Multiscale Models of Melting Arctic Sea Ice

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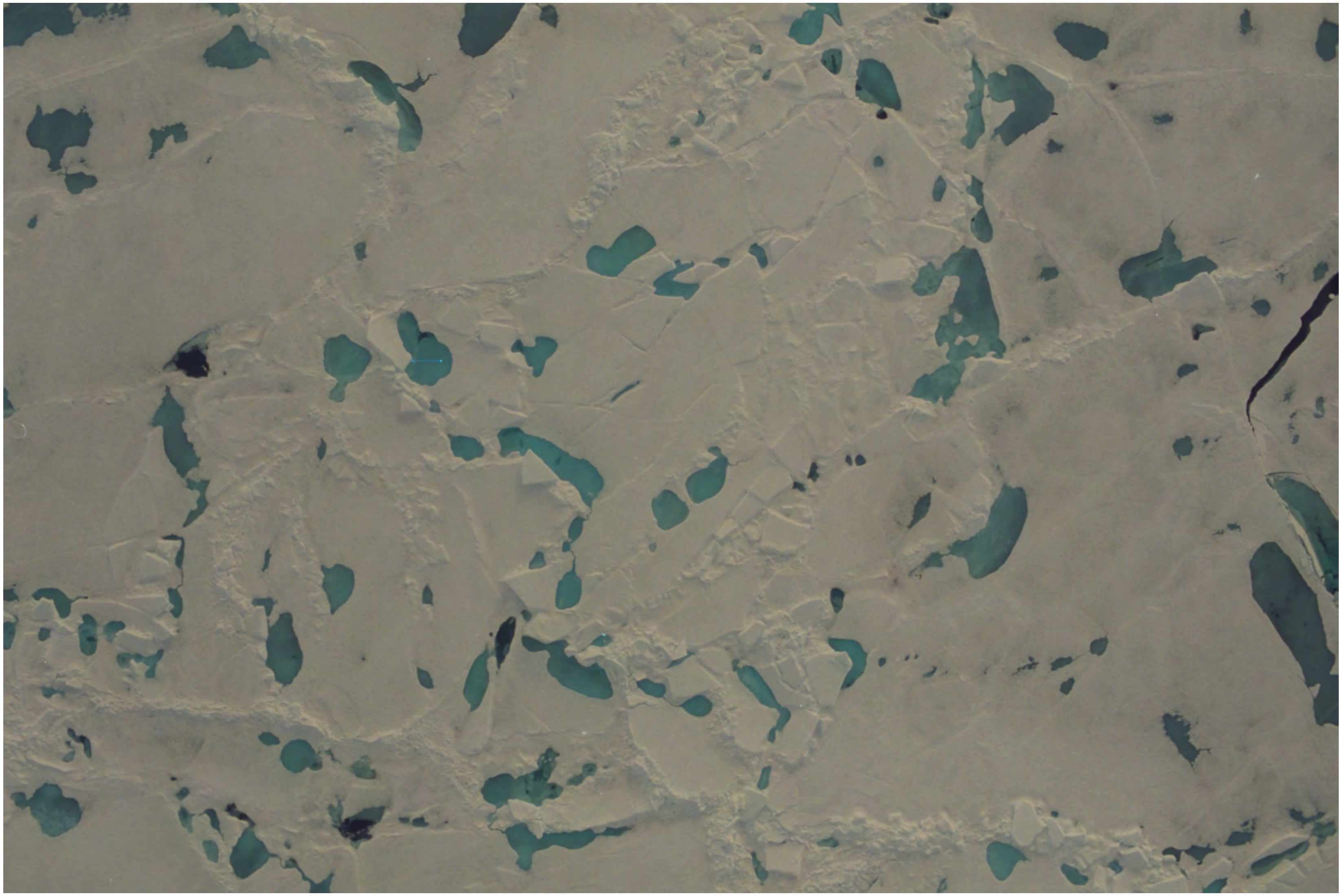
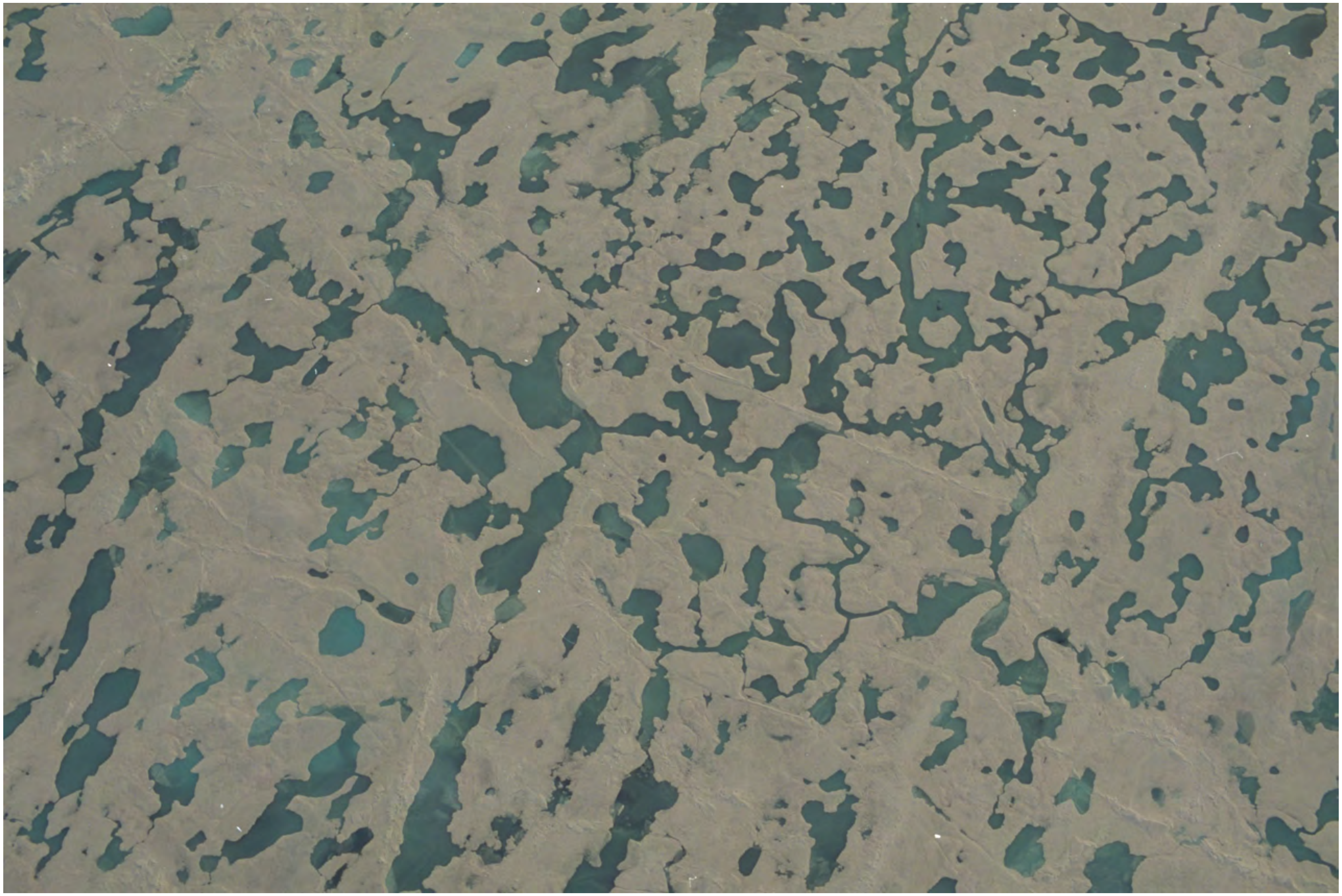
ONR Arctic Program Review
29 October 2014

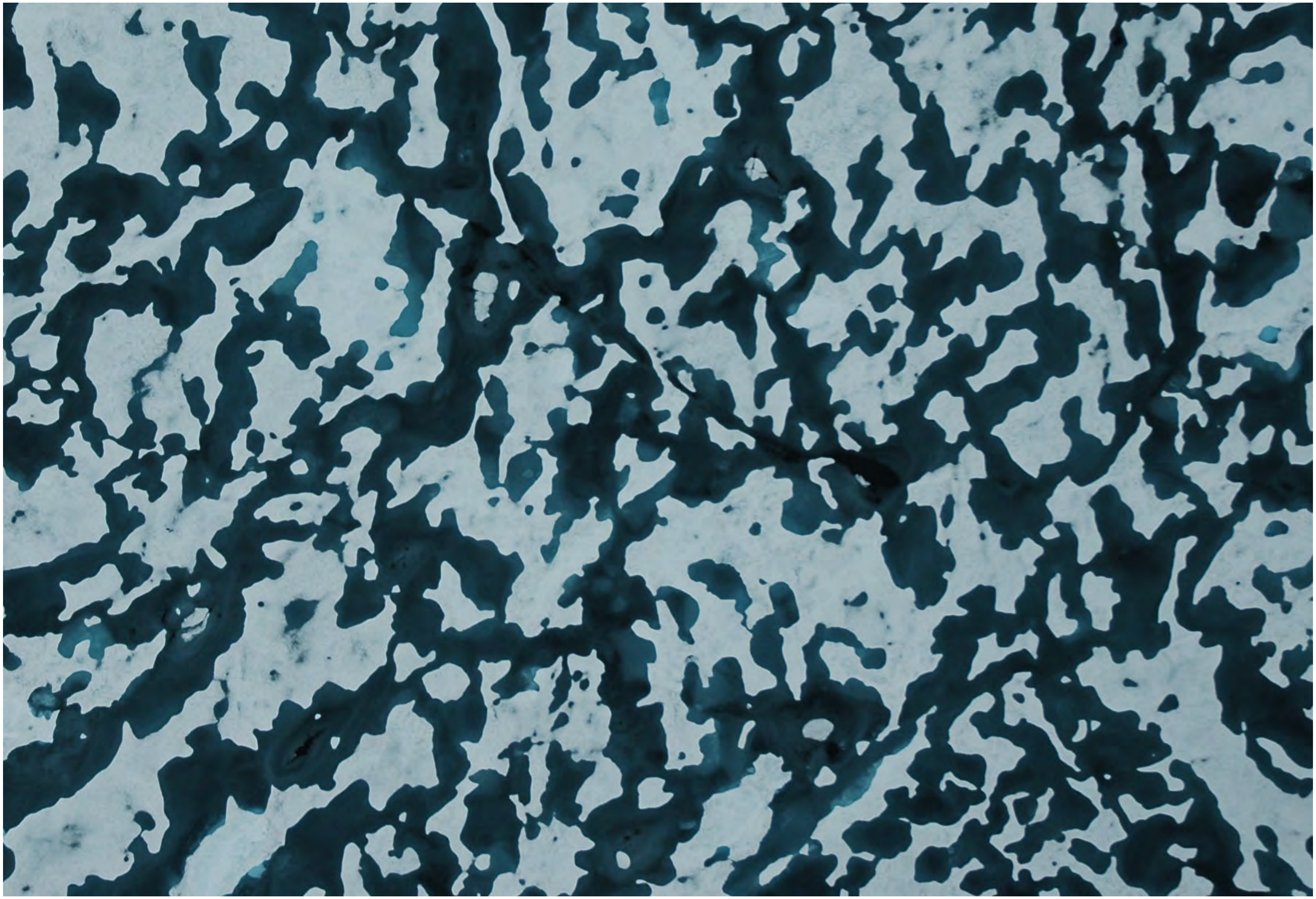
Perovich



Perovich



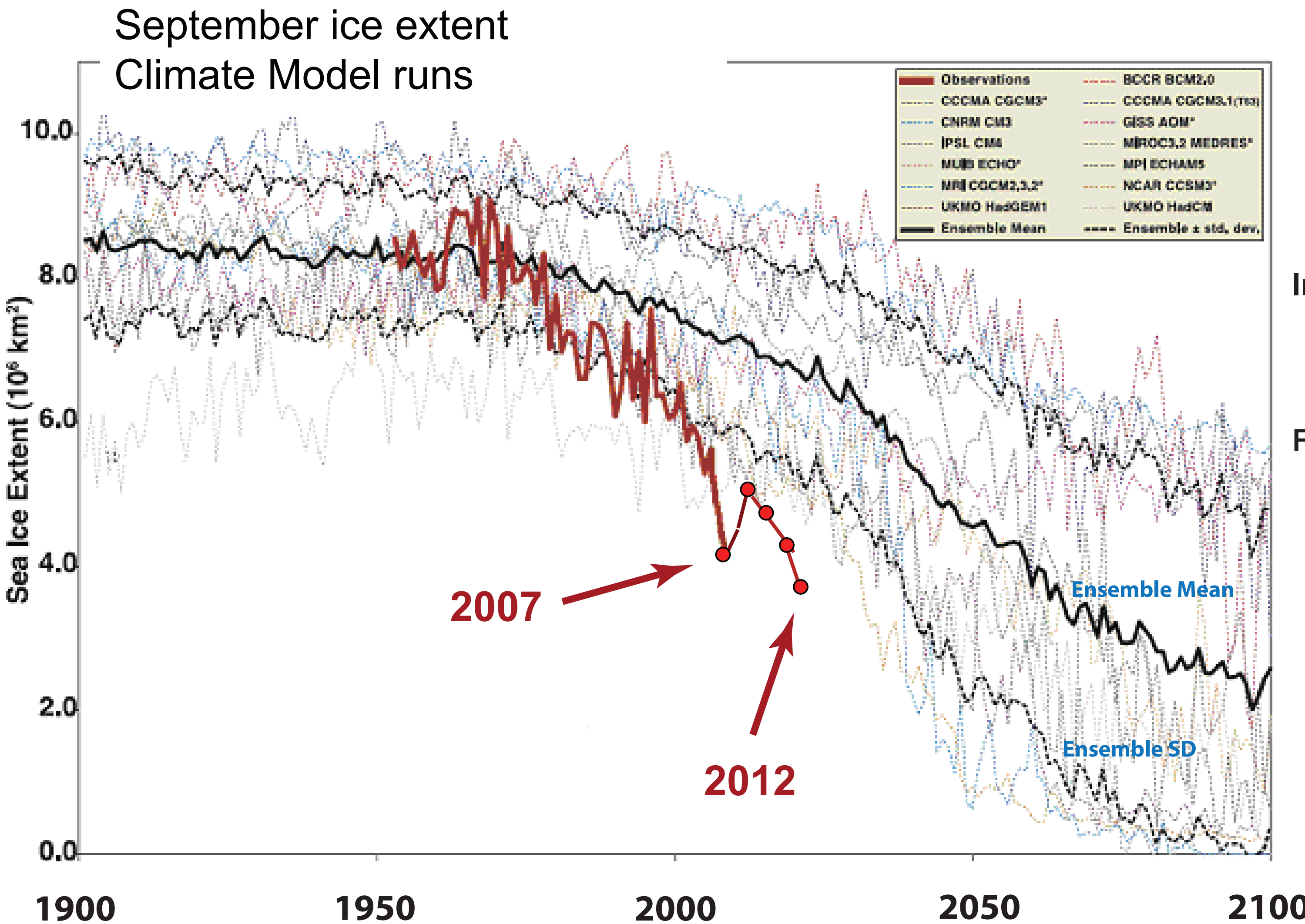
Perovich



Perovich

Arctic sea ice decline - faster than predicted by climate models

Stroeve et al., GRL, 2007



IPCC AR4 Models

Intergovernmental Panel on Climate Change (IPCC)

