**Mathematics 1220 PRACTICE EXAM III Spring 2013**

**ANSWERS**

1. $\ln(1+x) = x - x^2/2 + x^3/3 - x^4/4 + ...$, radius $= 1$
   $1/(1+x)^2 = 1 - 2x + 3x^2 - 4x^3 + ...$, radius $= 1$
2. $\frac{1}{4+x^2} = \frac{1}{4}\frac{1}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{4}\left(1 - \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^4 + \dots\right)$, radius $= 2$
3. $\sin^2 x = \frac{1}{2}\left(\frac{(2x)^2}{2!} - \frac{(2x)^4}{4!} + \frac{(2x)^6}{6!} - \dots\right)$, infinite radius
4. $\cosh x = 1 + x^2/2! + x^4/4! + x^6/6! + ...$, infinite radius
5. (a) $(-1, 1/3)$, conditionally convergent at $-1$, divergent at $1/3$
   (b) $(-1/\tau, 1/\tau),\ \tau = (1+\sqrt{5})/2$
6. (a) $|e^{-n}\sin n| \le e^{-n} \longrightarrow 0$
   (b) 1
   (c) Does not exist due to oscillation
   (d) $\displaystyle\int_0^1 e^{x^2}\,dx = 1 + \frac{1}{3} + \frac{1}{5\cdot 2!} + \frac{1}{7\cdot 3!} + ...$
7. 4 meters
8. (a) converges conditionally
   (b) $\sqrt{1 - \cos\left(\frac{1}{n}\right)} \sim \frac{1}{n} \Longrightarrow$ divergence
   (c) converges absolutely (ratio test)
   (d) diverges ($n^{th}$ term test)
   (e) converges absolutely (integral test)
   (f) converges absolutely (ratio test)
9. $y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots,\ \frac{d^2y}{dx^2} = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \dots,$ and we have $\frac{d^2y}{dx^2} = -y$ and the initial conditions $y(0) = a_0 = 0$ and $y'(0) = a_1 = 1$, so we find
   $$a_2 = \frac{-a_0}{2} = 0 \quad a_3 = \frac{-a_1}{6} = \frac{-1}{3!}$$
   $$a_4 = \frac{-a_2}{12} = 0 \quad a_5 = \frac{-a_3}{20} = \frac{1}{5!}$$
   and so on, so $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sin x$.
10. Finding the first four terms. $f(x) = \sqrt{x-2}, f'(x) = \frac{1}{2\sqrt{x-2}}, f''(x) = \frac{-1}{4(x-2)^{3/2}} f'''(x) = \frac{3}{8(x-2)^{5/2}}$ Thus $f(3) = 1, f'(3) = \frac{1}{2}, f''(3) = -\frac{1}{4}, f'''(3) = \frac{3}{8}$. Using taylor's theroem
    $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$ With $f(x) = \sqrt{x-2}$ and $a = 3$ we get
    $\sqrt{x-2} = 1 + \frac{1}{2}(x-3) - \frac{1}{8}(x-3)^2 + \frac{3}{48}(x-3)^3 + \dots$
    The radius of convergence is $[2,4]$ since we require $|x-3| < 1$ and we have an alternating series with limit 0 as $n \to \infty$ at each end point on $[2,4]$