Lec 31

8.2 z Tests for Hypotheses about a Population Mean

8.2.1 A normal Population distribution with Known σ

Null Hypothesis: $H_0: \mu = \mu_0$

Test statistic: Test stat. $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$

Significant level: α

Alternative Hypothesis:

$H_a: \mu > \mu_0$ reject H_0 if $Z > z_\alpha$ — critical value

$H_a: \mu < \mu_0$ reject H_0 if $Z < -z_\alpha$

$H_a: \mu \neq \mu_0$ reject H_0 if $Z > z_{\frac{\alpha}{2}}$ or $Z < -z_{\frac{\alpha}{2}}$

Assumptions:

Example 93. A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is 130°F. A sample of $n = 9$ systems yields a sample average activation temperature of 131.08°F. If the distribution of activation times is **normal with standard deviation** 1.5°F, does the data contradict the manufacturer's claim at significance level $\alpha = 0.01$?

Solution.

$H_0: \mu = 130$ $H_a: \mu \neq 130$

$n = 9$

$\sigma = 1.5$

$\alpha = 0.01$

$\bar{X} = 131.08$

$Z = \dfrac{131.08 - 130}{1.5/\sqrt{9}}$

$= 2.16$

$z_{0.005} = 2.58$

Q: $Z > 2.58$ or < -2.58

A: No. Fail to Reject H_0

Or: CI: $\bar{X} \pm 2.58 \cdot \dfrac{\sigma}{\sqrt{n}} = 131.08 \pm 2.58 \cdot \dfrac{1.5}{\sqrt{9}}$ 130 is inside

Example 94. The desired percentage of SiO_2 in a certain type of aluminous cement is 5.5. To test whether the true average percentage is 5.5 for a particular production facility, 16 independently obtained samples are analyzed. Suppose that the percentage of SiO_2 in a sample is **normally distributed** with $\sigma = 0.3$ and that $\overline{x} = 5.25$.

Does this indicate conclusively that the true average percentage differs from 5.5? Assume $\alpha = 0.05$

Solution.

$H_0: \mu = 5.5$ $H_a: \mu \neq 5.5$

$\bar{X} = 5.25$ $\sigma = 0.3$ $n = 16$

$z_{0.025} = 1.96$

$$z = \frac{5.25 - 5.5}{0.3/\sqrt{16}} = -3.33$$

Q. If $z > 1.96$ or < -1.96

A. yes. $\Rightarrow$ we reject H_0 in favor of H_a.

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w/ p-value: $N(0,1)$

$$\hookrightarrow = p(Z < -3.33 \text{ or } > 3.33)$$

$$= 2P(Z > 3.33)$$

$$= 2 * (1 - 0.9996) = 0.0008$$

8.2.2 Large-Sample Tests

When the sample size is large, the foregoing z tests are easily modified to yield valid test procedures without requiring either a normal population distribution or known σ.

When we have a large sample $n > 30$, $\overline{X}$ is approximately normal, σ is unknown, then we use the test statistic

$$Z = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

Example 95. (HW #6) The biological dessert in the Gulf of Mexico called the Dead Zone is a region in which there is very little or no oxygen. Most marine life in the Dead Zone dies or leaves the region. The area of this region varies and is affected by agriculture, fertilizer runoff, and weather. The long-term mean area of the Dead Zone is 5960 square miles. As a result of recent flooding in the Midwest and subsequent runoff from the Mississippi River, researchers believe that the Dead Zone area will increase. A random sample of 50 days was obtained and the sample mean area of the Dead Zone was 6759 mi^2 with a sample standard deviation of 1850 mi^2. Does the sample provide enough evidence to confirm the researchers' belief? Test using $\alpha = 0.025$.

Solution.

$H_0: \mu = 5960$

$H_a: \mu > 5960$

$n = 50$

$\overline{X} = 6759$

$S = 1850$

$\alpha = 0.025$

$z_{0.025} = 1.96$

$$Z = \frac{6759 - 5960}{1850/\sqrt{50}} = 3.05$$

Reject H_0 if $Z > 1.96$

Reject H_0

Example 96. Suppose that for a particular application it is required that the true average DCP value for a certain type of pavement be less than 30. The pavement will not be used unless there is conclusive evidence that the specification has been met. A descriptive summary obtained from a sample of $n = 52$ data shows that the sample mean $\overline{x} = 28.76$ and the sample sd $s = 12.2647$. Let's state and test the appropriate hypotheses for the use of the pavement.

Solution.