Fourth Assessment AR4, 2007

Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

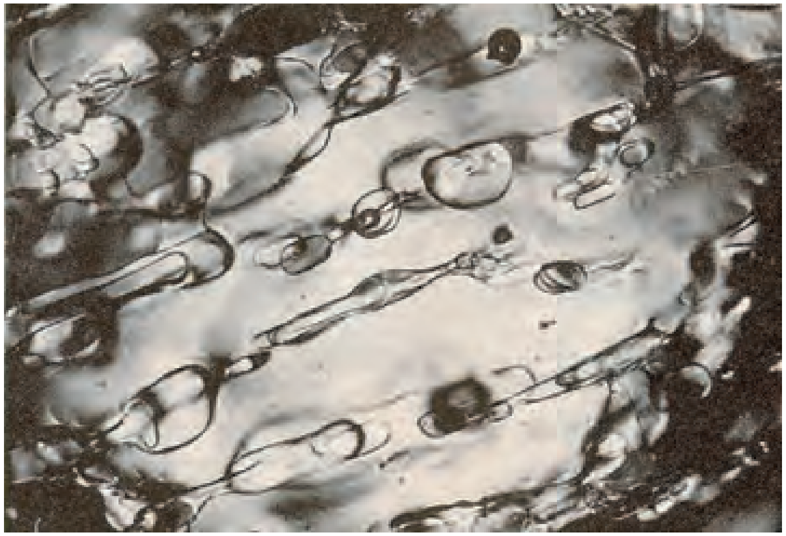
Flocco, Schroeder, Feltham, Hunke, JGR Oceans 2012

For simulations with ponds, the September ice volume is nearly 40% lower.

sea ice is a multiscale composite

structured on many length scales

brine inclusions

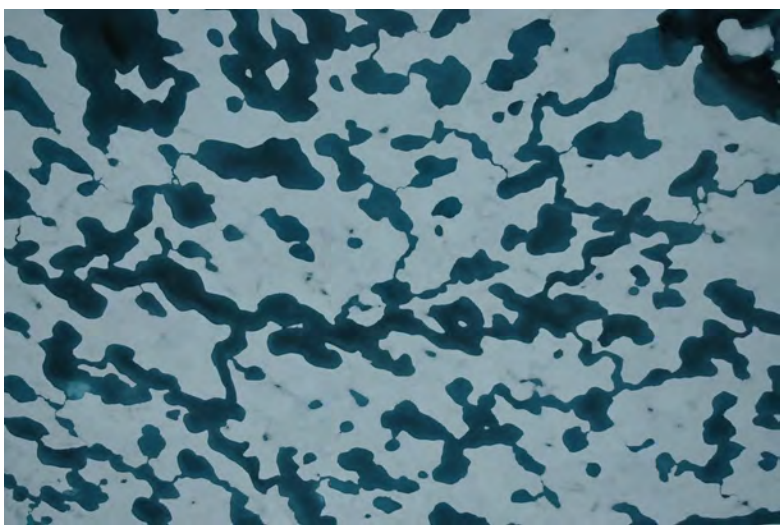


millimeters

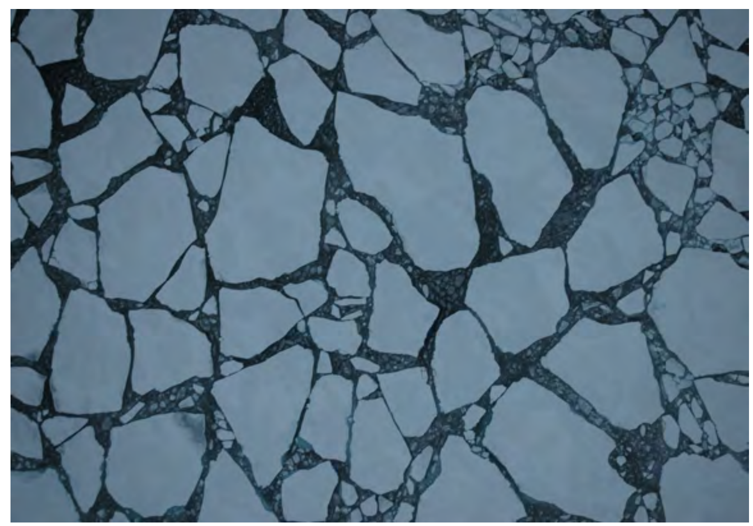
pancakes

centimeters

melt ponds



meters

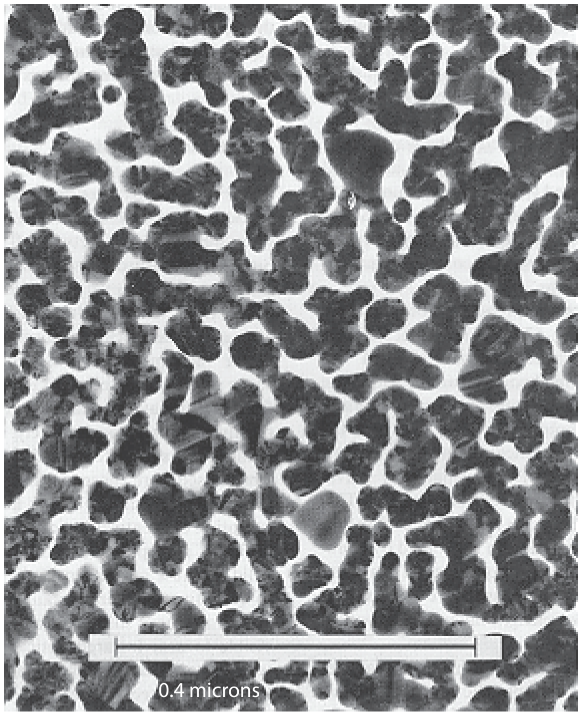


ice floes

kilometers

thin silver film

microns



(Davis, McKenzie, McPhedran, 1991)

Arctic melt ponds

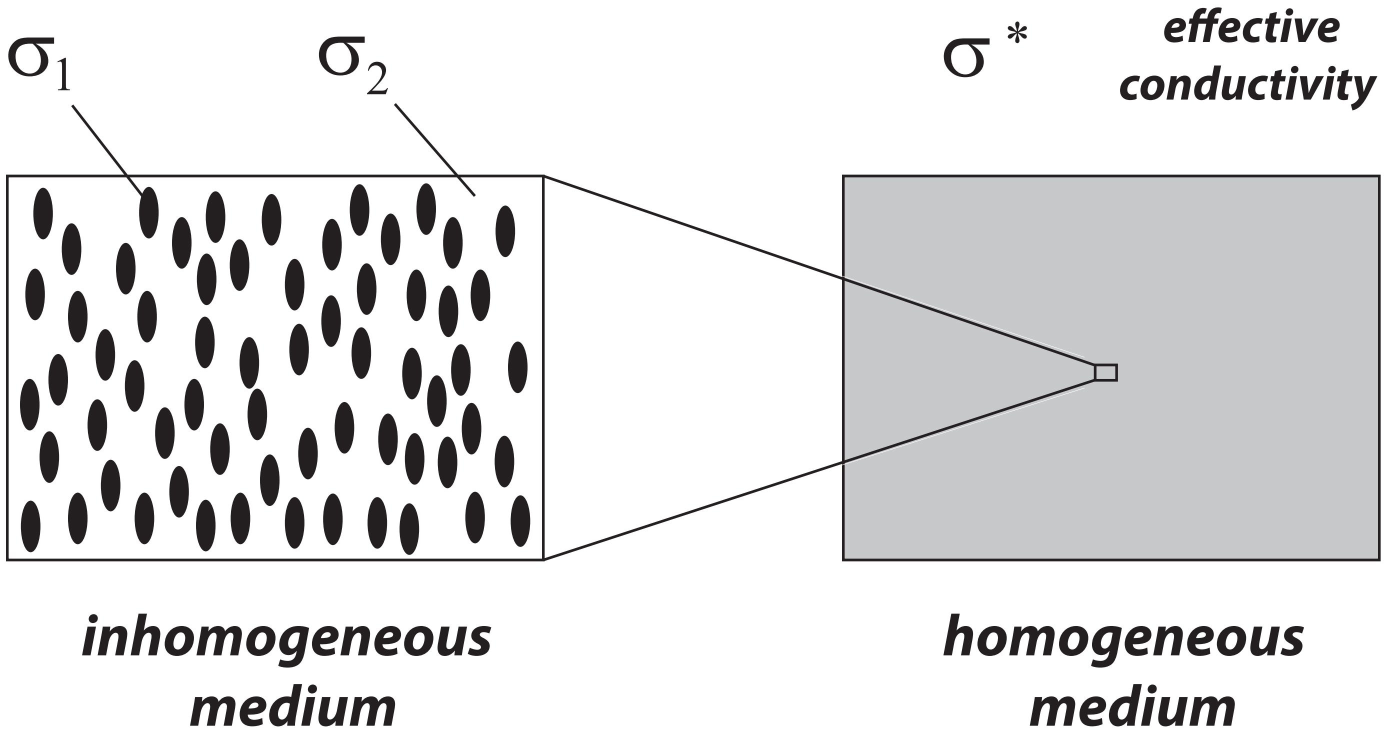
kilometers

(Perovich, 2005)

optical properties

composite geometry -- area fraction of phases, connectedness, necks

HOMOGENIZATION

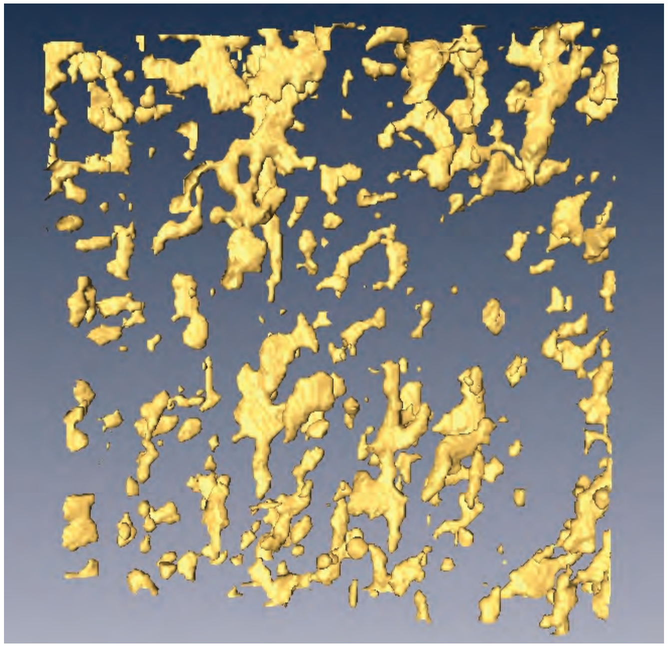


find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

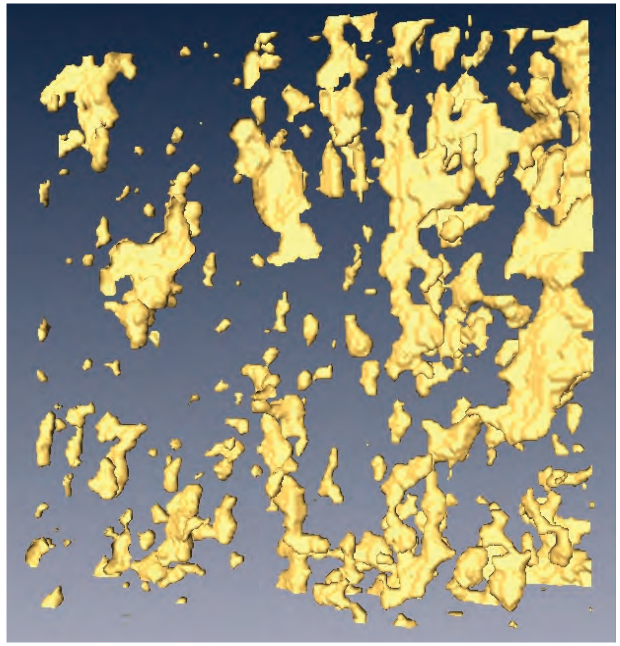
Maxwell 1873 : effective conductivity of a dilute suspension of spheres

Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

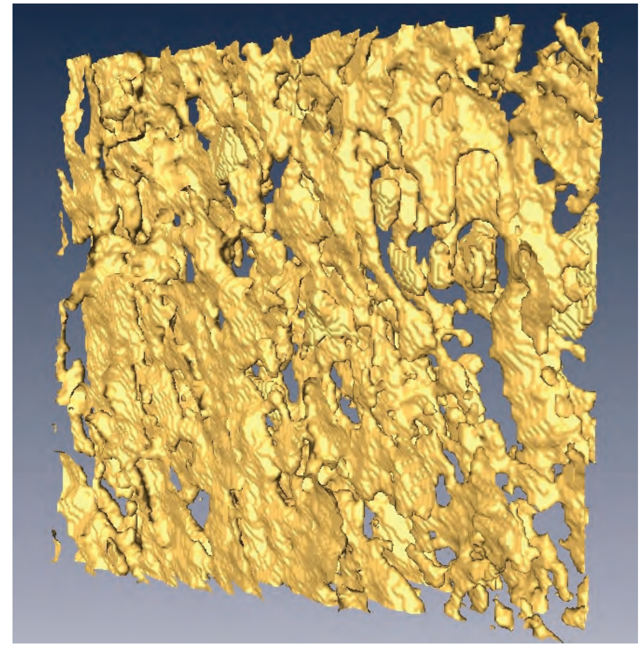
brine volume fraction and ***connectivity*** increase with temperature



