(d) $H_0 : p = 0.3$ vs $H_A : p = 0.5$ Not in this course

(e) $H_0 : \mu = 5$ vs $H_A : \mu < 5$ ✓

Step 2: Assume Ho is true

Suppose we want to test a population mean μ.

Example 88. We know that the national average on a standardized math exam is 50 points. The test average for a random sample of 100 second-graders is 54 points with a sd of 10 points.

μ = avg. among all second-graders

Question: Are the second-graders smarter than the national average?

Step 1: $H_0: \mu = 50$ $\quad H_a: \mu > 50$ ← second-graders are smarter

Step 2: Assume Ho is true.

Step 3: $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ $\quad \sigma$ = st. dev. in whole second-grader population

In order to find the ~~RR~~ p-value, we need:

(1) Sampling distribution of test statistic, (for our example, we need the distribution of $\overline{X}$).

(2) A significance level α, usually given to you. If α is not given, always assume __________. ← to decide what makes a small probability

Lec 30

Example 88 continued. Say 1% significant level,

p-value

$$P(\bar{X} \geq 54) = P\left(Z \geq \frac{54-50}{10/\sqrt{100}}\right)$$

probability that under the null we see data like ours.

$$= P(Z \geq 4) \leq 0.0002$$

↑ from table

Step 4: Conclude: since $0.0002 < 0.01$, reject H_0

the p-value < 0.01, it is unlikely to observe data like ours if H_0 were true.

Using CI instead: LCB $\rightarrow$ $\bar{X} - z_{0.01} \cdot \frac{s}{\sqrt{n}}$ z not t b/c n is large

$z_{0.01} = 2.33$ b/c testing $H_a: \mu > 50$

$54 - 2.33 \times \frac{10}{\sqrt{100}} = 51.67$ b/c $\alpha = 0.01$

Step 4: (using CI) since $51.67 > 50$ we are 99% confident H_a is true

The decision always has to be a statement about H_0. We reject H_0 or we fail to reject H_0 based on where our test statistic falls in relationship to our RR.

Example 88 continued.

Example 89. (From HW) Determine whether or not each of the following statements is correct.

(a) The value of the test statistic does not lie in the rejection region. Therefore, we accept the null hypothesis.

(b) The value of the test statistic lies in the rejection region. Therefore, there is sufficient evidence to suggest the alternative hypothesis is true.

(c) The value of the test statistic does not lie in the rejection region. Therefore, there is evidence to suggest the null hypothesis is true.

(d) The value of the test statistic does not lie in the rejection region. Therefore, there is insufficient evidence to suggest the alternative hypothesis is true.

8.1.2 Errors in Hypothesis Testing

A **type I error**: reject H_0 when it is true

A **type II error** fail to reject H_0 when it is false

Example 90.

1) $H_0 : \mu = 100$ (This is true) vs $H_A : \mu > 100$. If our test fails to reject H_0,

Good job!

2) $H_0 : \mu = 100$ (This is true) vs $H_A : \mu > 100$. If our test reject H_0,

Type I error

e.g. Reject H_0 if $P(\bar{X} \geq$ sample mean of our data$) \leq \alpha$ ← p-value
($H_0: \mu =$ value $H_a: \mu >$ value)

p-value = P(type I error) so α is an upper bound or P(type I error)

Usually α is given in our test. If α is not given, we use α=0.05.

3) $H_0 : \mu = 100$ vs $H_A : \mu > 100$ (This is true). If our test fails to reject H_0,

Type II error

β = P(type II error)
$1-\beta$ = power of test.

4) $H_0 : \mu = 100$ vs $H_A : \mu > 100$ (This is true). If our test reject H_0,
Good job.

Example 91. Water samples are taken from water used for cooling as it is being discharged from a power plant into a river. It has been determined that as long as the mean temperature of the discharged water is at most $150°F$, there will be no negative effects on the river's ecosystem. To investigate whether the plant is in compliance with regulations that prohibit a mean discharge water temperature above $150°F$, 50 water samples will be taken at randomly selected times and the temperature of each sample recorded. The resulting data will be used to test the hypotheses $H_0 : \mu = 150$ versus $H_A : \mu > 150$. In the context of this situation, describe type I and type II errors.

Solution.
Type I: We reject ~~H_0~~ H_0 in favor of $H_A: \mu > 150$
Type II: we fail to reject H_0 when it is true

Step 6: _______________ "Based on our evidence, at α significant level, we conclude that ... "

Example 92. (Exercise 19 on textbook page 333) The melting point of each of 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in $\overline{x} = 94.32$. Assume that the distribution of the melting point is normal with $\sigma = 1.2$.

Question: Test $H_0 : \mu = 95$ versus $H_A : \mu \neq 95$ using a two-tailed level 0.01 test.

Solution.

CI: $\bar{X} \pm z_{\frac{0.01}{2}} \frac{\sigma}{\sqrt{n}}$ $\quad \bar{X} \leftarrow 94.32$, $\sigma \leftarrow 1.2$, $n \leftarrow 16$ $\quad z_{.005} = 2.57$
b/c two-sided H_A

$94.32 \pm 2.57 \times 0.3$ $\quad (93.55, 95.09)$ $\quad$ 95 in inside so fail to reject w/ p-value: $P(|\bar{X}-95| > 0.68)$

$95 - 94.32 = 0.68$

$\bar{X} \sim N\left(95, \frac{1.2}{\sqrt{16}}\right)$