Mathematics 1220 PRACTICE EXAM I Spring 2013

1. Calculate the following limits.

   (a) $\lim_{n\to\infty} \left(\frac{n-1}{n}\right)^{2n}$

   (b) $\lim_{x\to 0} (1+4x)^{1/x}$

2. Calculate the following.

   (a) $\frac{d}{dx}(\ln(\tanh x))$

   (b) $\int \frac{z}{2z^2+8}\, dz$,

   (c) $\int \frac{\tan(\ln x)}{x}\, dx$,

   (d) $\int \frac{dx}{x(1-x)}$,

   (e) $\frac{dy}{dx}$, $y = \frac{(x^2+3)^{2/3}(3x+2)^2}{\sqrt{x+1}}$ (use logarithmic differentiation),

   (f) $\int \frac{e^x}{1+e^{2x}}$

3. Experiments show that the rate of change of the atmospheric pressure $P(x)$ with altitude x is proportional to the pressure. Find the differential equation for $P(x)$, and solve it, assuming that the pressure at 6000 meters is half its value P_0 at sea level.

4. Section 6.5 #18

5. Section 6.3 #40, 46

6. Section 6.6 #2, 4

7. Stewart wants to become a millionaire in 20 years by buying \$10,000 of a company's stock, which he wants to choose carefully. What must the sustained, annualized growth rate of the stock be in order to achieve his goal?

8. Newton's law of cooling states that the rate at which an object cools is proportional to the difference between the temperature $\theta(t)$ of the object and the constant ambient temperature T,

   $$\frac{d\theta}{dt} = -k(\theta - T),$$

   where $k > 0$ is a constant depending on the object. A corpse is discovered at 2 pm, and its temperature is found to be 85°F, with the ambient air temperature being 68°F. Assuming $k = 0.5$ hr^{-1}, find the time of death.

9. Section 6.8 #45, 51, 67

10. Section 6.9 #25, 33, 41, 45