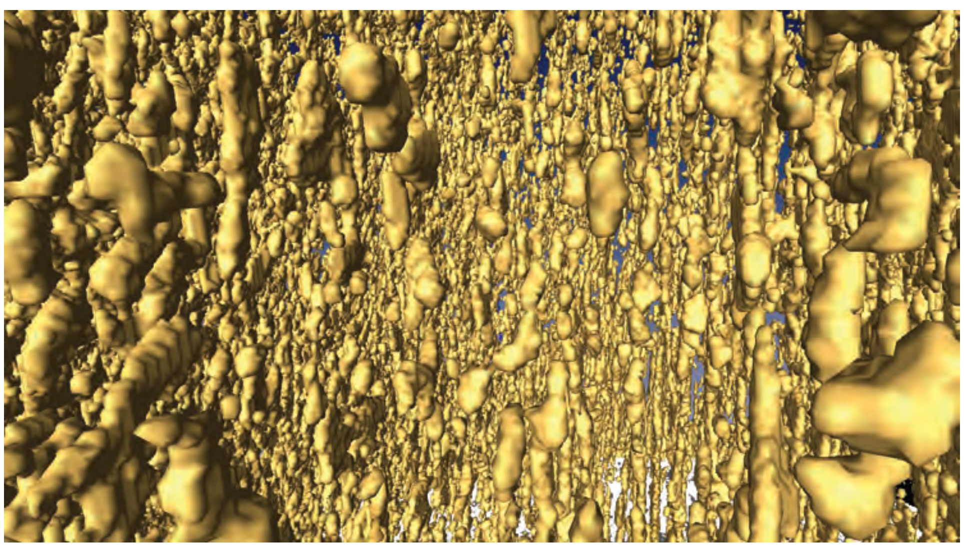
T = -15 °C, $\phi = 0.033$



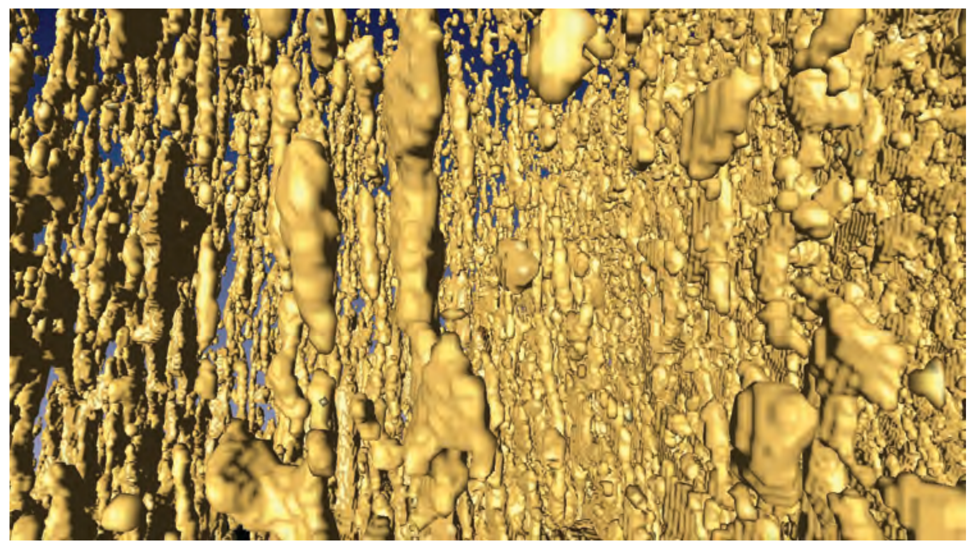
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$



T = -8 °C, $\phi = 0.057$



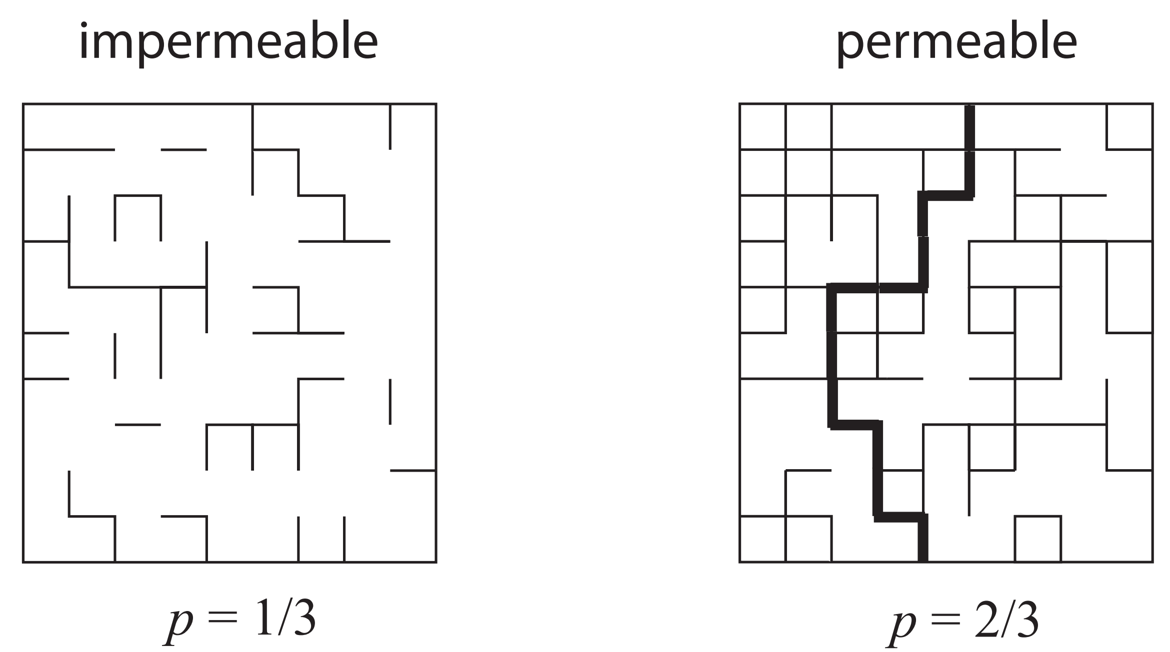
T = -4 °C, $\phi = 0.113$

X-ray tomography for brine in sea ice

Golden, Eicken, Heaton, Miner, Pringle and Zhu, *GRL*, 2007

percolation theory

mathematical theory of connectedness



bond → *open* *with probability p*
closed *with probability 1-p*

percolation threshold

$$p_c = 1/2 \quad \text{for} \quad d = 2$$

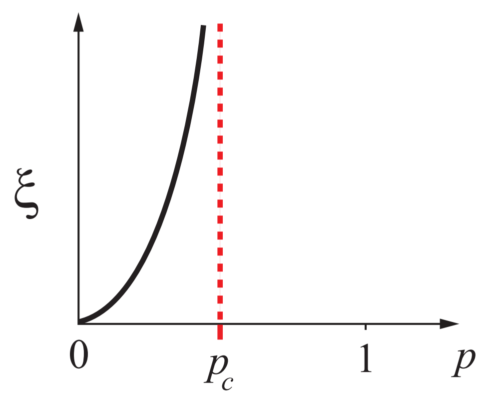
first appearance of infinite cluster ***"tipping point" for connectivity***

order parameters in percolation theory

geometry

correlation length

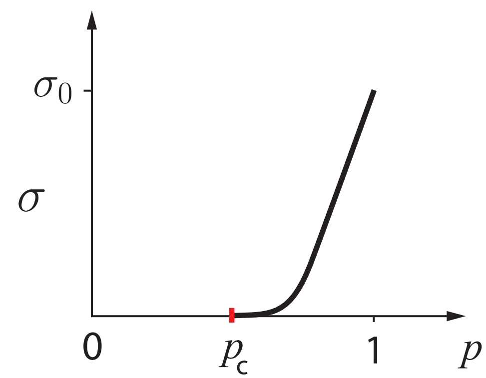
(characteristic scale of connectedness)



$$\xi(p) \sim |p - p_c|^{-\nu}$$

transport

effective conductivity or fluid permeability



$$\sigma(p) \sim \sigma_0 \, (p - p_c)^t$$

UNIVERSAL critical exponents for lattices -- depend only on dimension

(**1 ≤ *t* ≤ 2,** Golden, *Phys. Rev. Lett.* 1990 ; *Comm. Math. Phys.* 1992)

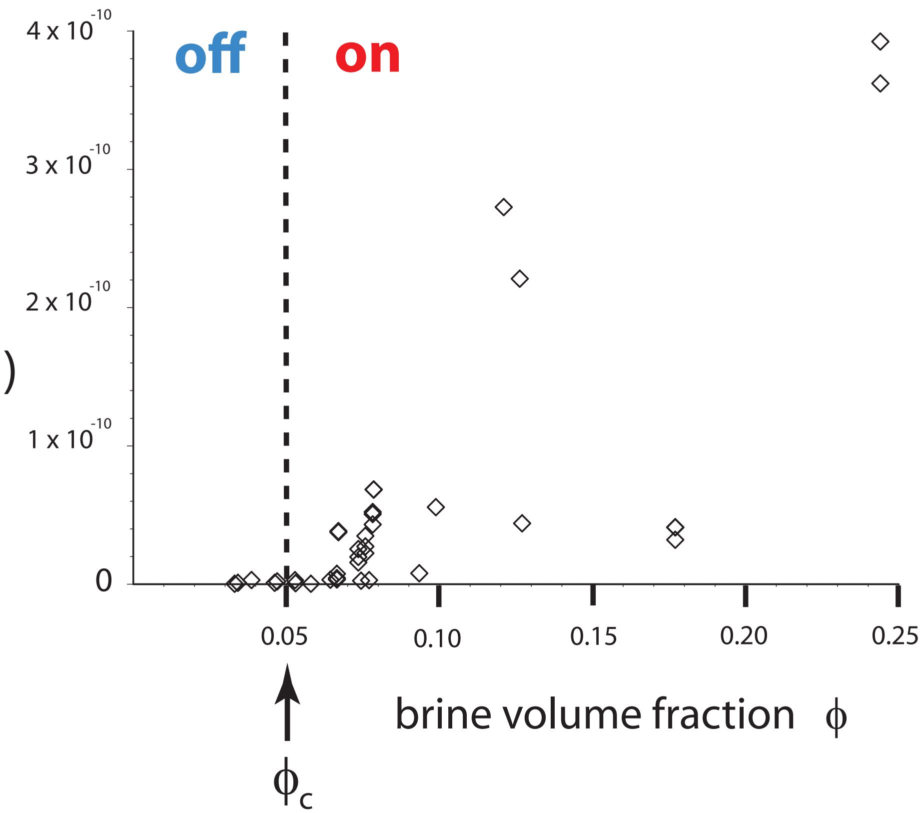
non-universal behavior in continuum

Critical behavior of fluid transport in sea ice

impermeable permeable

Arctic field data

vertical fluid permeability k (m^2)



"on - off" switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ C, \; S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998

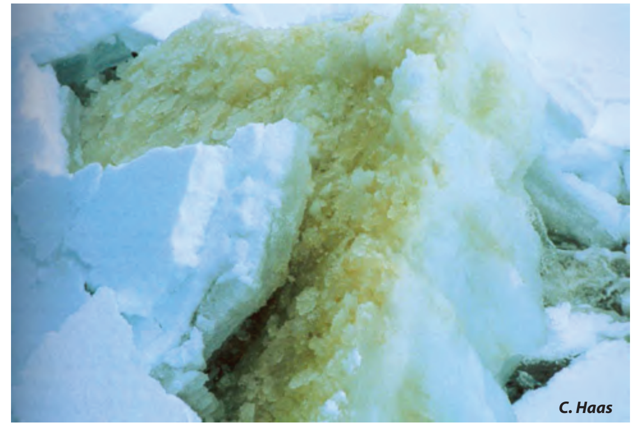
Golden, Eicken, Heaton, Miner, Pringle, Zhu Geophys. Res. Lett. 2007

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

rule of fives constrains key processes in sea ice physics and biology

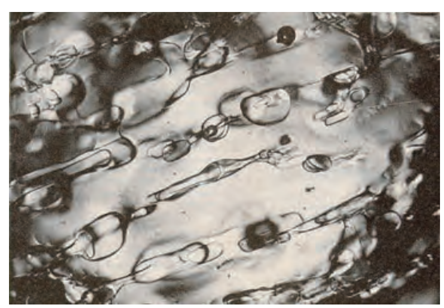
evolution of Arctic melt ponds and sea ice albedo

nutrient flux for algal communities



- *drainage of brine and melt water*
- *ocean-ice-air exchanges of heat, CO_2*
- *Antarctic surface flooding and snow-ice formation*
- *evolution of salinity profiles*

