Anisotropic sea ice mechanics in continuum models

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with thanks to Alexander Wilchinsky, Michel Tsamados

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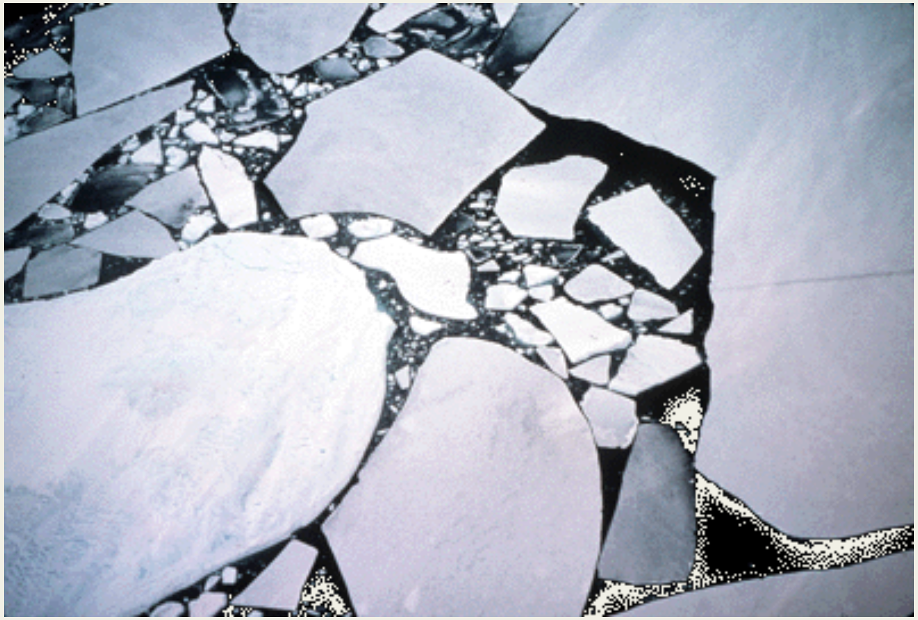
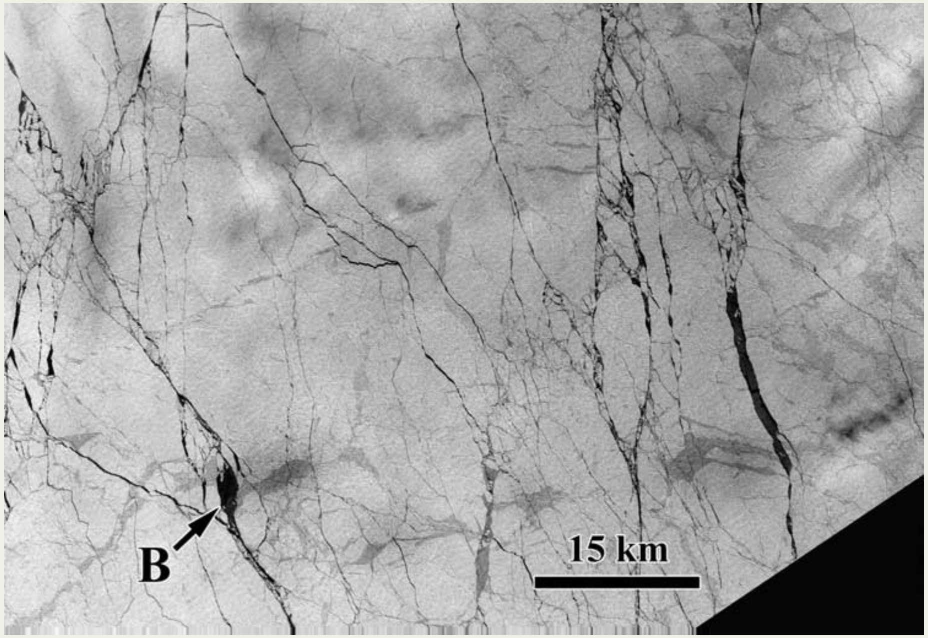
Sea ice floes



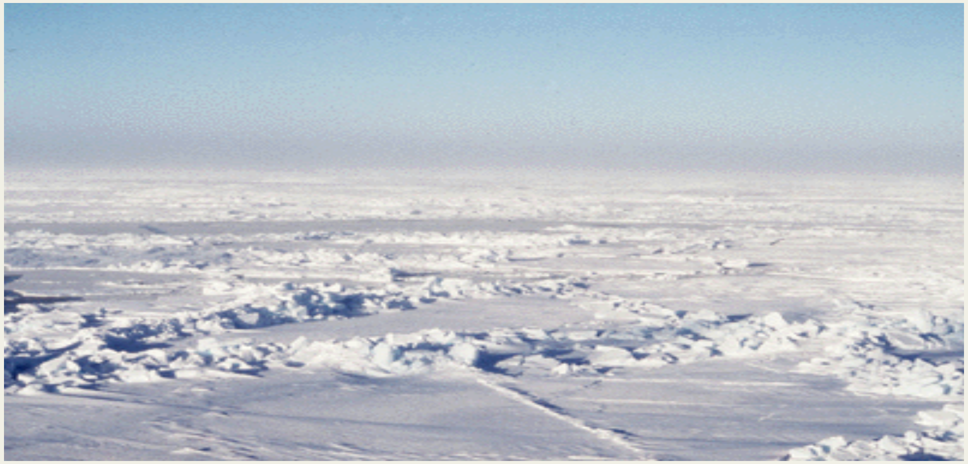


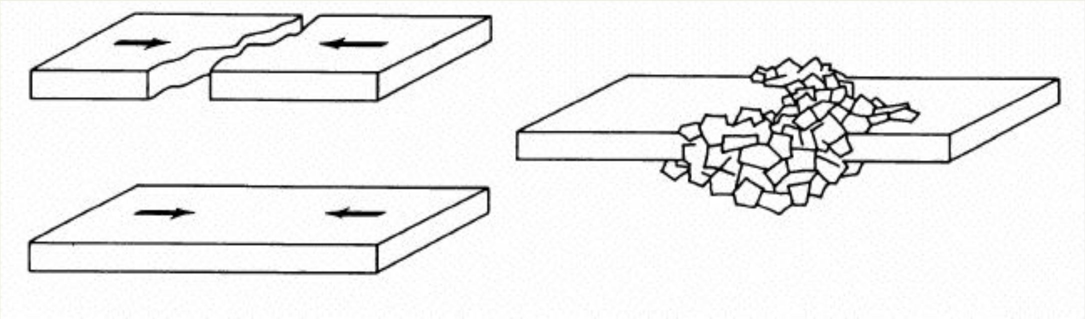
The sea ice cover consists of floes that are 0.1-10 km wide and 0.1-5 m thick; they are separated by cracks or leads.

The sea ice cover may look like a collection of granules (floes) or like a continuous cover with cracks.

Area concentration typically 0.90-0.99.

Pressure ridges





Pressure ridges form when floes collide with, or over ride, each other.
The ice sheet breaks up into blocks that are pushed into sails and keels.

Pressure ridges can be many kilometres in length and the sails and keels are approximately triangular.

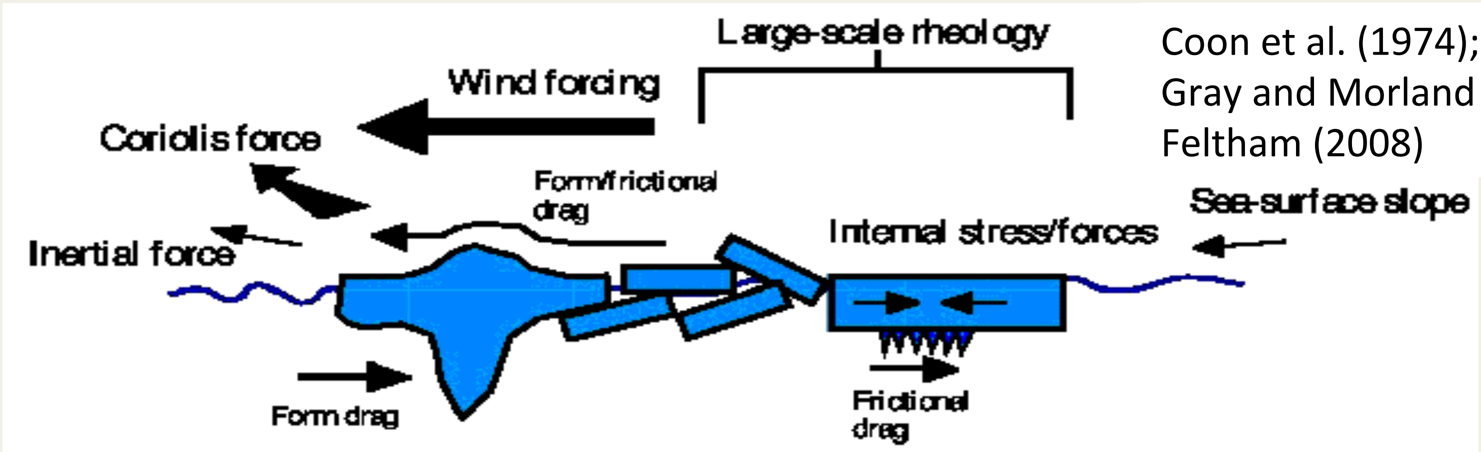
Leads

Leads are narrow regions of open ocean or thin ice separating floes.

Typically 0.1-1 km wide, with lengths spanning many floes.

In central Arctic, typically 1km of leads/km^2.

Local momentum balance of sea ice



Coon et al. (1974);
Gray and Morland (1994);
Feltham (2008)

Vertically-integrated (i.e. horizontal) momentum balance is:

$$m\frac{D\mathbf{u}}{Dt} = \boldsymbol{\tau}_{\mathbf{a}} + \boldsymbol{\tau}_{\mathbf{w}} + \nabla\cdot\boldsymbol{\sigma} - mf_C\mathbf{k}\times\mathbf{u} - mg\nabla H$$

mass X acceleration = air drag + ocean drag + ice-ice force + Coriolis "force" + gravity force from sea surface tilt

$\boldsymbol{\sigma} \equiv \int \boldsymbol{\sigma}' dz - \frac{1}{2}\rho_{\mathrm{ice}}\mathbf{g}h^2$ is the stress caused by frictional sliding between floes, pressure ridging, and collisions.

The dominant forces are air drag, ocean drag, and the ice-ice force.

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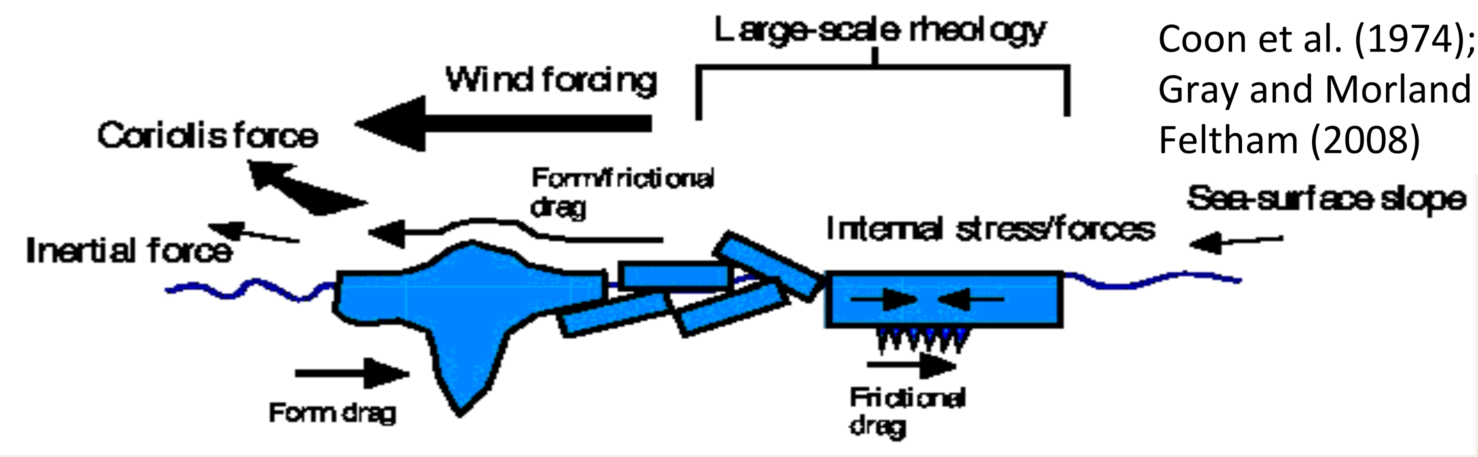

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Sea ice rheology

Sea ice rheology describes the forces (stresses) necessary to create a deformation of the sea ice cover.

Early sea ice modellers supposed that ice stress depends only upon rate of deformation and scalars:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}\left(\frac{\partial u_i}{\partial x_j}, \text{scalars}\right)$$

Material frame indifference then leads to the Reiner-Rivlin form:

$$\sigma_{ij} = 2\eta\dot{\varepsilon}_{ij} + [\varsigma - \eta]\dot{\varepsilon}_{kk}\delta_{ij} - \frac{P}{2}\delta_{ij} + \gamma\dot{\varepsilon}_{is}\dot{\varepsilon}_{sj}$$

which depends on the symmetric part of the deformation rate, i.e. the strain rate

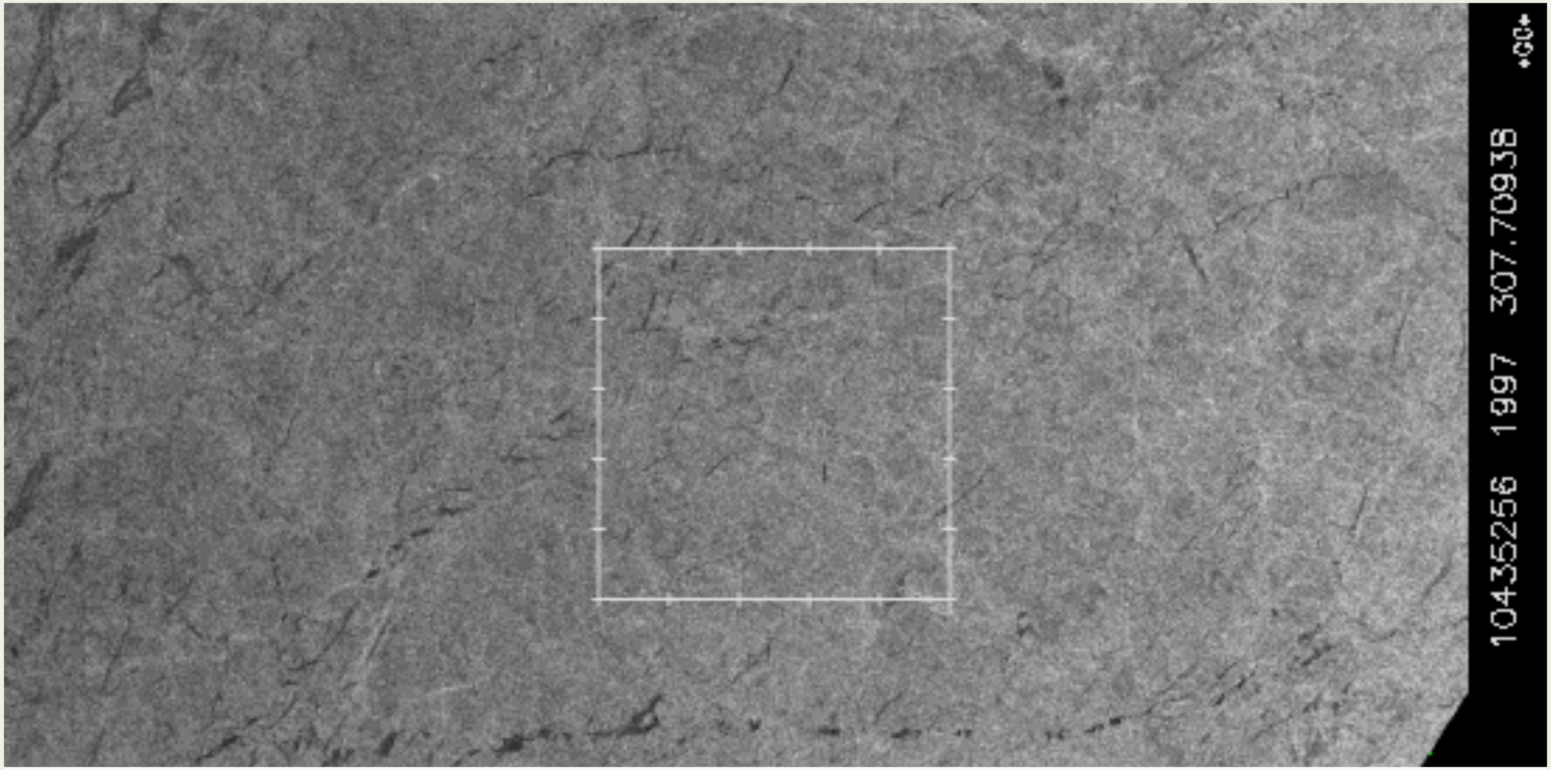
$$\dot{\varepsilon}_{ij} \equiv \tfrac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

and η, ς, γ, P are functions of the scalars and the invariants $\dot{\varepsilon}_I$, $\dot{\varepsilon}_{II}$

The RR equation is quite general; certain choices of the scalar functions lead to Newtonian fluid flow or plastic flow. Typically set $\gamma = 0$.

The RR description treats sea ice as an isotropic continuum.

Sea ice deformation



Initial square is 50 km x 50 km.
Deformation from SAR imagery.

Some features of sea ice deformation

- Sea ice cover is permeated with **cracks**, e.g. uneven isostatic loading, long-period ocean waves, thermal expansion.

- Deformation occurs at cracks but ice between cracks is **rigid**.

- A crack may open to form a lead or close to form a pressure ridge with or without shear.

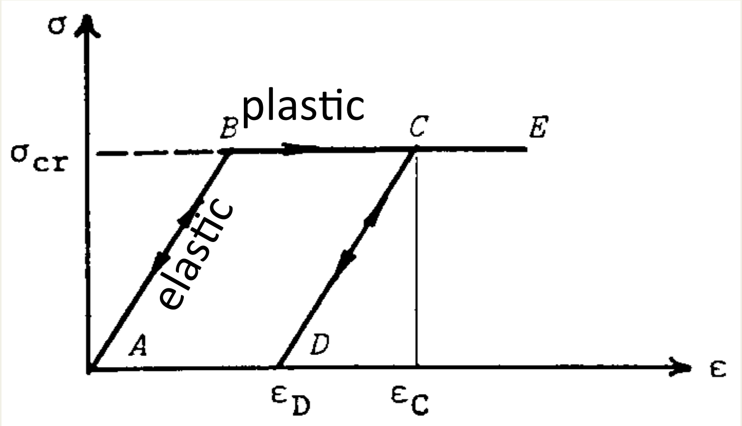
- Deformation occurs sporadically *as though* a **critical stress state** had been reached. (Air and ocean tractions are **smooth**.)

These features suggest sea ice is a PLASTIC material.

Other plastic materials are glass, metal, wood, and.. plastic.

Plasticity

- The response of sea ice is elastic and/or viscous until the stress is great enough to fulfil a **yield criterion**.
- Once this criterion is met, further deformation is **plastic** i.e. non-recoverable and independent of the magnitude of strain rate. Other names: brittle failure, decohesive failure.



