Mathematics 1220 PRACTICE EXAM III Spring 2013

1. Use the geometric series to find the Taylor series for $f(x) = \ln(1+x)$ around $x = 0$. Determine the radius of convergence of this series. Explain your result for the radius in terms of the singularity of f. Do the same for $f(x) = 1/(1+x)^2$.

2. Use the geometric series to find the Taylor series for $f(x) = \frac{1}{4+x^2}$ around $x = 0$. Determine the radius of convergence for this series.

3. Use the Taylor series around $x = 0$ for $\sin x$ and the trigonometric identity $2 \sin x \cos x = \sin(2x)$ to determine the Taylor series around $x = 0$ for $\sin^2 x$. Also determine the radius of convergence.

4. Find the Taylor series for $\cosh x$ around $x = 0$ by using the series for e^x. What is its radius of convergence?

5. Find the convergence set for the following power series. For (a), also analyze the type of convergence (or divergence) at the endpoints of the convergence set.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n\, 2^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} f_n\, x^n$, where $\{f_n\}$ is the Fibonacci sequence

6. Find the following limits. Be sure to fully justify your answers.

   (a) $\displaystyle\lim_{n\to\infty} e^{-n} \sin n$ (b) $\displaystyle\lim_{n\to\infty} (2n)^{1/2n}$

   (c) $\displaystyle\lim_{n\to\infty} \left(1 - \frac{1}{n}\right) \cos n\pi$ (d) $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \left(\exp\left(\frac{k}{n}\right)^2\right) \frac{1}{n}$

7. A (zero dimensional) bull frog initially jumps a meter. On each successive jump, he can only go $\frac{3}{4}$ of the distance of the previous jump. If he takes infinitely many jumps, how far does he travel?

8. Determine whether the following infinite series converge or diverge. If a series converges, determine whether the convergence is absolute or conditional. Be sure to justify your answers completely.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\tan^{-1} n}{1+n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \sqrt{1 - \cos\left(\frac{1}{n}\right)}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{100}}{n!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$ (e) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\pi}}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$

9. Use a power series to find the solution $y(x)$ to the differential equation

   $$\frac{d^2y}{dx^2} = -y$$

   with initial conditions $y(0) = 0$ and $y'(0) = 1$.

10. Find the Taylor series for $f(x) = \sqrt{x-2}$ around the point $x = 3$. Find the radius of convergence of this series.