linkage of scales

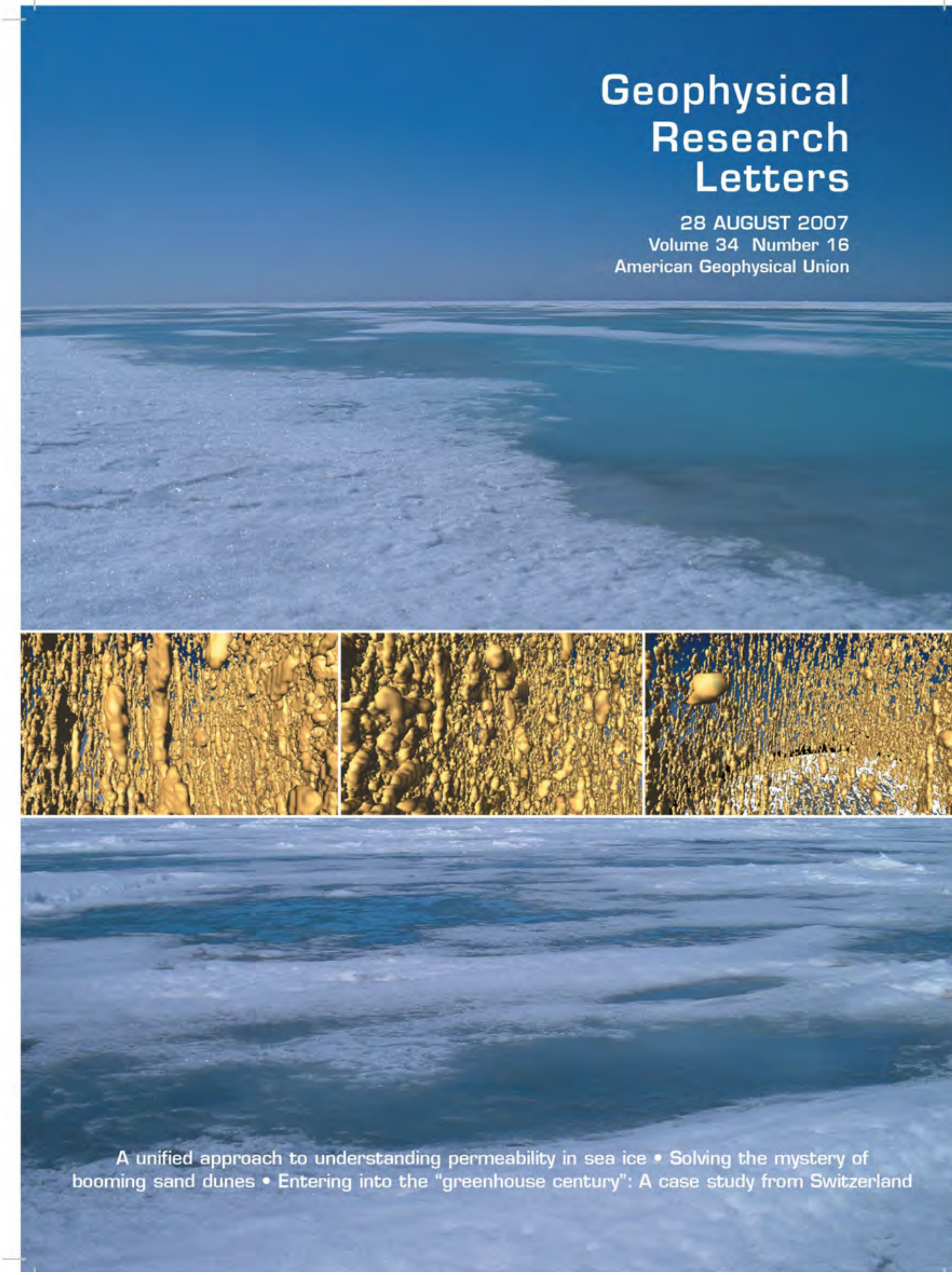


Thermal evolution of permeability and microstructure in sea ice **Golden, Eicken, Heaton, Miner, Pringle, Zhu**

micro-scale

controls

macro-scale processes



rigorous bounds
percolation theory
hierarchical model
network model

field data

X-ray tomography for brine inclusions

unprecedented look at thermal evolution of brine phase and its connectivity

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

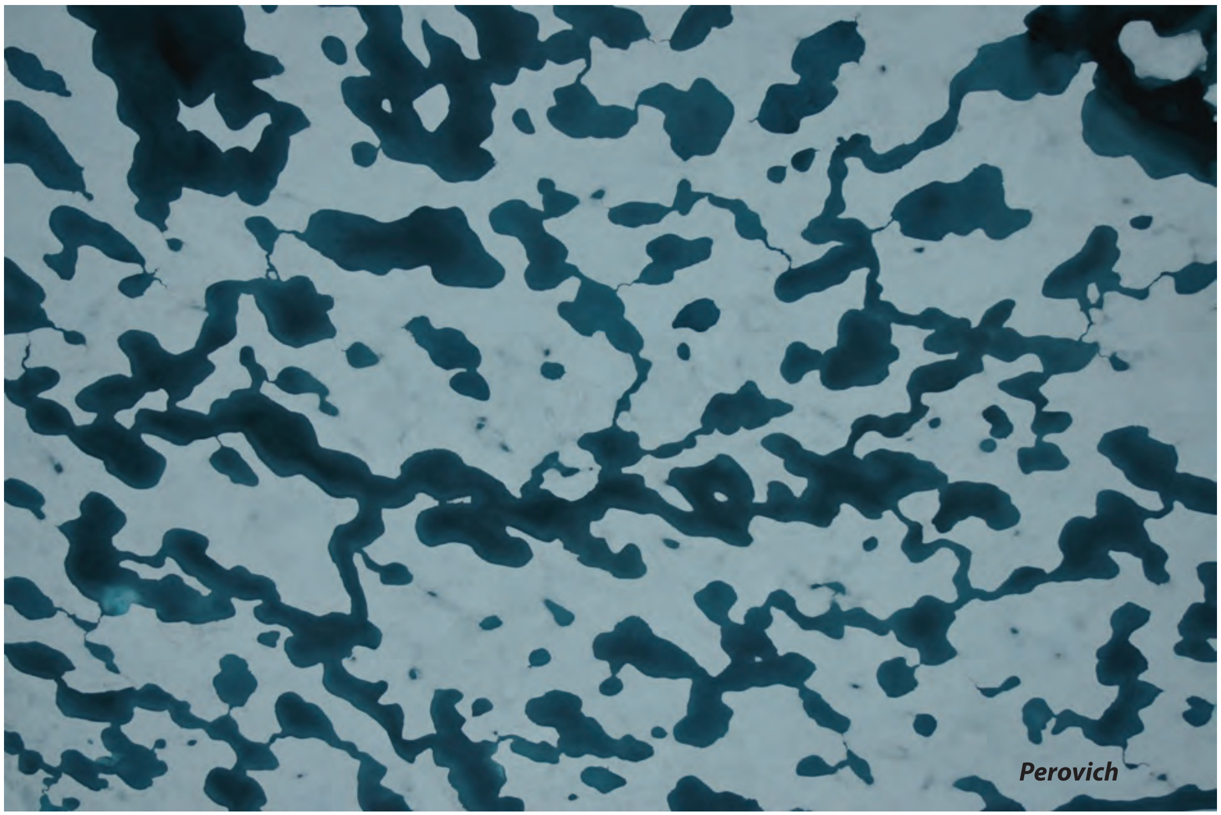
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

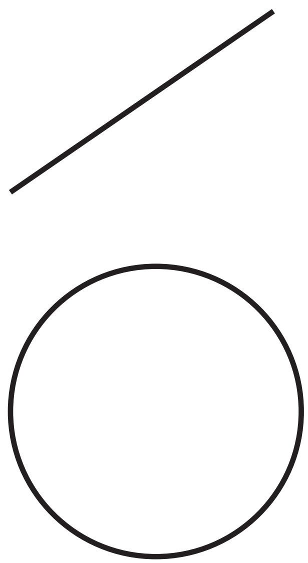
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

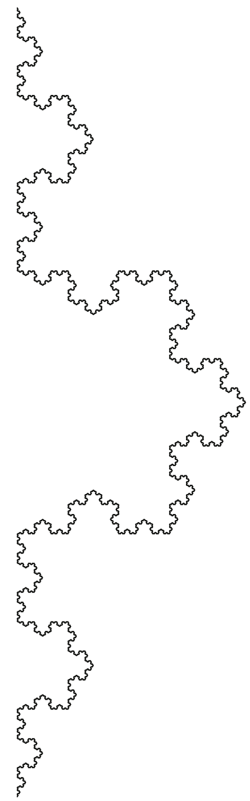
fractal curves in the plane

they wiggle so much that their dimension is >1



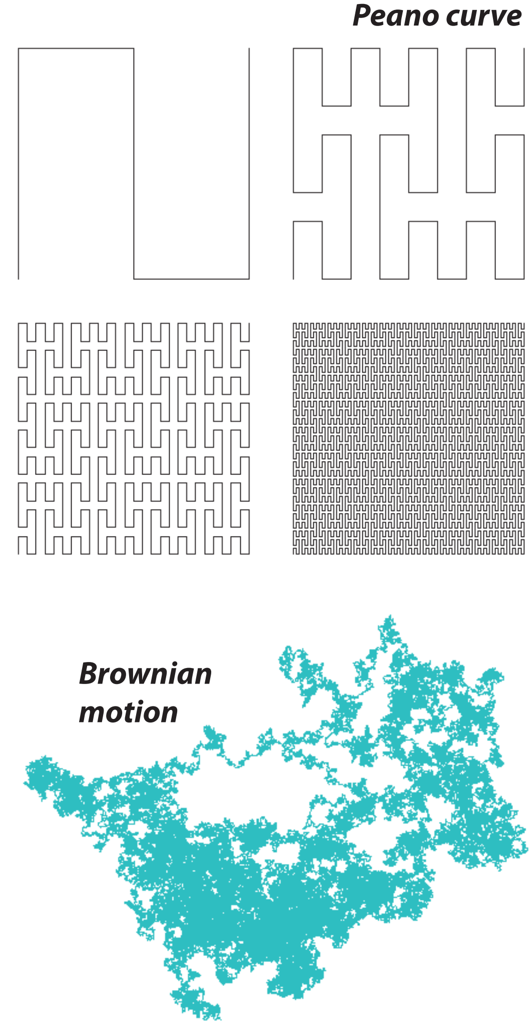
simple curves

***D* = 1**



Koch snowflake

***D* = 1.26**



space filling curves

***D* = 2**

clouds exhibit fractal behavior from 1 to 1000 km

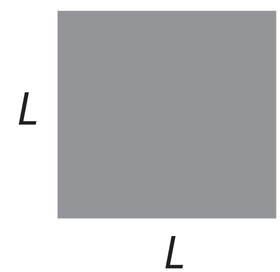
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

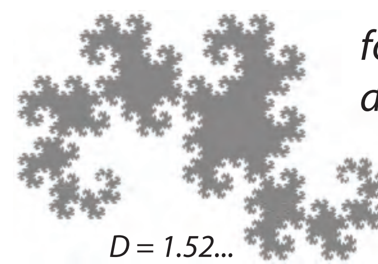
simple shapes



$A = L^2$

$P = 4L = 4\sqrt{A}$

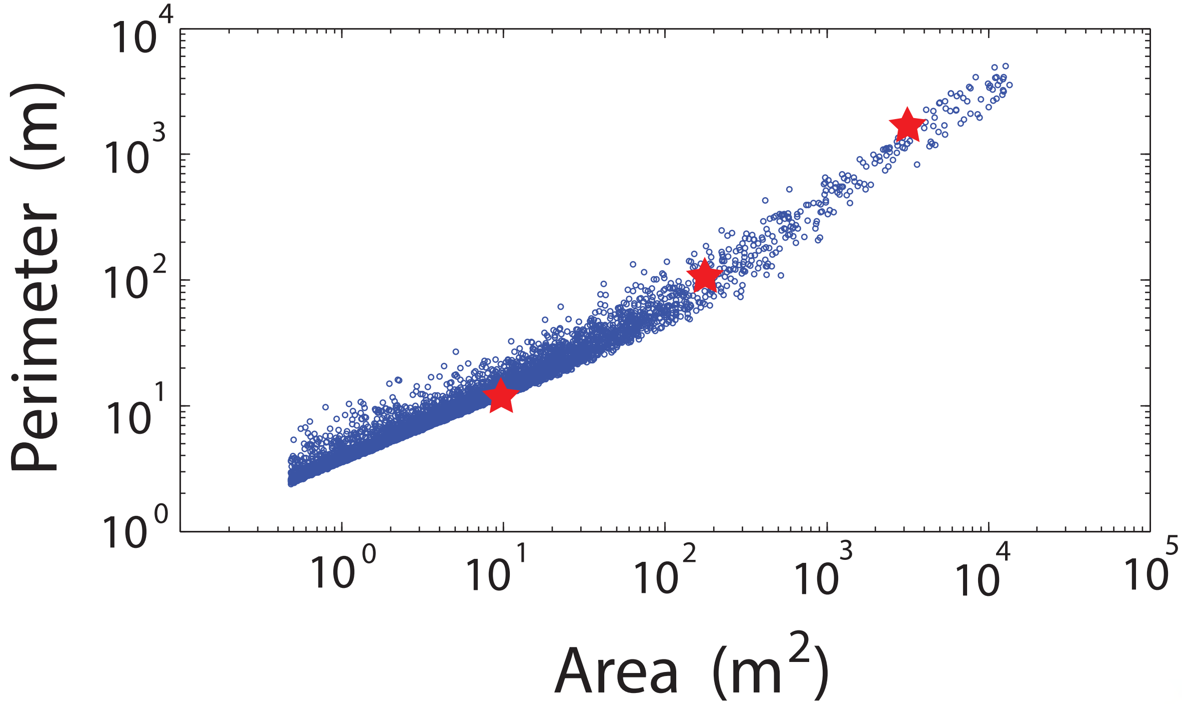
$$P \sim \sqrt{A}^{\,D}$$



for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds *The Cryosphere, 2012*