- Due to **isotropy**, the yield criterion may be expressed as a function of the principal invariants of stress and scalars. For 2-dimensional deformation, the failure surface is

$$F\left(\sigma_I,\ \sigma_{II},\ \text{scalars}\right) = 0$$

$$\sigma_I \equiv \tfrac{1}{2}\sigma_{ii} = \tfrac{1}{2}\left(\sigma_1 + \sigma_2\right),$$ Negative pressure

$$\sigma_{II} \equiv \sqrt{-\det\left(\sigma_{ij} - \delta_{ij}\sigma_I\right)} = \tfrac{1}{2}\left(\sigma_2 - \sigma_1\right)$$ Max. shear stress

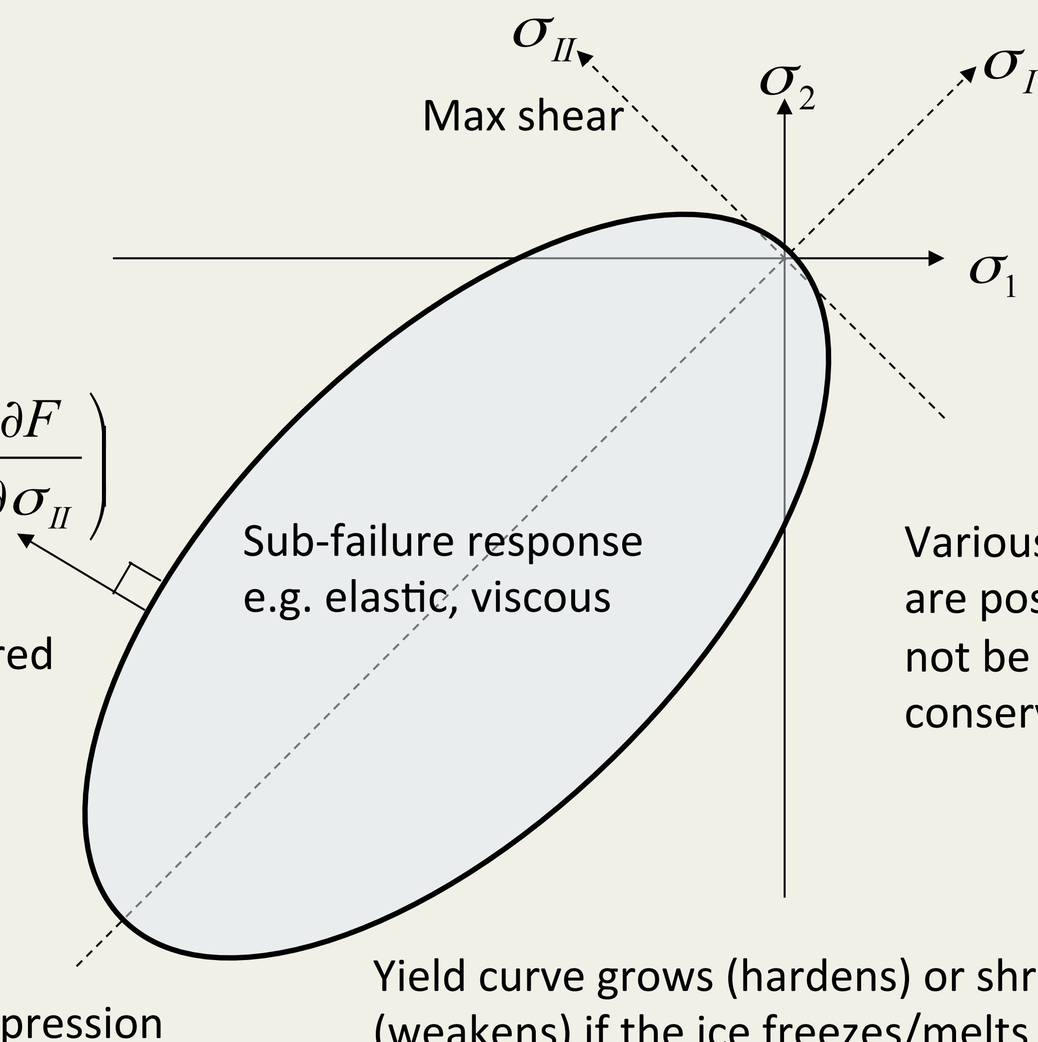
- Determination of sea ice rheology is complicated and ongoing [reviews: Hibler, 2001; Feltham, 2008]
- Simulations of the ice cover (e.g. climate forecasts) are sensitive to rheology

Plastic yield curve $F(\sigma_I,\ \sigma_{II},\ \text{scalars}) = 0$ for sea ice

Plastic flow occurs **on** the yield curve:

$$\left(\dot{\varepsilon}_I,\ \dot{\varepsilon}_{II}\right) = \lambda\left(\frac{\partial F}{\partial \sigma_I},\ \frac{\partial F}{\partial \sigma_{II}}\right)$$

Plastic flow is any deformation not recovered when the stresses are removed, e.g. pressure ridging, sliding of floes past each other.



Various yield curves are possible but must not be concave to satisfy conservation of energy.

Yield curve grows (hardens) or shrinks (weakens) if the ice freezes/melts or increases/decreases concentration.

Aside: Strain rate invariants in 2D

$$\dot{\varepsilon}_I \equiv \dot{\varepsilon}_{kk} = \nabla \cdot \mathbf{u} = |\dot{\varepsilon}|\cos\theta,$$ divergence

$$\dot{\varepsilon}_{II} \equiv 2\sqrt{-\det\left(\dot{\boldsymbol{\varepsilon}} - \tfrac{1}{2}\boldsymbol{\delta}\dot{\varepsilon}_I\right)} = |\dot{\varepsilon}|\sin\theta$$ maximum shear rate

where $$|\dot{\varepsilon}| = \sqrt{\dot{\varepsilon}_I^2 + \dot{\varepsilon}_{II}^2}, \quad \theta = \tan^{-1}\left(\frac{\dot{\varepsilon}_{II}}{\dot{\varepsilon}_I}\right)$$

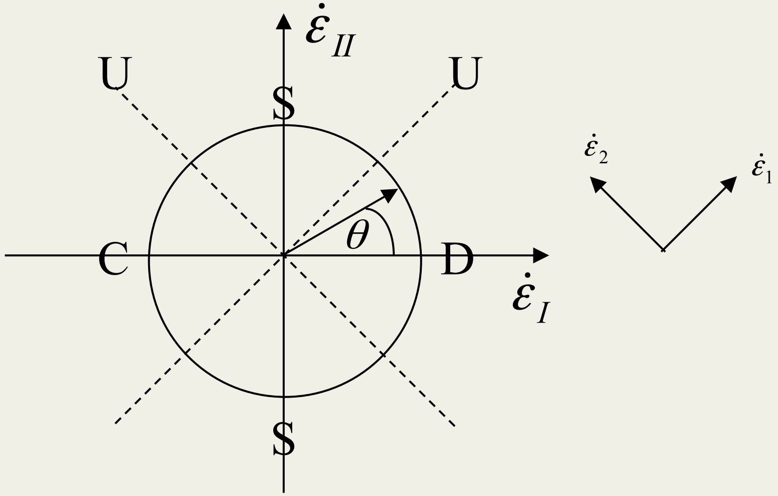
$\theta = 0,$ pure divergence,

$\frac{1}{4}\pi,$ uniaxial extension,

$\frac{1}{2}\pi,$ pure shear,

$\frac{3}{4}\pi,$ uniaxial extension,

π pure convergence

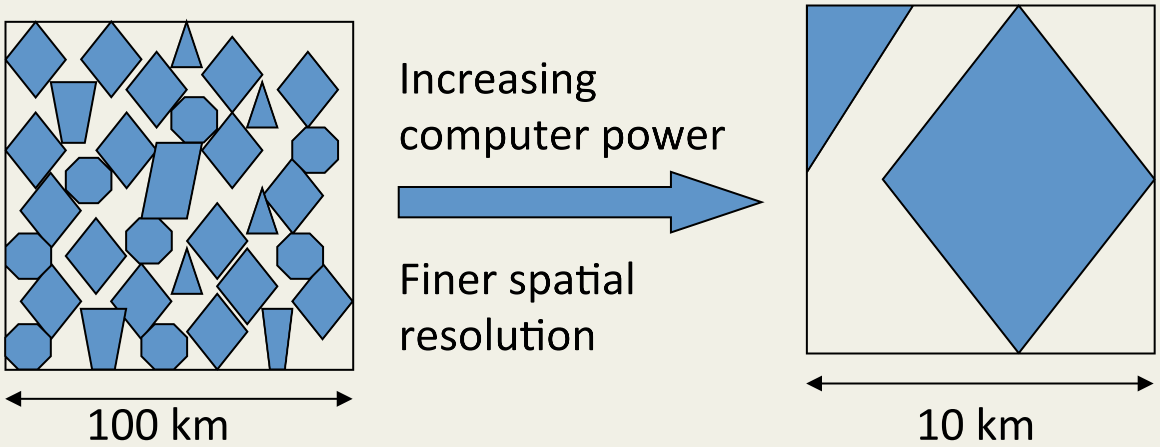


Continuity and scaling of sea ice rheology

Sea ice rheology at the grid size of climate models is a continuum rheology, describing the combined behaviour of many sea ice floes and leads (c.f. Navier-Stokes eqns).

We do not resolve individual floes or leads and assume we can represent their “average” response in a model.

As computer resolution is improving, the grid size can approach the size of a floe and so the continuum hypothesis may become invalid.



This cell only contains one complete floe.
A continuum model prediction may not be valid for this floe.

Lack of continuity becomes important if the rheology of a single floe or lead differs from that of an ensemble of floes and leads.

AIDJEX sea ice rheology [Coon et al, 1974]

1. Sea ice is plastic. (Irreversible deformation occurs at a **critical stress state**.)

2. Sea ice cannot withstand tensile stress.

3. Sea ice is isotropic at timescales of ~1 day and lengthscales of ~100 km.

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JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 112, C11S90, doi:10.1029/2005JC003393, 2007

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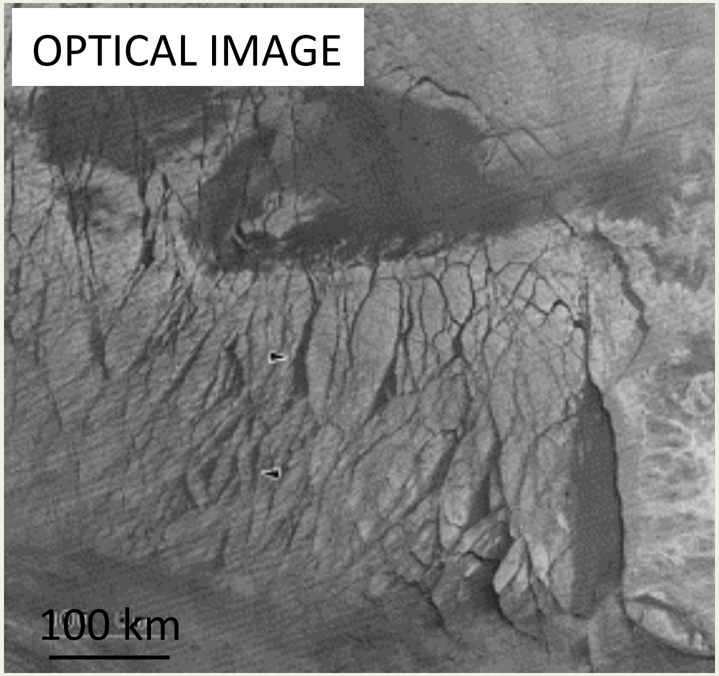
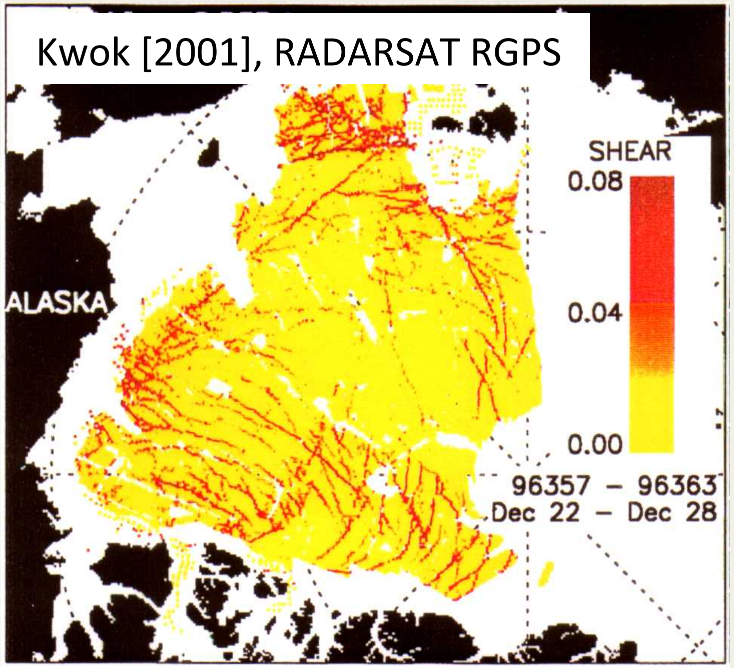
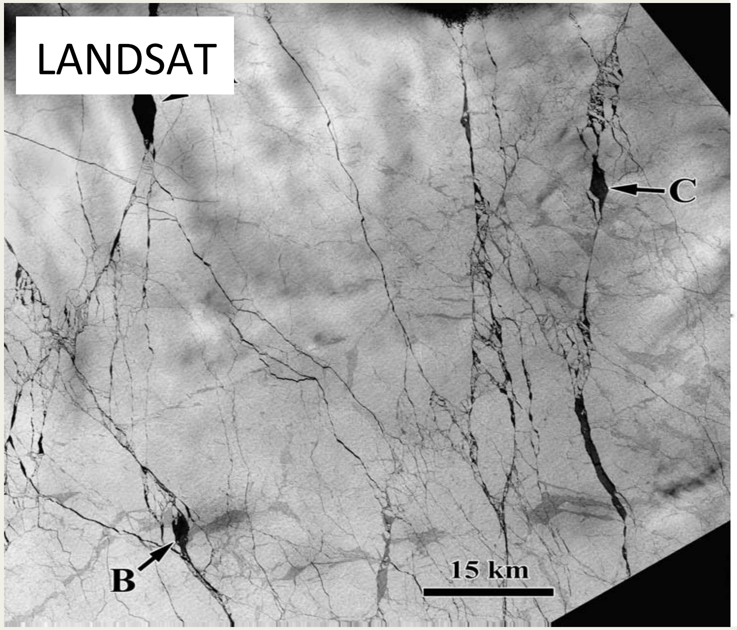
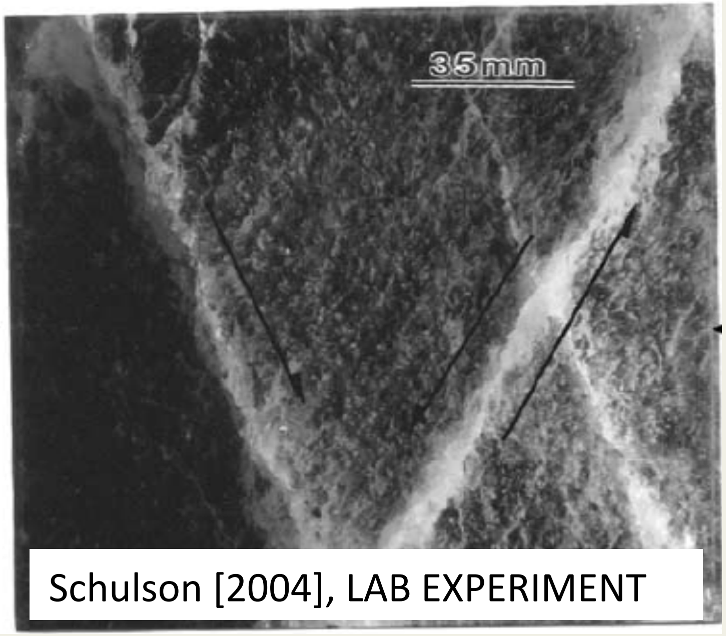




Linear Kinematic Features





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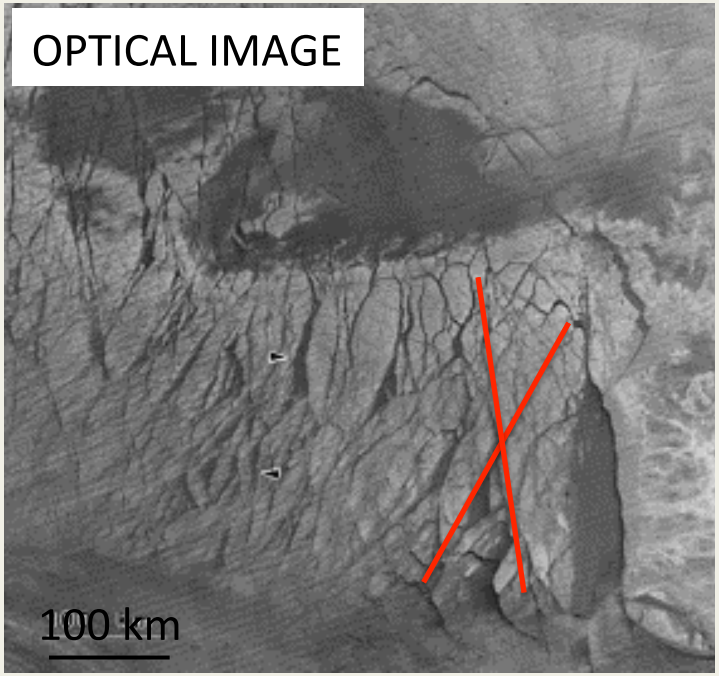

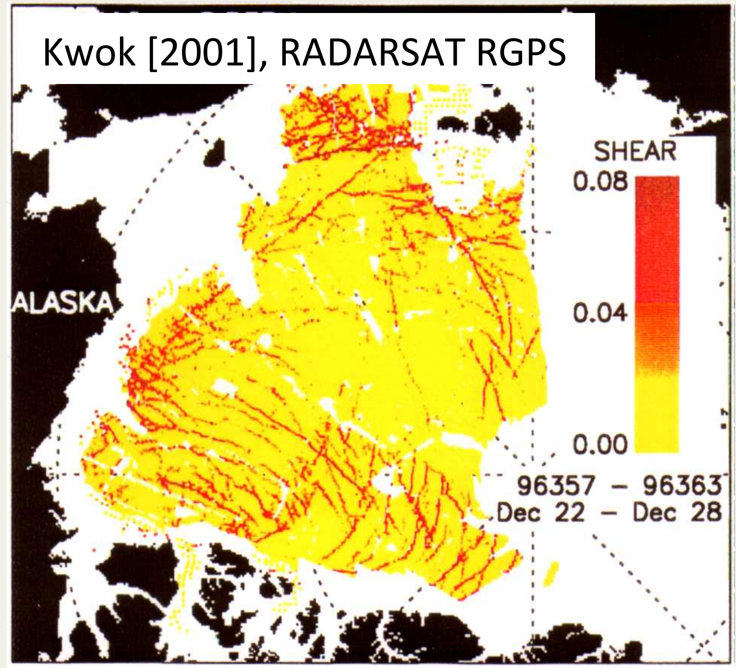

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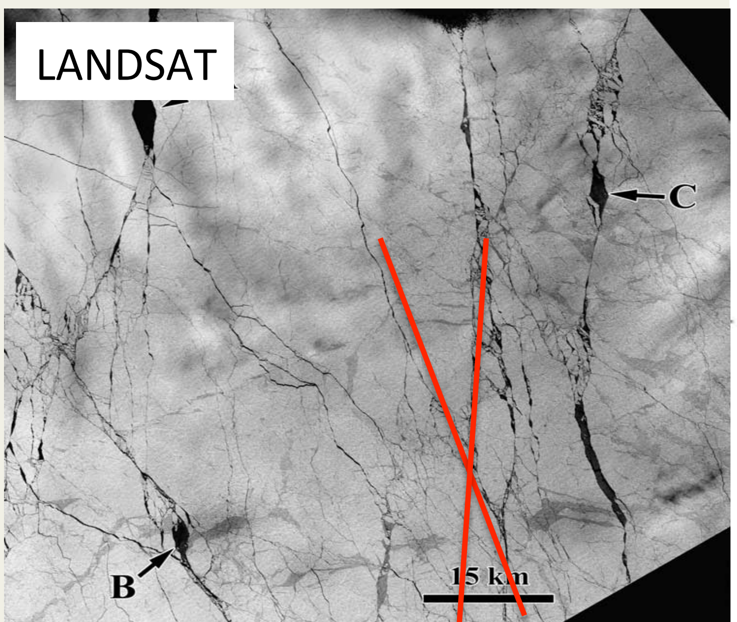

