Mathematics 1220 PRACTICE EXAM I Spring 2002

ANSWERS

1. a. $\dfrac{1}{e^2}$ (Hint: first show $e^x = \lim_{h\to 0}(1+xh)^{1/h}$.)

   b. 1 c. 0

2. a. $(\sinh x \cosh x)^{-1}$ b. $\dfrac{1}{4}\ln(2z^2+8)+C$

   c. $\ln|\sec(\ln x)|+C$ d. $\ln\left|\dfrac{x}{x-1}\right|+C$

   e. $y(x)\left(\dfrac{4x}{3x^2+9}+\dfrac{6}{3x+2}-\dfrac{1}{2x+2}\right)$

   f. $\arctan(e^x)+C$

3. $\dfrac{dP}{dx}=-kP,\ \ k>0;\quad P(x)=P_0e^{-kx},\ \ k=(\ln 2)/6000$

4. 5,565 years ago

5. 32. $\ln|e^x-1|+\mathrm{C}$

   38. $V=2\pi\int_0^1 xe^{-x^2}\,dx=\pi\left(1-\dfrac{1}{e}\right)$

6. $y(x)=\dfrac{\frac{1}{3}x^3-x}{x-1}+\dfrac{C}{x-1}$

7. See class notes.

8. 70%

9. $\theta(t)=T+(\theta_0-T)e^{-kt}$. The corpse is discovered at $t=0$ (or 2 pm), and has a temperature $\theta_0=\theta(0)=85°$F. Assume that t_d is the time of death and that $\theta(t_d)=\theta_d=98.6°$F. Thus

$$t_d=-\frac{1}{k}\ln\left(\frac{\theta_d-T}{\theta_0-T}\right)=-1.176$$

or about 12:49 pm.