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

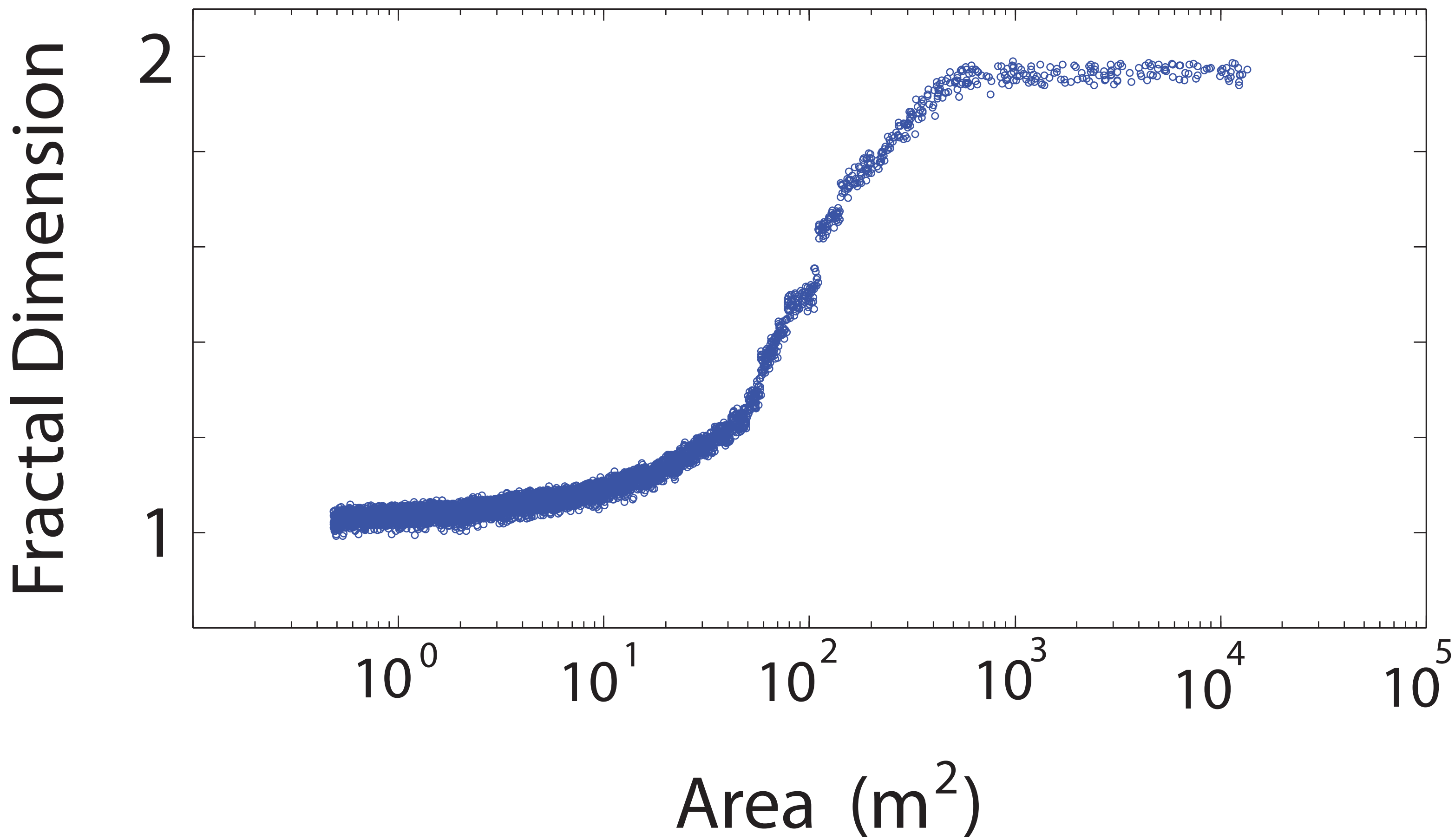


data from 5269 Arctic melt ponds

simple pond ***transitional pond*** ***complex pond***

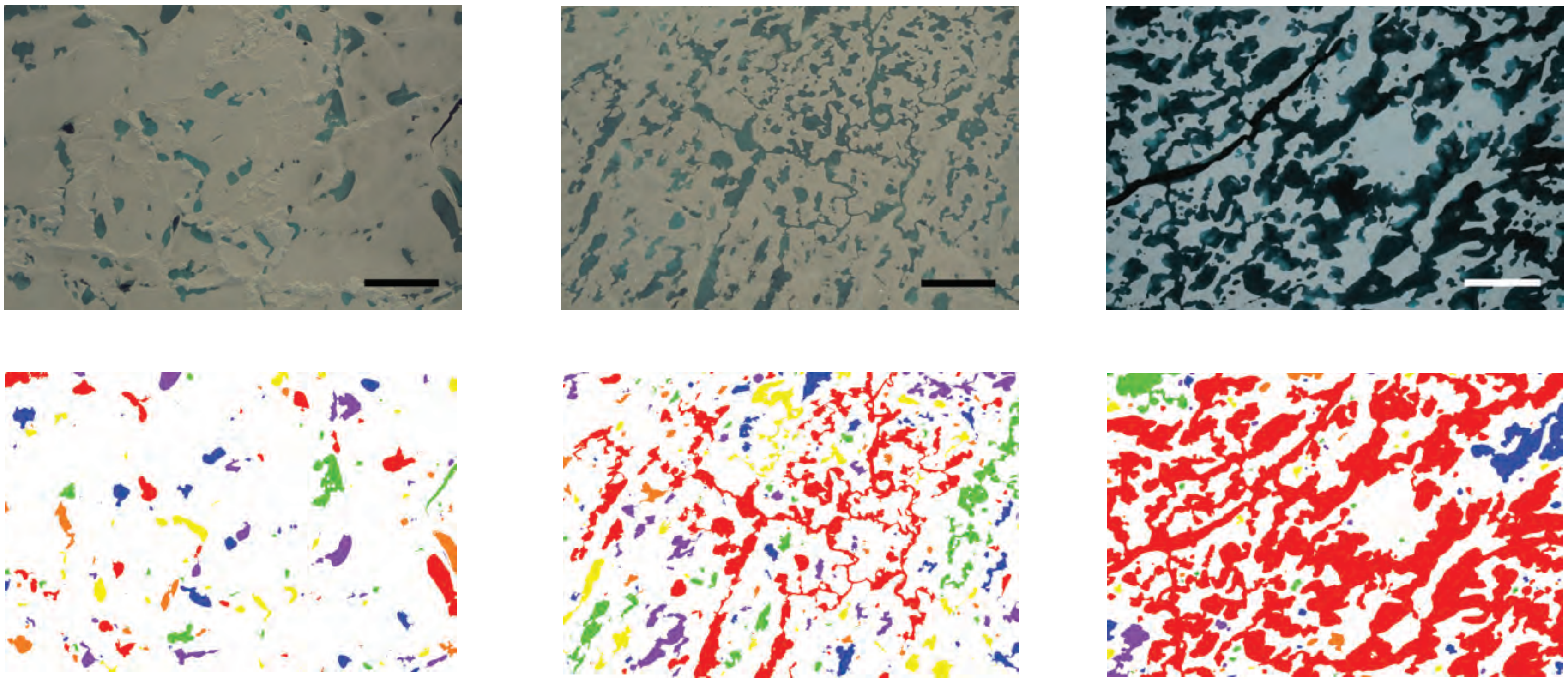
transition in the fractal dimension

complexity grows with length scale



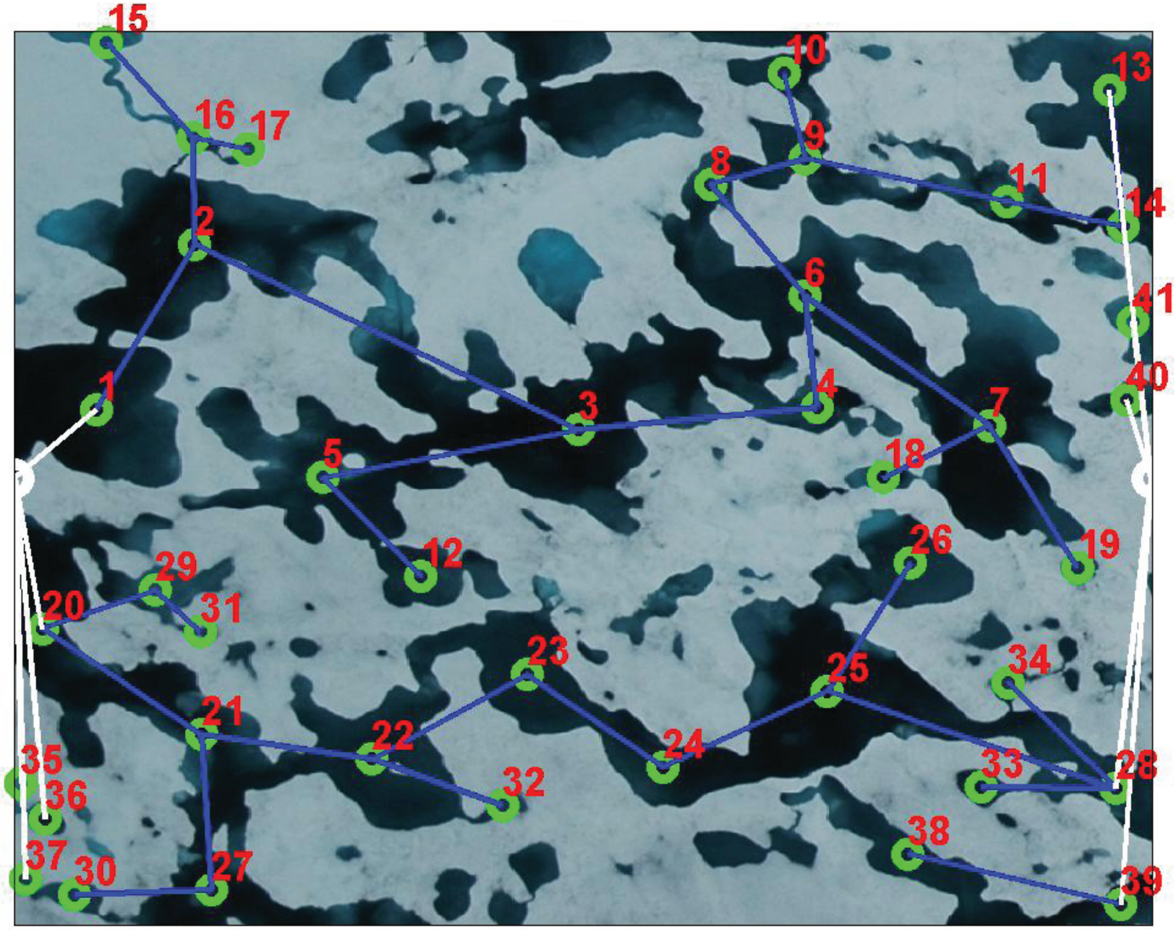
compute "derivative" of area - perimeter data

small simple ponds coalesce to form
large connected structures with complex boundaries



melt pond percolation

map melt pond configurations onto resistor networks
compute horizontal fluid permeability



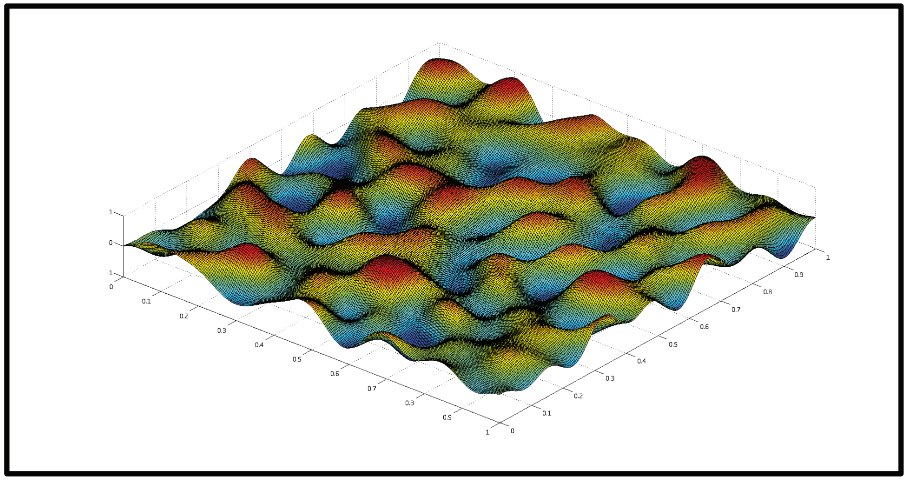
4 August 2005, Healy–Oden Trans Arctic Expedition (HOTRAX)

Network modeling of Arctic melt ponds
Barjatia, Tasdizen, Song, Golden 2014

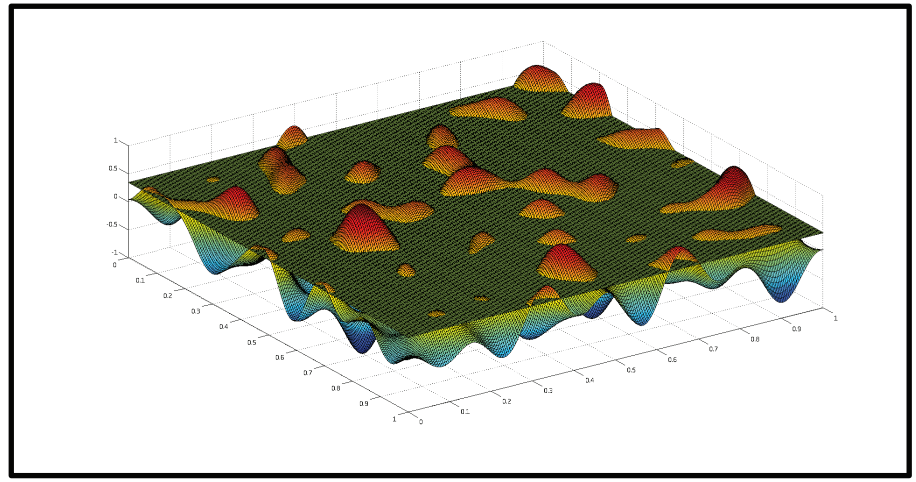
SCI and Math, U. of Utah

Continuum percolation model for melt pond evolution

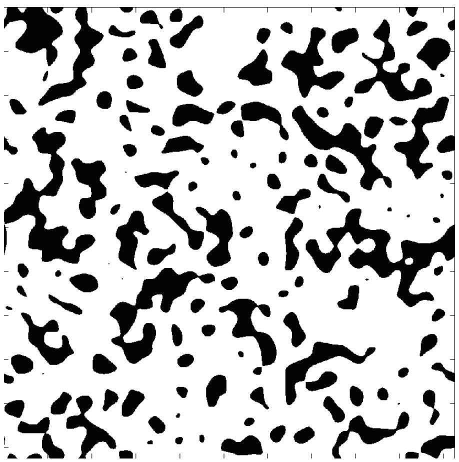
Brady Bowen, Court Strong, Ken Golden, 2014

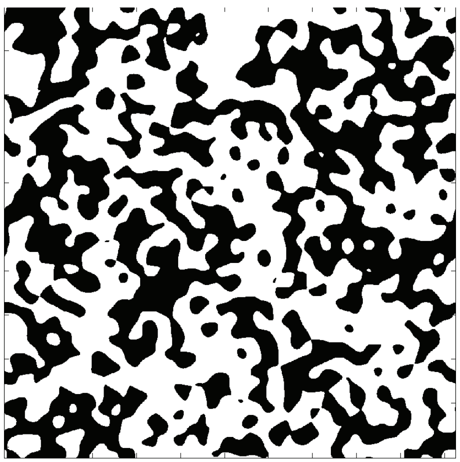


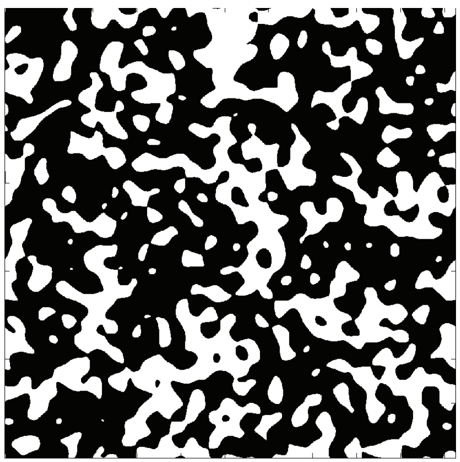
random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds