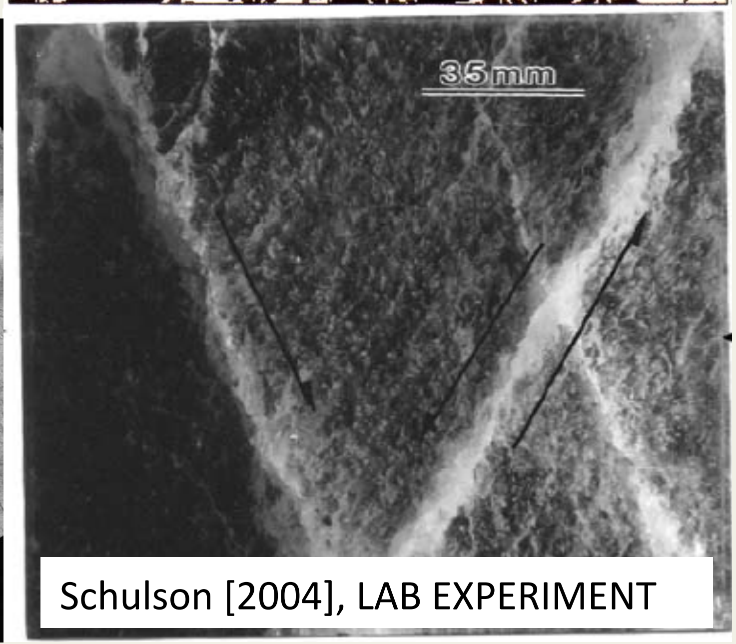

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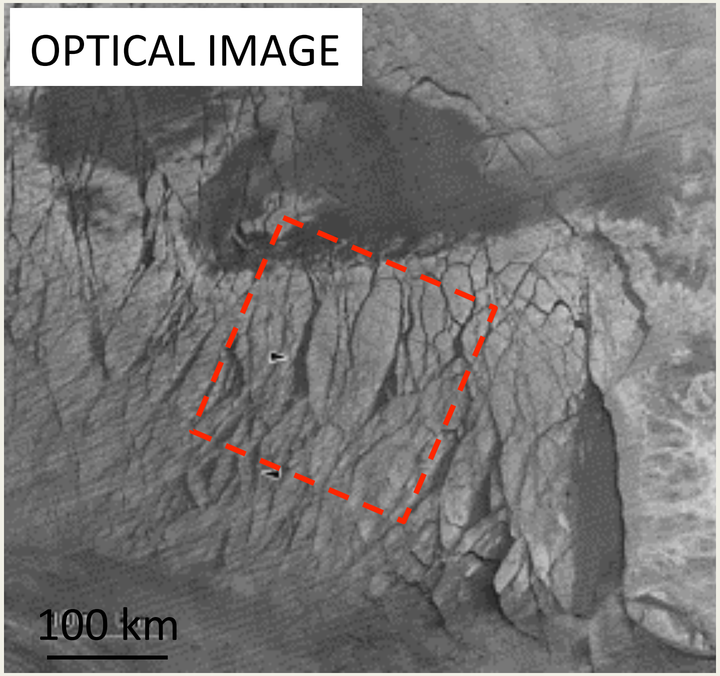

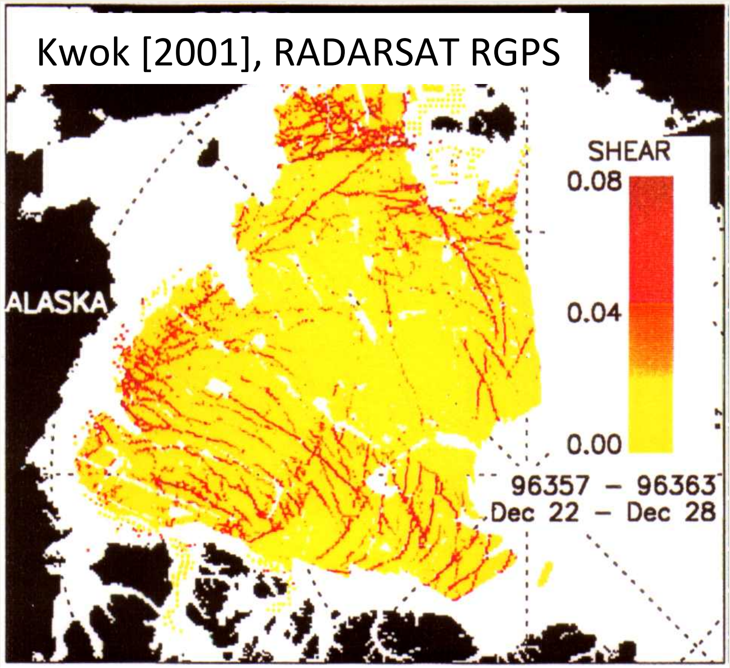

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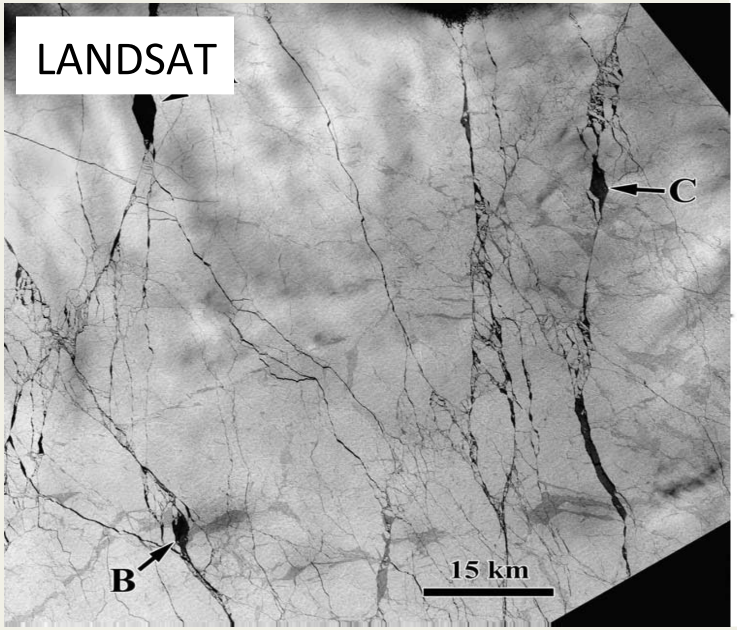

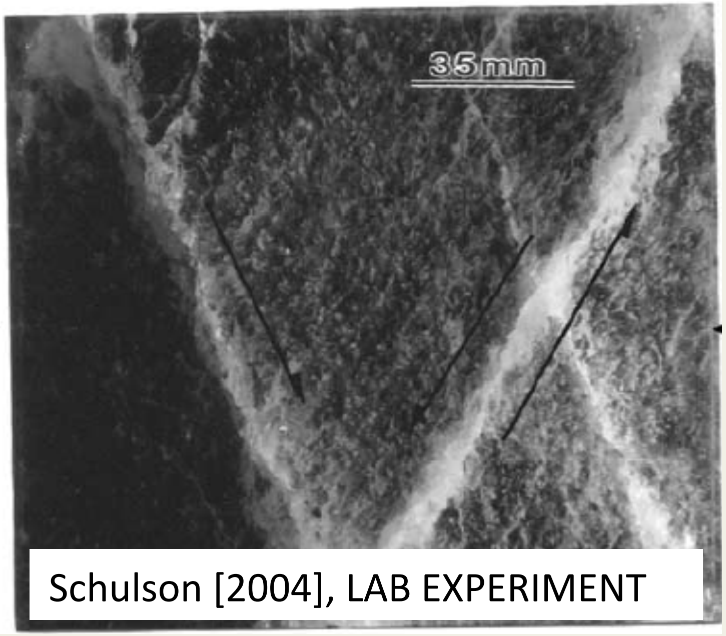

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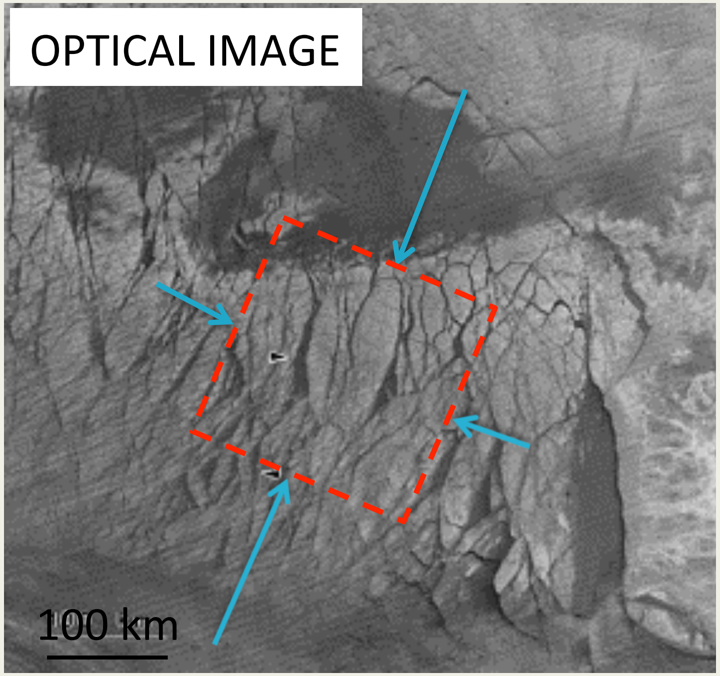

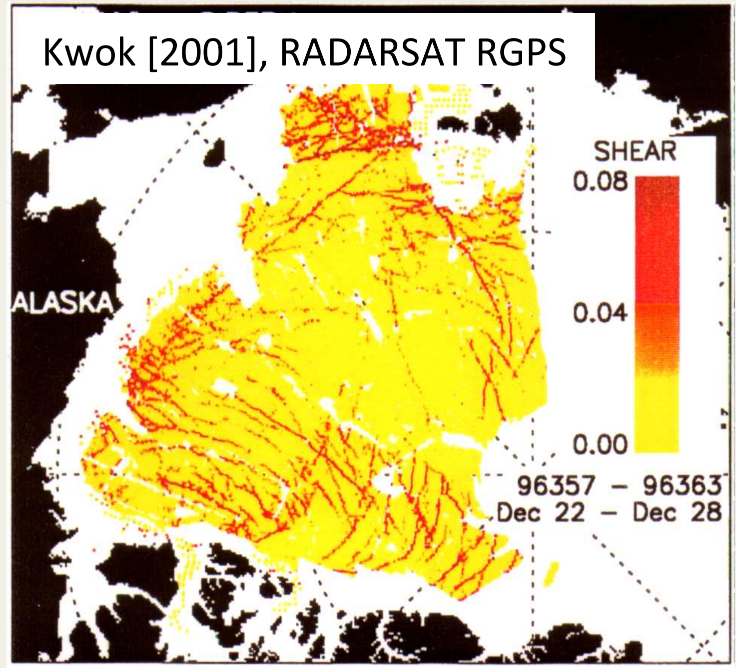

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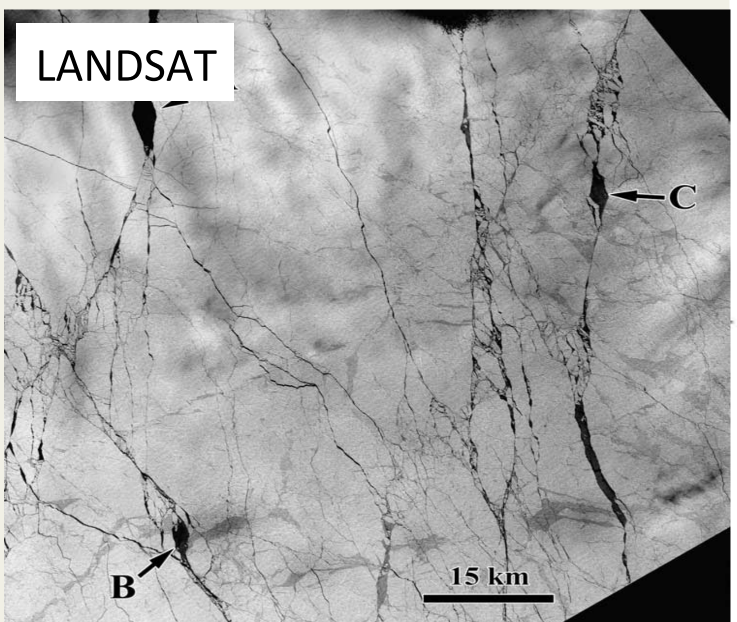
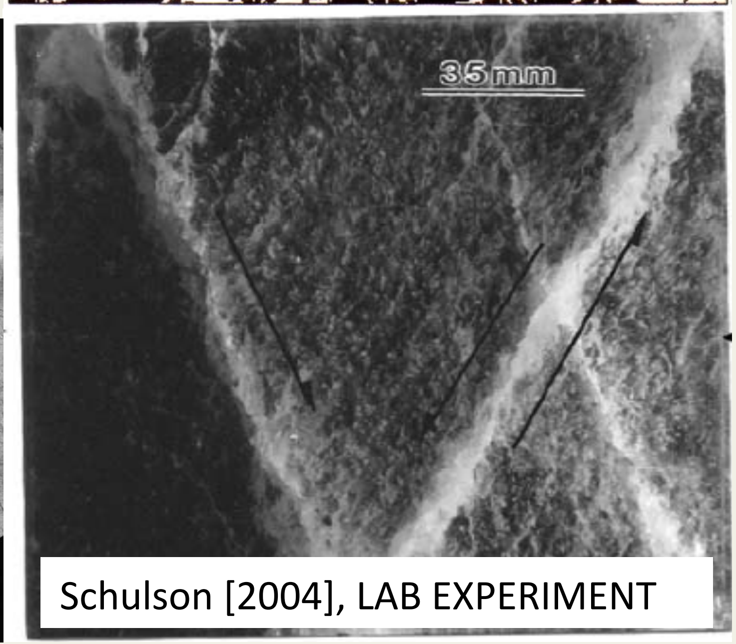
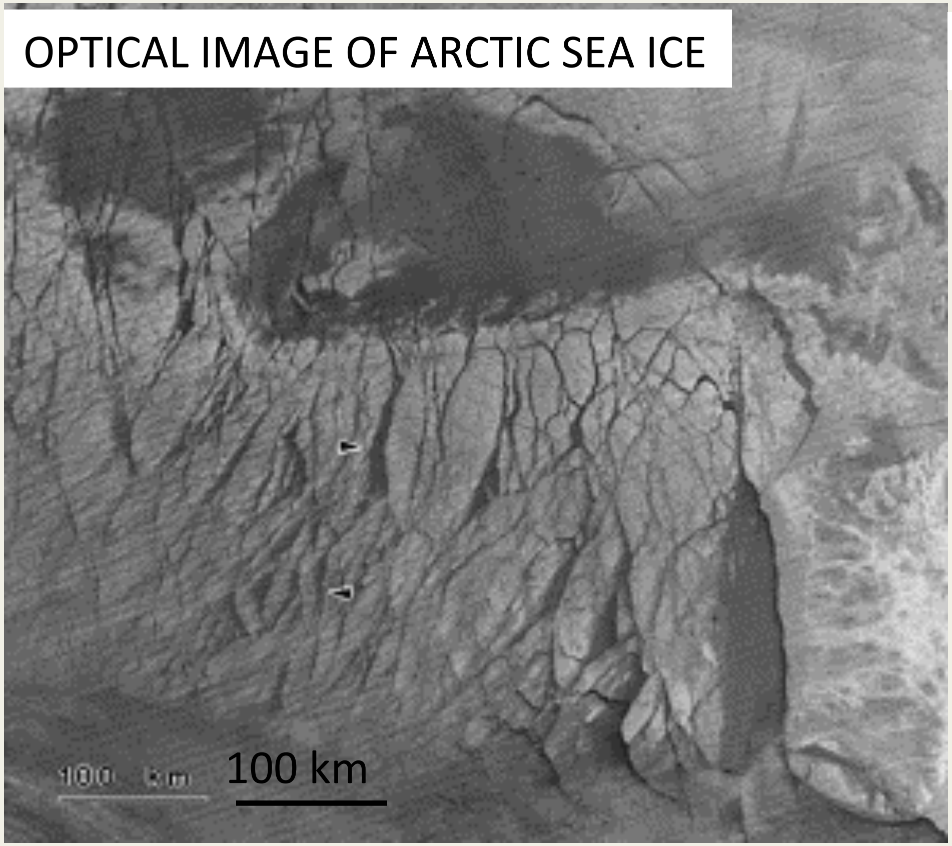


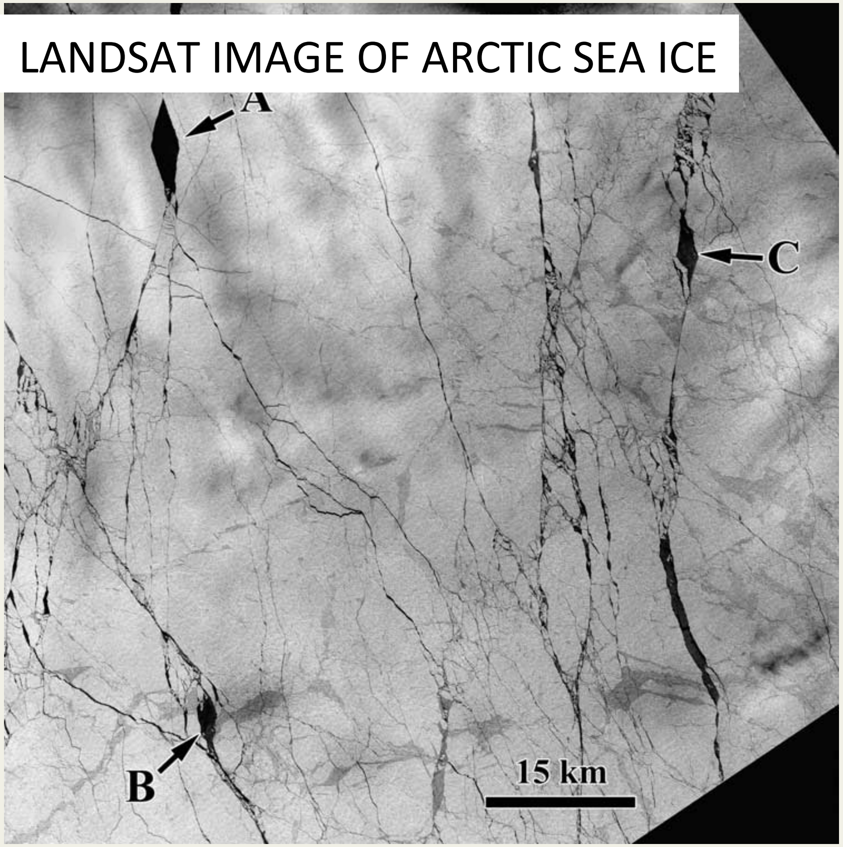


Investigation of sea ice failure with a discrete element sea ice model

Wilchinsky, Feltham, Hopkins [2010; 2011]

Observations: sea ice floe shape





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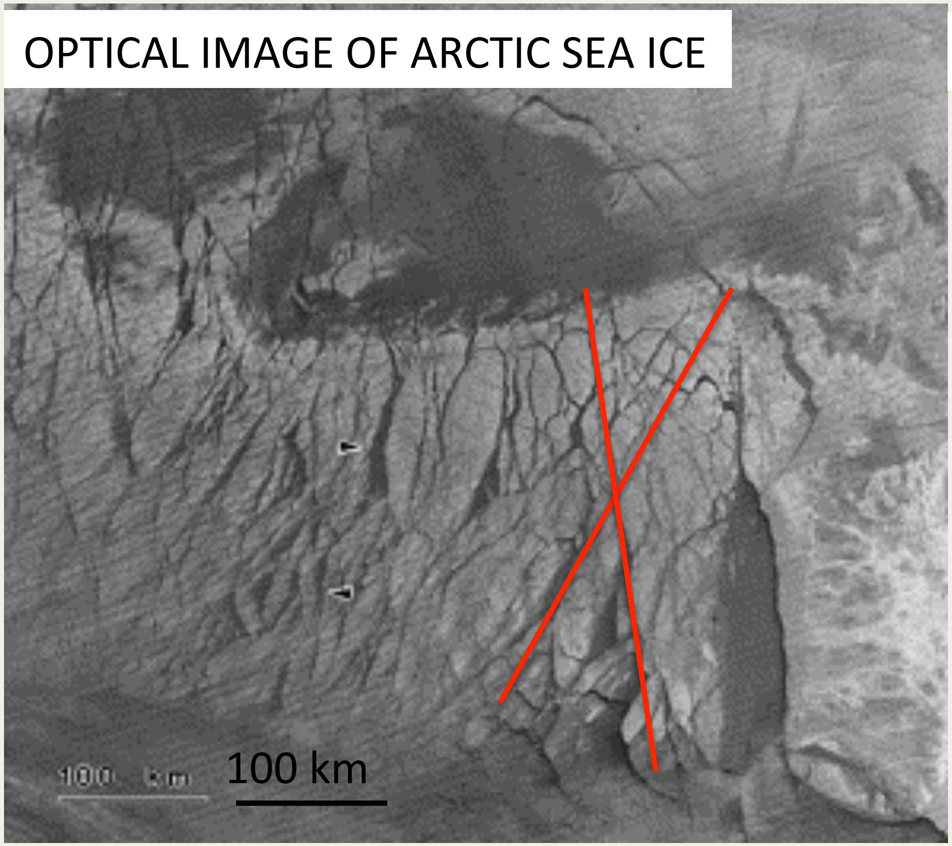

