Mathematics 1220 PRACTICE EXAM I Spring 2017

ANSWERS

1. Calculate the following limits.

   (a) $\lim_{n\to\infty}\left(\frac{n-1}{n}\right)^{2n} = e^{-2}$

   (b) $\lim\limits_{x\to 0}(1+4x)^{1/x} = e^4$

2. Calculate the following.

   (a) $\dfrac{d}{dx}(\ln(\tanh x)) = (\sinh x \cosh x)^{-1}$

   (b) $\displaystyle\int \frac{z}{2z^2+8}\,dz = \frac{1}{4}\ln(2z^2+8) + C$

   (c) $\displaystyle\int \frac{\tan(\ln x)}{x}\,dx = \ln|\sec(\ln x)| + C,$

   (d) $\displaystyle\int \frac{dx}{x(1-x)} = \ln\left|\frac{x}{x-1}\right| + C,$

   (e) $\dfrac{dy}{dx}$, $\quad y = \dfrac{(x^2+3)^{2/3}(3x+2)^2}{\sqrt{x+1}} \qquad \dfrac{dy}{dx} = \left(\dfrac{4x}{3x^2+9} + \dfrac{6}{3x+2} - \dfrac{1}{2x+2}\right)$

   (f) $\displaystyle\int \frac{e^x}{1+e^{2x}} = \arctan(e^x) + C$

3. Experiments show that the rate of change of the atmospheric pressure $P(x)$ with altitude x is proportional to the pressure. Write down the resulting differential equation for $P(x)$, and solve it, assuming that the pressure at 6000 meters is half its value P_0 at sea level.

   $\frac{dP}{dx} = -kP, \quad k > 0; \qquad P(x) = P_0 e^{-kx}, \quad k = (\ln 2)/6000$

4. Section 6.5 #18

   5,565 years ago

5. Section 6.3

   40. $\ln|e^x - 1| + C$

   46. $V = 2\pi\int_0^1 xe^{-x^2}\,dx = \pi\left(1 - \frac{1}{e}\right)$

6. Section 6.6

   2. $y(x) = \frac{\frac{1}{3}x^3 - x}{x-1} + \frac{C}{x-1}$

   4. $y(x) = \sin x + C\cos x$

7. Stewart wants to become a millionaire after 10 years by buying \$5,000 worth of a company's stock, which he wants to choose carefully. What must the sustained, annualized growth rate of the stock be in order to achieve his goal?
   $1,000,000 = 10,000(1+r)^{20}$ or $r = 0.2588...$, or about 26%

8. Newton's law of cooling states that the rate at which an object cools is proportional to the difference between the temperature $\theta(t)$ of the object and the constant ambient temperature T,

$$\frac{d\theta}{dt} = -k(\theta - T),$$

where $k > 0$ is a constant depending on the object. A corpse is discovered at 2 pm, and its temperature is found to be 85°F, with the ambient air temperature being 68°F. Assuming $k = 0.5$ hr^{-1}, find the time of death.

$\theta(t) = T + (\theta_0 - T)e^{-kt}$. The corpse is discovered at $t = 0$ (or 2 pm), and has a temperature $\theta_0 = \theta(0) = 85$°F. Assume that t_d is the time of death and that $\theta(t_d) = \theta_d = 98.6$°F. Thus

$$t_d = -\frac{1}{k}\ln\left(\frac{\theta_d - T}{\theta_0 - T}\right) = -1.176$$

or about 12:49 pm.

9. Section 6.8

   45. $x^2\left[\frac{xe^x}{1+e^{2x}} + 3\arctan(e^x)\right]$

   51. $\frac{3(1+\arcsin(x))^2}{\sqrt{1-x^2}}$

   67. $\frac{1}{3}\arcsin\left(\frac{\sqrt{3}}{2}x\right) + C$

10. Section 6.9

   25. $2\tanh(x)\cosh(2x) + \sinh(2x)\text{sech}^2(x)$

   33. $\frac{1}{\sqrt{x^2-1}\cosh(x)}$

   41. $2\cosh(2z^{\frac{1}{4}}) + C$

   45. $\frac{1}{4}\left[\ln(\sinh(x^2))\right]^2 + C$