Math 5470 Spring 2019 Practice Final

1. Consider the system

$$\begin{aligned}\dot{x} &= -x + 4y, \\ \dot{y} &= -x - y^3.\end{aligned}$$

   (a) What does it mean for a function $V(x, y)$ to be a Liapunov function for this system?

   (b) Determine whether the function $V(x, y) = -2x^2 - 8y^4$ is a Liapunov function for this system.

   (c) If $V(x, y)$ is a Liapunov function, what can it be used to show about the fixed point at the origin? What can it be used to show about the existence and stbility of limit cycles?

2. For the system $\dot{x} = \mu x - x^3$, $\dot{y} = -y$, find the fixed points and classify their stability as functions of μ. Find the eigenvalues at the stable fixed points as functions of μ. Show that one of the eigenvalues tents to 0 as $\mu \to 0$. Does an eigenvalue of the Jacobian always go to zero at a bifurcation point?

3. Use the Poincare Bendixson theorem to show that the system below has a periodic solution.

$$\begin{aligned}\dot{x} &= x - y - x^3, \\ \dot{y} &= x + y - y^3.\end{aligned}$$

4. Analyze the following equation graphically. Find all fixed points, classify their stability, graph the phase line and sketch the time course of solutions for different initial conditions.

$$\dot{x} = 4x - 5x^2 + x^3.$$

5. For the given flow on the circle

$$\dot{\theta} = \mu - \sin\theta$$

   (a) Draw all qualitatively different scenarios in the phase space as parameter μ is varied.

   (b) For which values of μ does this problem have a periodic solution?

   (c) Name the bifurcations that occur and sketch a bifurcation diagram.

   (d) For $\mu = 1/2$, sketch the time course (in the t-θ plane) of the solution with $\theta(0) = 0$.

   (e) For $\mu = 1.01$, sketch the time course (in the t-θ plane) of the solution with $\theta(0) = 0$.