Math 5470 Spring 2019 Practice Midterm 1

1. Analyze the equation graphically. Find all fixed points, classify their stability, graph the phase line, and sketch the time course x *vs.* t for different initial conditions.

$$\dot{x} = x(1-x)(2-x).$$

2. For the given flow on the circle

$$\dot{\theta} = \mu - 2\sin(\theta),$$

a) draw all qualitatively different scenarios in the phase space as μ is varied,
b) name the bifurcations that occur,
c) sketch a bifurcation diagram,
d) sketch the time course of the solution which starts with $\mu = 1, \theta(0) = 0$, and for which, starting at $t = 1$, μ slowly increases to 3 and stays there.

3. Find and classify the fixpoints. Sketch the nullclinesm the vectior field, and a plausible phase portrait.

$$\begin{aligned} \dot{x} &= x(2-x-y), \\ \dot{y} &= x-y. \end{aligned}$$

4. Consider

$$\ddot{x} = x - x^4.$$

a) Find and classify the equilibrium points.
b) Find a conserved quantity.
c) Sketch a phase portrait.
d) What is different about the solutions that start inside *vs.* outside of the homoclinic orbit in the phase portrait?

5. Show that closed orbits are impossible for the system:

$$\begin{aligned} \dot{x} &= x(6-x-4y), \\ \dot{y} &= y(2-x-y). \end{aligned}$$