Lec 36

9.2 The Two-Sample t Test and Confidence Interval

$X_1^1 \cdots X_{n_1}^1$

$X_1^2 \cdots X_{n_2}^2$

Pooled t Procedures

- Consider 2 independent samples from 2 normally dist. populations.
- with small sample size for at least one sample.
- Population variances $\sigma_1^2 = \sigma_2^2 = \sigma^2$ is unknown

$$\frac{(X_1^1 - \bar{X}_1)^2 + \cdots + (X_{n_1}^1 - \bar{X}_1)^2 + (X_1^2 - \bar{X}_2)^2 + \cdots + (X_{n_2}^2 - \bar{X}_2)^2}{n_1 + n_2 - 2}$$

The pooled test statistic uses a weighted average of the two sample variances:

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

Null Hypothesis: $H_0: \mu_1 - \mu_2 = \Delta_0$

Test statistic:

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\underbrace{\mu_1 - \mu_2}_{\Delta_0})}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

a t-dist. w/ $n_1 + n_2 - 2$ df

Significant level:

Alternative Hypothesis:

$H_a: \mu_1 - \mu_2 > \Delta_0$ reject if $T > t_{\alpha, n_1+n_2-2}$

$H_a: \mu_1 - \mu_2 < \Delta_0$ reject if $T < -t_{\alpha, n_1+n_2-2}$

$H_a: \mu_1 - \mu_2 \neq \Delta_0$ reject if $|T| > t_{\frac{\alpha}{2}, n_1+n_2-2}$

Assumptions:

- pop. normal
- two indep. samples
- one has small size
- $\sigma_1^2 = \sigma_2^2$ unknown

Provided the above assumptions in a pooled t test, a CI for $\mu_1 - \mu_2$ with a confidence level of $100(1-\alpha)\%$ is

$$\bar{X}_1 - \bar{X}_2 \pm t_{\frac{\alpha}{2}, n_1+n_2-2} \sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Example 103.

Many homeowners use tiki torches for outside decoration and to burn special oil to repel insects. Independent random samples of two types of oil were obtained and the burn time for 3 ounces of each was recorded (in hours). The summary statistics are given in the following table. Assume the underlying populations of burn times are normal.

Assume $\sigma_1^2 = \sigma_2^2$.

Oil	Sample Size	Sample Mean	Sample Variance
Citronella Torch Fuel	18	6.25	1.04
Black Flag Mosquito Control	24	5.98	0.77

$n_1 = 18$

$n_2 = 24$

Based on the sample variances, do you think the assumption of equal population variances is reasonable? Why or why not?

Want to test if mean burning time differ.

use $\alpha = 0.01$.

$H_0: \mu_1 = \mu_2$ $(\mu_1 - \mu_2 = 0)$

$H_a: \mu_1 \neq \mu_2$ $(\mu_1 - \mu_2 \neq 0)$

$s_1^2 = 1.04$

$s_2^2 = 0.77$

$$T = \frac{(6.25 - 5.98) - 0}{\sqrt{✿ \cdot \left(\frac{1}{18} + \frac{1}{24}\right)}} = 0.922$$

$$S_p^2 = \frac{17 \times 1.04 + 23 \times 0.77}{18 + 24 - 2} = ✿$$

$$t_{0.005,\ \underbrace{18+24-2}_{40}} = 2.704$$

p-value

0.922

reject if $|0.922| > 2.704$

~~go~~ fail to reject

Example 104.

Frequently, patients must wait a long time for elective surgery. Suppose the wait time for patients needing a knee replacement at two hospitals was investigated. Independent random samples of patients were obtained and the wait time for each (in weeks) was recorded. The resulting summary statistics are given in the following table.

Hospital	Sample Size	Sample Mean	Sample Variance
Hospital 1	15	17.4	34.81
Hospital 2	17	12.1	46.24

$n_1 = 15$ $\quad S_1^2 = 34.81$

$n_2 = 17$ $\quad S_2^2 = 46.24$

$\bar{X}_1 = 17.4$

$\bar{X}_2 = 12.1$

(a) Do the samples give enough evidence to conclude that the mean wait time at hospital 1 is longer than that of hospital 2? Assume that the unknown population variances are equal and test using $\alpha = 0.05$. [12 points] and populations are normal

(b) Find bounds on the p-value associated with the test. [2 points]

(c) How much is the mean wait time for hospital 1 longer than the mean for hospital 2? Calculate a 90% confidence interval to estimate the true value of $\mu_1 - \mu_2$. [4 points]

(d) Besides the population variances being equal, what else must be true (or what else must we assume) about the populations for parts (a) and (b) to be valid? [2 points]

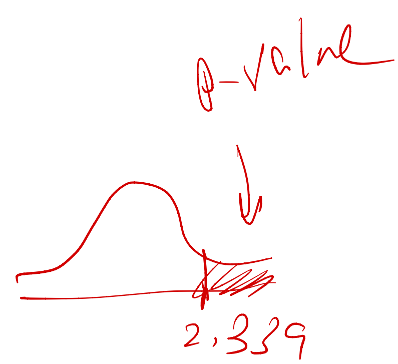
$$H_0: \mu_1 = \mu_2 \quad (\mu_1 - \mu_2 = 0) \qquad H_a: \mu_1 > \mu_2$$

$$t = \frac{17.4 - 12.1 - 0}{\sqrt{S_p^2 \cdot \left(\frac{1}{15} + \frac{1}{17}\right)}} = 2.339$$

$$S_p^2 = \frac{14 \times 34.81 + 16 \times 46.24}{15 + 17 - 2} = \ldots$$

$$t_{0.05,\, \underbrace{15+17-2}_{30}} = 1.697$$

reject if $2.339 > 1.697$



we do reject