Lec 29

8 Tests of Hypotheses Based on a Single Sample

A parameter can be estimated from sample data either by a single number (a point estimate) or an entire interval of plausible values (a confidence interval). Frequently, however, the objective of an investigation is not to estimate a parameter but to decide which of two contradictory claims about the parameter is correct. Methods for accomplishing this is called ______________________________.

8.1 Hypotheses and Test Procedures

A hypothesis is a claim about the value of a population parameter.

Examples:

- The claim $\mu = 0.75$, where μ is the true average inside diameter of a certain type of PVC pipe.
- The statement $p = 0.1$, where p is the proportion of defective circuit boards among all circuit boards produced by a certain manufacturer.

In any hypothesis-testing problem, there are two contradicting hypotheses under consideration.

- One hypothesis might be the claim $\mu = 0.75$ and the other $\mu \neq 0.75$.
- One hypothesis might be the claim $p = 0.1$ and the other $p < 0.1$.

8.1.1 Test Procedures

The objective is to decide, based on sample information, which of the two hypotheses is correct.

Null hypothesis H_0 is the claim that is initially assumed to be true.

Alternate hypothesis H_a is the assertion that is contradictory to H_0.

The null hypothesis will be rejected in favor of the alternative hypothesis only if sample evidence suggests that H_0 is false. If the sample does not strongly contradict H_0, we will continue to believe in the plausibility of the null hypothesis.

The two possible conclusions from a hypothesis-testing analysis are_______________

______________________________.

A test of hypothesis is a method for using sample data to decide whether the null hypothesis should be rejected.

Step 1: State the opposing hypotheses

The null hypothesis, H_0, (in STAT 300)

- statements pop. parameter
- always equality $\mu =$ value or $p =$ value

The alternative hypothesis, H_A,

- statement about pop. parameter
- what we are trying to prove

For example, suppose the true average time to pain relief for the current best-selling pain reliever is known to be 15 minutes. A new formulation has been developed that it is hoped will reduce this time. The relevant hypotheses are $H_0 : \mu = 15$ versus $H_A : \mu < 15$, where μ is the true average time to relief using the new formulation.

- Three forms
  - par. > value
  - par. < value
  - par $\neq$ value

NOTE:

1. The only difference between the null hypothesis H_0 and H_A is the sign in the middle.

2. Only one hypothesis can be true in a given situation.

Example 87. (From HW) Determine whether or not each of the following is a valid pair of hypotheses.

(a) $H_0 : \overline{x} = 5$ vs $H_A : \overline{x} < 5$ $\bar{x}$ is a sample mean

(b) $H_0 : p = 0.7$ vs $H_A : p \neq 0.7$ ✓

(c) $H_0 : \mu = 5$ vs $H_A : \mu \geq 5$ ✗ should be $\mu > 5$

(d) $H_0 : p = 0.3$ vs $H_A : p = 0.5$ Not in this course

(e) $H_0 : \mu = 5$ vs $H_A : \mu < 5$ ✓

Step 2: Assume Ho is true

Suppose we want to test a population mean μ.

Example 88. We know that the national average on a standardized math exam is 50 points. The test average for a random sample of 100 second-graders is 54 points with a sd of 10 points.

μ = avg. among all second-graders

Question: Are the second-graders smarter than the national average?

Step 1: $H_0: \mu = 50$ $H_a: \mu > 50$ ← second-graders are smarter

Step 2: Assume Ho is true.

Step 3: $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ σ = std dev. in whole second-grader population

In order to find the ~~RR~~ p-value, we need:

(1) Sampling distribution of test statistic, (for our example, we need the distribution of $\bar{X}$).

(2) A significance level α, usually given to you. If α is not given, always assume __________. ← to decide what makes a small probability

Example 88 continued. Say 1% significant level,

p-value

$$P(\bar{X} \geq 54) = P\left(Z \geq \frac{54-50}{10/\sqrt{100}}\right)$$

probability that under the null we see data like ours.

$$= P(Z \geq 4) \leq 0.0002$$

← from table

Step 4: Conclude: since $0.0002 < 0.01$, reject H_0

the p-value < 0.01, it is unlikely to observe data like ours if H_0 were true.