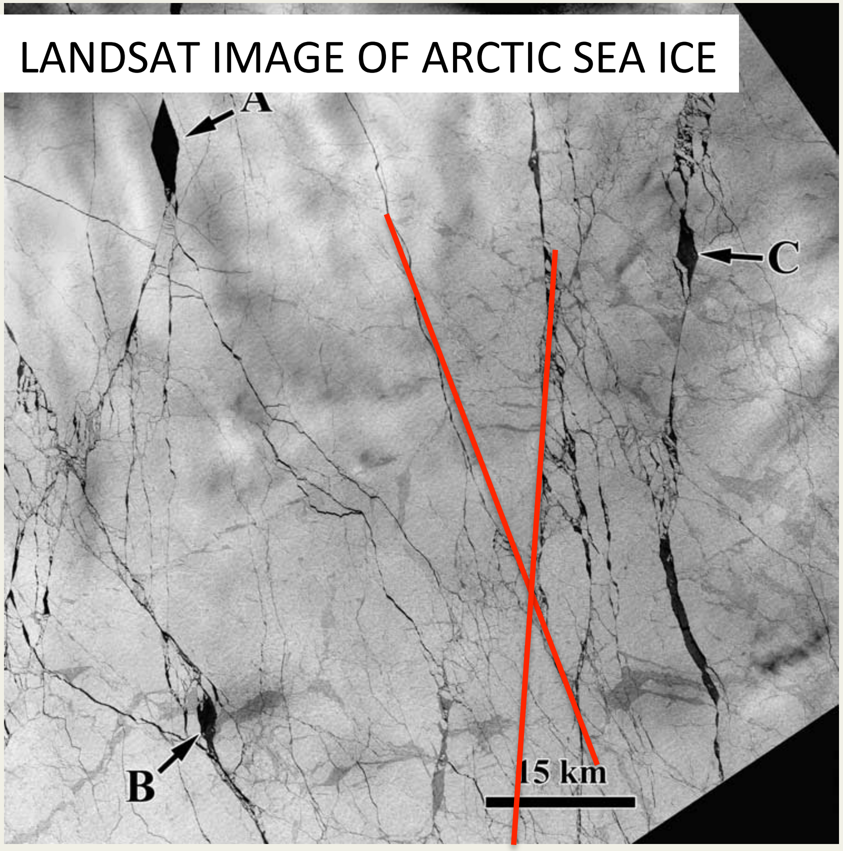

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"Classical" Coloumb failure theory

c.f. Wilchinsky and Feltham [2011; 2012]

Assume that traction components along a putative failure line satisfy the Coulomb criterion

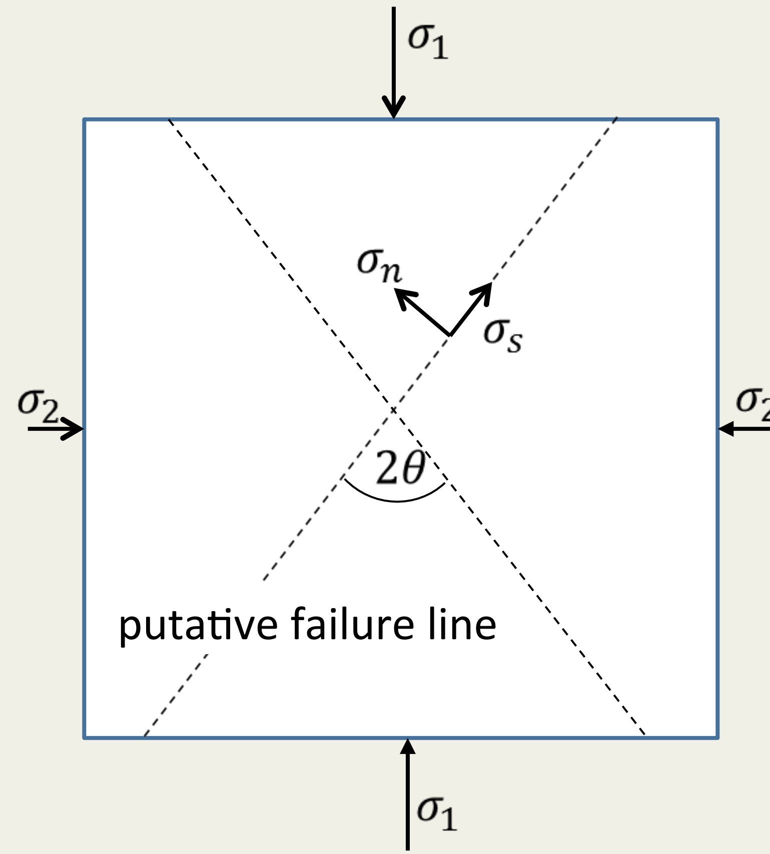
$$|\sigma_s| - \mu\sigma_n = \sigma_c$$

Tangential stress along failure line

Shear rupture coefficient

Normal stress across failure line

Cohesion



Coulomb theory says that the failure line angles are those which maximise the left-hand side.

The predicted angle of intersection of the conjugate failure lines is 2θ given by $\tan 2\theta = 1/\mu$

NOTE: We are talking here about the formation of new failure lines (cracks) and NOT failure along existing cracks.

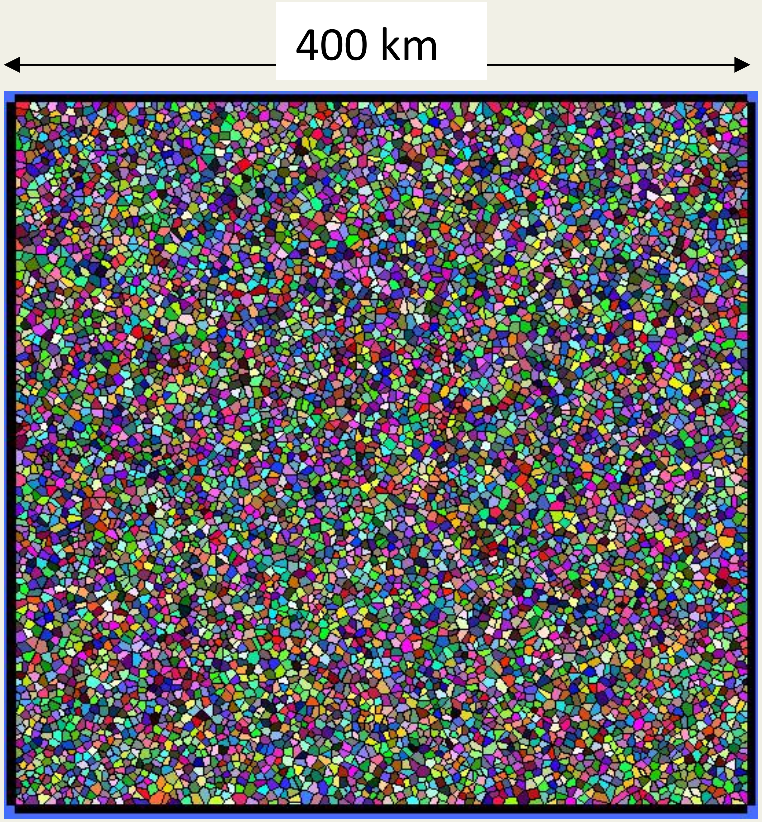
Modelling granularisation using a Discrete Element Model

Granularisation is the breakup of an intact ice sheet, comprising floes frozen together, into **floe aggregates** (sub regions of floes frozen together).

A **discrete element model** [Hopkins et al, 2004] considers individual floes (the "discrete elements") and calculates their motion from a local force balance.

The floes are rigid bodies joined together with visco-elastic "glue" representing thinner ice.

When the joints/"glue" undergo sufficient stretching or compression they break to form either leads or pressure ridges.

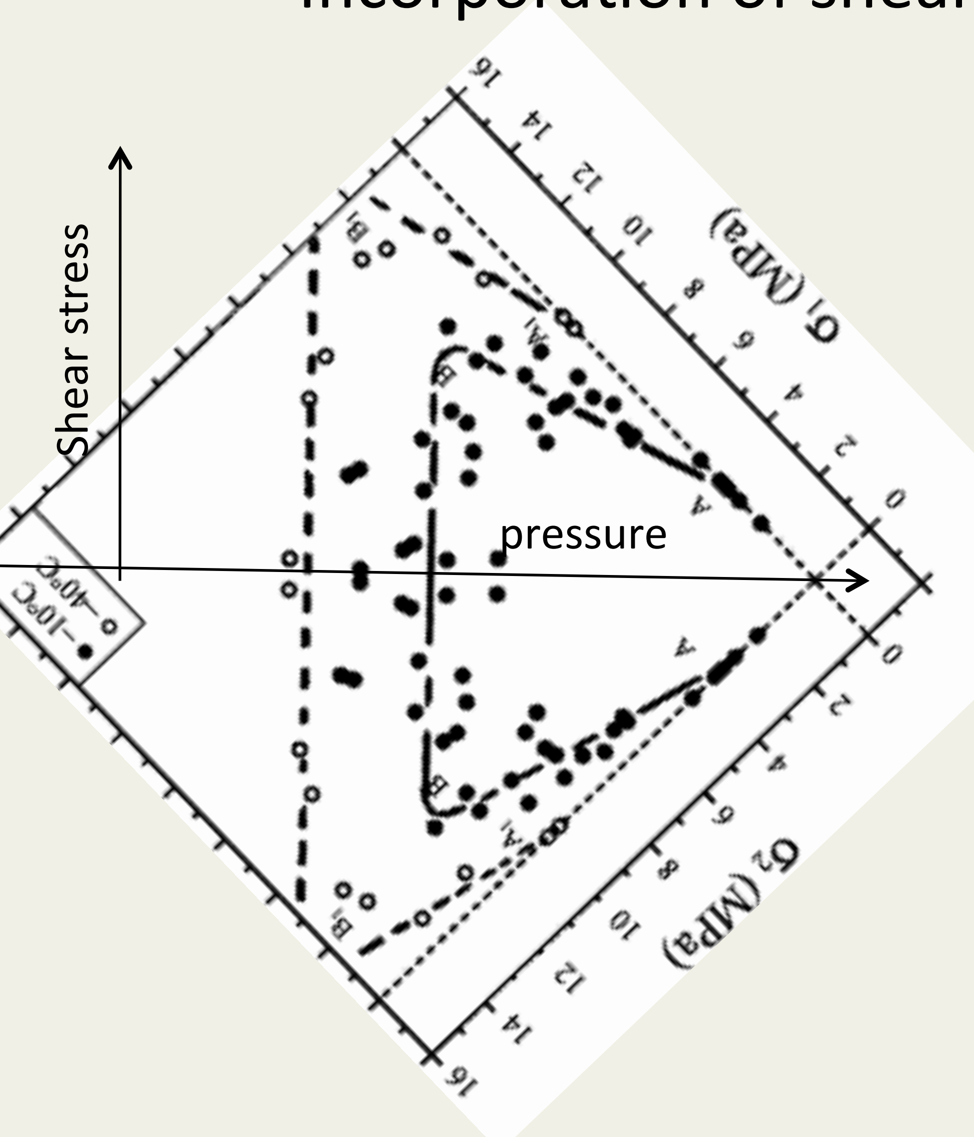


Additionally, we have allowed the "glue" (thin ice) to fail by the **shear failure mode**.

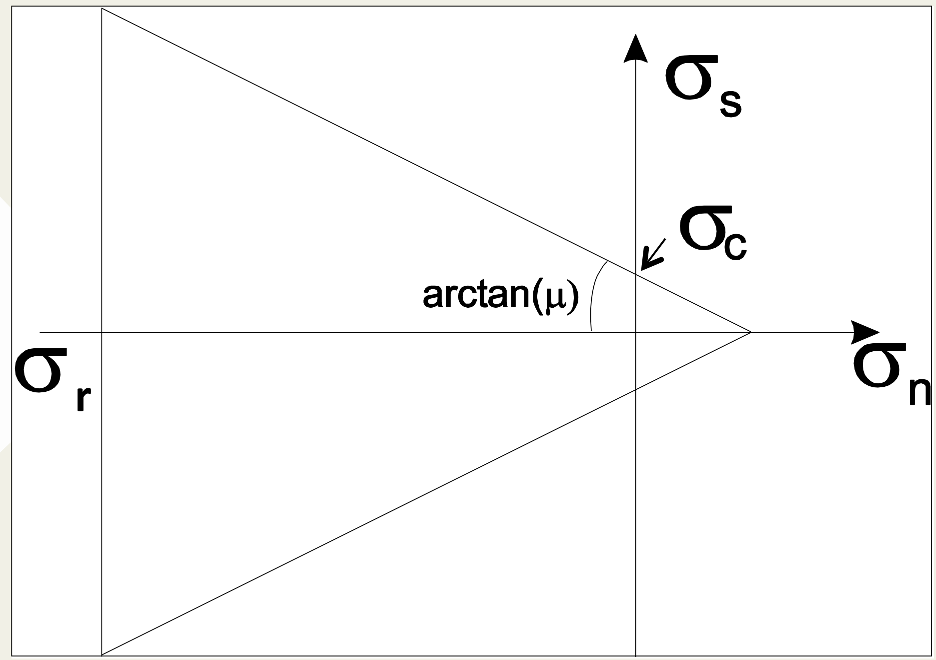
<u>Modelling granularisation with a discrete element model:</u>

1. At time t=0 the floes are frozen together. We have a **heterogeneous** continuum.
2. We apply a wind stress and observe how the ice cover breaks up into floe aggregates.

Incorporation of shear failure into the DEM



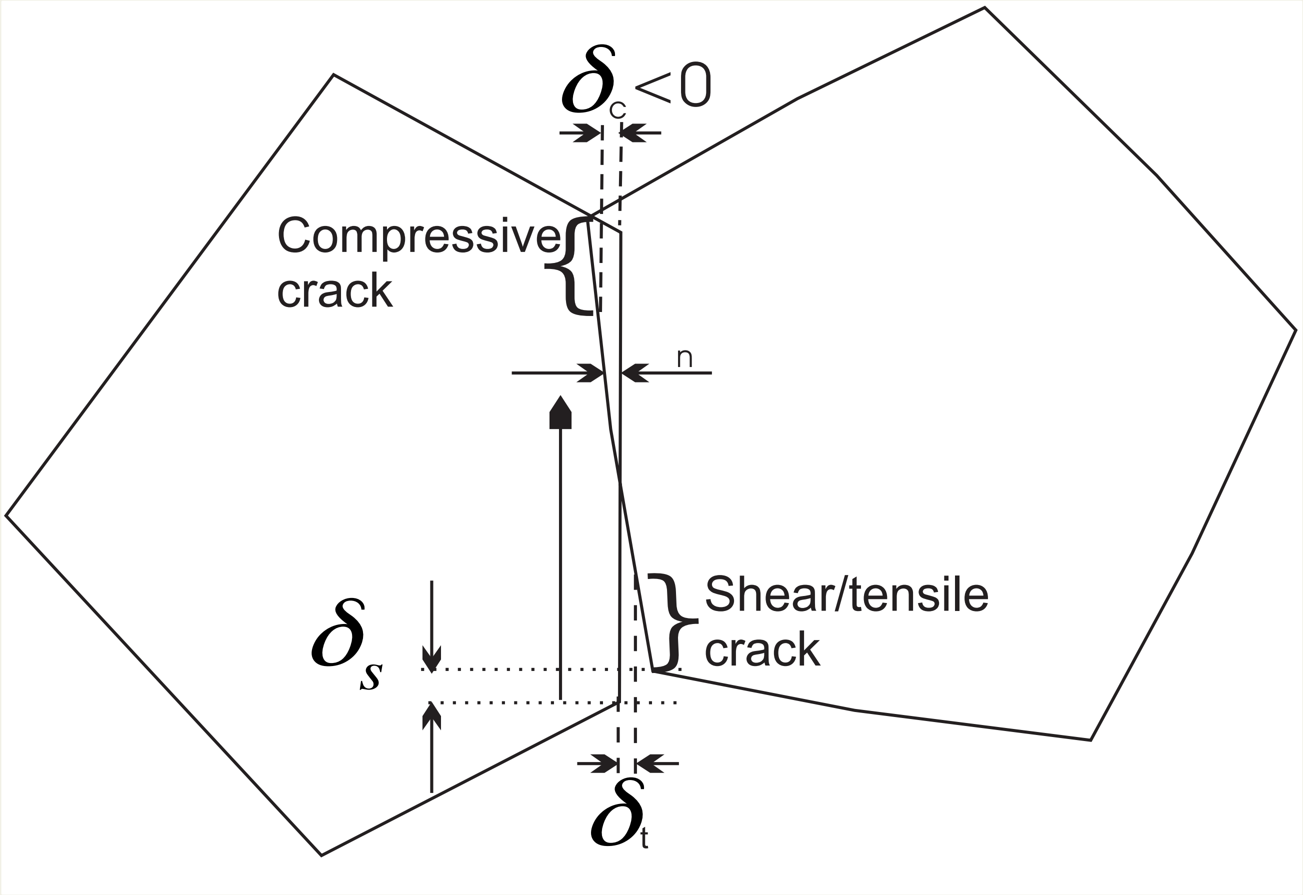
Brittle compressive failure envelopes for S2 ice [Schulson, 2001]. Mohr-Coulomb friction coefficient μ~0.6.



Simplified Coulombic yield curve of ice **in the joints** connecting the floes together.

Shear failure can occur at sub-critical compressive/tensile stresses.

Failure modes in joints connecting floes



<u>Failure stress</u> reached at a <u>critical displacement</u>. Failure regimes are:

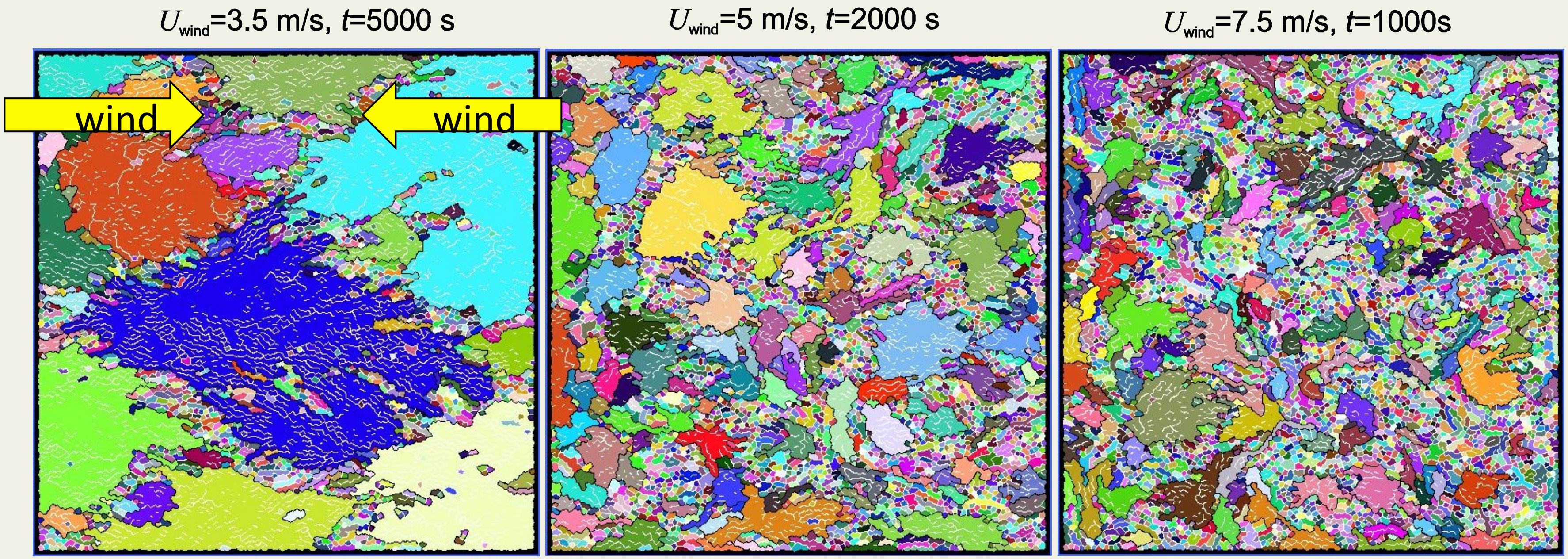
Contractive (compressive) ➔ ridge formation

Extensional (tensile) ➔ lead formation

Shear (with/without contraction/extension) ➔ shear rupture

Uniaxial wind stress applied to 400x400km^2 domain with 4km floes

[Wilchinsky, Feltham, Hopkins, 2010]

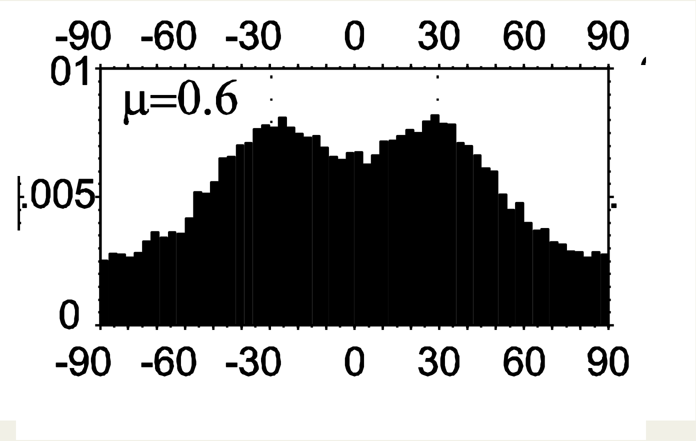


NEW: SHEAR FAILURE MODE

At moderate uniaxial wind stress, the ice cover fragments into noticeably diamond-shaped floe aggregates, separated by damage zones.