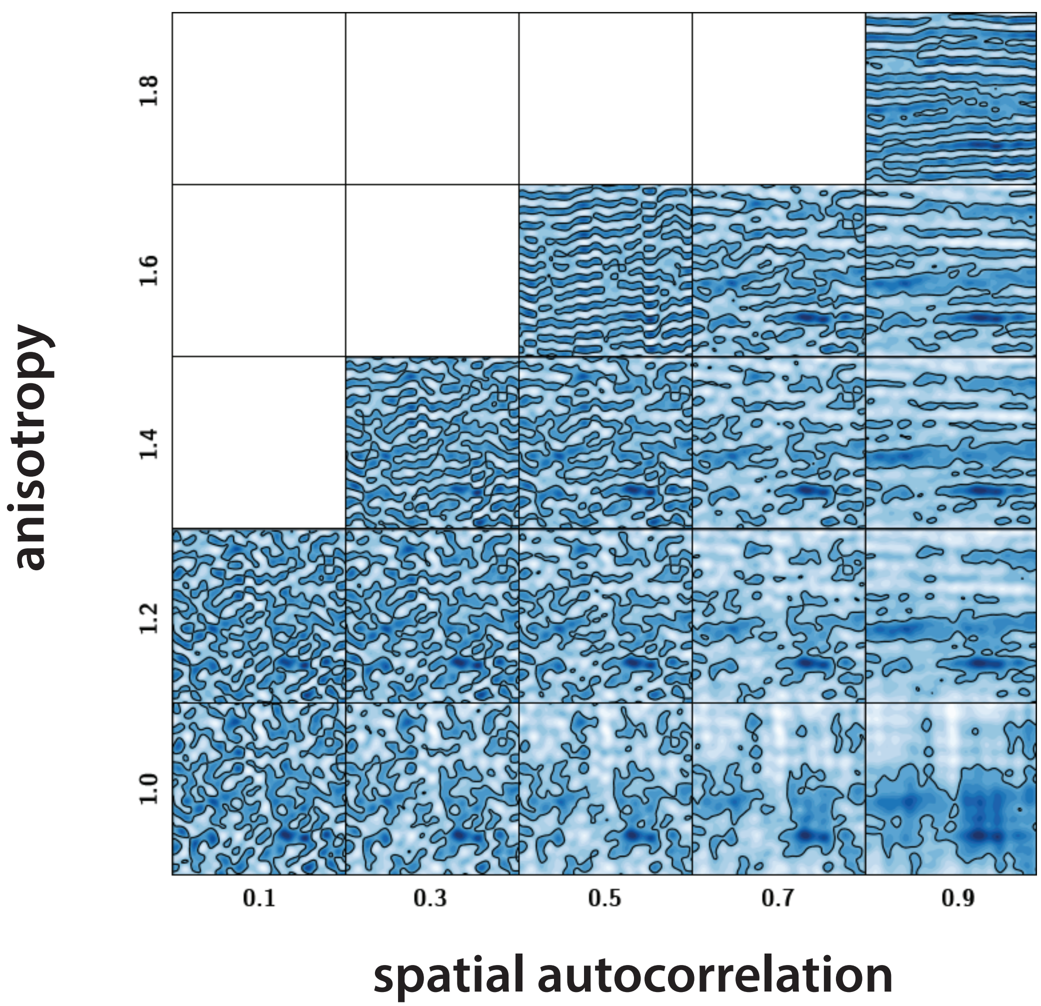




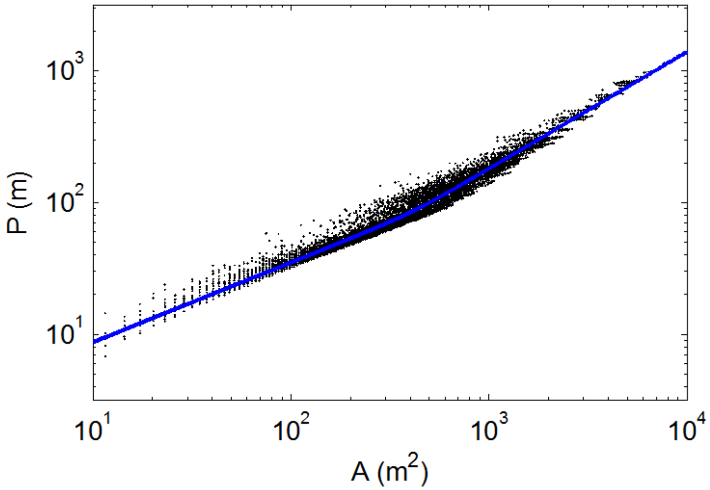


electronic transport in disordered media *diffusion in turbulent plasmas* *(Isichenko, Rev. Mod. Phys., 1992)*

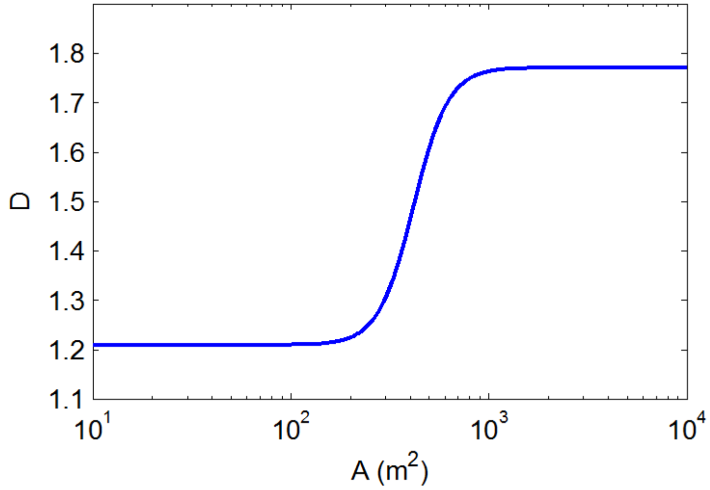
Coefficients of Fourier surface chosen to produce topography with given autocorrelation and anisotropy



Fractal properties of simulated ponds



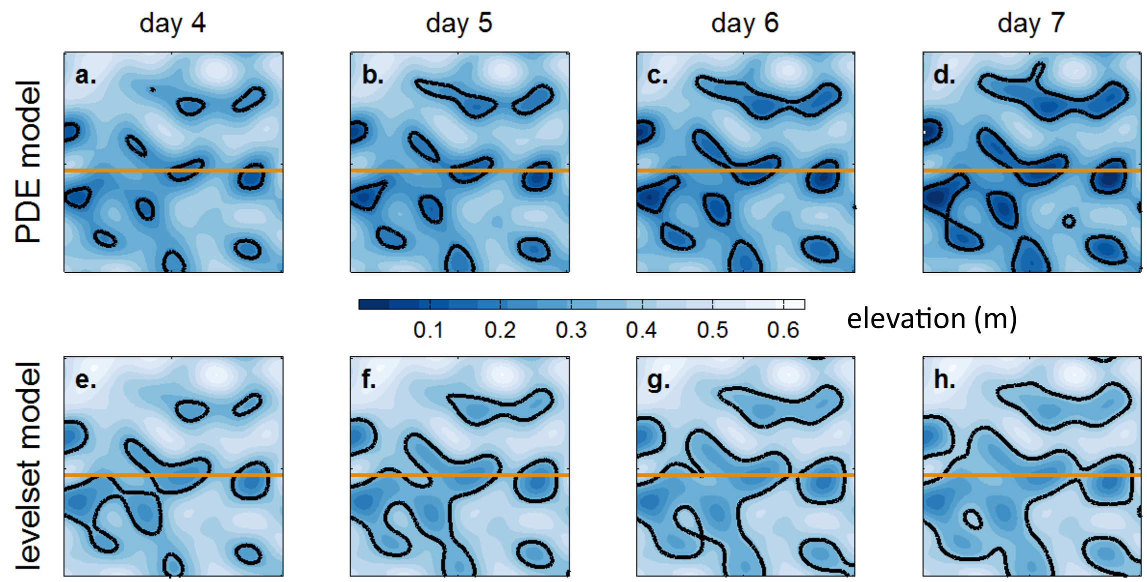
Example for surface
autocorrelation 0.5
anisotropy 1.4



Comparison with PDE Model

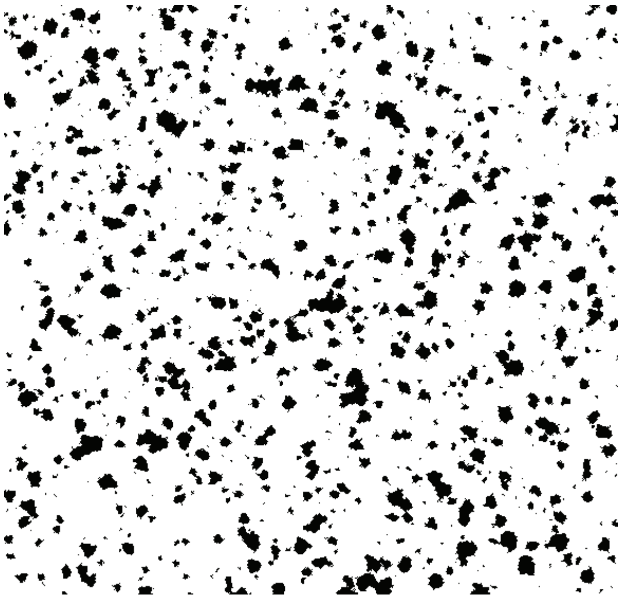
- Physically-based partial differential equation (PDE) melt pond model that includes seepage, pond-enhanced albedo, and horizontal transport (Lüthje et al. 2006).

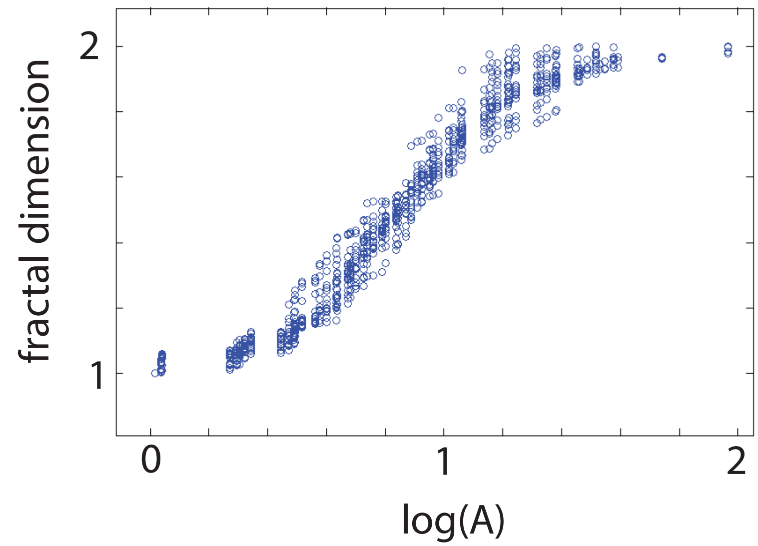
- Pond deepening by downward melting slows growth of pond area, **enhancing fractal dimension shift**.

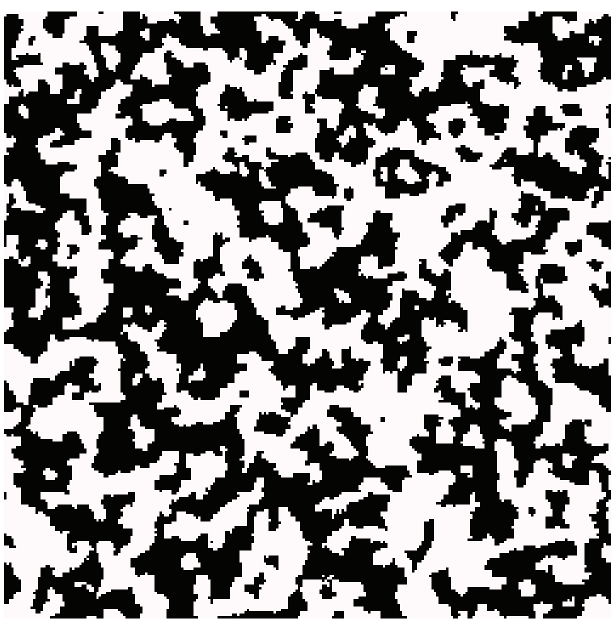


Lüthje, M., D. L. Feltham, P. D. Taylor, and M. G. Worster (2006), Modeling the summertime evolution of sea-ice melt ponds, J. Geophys. Res., 111, C02001, doi:10.1029/2004JC002818.

simple stochastic growth model of melt pond evolution







voter model

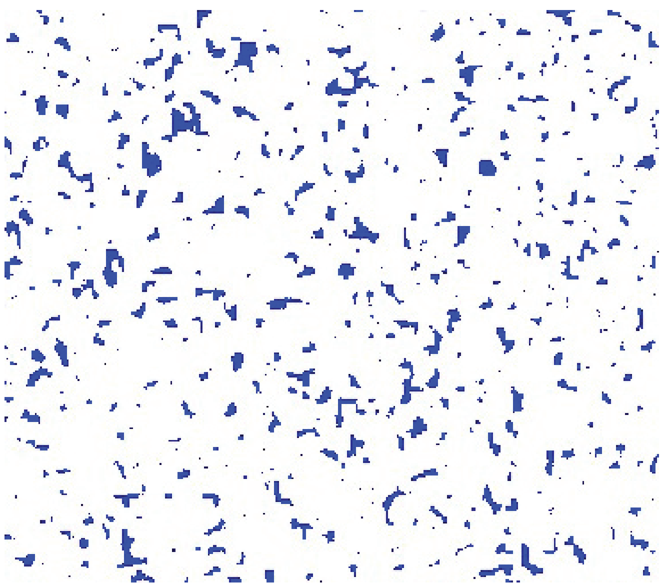
a square is more likely to melt if its neighbors have melted

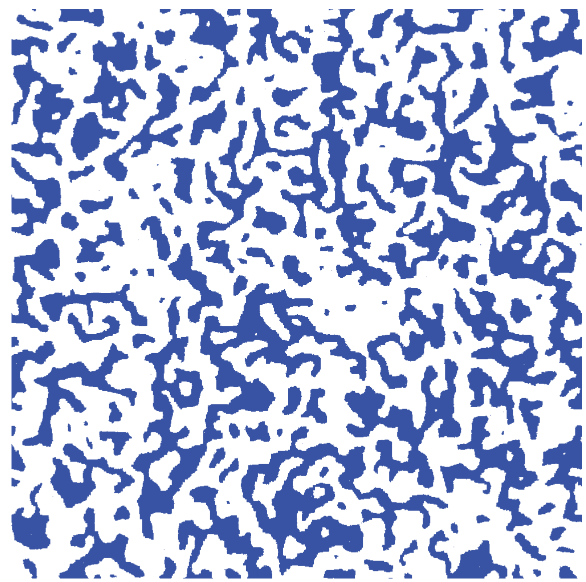
Rebecca Nickerson (West HS, Salt Lake City) and Ken Golden

