Mathematics 1220 COMPUTER EXERCISES Spring 2002

Complete the following exercises on a computer (except where otherwise indicated, where you will need to do it by hand on regular paper). You are advised to use Maple in solving the problems. The assignment is due Friday, April 26.

1. Find the Taylor series expansion to order 10 of $f(x) = 1/(1-x)$ around $x = 0$. On the same axes, plot the graphs of $1/(1-x)$ and the Taylor polynomials $P_1(x), P_2(x)$, and $P_3(x)$. Do the same for $g(x) = \sin x$, and plot the simultaneous graphs of $\sin x, P_1(x), P_3(x)$, and $P_5(x)$. Be sure to display these graphs on an appropriate scale, and indicate on your plots which curves correspond to which Taylor polynomials.

2. Use the trapezoidal rule with various values for n to approximate $I = \int_0^1 x^3\, dx$. You should demonstrate how, as n increases, the accuracy improves. Find the smallest n such that the approximation I_n is accurate to within four decimal places (that is, the smallest n such that if I_n is rounded off to four decimal places, then the result is equal to the actual value $I = 1/4$). Also, find an accurate numerical value for $\int_{-\infty}^{-\infty} e^{-x^2/2}\, dx$.

3. Let $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ +1, & 0 < x < \pi \end{cases}$. As in problem #31 of section 8.2, let $\mathcal{F}_N$ be the N^{th} Fourier approximant to $f(x)$, given by

$$\mathcal{F}_N(x) = \sum_{n=1}^{N} a_n \sin{(nx)}, \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin{(nx)}\, dx.$$

   Have the computer calculate a_1 through a_{10}, and then make three separate graphs, each having $f(x)$ plotted simultaneously with $\mathcal{F}_N(x)$, for $N = 1, 3, 5$. Check your results by calculating a_n analytically.

4. Use Newton's method for the equation $f(x) = x^3 - 6 = 0$ to approximate $\sqrt[3]{6}$ to five decimal places. Plot $f(x)$ in an appropriate range to obtain a reasonable first guess to start the iteration.

5. Consider the local extrema of $f(x) = (\sin x)/x$. Find the location of the minimum of $f(x)$ which is nearest the origin, with $x > 0$, by finding a fixed point solution of $x = \tan x$ through iteration. (Hint: the fixed point method cannot be directly applied in this case – you must first modify the problem.)

6. p. 515, Exercise 4 (a-f): the period doubling route to chaos.

7. EXTRA CREDIT: Create a plot of the pitchfork diagram associated with the previous problem on the period doubling route to chaos, i.e., a graph of the values in the attractor vs. the parameter λ.