At higher wind speeds, the aggregates are smaller but the crack angle is still bi-modally distributed around the conjugate failure lines.

$$\tan 2\theta = 1/\mu$$



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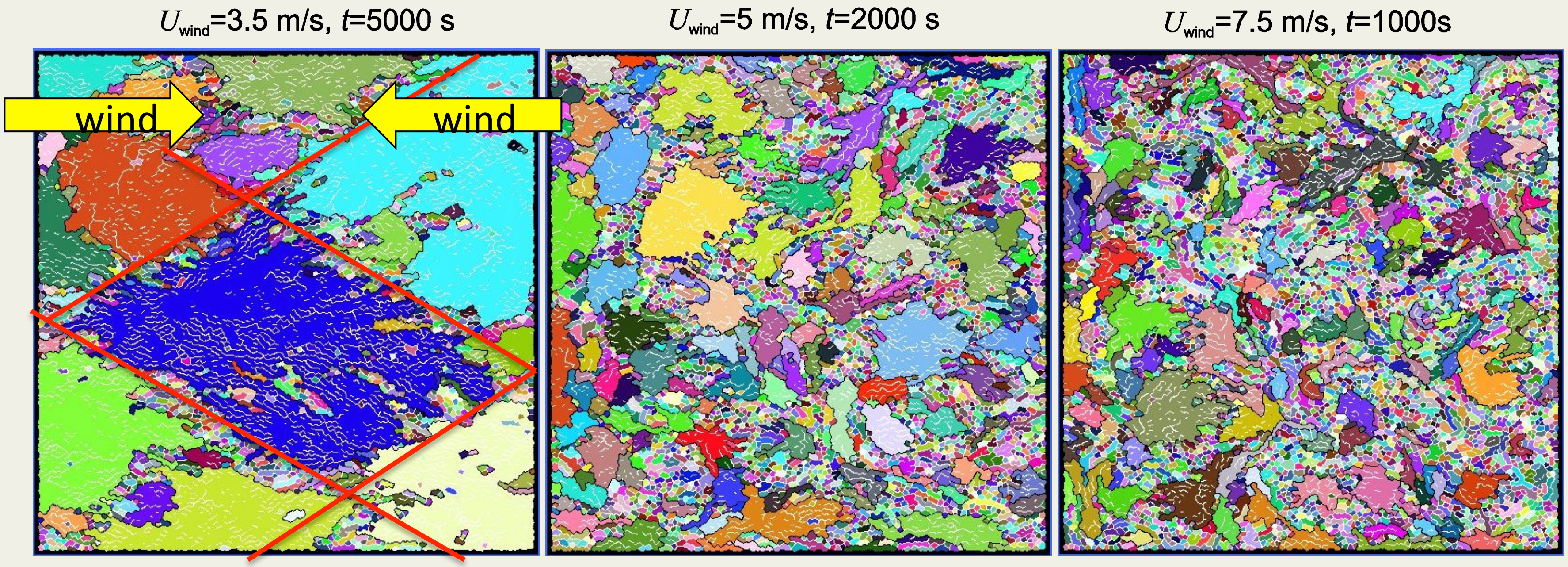

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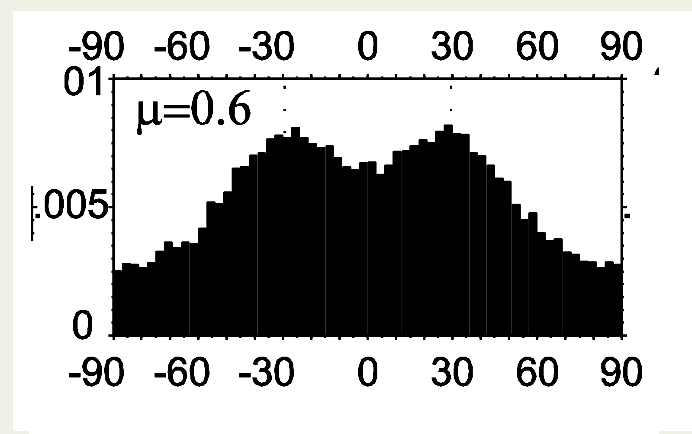


Determining the dominant failure mode

Theory:

- Work required to deform a unit length joint is

$$W = \tfrac{1}{2} k_n \delta_n^2 + \tfrac{1}{2} k_s \delta_s^2 \qquad (1)$$

Elastic work   Shear elastic work

- Mohr-Coulomb failure criterion, translated into displacements, is

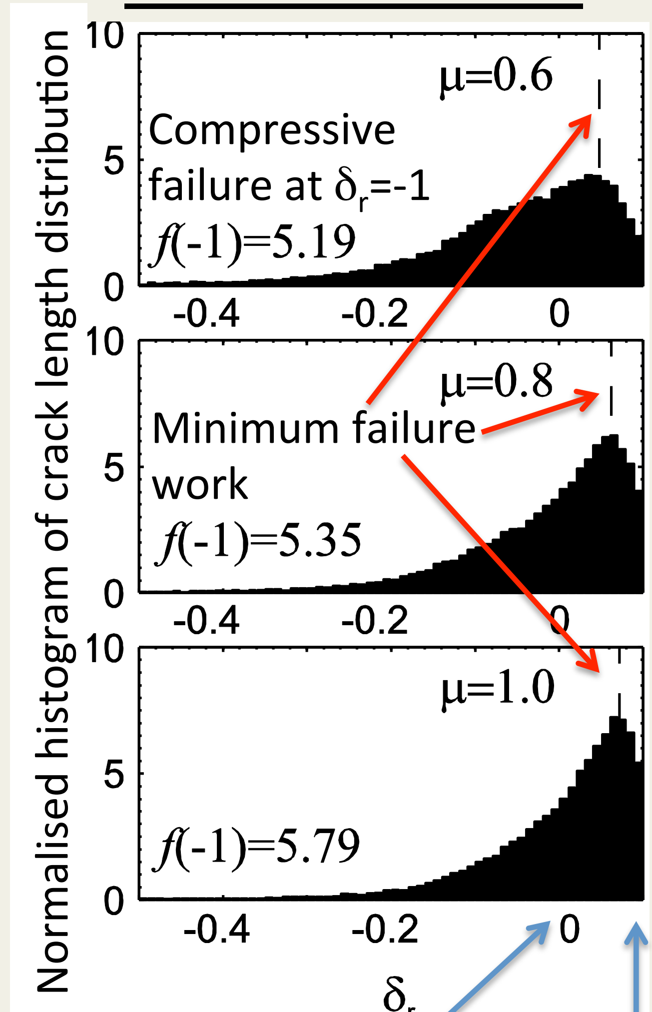
$$\delta_s = \mu\left(\sigma_t - k_n \delta_n\right)/k_s \qquad (2)$$

- Substituting (2) into (1) determines the failure work $W=W_f$. Minimising W_f yields the favoured failure mode:

$$\delta_n^{\min} = \frac{2(1+\nu)\mu^2}{1+2(1+\nu)\mu^2}\frac{\sigma_t}{k_n}; \qquad \delta_s^{\min} = \frac{1}{\mu}\delta_n^{\min}$$

Theory and simulations show that the optimal failure mode is SHEAR RUPTURE + DILATION

DEM Simulations:

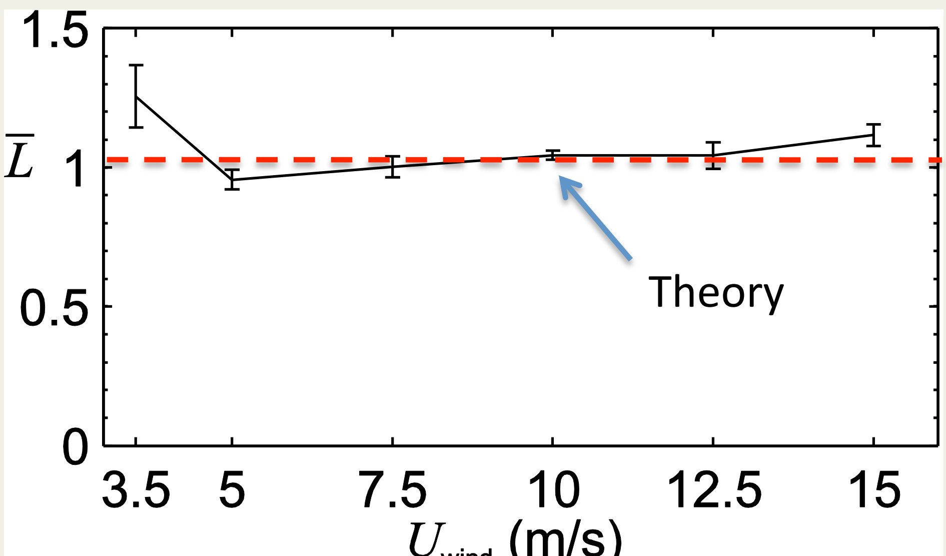


Pure shear failure   Tensile failure

Estimation of floe aggregate size

c.f. Hopkins, Frankenstein, Thorndike [2004]

- Wind drag does work to increase the elastic strain energy between floes.
- Crack formation transforms the strain energy into surface energy. The stress in the surrounding ice reduces via the propagation of an elastic wave.
- When the rate of crack formation is equal to the rate of elastic stress reduction, the cracks can delineate a floe aggregate/block without breaking it up further.

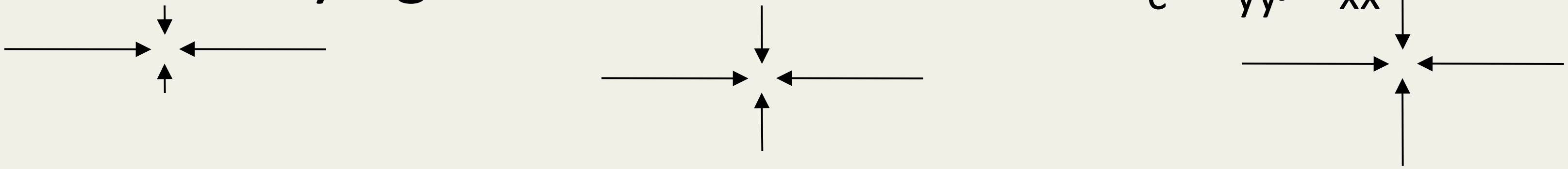


$$L_s^b = \left[\frac{(1+\nu)\mu}{1+2(1+\nu)\mu^2} \frac{\sigma_t L^2}{\nabla \tau} \right]^{1/4}$$

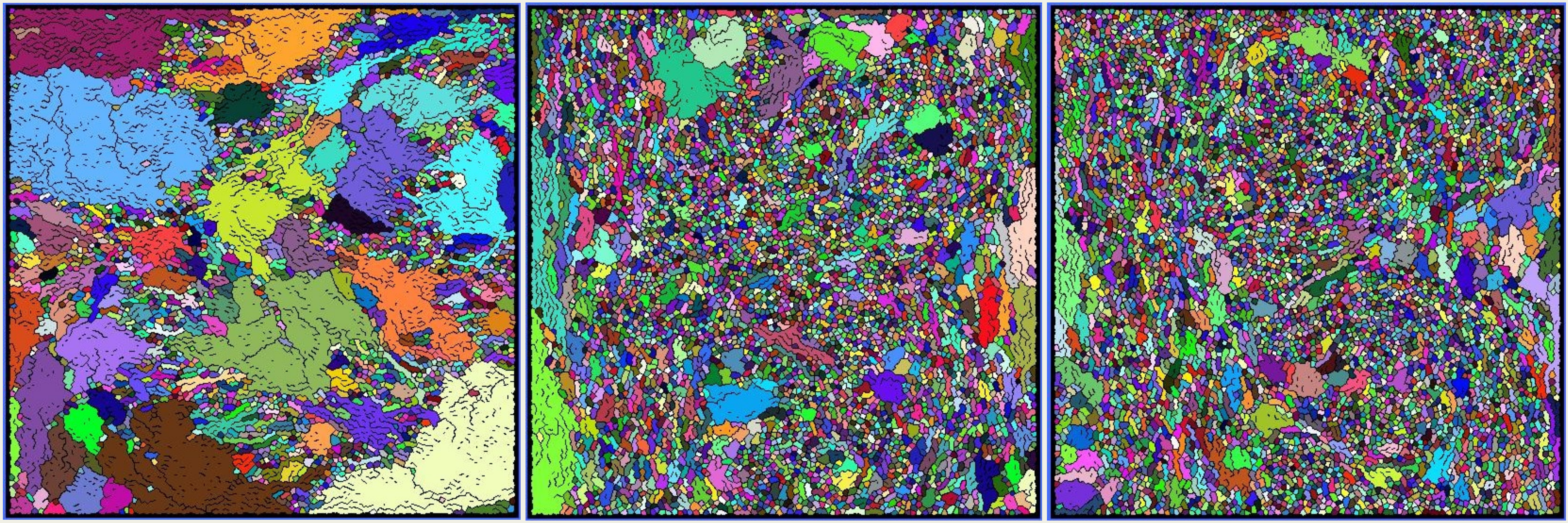
L is joint length

Wind drag

Varying the confinement ratio $R_c = \tau_{yy}/\tau_{xx}$



R_c=0.2, U_{wind}=3.8 m/s R_c=0.4, U_{wind}=5.2 m/s R_c=0.6, U_{wind}=5.2 m/s



Theory and experiments [e.g. Schulson, 2001] show that brittle compressive shear failure is prevented when the confinement ratio reaches the critical limit

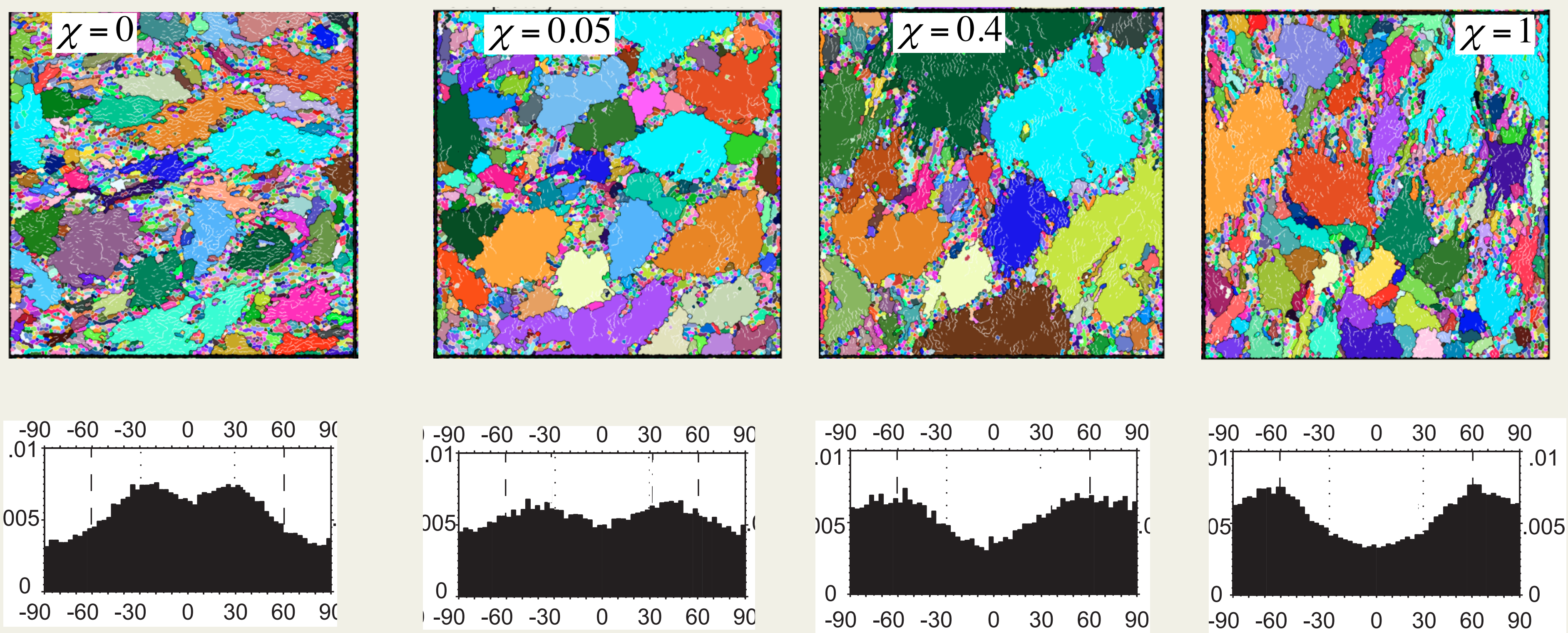
$$R_c{}^* = \frac{\sqrt{\mu^2+1}-\mu}{\sqrt{\mu^2+1}+\mu} = 0.32 \text{ for } \mu = 0.6$$

Above this confinement ratio, failure is no longer controlled by the large-scale wind field, but by local shear at the floe joints, leading to a random break up into individual floes.

Changing the wind direction after refreezing of floe joints

wind → ← wind + Partial refreezing of joints + ↓ wind ↑ wind →

Increasing amounts of joint refreezing



For little joint refreezing, the strain induced by the new wind stress direction is accommodated in existing cracks.

[Wilchinsky, Feltham, Hopkins, 2011]

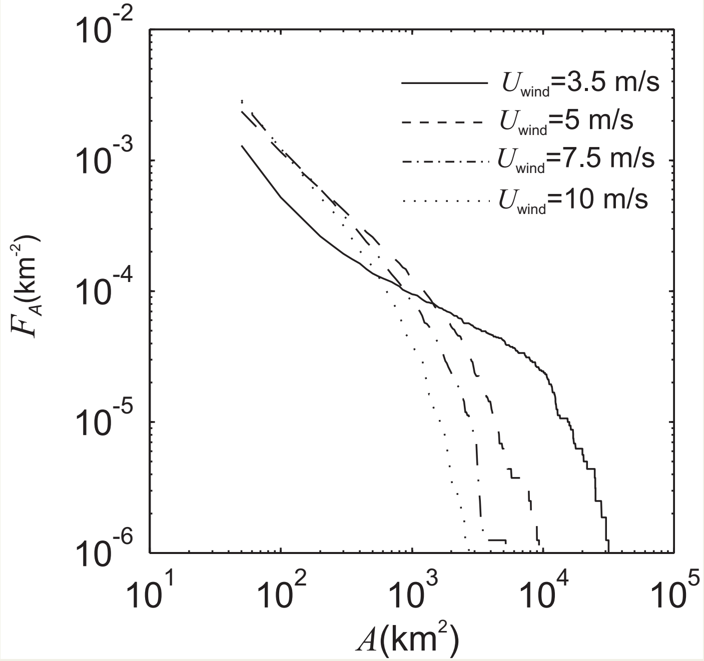
For increased joint refreezing, the new wind stress direction induces new cracks.

Summary remarks

- Failure modes imposed at the floe scale (4 km).
  Model calculates floe scale dynamics, and the large scale (400 km) failure pattern is emergent.
- At the large scale:
  1. Flaw orientation is determined by Coulomb failure;
  2. Flaw spacing is determined by elastic redistribution of stress.
- The most energetically favourable failure mode involves shear rupture and dilation (opening).
- Existing flaws, even if not favourably oriented with the large scale stress, affects whether new flaws will form [W&F, 2011,2012].
- Large scale failure related to the Linear Kinematic Features seen in satellite data (but LKFs show deformation rather than failure).
- (Not discussed.) Cumulative PDF of floe aggregate size follows a power law $F(A) \sim A^{-1}$.

