So the pavement will not be used unless the null hypothesis is rejected.

Step 2: Assume the H_0 is true.

Step 3: The significant level is not specified, so we use $\alpha = 0.05$. Since we have a lower-tail test and $\Phi(1.645) = 0.95$, the critical value is $-z_{0.05} = -1.645$. RR: we reject H_0 if $Z \leq -1.645$.

Step 4: Test statistics

$$Z = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{28.76 - 30}{12.2647/\sqrt{52}} = -0.73$$

Step 5: Decision: since $Z = -0.73 > -1.64$, we fail to reject H_0.

Step 6: Conclusion: Based on our evidence at α= 0.05, we conclude that the use of the pavement is not justified.

Let's achieve the same conclusion by calculating the P-value.

Lec 34

Example 97 continued : Light bulbs of a certain type are advertised as having an average lifetime of 750 hours. The price of these bulbs is very favorable and so a potential customer has decided to go ahead with the purchase unless it can be conclusively demonstrated that the true average lifetime is smaller than what is advertised. A random sample of 20 bulbs was selected and the lifetime of each was recorded. Suppose that the sample mean was 738.4 hours with a sample standard deviation of 41.2 hours. Does the sample provide evidence that the true mean lifetime is less than 750? Assume lifetimes vary according to a normal distribution and test using $\alpha = 0.10$.

Solution. Given information $\mu = 750$, $n = 20$, $\overline{x} = 738.4$, $s = 41.2$, $\alpha = 0.1$.

Step 1: $H_0 : \mu = 750$ vs $H_A : \mu < 750$ (lower-tail test)

$$T = \frac{738.4 - 750}{41.2/\sqrt{20}} = -1.259$$

Step 2: Assume H_0 is true.

Step 3: From t-table, we have $t_{n-1,\alpha} = t_{19,0.1} = 1.328$. Since we have a left tail test, so the critical value is $-t_{19,0.1} = -1.328$. We will reject H_0 if T is smaller than -1.328. RR: $T \leq -1.328$

$t_{19,0.1} = 1.328$

$-1.259 > -1.328$

fail to reject $\quad -t_{\alpha, n-1}$

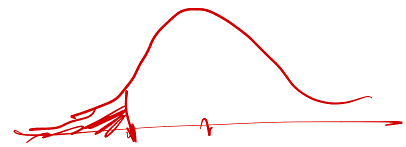
Step 4:

$$t = \frac{\overline{x} - \mu_{\overline{X}}}{s/\sqrt{n}} = -1.259 > -1.328$$

Step 5: Decision: we fail to reject H_0.

Step 6: Conclusion: Based on our evidence at $\alpha = 0.1$, we conclude that the sample does not have enough evidence to show the true mean bulb lifetime is less than 750 hours.

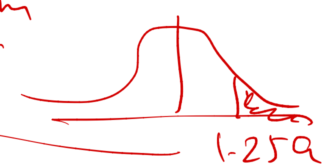
Let's achieve the same conclusion by calculating the P-value.



P-value > 0.1

-1.259

t-table w/ 19 deg. of freedom



fail to reject

1.259

Example 98 continued : After a 24-hour smoking abstinence, each of 20 smokers was asked to estimate how much time had elapsed during a 45-second period. The collected elapsed time gives sample mean $\overline{x} = 59.3$ sec and sample sd $s = 9.84$ sec. Assume the data follows a normal distribution. Let's carry out a test of hypotheses at significance level 0.05 to decide whether true average perceived elapsed time differs from the known time 45 sec.

Solution. μ = true average perceived elapsed time for all smokers.

$n=20$
$\overline{X}=59.3$
$s=9.84$
$\alpha=0.05$

Hypothesis: $\mu = 45$ vs $\mu \neq 45$

Test statistic:

$$T = \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{59.3 - 45}{9.84/\sqrt{20}} = 6.5$$

Rejection Region: Since we have a two-tailed test , so we reject H_0 if $|T| \geq t_{n-1,\alpha/2} = t_{19,0.025} = 2.093$.

Decision: Since $T = 6.5 \geq 2.093$, we reject H_0.

Conclusion: Based on our evidence at $\alpha = 0.05$, we conclude that the true average perceived elapsed time is evidently something other than 45.

Let's achieve the same conclusion by calculating the P-value.