Ising model for ferromagnets → Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow & -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

magnetization $M = \lim_{N\to\infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$ **pond coverage** $\frac{(M+1)}{2}$





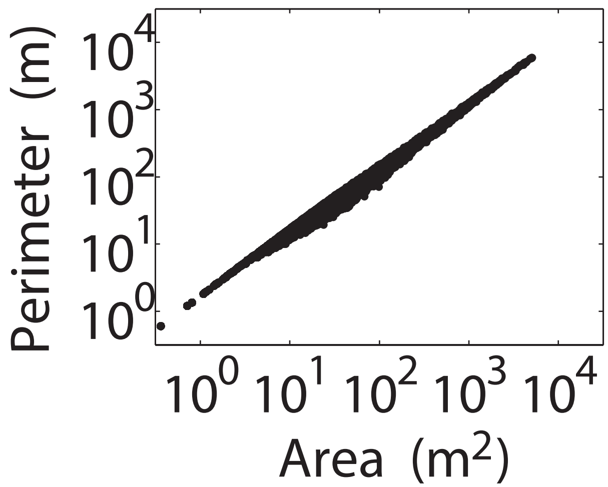
"melt ponds" are clusters of magnetic spins that align with the applied field

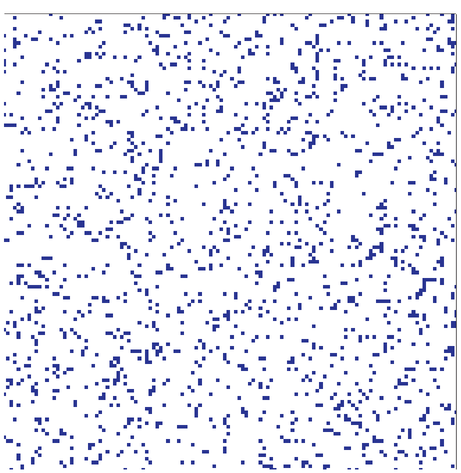
Ma, Sudakov, Golden 2014
(Thekkedath, Alali, Strong, Golden)

Melt Pond Ising Model

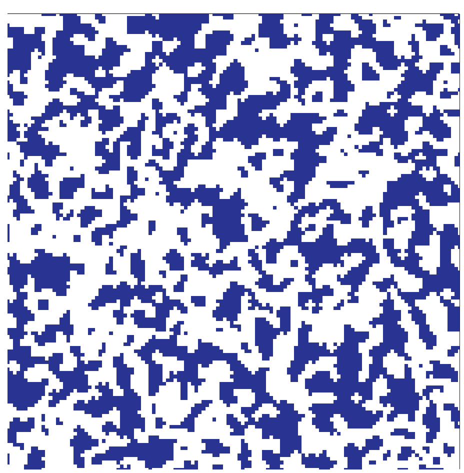
- Minimize an Ising Hamiltonian
  random magnetic field represents the initial ice topography
  interaction term represents horizontal heat transfer

- Ice-albedo feedback incorporated by taking coupling constant in interaction term to be proportional to the pond coverage





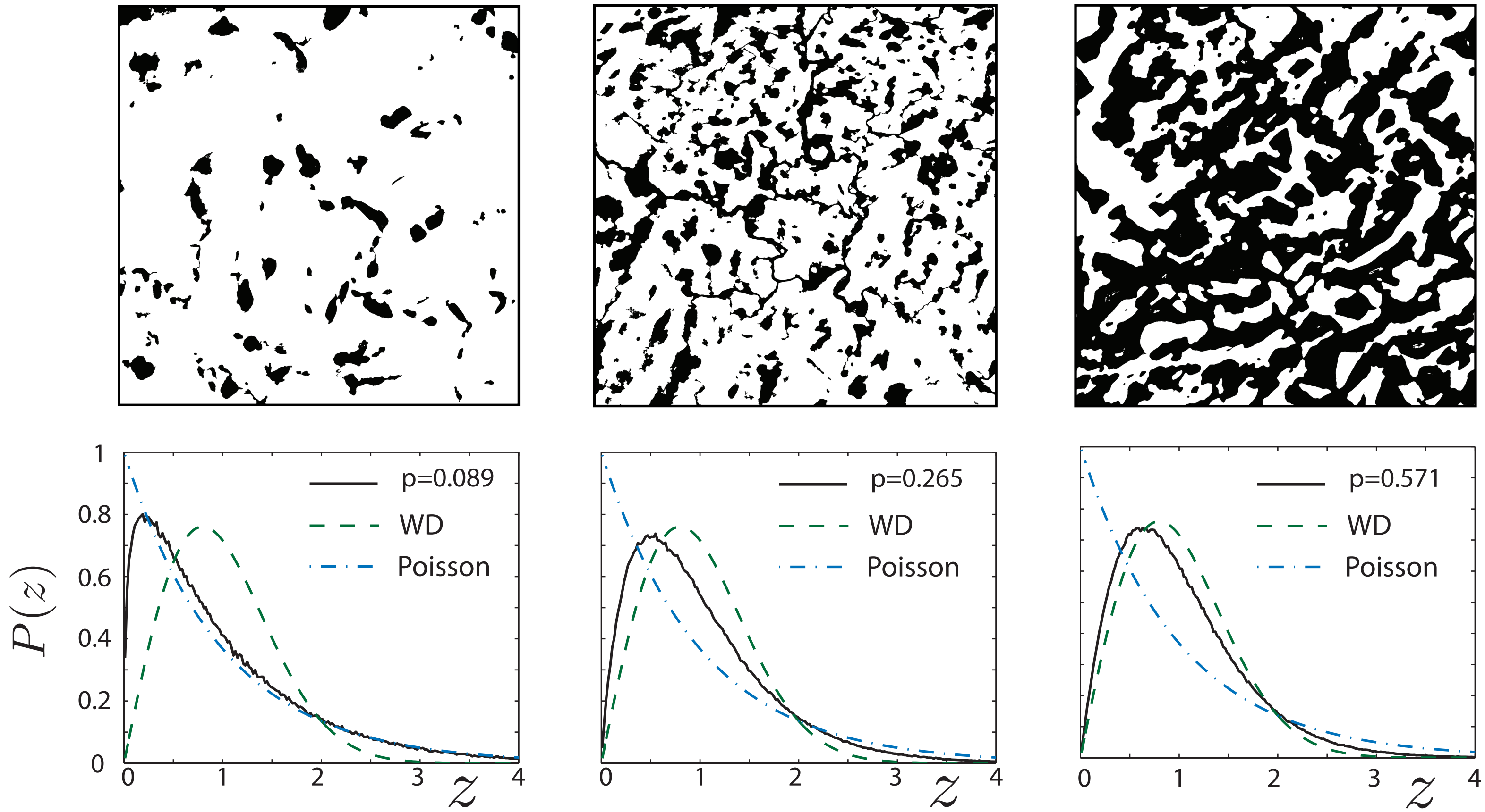
predicted length scale of fractal transition agrees well with data



random matrix characterization of connectedness transition -- discretization of $\chi\Gamma\chi$

Unfolded Eigenvalue Spacing Distribution

ARCTIC MELT PONDS



eigenvalue statistics for transport tend toward the UNIVERSAL Wigner-Dyson distribution as the "conducting" phase becomes connected over large scales

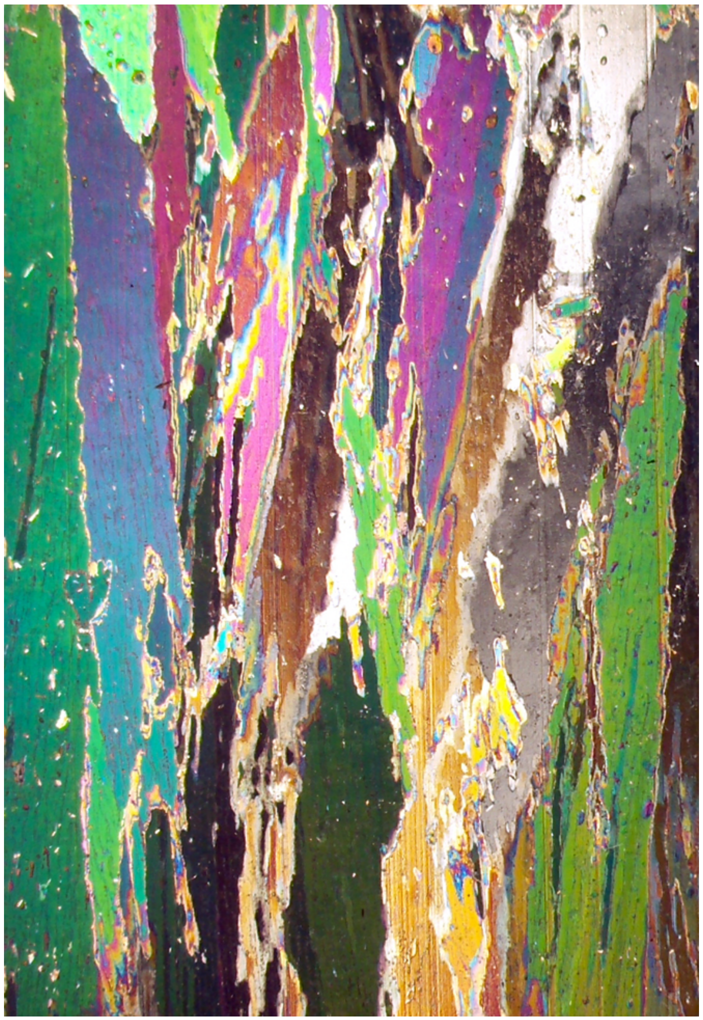
uncorrelated → *"level repulsion"*

N. B. Murphy, K. M. Golden 2014

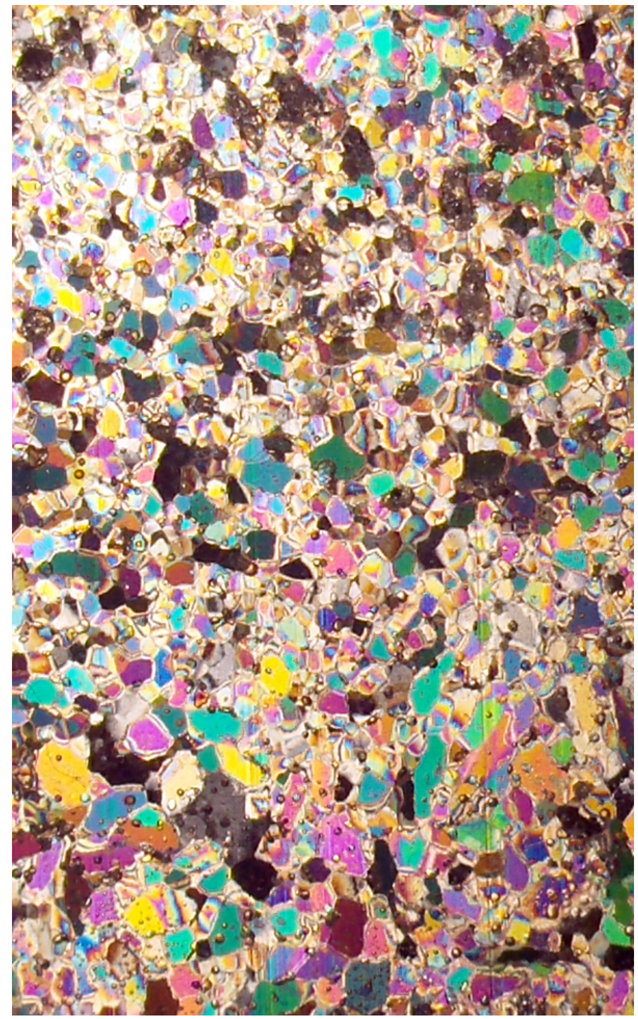
a few results related to melt pond evolution