Cumulative floe aggregate area distribution function

$$F_A = \frac{N}{A_d}\int_A^{\infty} P(A)\,dA$$ is the number of floe aggregates whose area is ≥ *A*

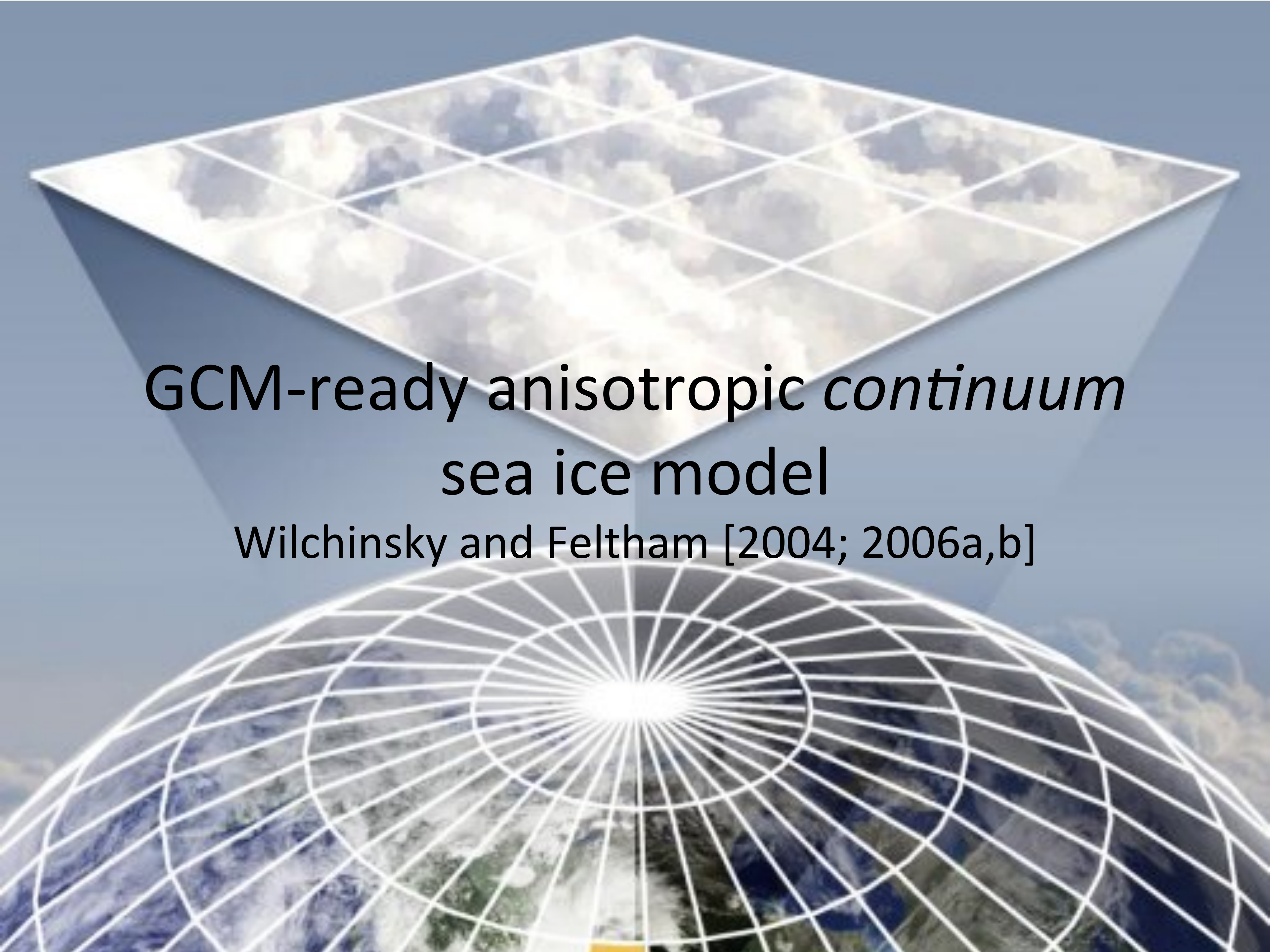


We find $F_A \sim A^{-1}$ for areas less than about 300 km^2.

Hopkins and Thorndike [2006] using the DEM with no shear failure found exponent -0.68.

Observations suggest -0.65 [*Rothrock and Thorndike*, 1984], -0.5 to -0.9 [*Weiss*, 2003] and -0.65 and -0.77 [*Weiss and Marsan*, 2004].

The difference between the -1 power law we find and the power law found by Hopkins and Thorndike [2006] can be explained in terms of the additional shear failure mode present in our model and the tendency for the ice cover to break so as to minimise failure energy [see paper for more details].



GCM-ready anisotropic *continuum* sea ice model

Wilchinsky and Feltham [2004; 2006a,b]

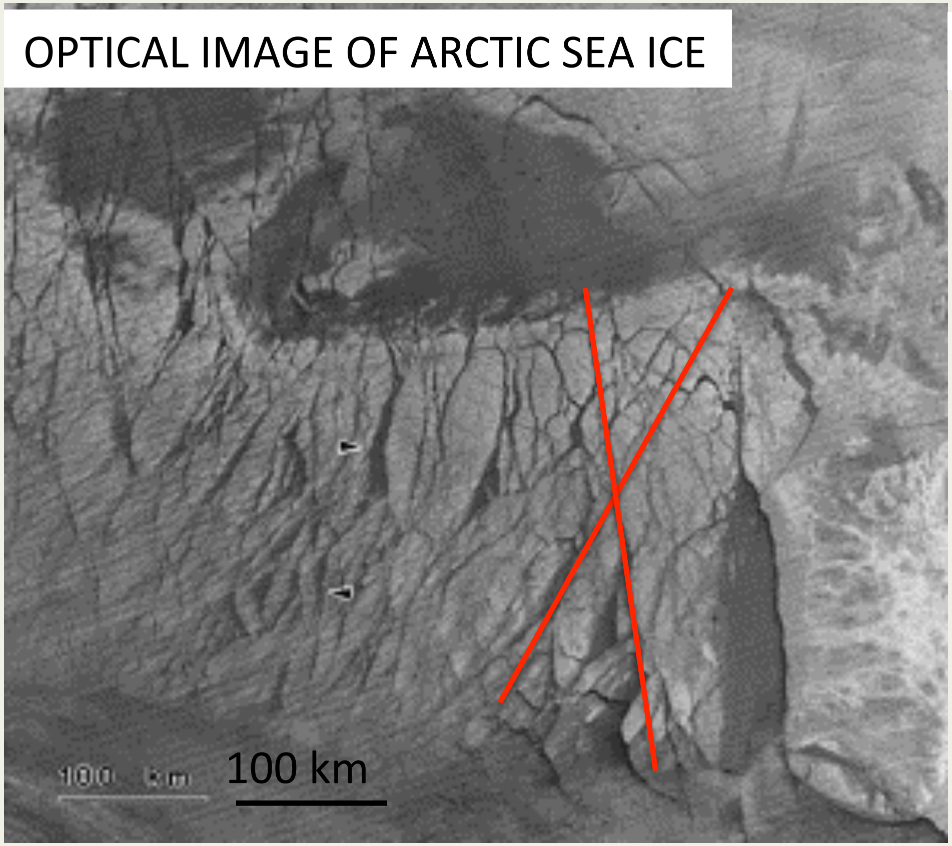
Continuum anisotropic sea ice model

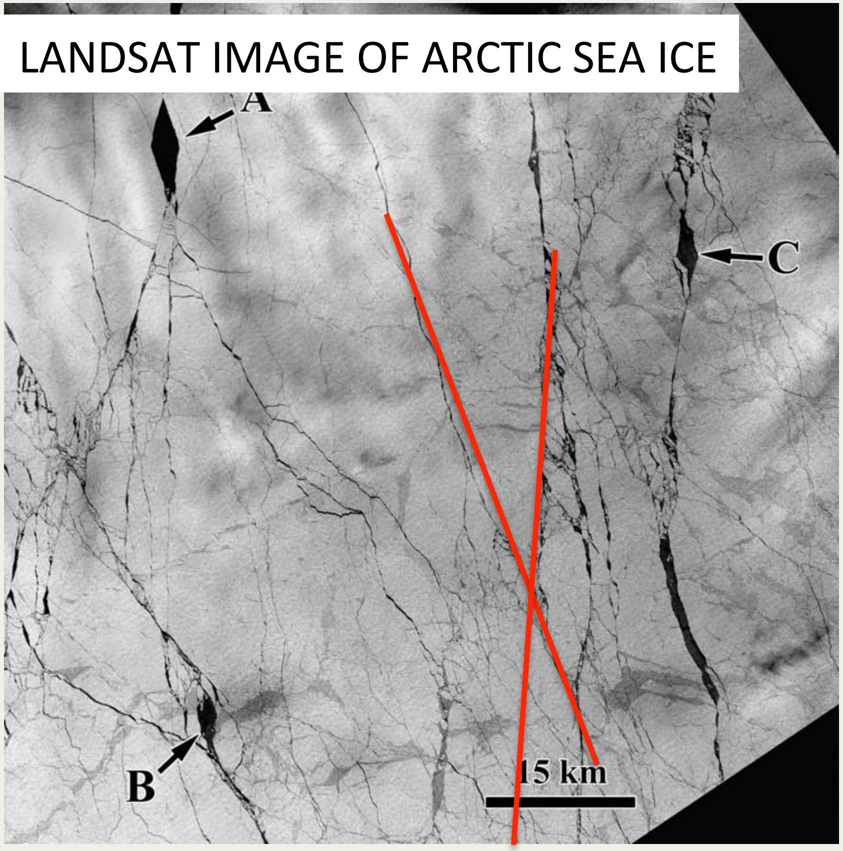
- For climate modelling work, **continuum** sea ice models are a **computational necessity**.
- However, (almost all) existing continuum sea ice rheology models assume isotropy, which **observations show is wrong**.
- In addition to the requirements of a standard isotropic sea ice model, an anisotropic sea ice model needs to:
    1. Describe anisotropy and its evolution;
    2. Relate the sea ice stress to anisotropy (rheology).

Various approaches have been formulated. We have taken the approach of **parameterising** the anisotropy, rather than seeking to represent it directly.

Here, we describe the **Elastic-Anisotropic (EA)** continuum model of Wilchinsky and Feltham [2006] that assumes the existence of diamond-shaped floe aggregates. Now in CICE 5.0.

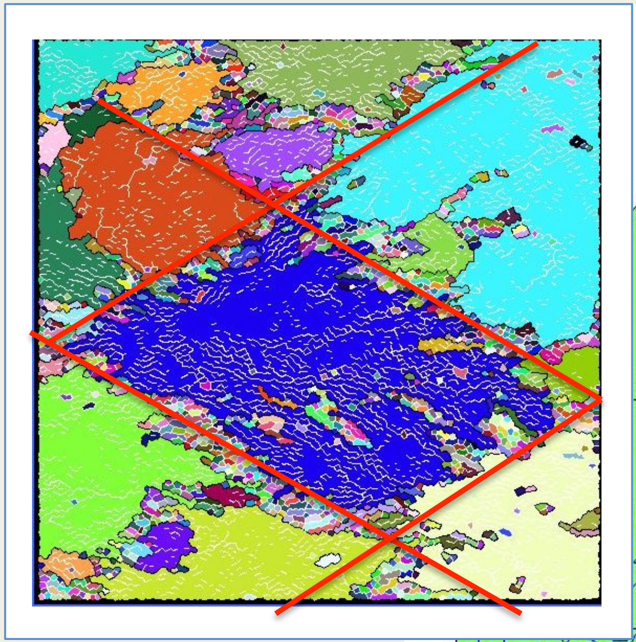
Observations: sea ice floe shape

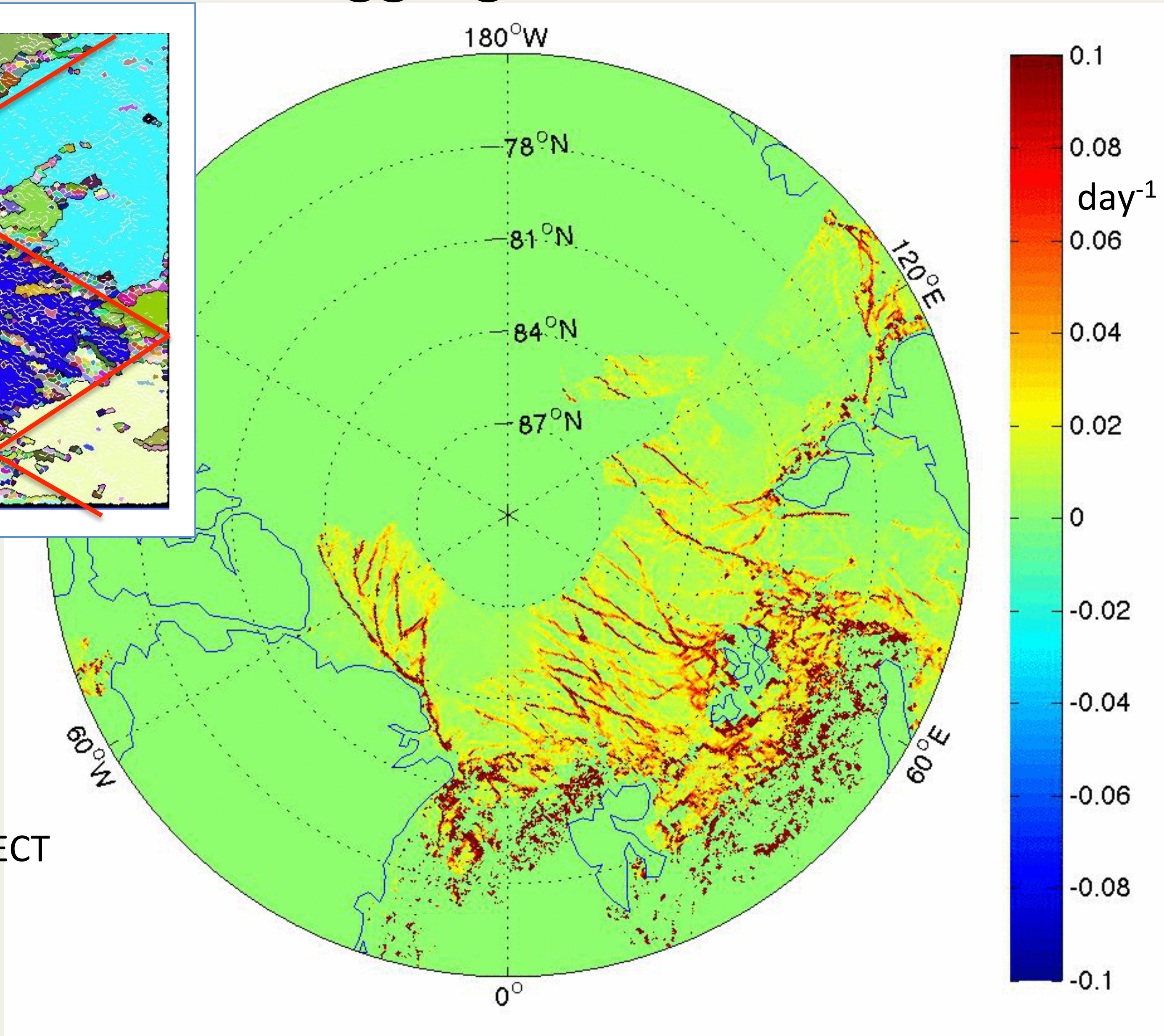




Leads delineate a mosaic of anisotropic ice floes, with a predominantly diamond shape. Inner angles of 30-40^{o} or 40-70^{o}. Possibly scale invariant between 10-150 km (e.g. Schulson, 2004).

Motivation from observations: diamond-shaped floe aggregates





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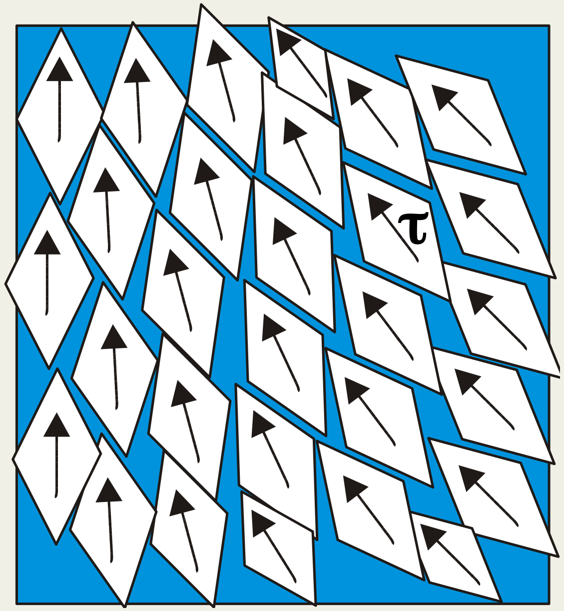
Simple (practical) representation of anisotropy

c.f. Wilchinsky and Feltham [2004; 2006a,b]

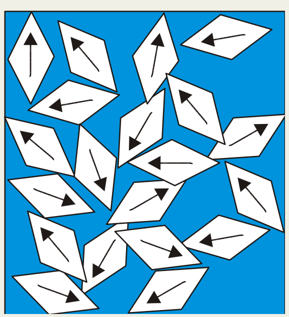


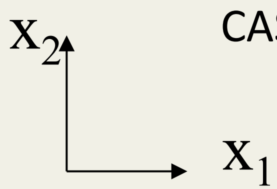
- Distribution of ice floes is given by a probability density function $\Psi(h, w, \boldsymbol{\tau})$

- Use **internal** variables to treat anisotropy. Introduce the **structure tensor**:

$$\mathbf{A} \equiv \langle \boldsymbol{\tau} \otimes \boldsymbol{\tau} \rangle = \iiint \Psi \tau_i \tau_j \, dh\, dw\, d\boldsymbol{\tau}$$

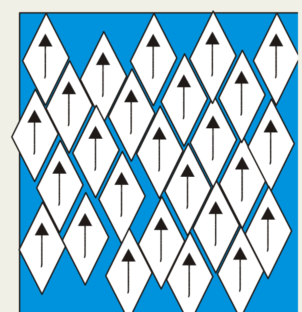
ISOTROPIC CASE:

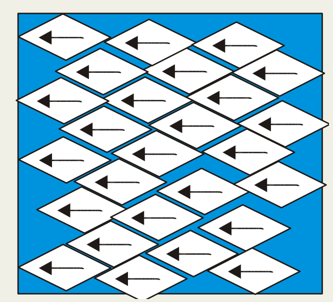




$$\mathbf{A} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

TOTALLY ANISOTROPIC CASES:





$$\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Main processes of floe orientation change 1/2

Evolution equation for the anisotropy **A**:

$$\frac{D\mathbf{A}}{Dt} = \mathbf{F}_{therm} + \mathbf{F}_{frac}$$

Co-rotational derivative
(includes rigid body rotation)

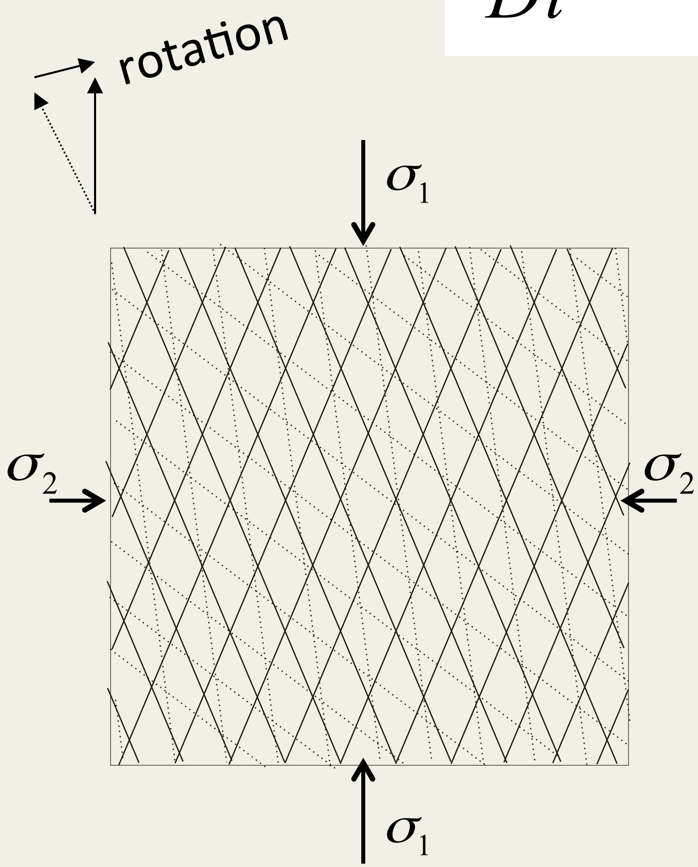
As floes freeze together, or melt apart, the ice cover becomes more isotropic:

$$\mathbf{F}_{therm} = -k_{therm}\left(\mathbf{A} - \frac{1}{2}\mathbf{1}\right)$$

Main processes of floe orientation change 2/2

$$\frac{DA}{Dt} = \mathbf{F}_{therm} + \mathbf{F}_{frac}$$

Schulson [2001]



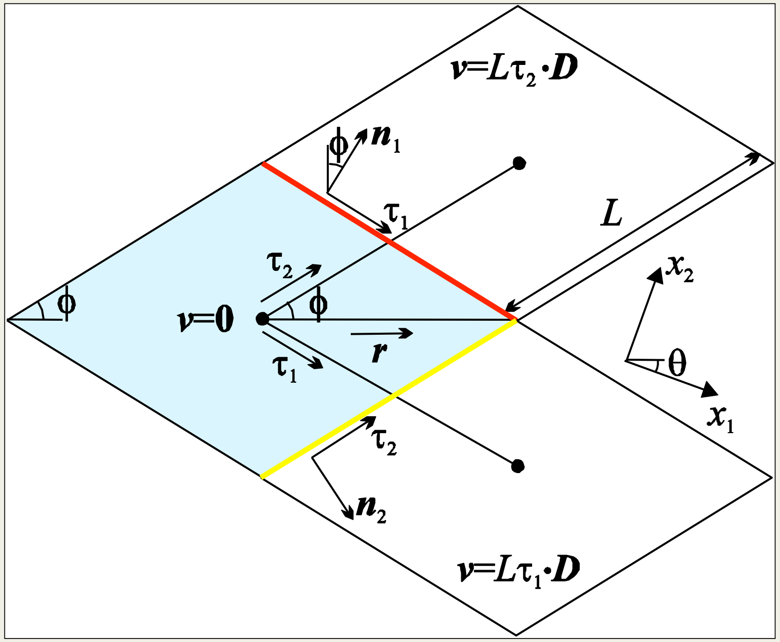
Under biaxial tension ($\sigma_1>0$, $\sigma_2>0$) and strong biaxial compression ($\sigma_1<0$, $\sigma_2<0$), failure is isotropic, $\mathbf{F}_{frac} = \mathbf{0}$

- TYPICAL: Biaxial compression ($\sigma_1 \leq \sigma_2 < 0$) with low confinement ratio ($\sigma_1/\sigma_2 < R$) leads to conjugate fault formation → diamond-shaped floes
- Compressive in one direction, tensile in the other (e.g. $\sigma_2<0$, $\sigma_1 \geq 0$) leads to axial splitting – cracks aligned with compressive stress direction.
- In both cases

$$\mathbf{F}_{frac} = -k_{mech}\left(\mathbf{A} - \mathbf{S}\right)$$

where **S** reflects the new preferred orientation of cracks.