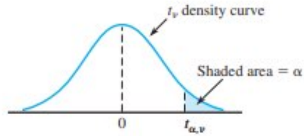
< 0.0005

-6.5 $\quad$ 6.5

P-val. < 0.001

< 0.05
so reject H_0

Table A.5 Critical Values for t Distributions



	α						
ν	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Example 99 continued : The use of a phone to text during an exam is a serious breach of conduct. One article reported that 27 of the 267 students in a sample admitted to doing this. Can it be concluded at significance level 0.001 that more than 5% of all students in the population sampled had texted during an exam?

Solution. The parameter of interest is the proportion p of the sampled population that has texted during an exam.

Step 1: Hypothesis: $H_0 : p = 0.05$ vs $H_A : p > 0.05$

Step 2: Assume H_0 is true and check conditions:

$$np_0 = (267)(0.05) = 13.35 \geq 10 \quad \text{and} \quad n(1-p_0) = (267)(0.95) = 253.65 \geq 10$$

the large-sample z test can be used.

Step 3: Since it is the upper-tail test, from the z-table we have the critical value is $z_{0.001} = 3.1$ which give RR : reject H_0 if $Z \geq 3.1$.

Step 4: Since $\hat{p} = \frac{27}{267} = 0.1011$, then the test statistic

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.1011 - 0.05}{\sqrt{\frac{(0.05)(0.95)}{267}}} = 3.84$$

Step 5: Decision: we reject H_0.

Step 6: Conclusion: Based on our evidence at $\alpha = 0.001$, we conclude that the evidence for concluding that the population percentage of students who text during an exam exceeds 5% is very compelling.

Let's achieve the same conclusion by calculating the P-value.

$$\text{P-Val.} = 1 - \underbrace{\Phi(3.84)}_{> 0.9998} < 0.0002$$

< 0.001

Reject H_0

Example 100 continued : A plan for an executive travelers' club has been developed by an airline on the premise that 5% of its current customers would qualify for membership. A random sample of 500 customers yielded 40 who would qualify. Using this data, test at level 0.01 the null hypothesis that the company's premise is correct against the alternative that it is not correct.

Solution.

Hypothesis: $H_0 : p = 0.05$ vs $H_A : p \neq 0.05$

Test statistic:

$$\hat{p} = \frac{40}{500} = 0.08$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.08 - 0.05}{\sqrt{\frac{(0.05)(0.95)}{500}}} = 3.0779$$

Rejection Region: $\alpha = 0.01$ and we have a two tailed test here, so the critical value is

$$z_{\alpha/2} = z_{0.005} = 2.575$$

So we reject H_0 when $|Z| \geq 2.575$.

Decision: since $Z = 3.0779 \geq 2.575$, we reject H_0.

Conclusion: Based on our evidence at $\alpha = 0.01$, we conclude that the company's premise is not correct.

Let's achieve the same conclusion by calculating the P-value.

9 Inferences Based on Two Samples

Chapters 7 and 8 presented confidence intervals (CI's) and hypothesis-testing procedures for a single mean μ, and a single proportion p. Chapter 9 extend these methods to situations involving the means, proportions, and variances of two populations.

9.1 z Tests and Confidence Intervals for a Difference Between Two Population Means

The inferences discussed in this section concern difference $\mu_1 - \mu_2$.

Basic Assumptions:

- Suppose we have a random sample from population 1 with mean μ_1, population standard deviation σ_1, sample size n_1, sample mean $\overline{X}_1$, and sample standard deviation S_1.
- Suppose we have a random sample from population 2 with mean μ_2, population standard deviation σ_2, sample size n_2, sample mean $\overline{X}_2$, and sample standard deviation S_2.
- The two samples are independent of one another.

The natural estimator of $\mu_1 - \mu_2$ is $\overline{X}_1 - \overline{X}_2$, the difference between the corresponding smaple means. Inferential procedures are based on standardizing this estimator, so we need expressions for the expected value and standard deviation of $\overline{X}_1 - \overline{X}_2$.

Let the random variable $Y = \overline{X}_1 - \overline{X}_2$, then

$$Y \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$\boxed{Z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)}$$

$$\overline{X}_1 \sim N\left(\mu_1, \frac{\sigma_1^2}{n_1}\right)$$

$$\overline{X}_2 \sim N\left(\mu_2, \frac{\sigma_2^2}{n_2}\right)$$

$\overline{X}_1 - \overline{X}_2$ has variance

$$\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$