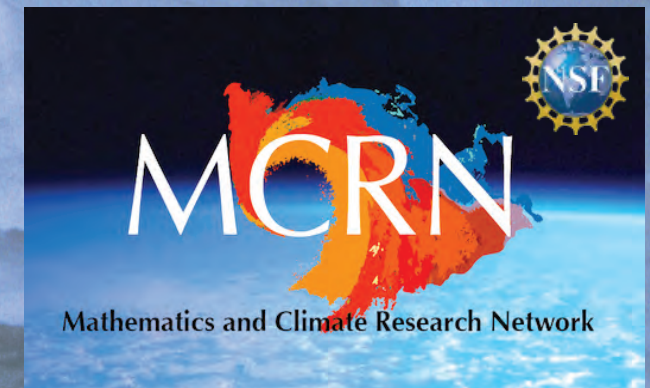
Division of Mathematical Sciences

Division of Polar Programs

Office of Naval Research

Arctic and Global Prediction Program

Applied and Computational Analysis Program



Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999