Determination of anisotropic sea ice rheology σ(*D*, *h*; *A*) 1/2



- Determination of mean floe stress from edge tractions due to ridging or sliding.

Edge tractions on floe:

$$\mathbf{F}_1 = -\mathrm{He}\left(-\mathbf{D}:\mathbf{n}_1\boldsymbol{\tau}_2\right)F_r\mathbf{n}_1 + \mathrm{sgn}\left(-\mathbf{D}:\boldsymbol{\tau}_2\boldsymbol{\tau}_1\right)\mathrm{He}\left(-\mathbf{D}:\boldsymbol{\tau}_2\mathbf{n}_1\right)F_s\boldsymbol{\tau}_1$$

$$\mathbf{F}_2 = -\mathrm{He}\left(-\mathbf{D}:\mathbf{n}_2\boldsymbol{\tau}_1\right)F_r\mathbf{n}_2 + \mathrm{sgn}\left(-\mathbf{D}:\boldsymbol{\tau}_2\boldsymbol{\tau}_1\right)\mathrm{He}\left(-\mathbf{D}:\boldsymbol{\tau}_1\mathbf{n}_2\right)F_s\boldsymbol{\tau}_2$$

F_r is the normal ridging force; F_s is the tangential sliding force. Forces only active when edges are compressed (the Heaviside functions become unity).

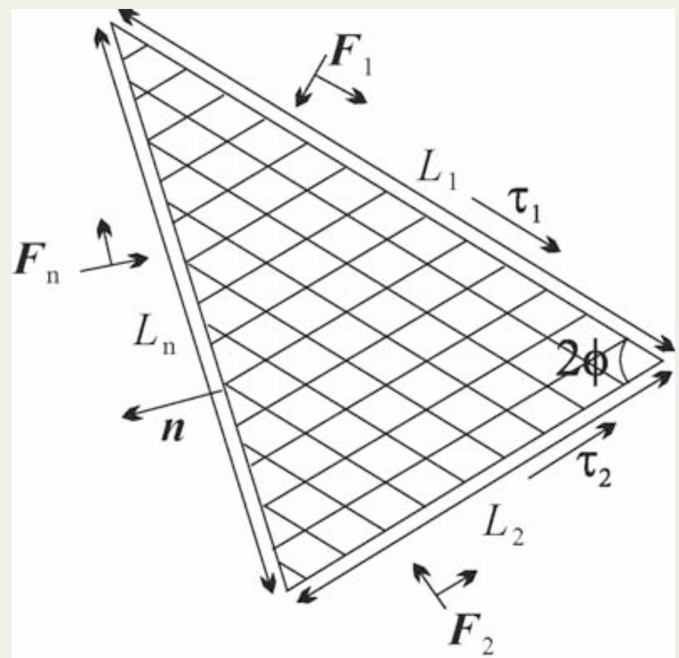
Mean stress theorem yields

$$L_1\mathbf{F}_1 + L_2\mathbf{F}_2 + L_n\mathbf{F}_\mathbf{n} = \mathbf{0} + \underset{\approx\,\mathbf{0}}{\mathbf{O}\left(L_1 L_2\right)}$$

Edge tractions dominate body/inertial forces

This yields the normal traction (after algebra)

$$\mathbf{F}_\mathbf{n} = \frac{1}{\sin\phi}\left[\mathbf{F}_1\boldsymbol{\tau}_2 + \mathbf{F}_2\boldsymbol{\tau}_1\right]\cdot\mathbf{n} \equiv \boldsymbol{\sigma}_\mathbf{a}\cdot\mathbf{n}$$



Determination of anisotropic sea ice rheology $\boldsymbol{\sigma}(D, h; A)$ 2/2

$$\mathbf{F_n} = \frac{1}{\sin\phi}\left[\mathbf{F}_1\boldsymbol{\tau}_2 + \mathbf{F}_2\boldsymbol{\tau}_1\right]\cdot\mathbf{n} \equiv \boldsymbol{\sigma}_\mathbf{a}\cdot\mathbf{n}$$

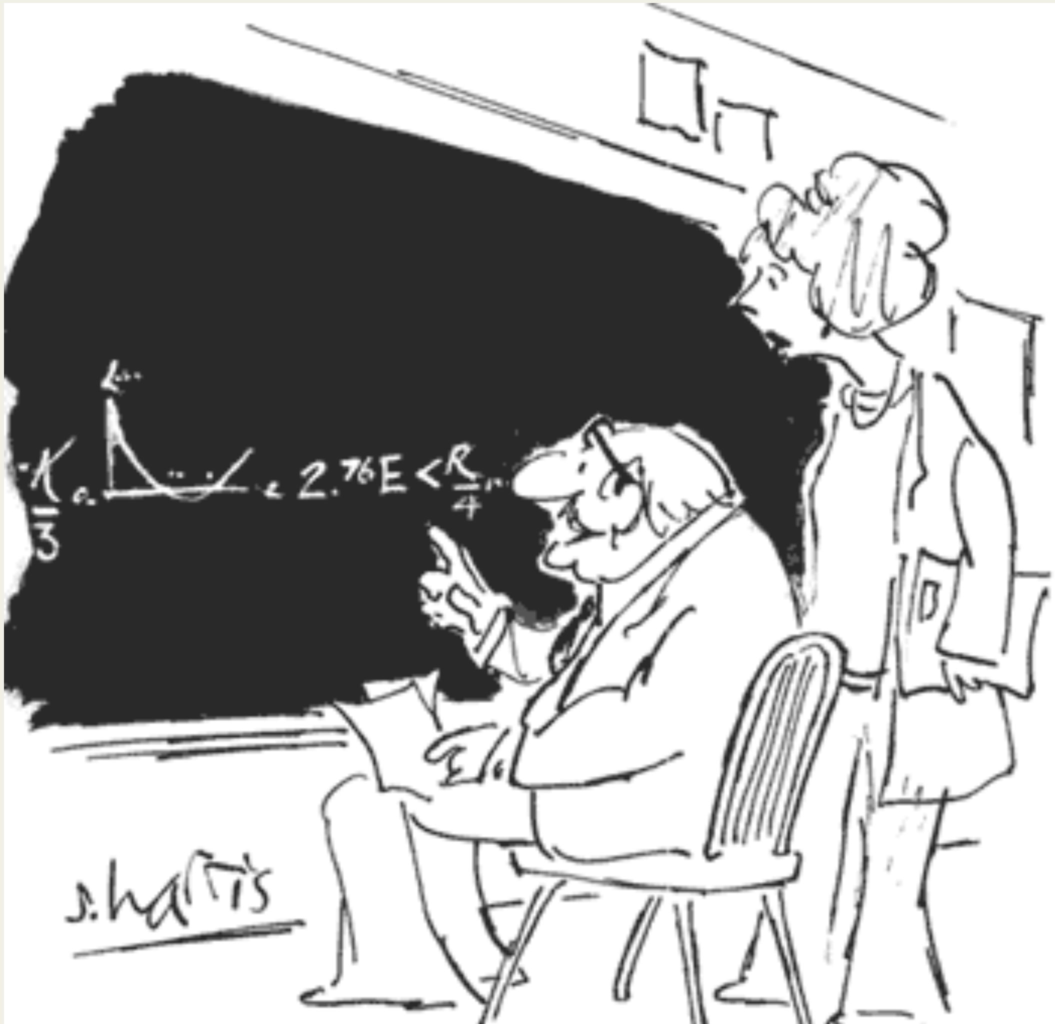
- The derived stress $\boldsymbol{\sigma}_\mathbf{a}$ is generally asymmetric. Unopposed, this would cause the floes to rotate, which is not observed.

  We conclude that floe interaction prevents spin and parameterise this by taking only the symmetric part of the stress.

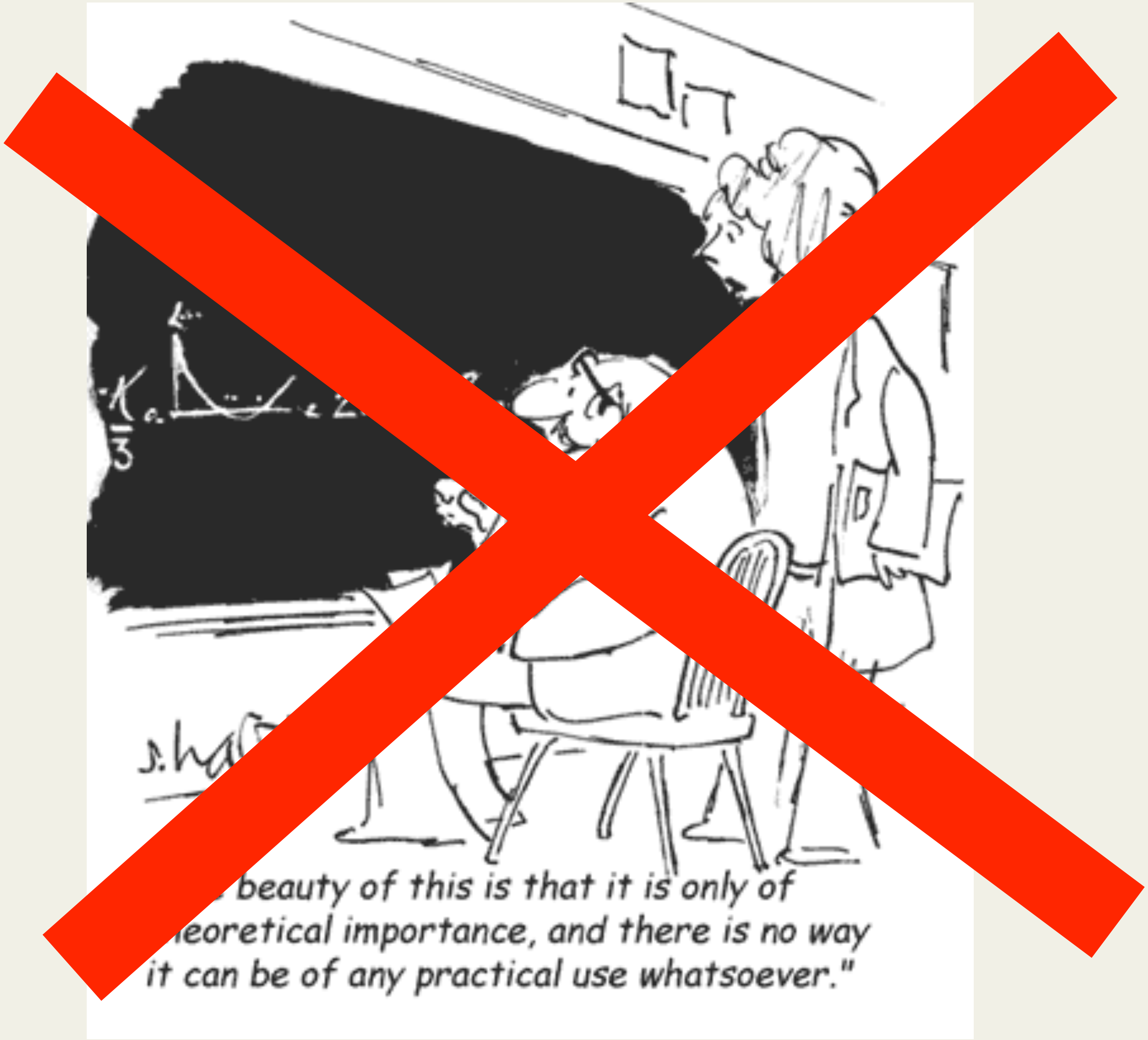
- Average over all orientations of the floe, weighted by $\psi(\theta)$, denoted by $<\cdot>$.

$$\begin{aligned}
\boldsymbol{\sigma} &= F_r\boldsymbol{\sigma}_r + F_s\boldsymbol{\sigma}_s \\
\boldsymbol{\sigma}_r &= -\frac{1}{\sin 2\phi}\left\langle\left[\mathrm{He}(-\mathbf{D}:\mathbf{n}_1\boldsymbol{\tau}_2)\mathbf{n}_1\boldsymbol{\tau}_2 + \mathrm{He}(-\mathbf{D}:\mathbf{n}_2\boldsymbol{\tau}_1)\mathbf{n}_2\boldsymbol{\tau}_1\right]^{sym}\right\rangle \\
\boldsymbol{\sigma}_s &= \frac{1}{\sin 2\phi}\left\langle \mathrm{sgn}(\mathbf{D}:\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\left[\mathrm{He}(-\mathbf{D}:\mathbf{n}_1\boldsymbol{\tau}_2)\boldsymbol{\tau}_1\boldsymbol{\tau}_2 + \mathrm{He}(-\mathbf{D}:\mathbf{n}_2\boldsymbol{\tau}_1)\boldsymbol{\tau}_2\boldsymbol{\tau}_1\right]^{sym}\right\rangle
\end{aligned}$$

- The total sea ice stress $\boldsymbol{\sigma}$ may be expressed in terms of the structure tensor **A**, strain rate and ice thickness using fourth order tensor closures. THIS IS THE ANISOTROPIC RHEOLOGY WE HAVE BEEN LOOKING FOR.



"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."


beauty of this is that it is only of
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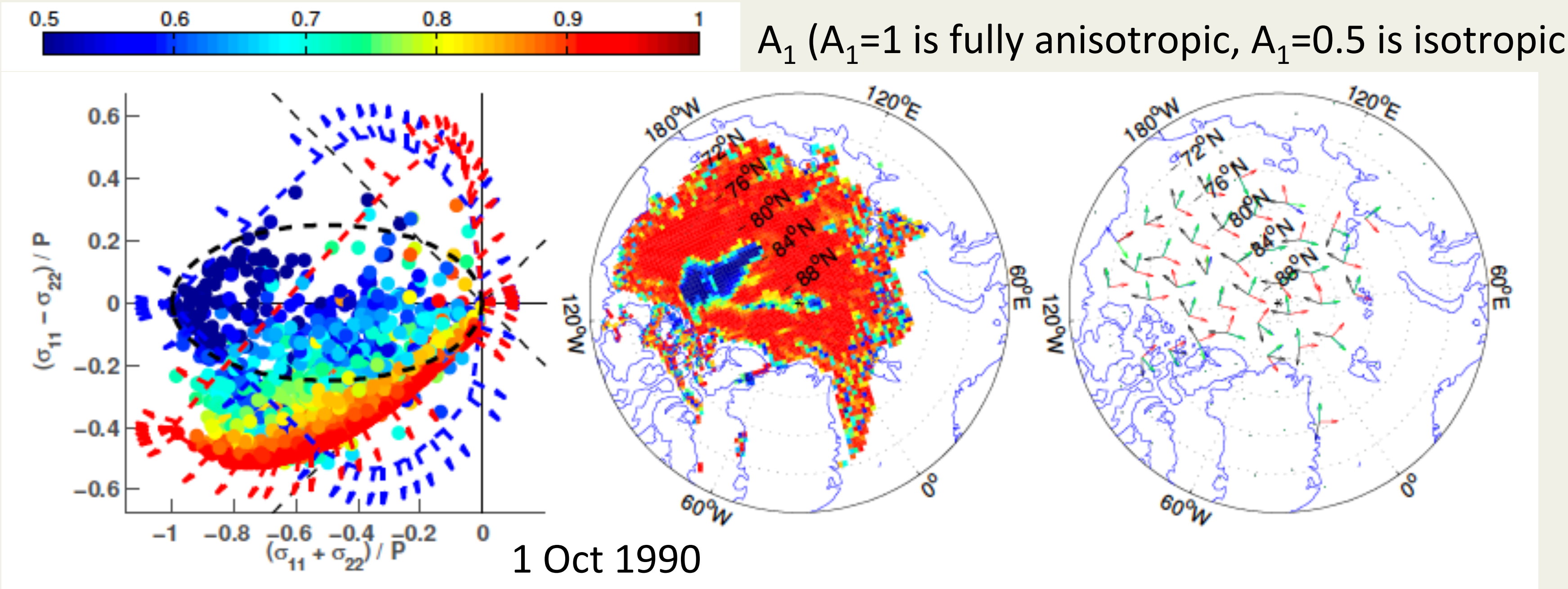
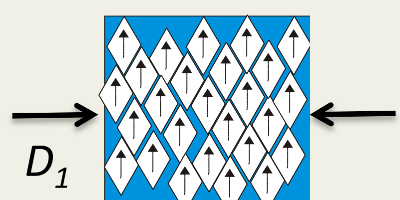
it can be of any practical use whatsoever."

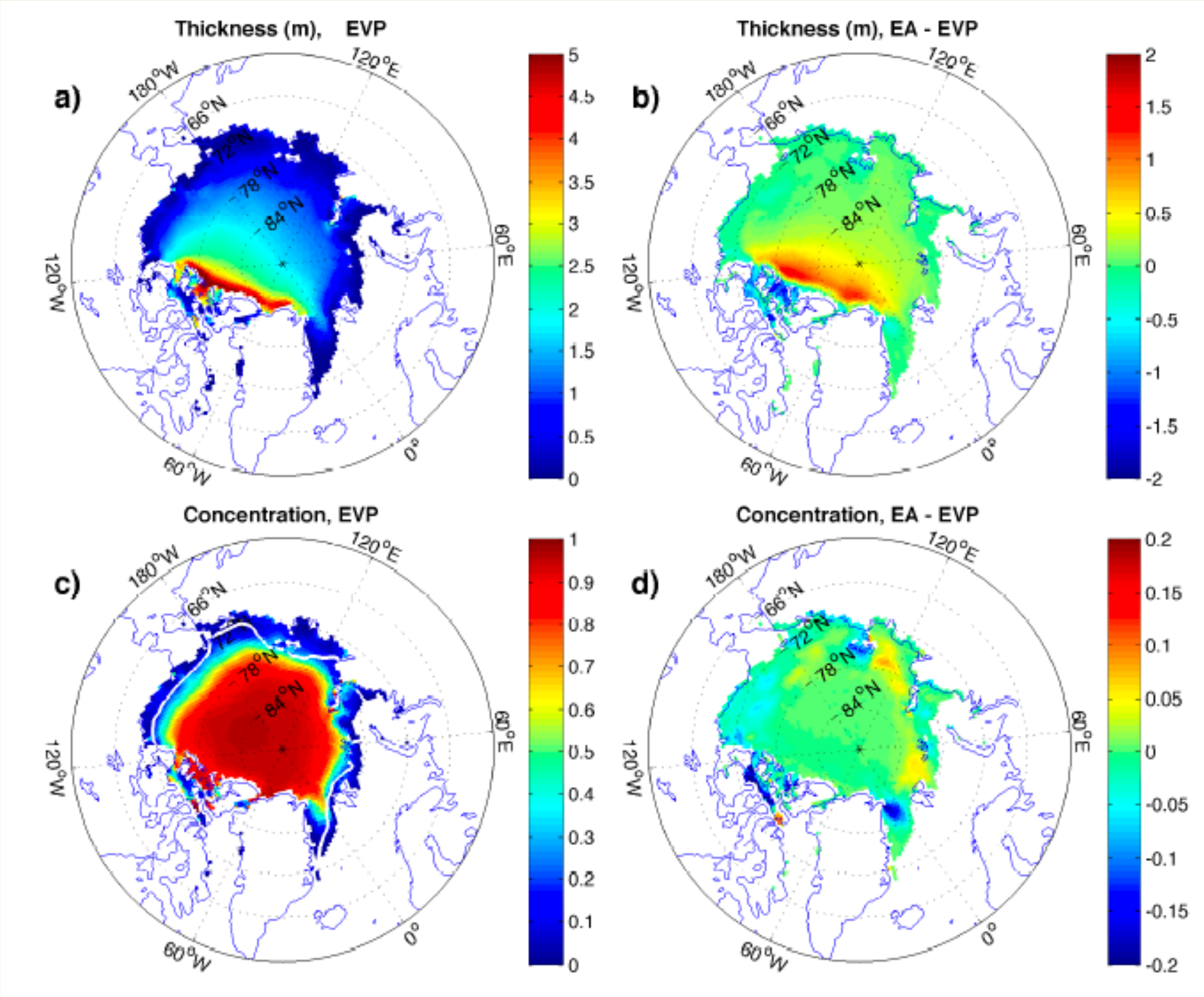
First anisotropic climate sea ice model



- Anisotropy produces **large shear stresses** (“fat” yield curve)
- Under realistic forcing, ice cover is **mainly anisotropic** and this evolves on the **wind pattern timescale**
- Major principal axes of structure tensor and deformation rate are orthogonal