higher threshold for fluid flow in Antarctic granular sea ice

columnar



granular

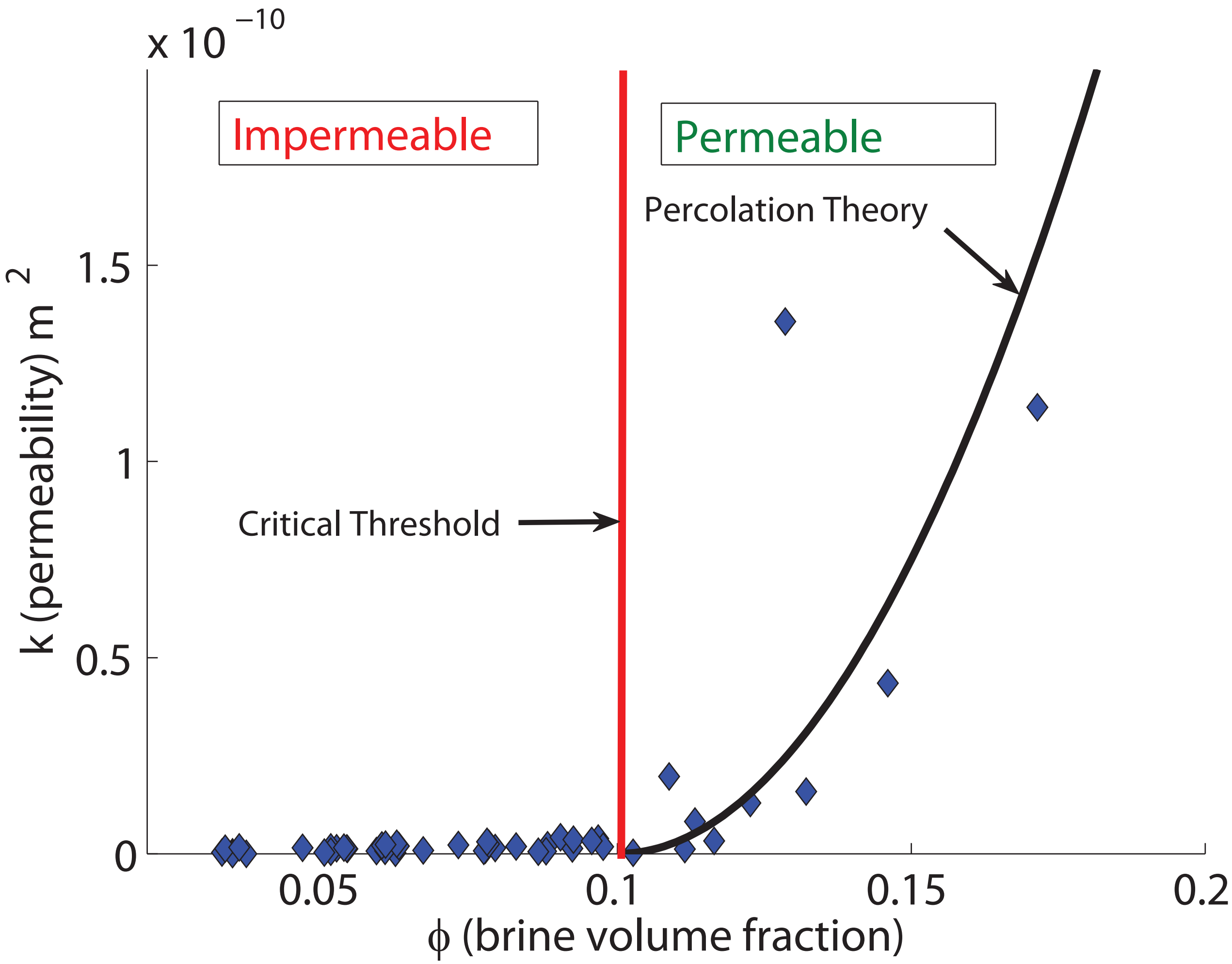


5%

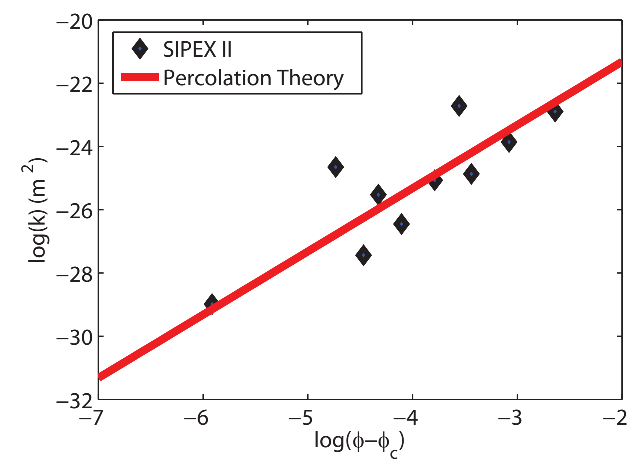
10%

Golden, Gully, Sampson, Lubbers, Tison 2014

SIPEX II vertical permeability data



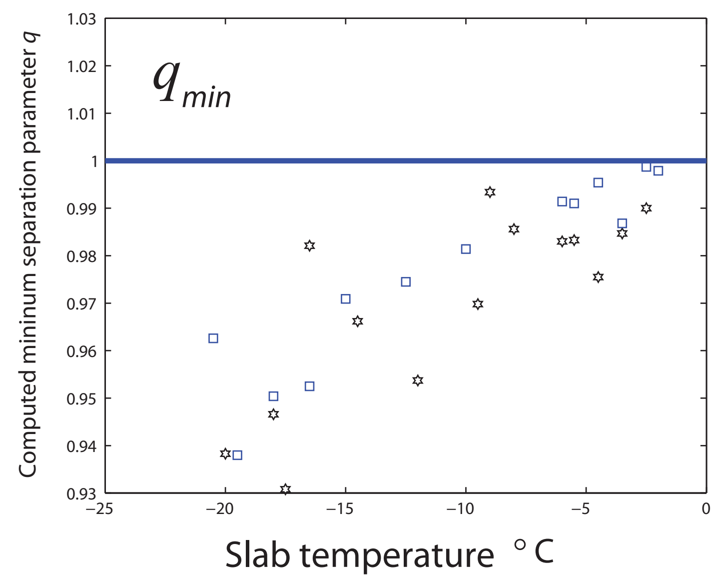
data above threshold



forward and inverse bounds on the complex permittivity of sea ice

inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012



polycrystalline bounds
two-scale homogenization

Gully, Lin, Cherkaev, Golden, 2014

electromagnetically distinguish between granular and columnar sea ice

Arctic melt ponds and bifurcations in the climate system

I. Sudakov, S. A. Vakulenko, and K. M. Golden

Communications in Nonlinear Science and Numerical Simulation, in press, 2014.

investigate effect of fractal transition in melt pond geometry on conceptual climate models, bifurcations, multiple equilibria, etc.

The Conundrum of Melt Pond Formation:

How can ponds form on top of sea ice that is highly permeable?

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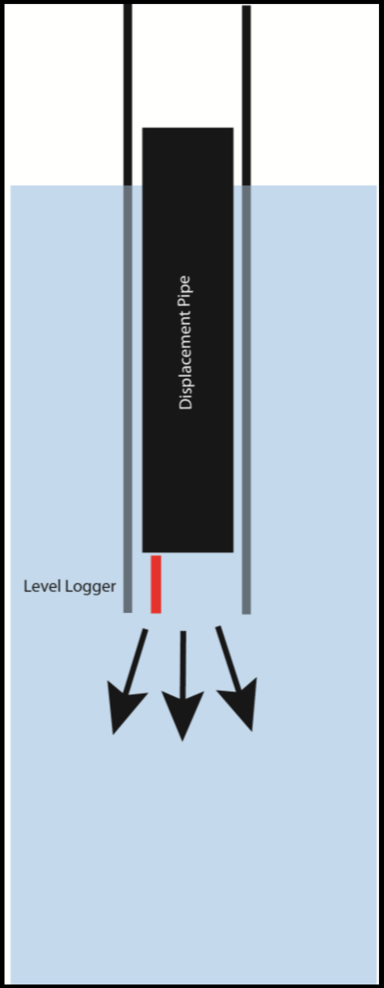
2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE)

aboard USCGC Healy

Hypothesis – Freshwater re-seals ice

Borehole test with varying salinity



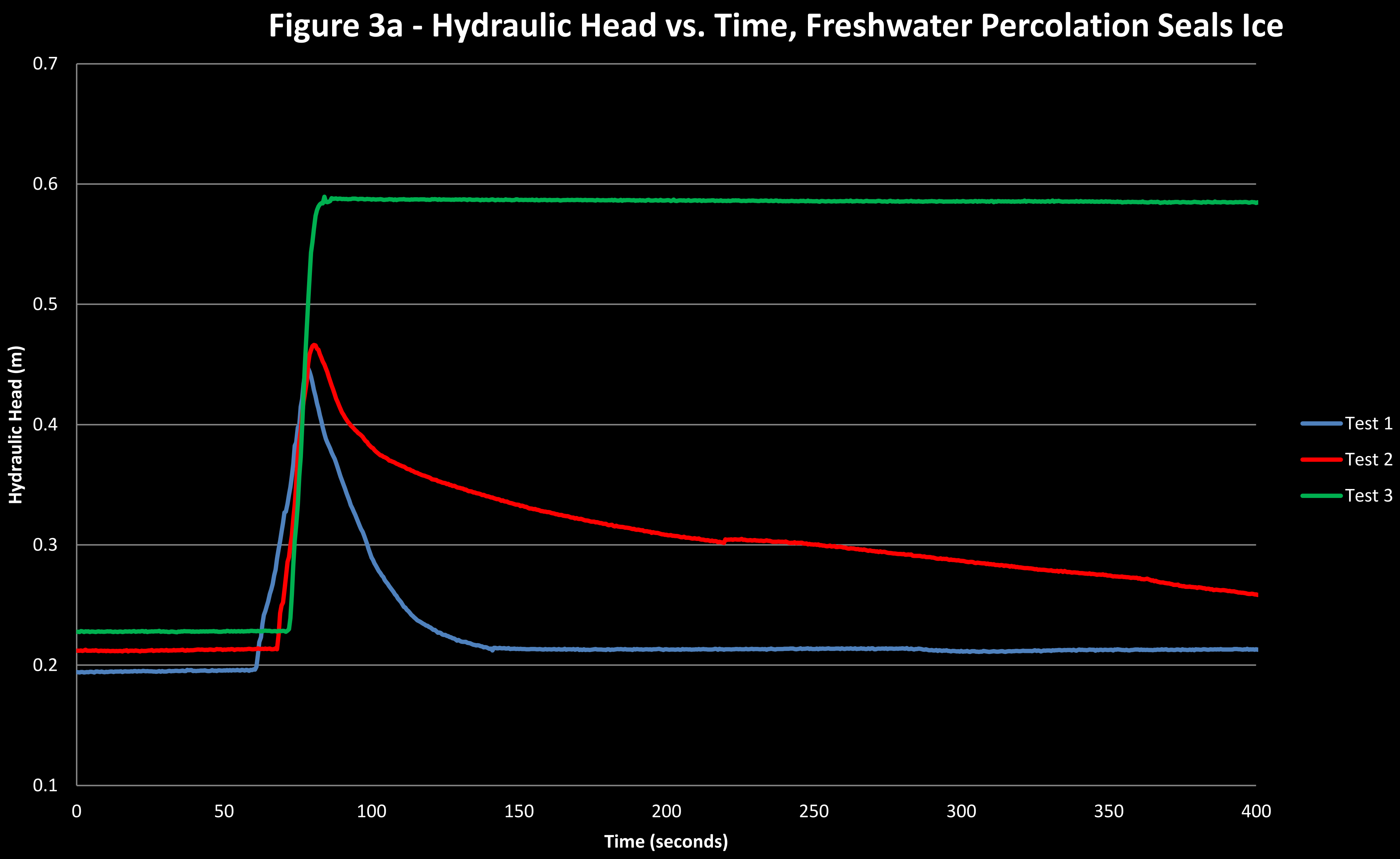
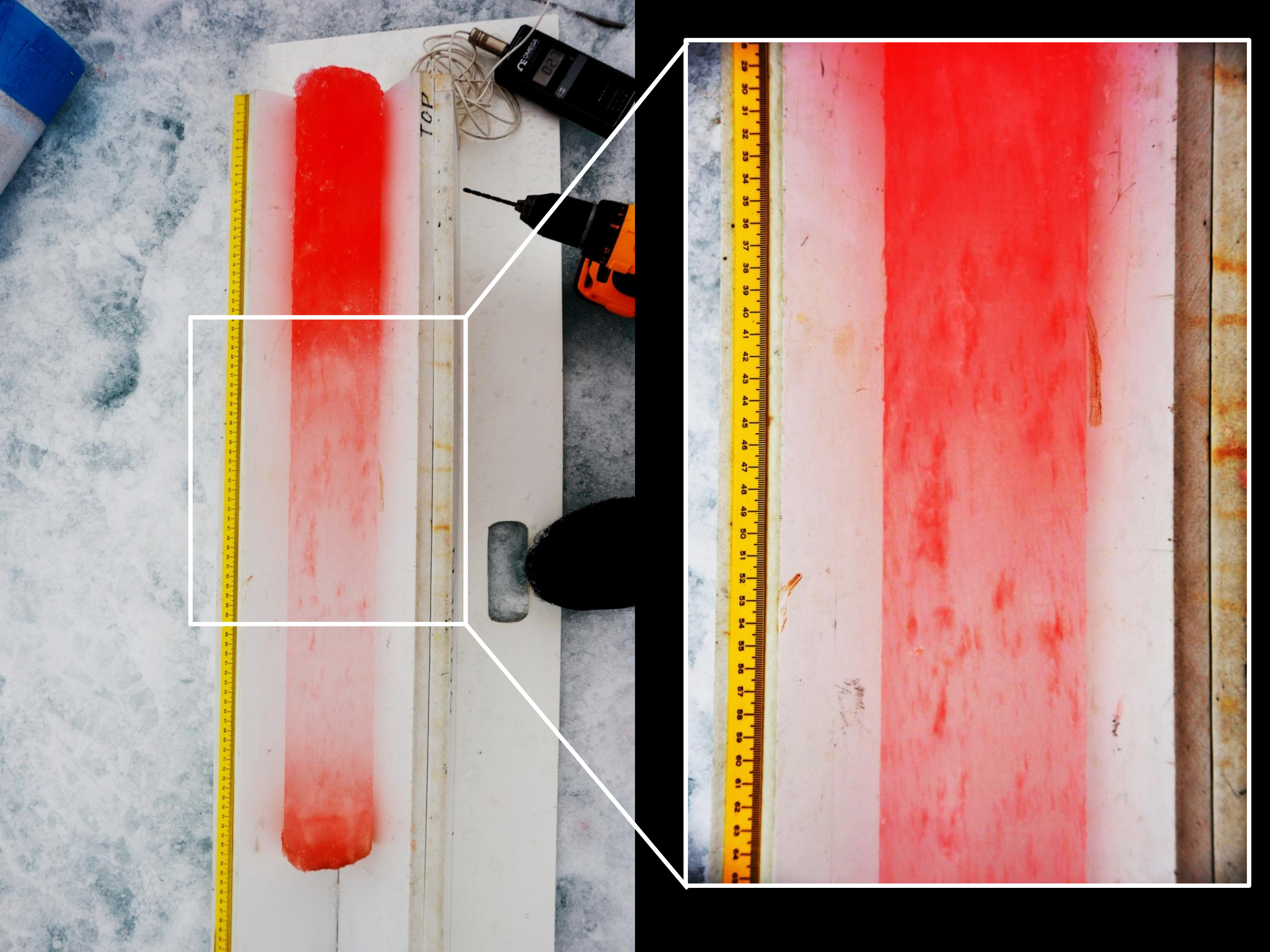


Figure 3a - Hydraulic Head vs. Time, Freshwater Percolation Seals Ice
Hydraulic Head (m)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
50
100
150
200
250
300
350
400
Time (seconds)
Test 1
Test 2
Test 3

Dyed tracer confirms that ice was permeable and blockage occurs within the ice matrix


OMEGA
TOP

Summary

1. Summer Arctic sea ice is melting rapidly, and understanding melt pond evolution is critical to improving projections.

2. Melt ponds exhibit a fundamental transition in fractal geometry around a critical length scale.

3. Mathematical models of composite materials and statistical physics help unravel the complexities of sea ice structure and processes, and provide a path toward rigorous representation of sea ice in climate models .

THANK YOU

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THE UNIVERSITY OF UTAH

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999