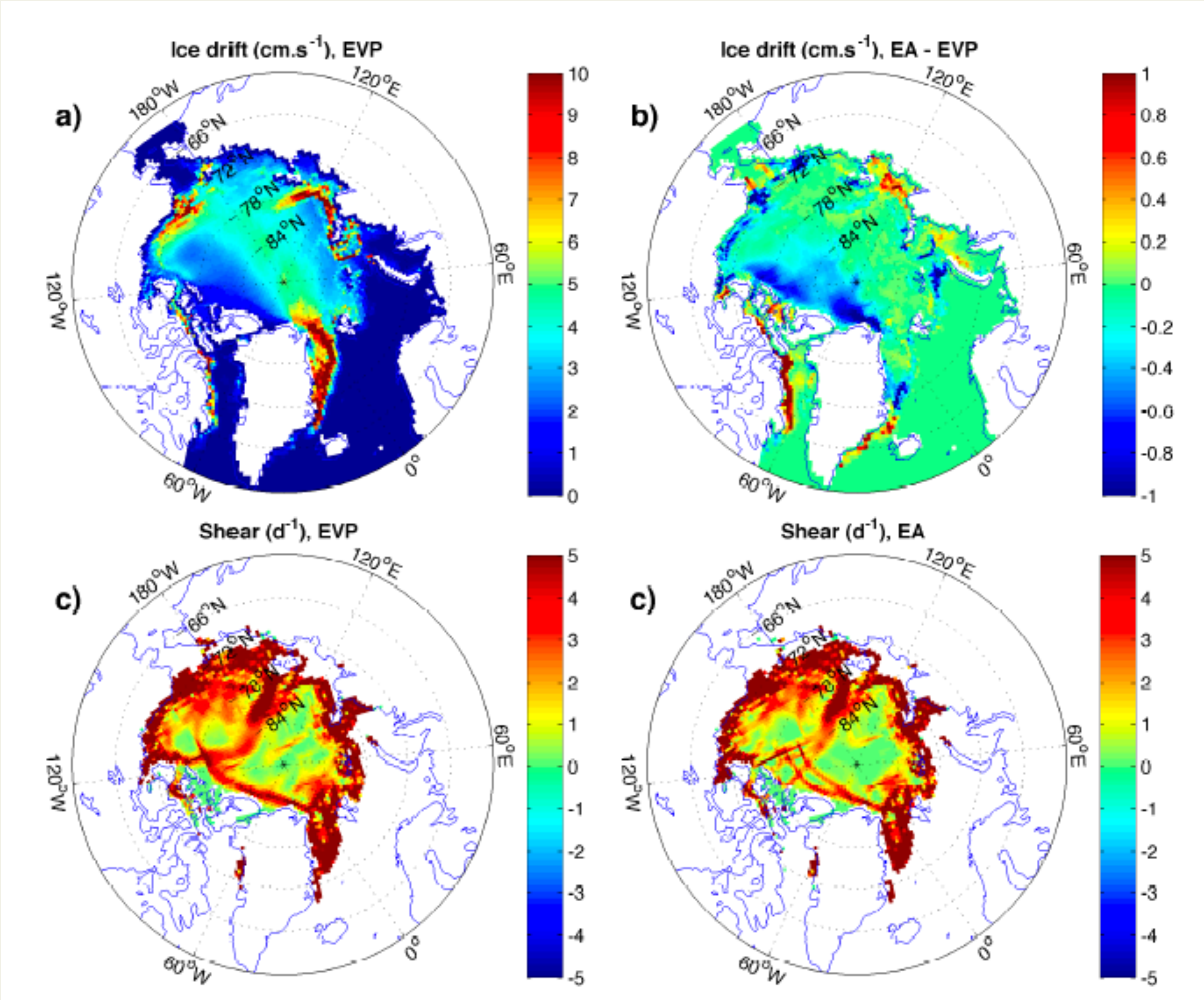
Arctic sea ice state – September average 1990-2007



The Elastic Anisotropic (EA) model is tuned to have same compressive isotropic strength as the standard (EVP) isotropic model

30% more ice with EA

Arctic sea ice dynamics – March average 1990 to 2007



Concluding remarks 1/2

- **Observations** show the sea ice cover is **anisotropic**.
- A **discrete element model** of sea ice failure gives insight into the **mechanisms** of sea ice failure and **anisotropy generation**.
- But climate models require **continuum models** of sea ice rheology, for computational reasons.

Concluding remarks 2/2

- By **assuming** the sea ice cover comprises a **mosaic** of **diamond-shaped floe aggregates**, we have been able to formulate a **practical** description of **anisotropy evolution** and **rheology**.
- Including anisotropy accounts for a **range** of yield behaviours (yield curves) and **affects the mass budget and flow to leading order**.
- We believe anisotropy results in a more realistic **thickness distribution** and **mass flux** to lower latitudes.
- The model contains some parameters that we hope to constrain with process modelling, data assimilation, and high resolution SAR imagery (proposal submitted).


Questions?

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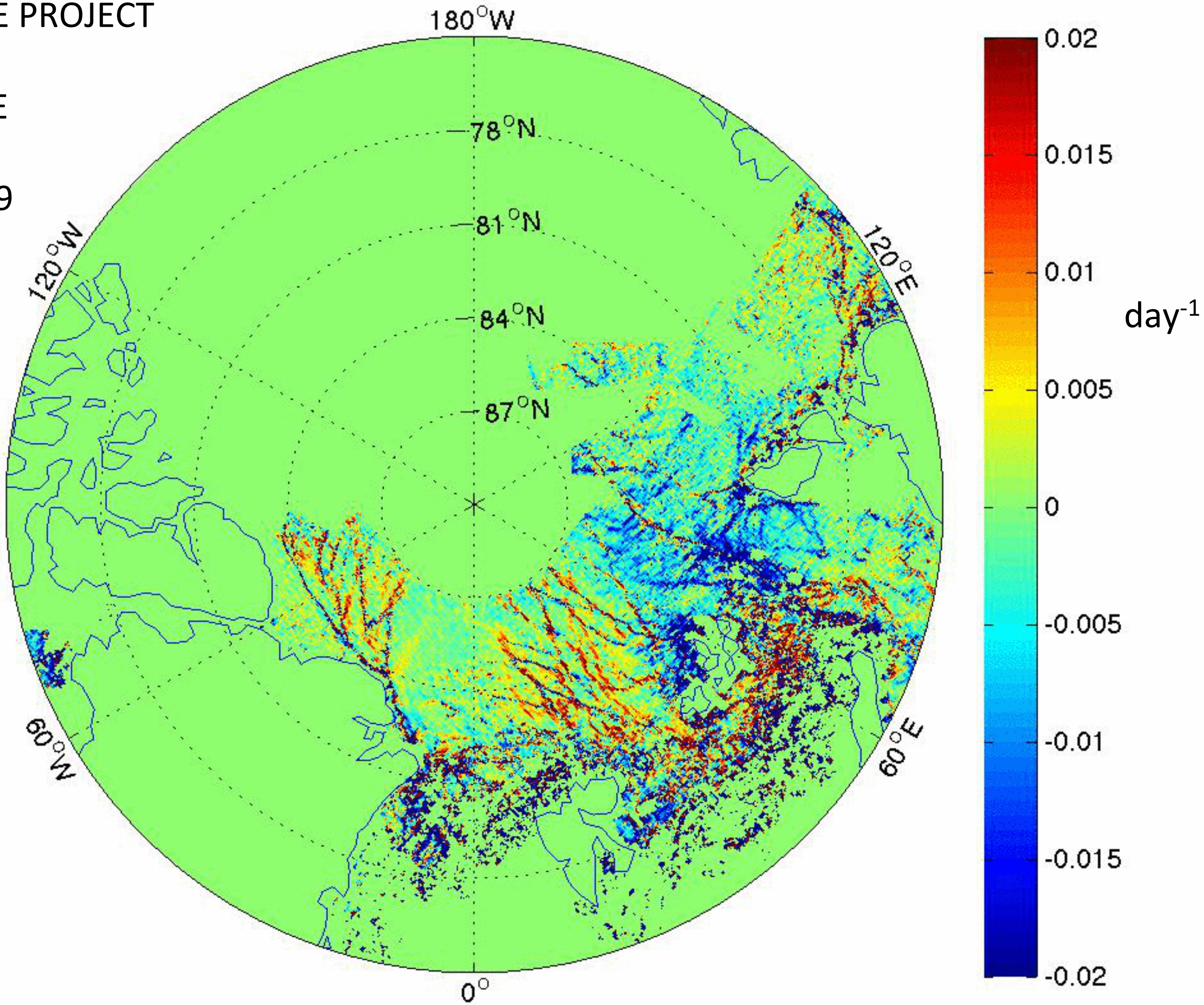

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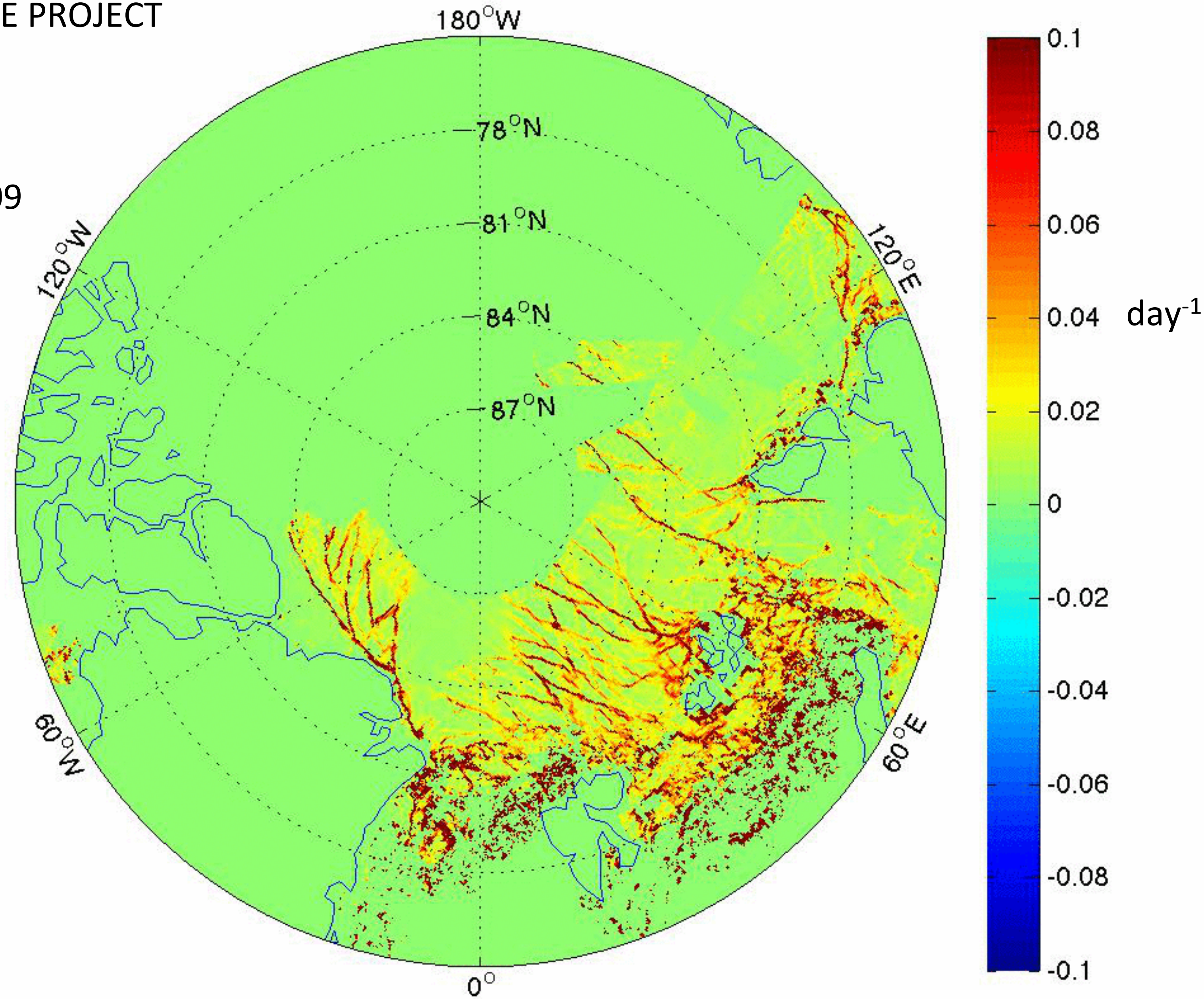

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VORTICITY

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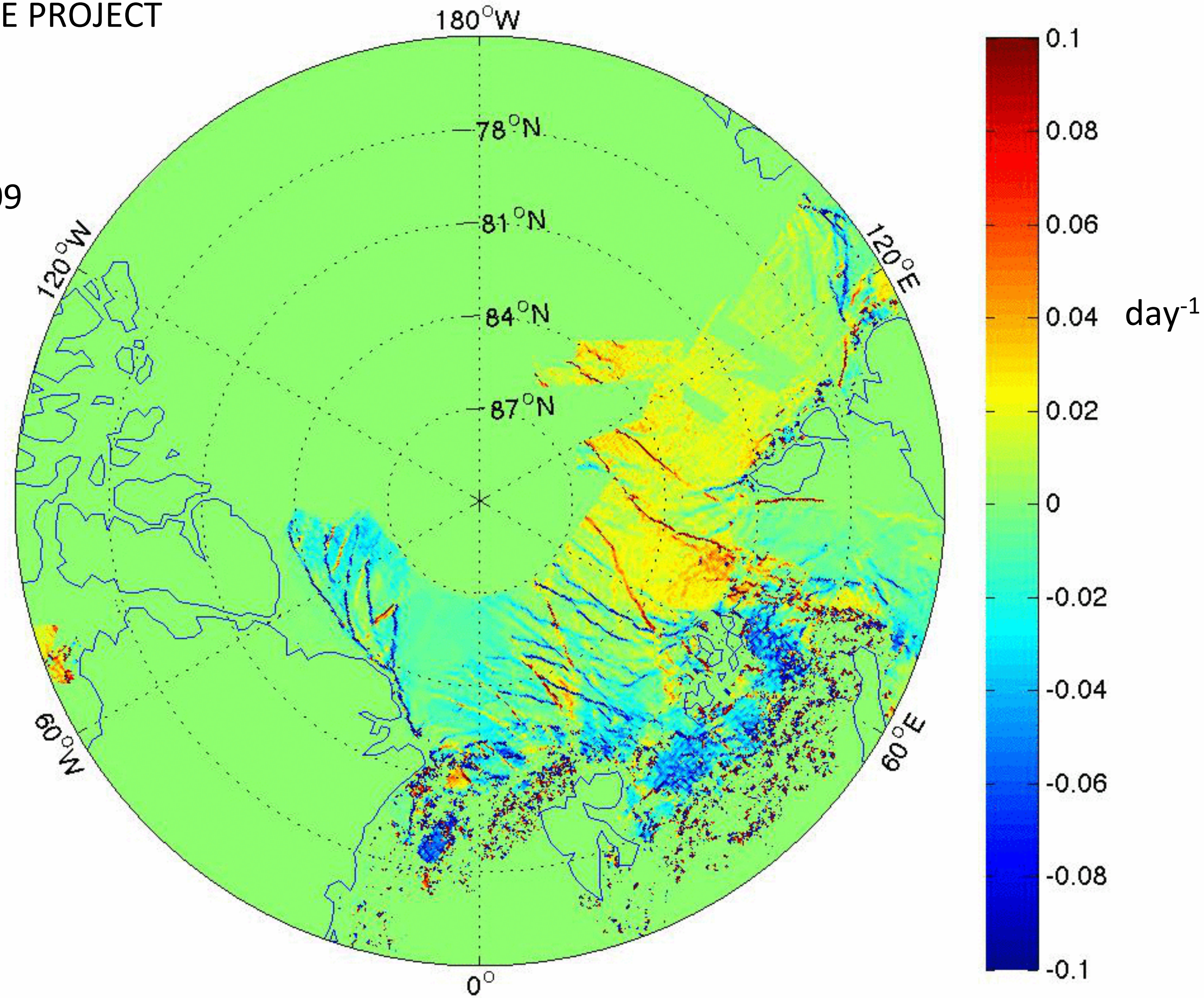
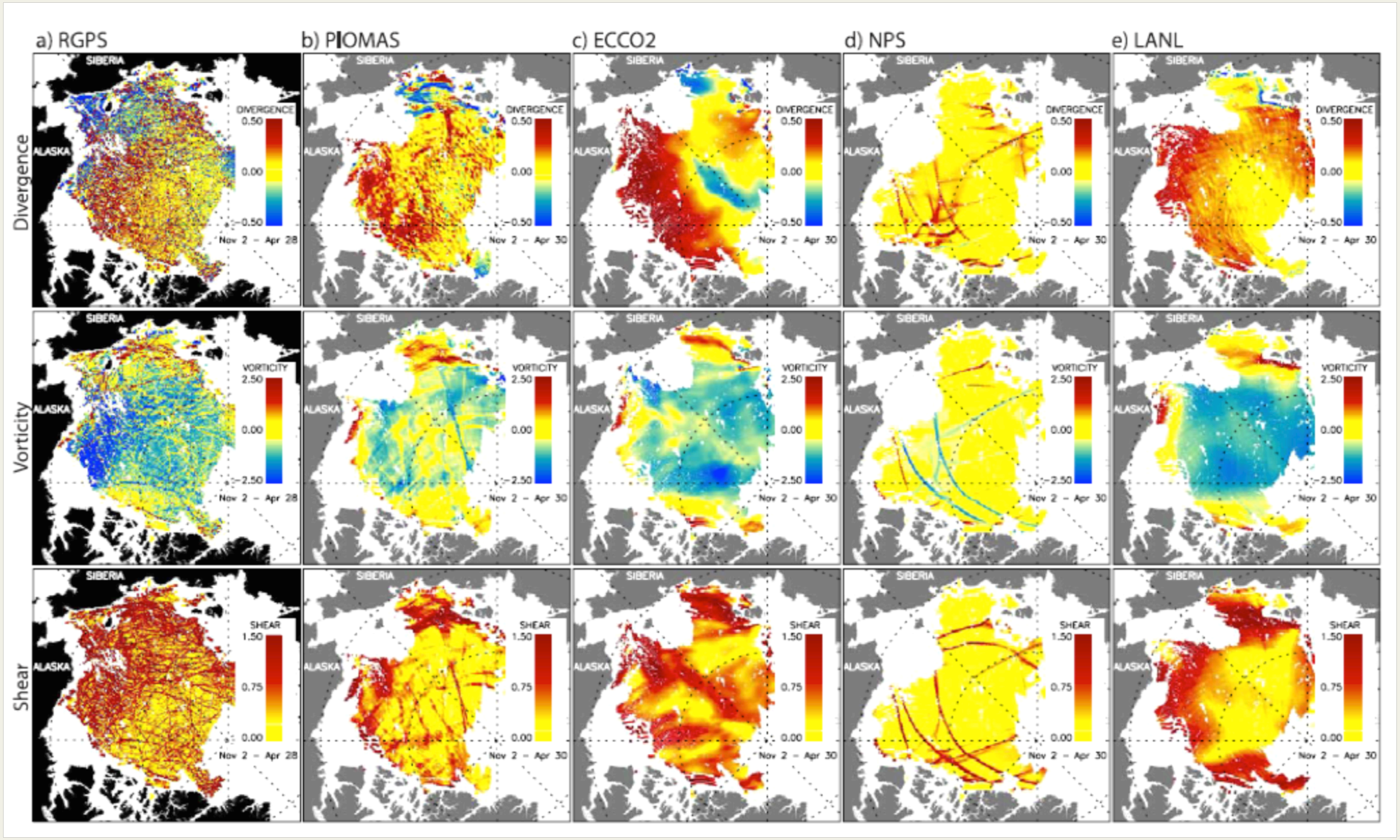


Comparison of various sea ice—ocean models with RGPS data on LKFs [Kwok et al, 2008]



All the sea ice models are continuum models with an isotropic, plastic rheology, none of which do a good job of simulating the LKFs.