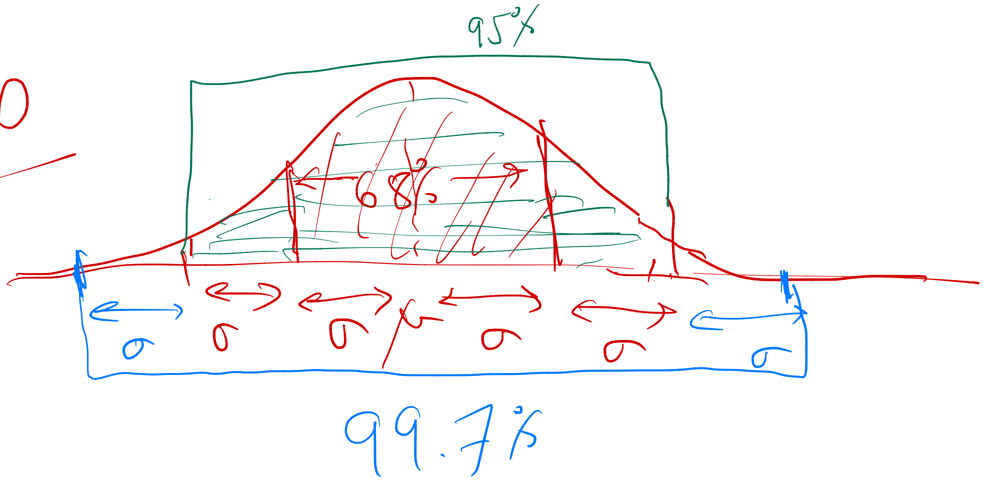
Part (c) in Example 64 can be answered without knowing either μ or σ, as long as the distribution is known to be normal, the answer is the same for any normal distribution:

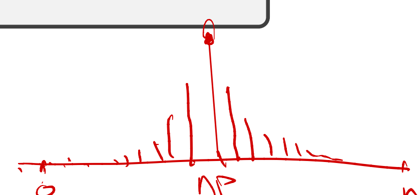
Example 65. The breakdown voltage of a randomly chosen diode of a particular type is known to be normally distributed. What is the probability that a diode's breakdown voltage is within 1 standard deviation of its mean value?

Solution.

Lec 20



If the population distribution of a variable is (approximately) normal, then
1. Roughly <u>68%</u> of the values are within 1 SD of the mean.
2. Roughly <u>95%</u> of the values are within 2 SDs of the mean.
3. Roughly <u>99.7%</u> of the values are within 3 SDs of the mean.



4.3.5 Normal Approximation to the Binomial Distribution

max, fn for Bin(n,p) looks like normal:

- Let X be a binomial random variable based on n trials with success probability p. So $X \sim b(n,p)$.
- If the binomial probability histogram is not too skewed, and both np and $n(1-p)$ are ≥ 10.

Then X has approximately a normal distribution with $\mu = np$ and $\sigma = \sqrt{np(1-p)}$. Then

$$P(X \le b) \approx P(N(np, np(1-p)) \le b + 0.5)$$

$$P(X \ge a) \approx P(N(np, np(1-p)) \ge a - 0.5)$$

$$P(a \le X \le b) \approx P(a - 0.5 \le N(np, np(1-p)) \le b + 0.5)$$

Note: 0.5 is the correction for continuity. Let's see why we want this correction in the next example.

Example 66. If $X \sim b(25, 0.6)$, We can approximate X with

$\underbrace{25 \times 0.6}_{np} = 15 \geq 10$ ✓

$N(\underset{np}{15}, 6)$

$np(1-p) = 15 \times 0.4 = 6$

Therefore,

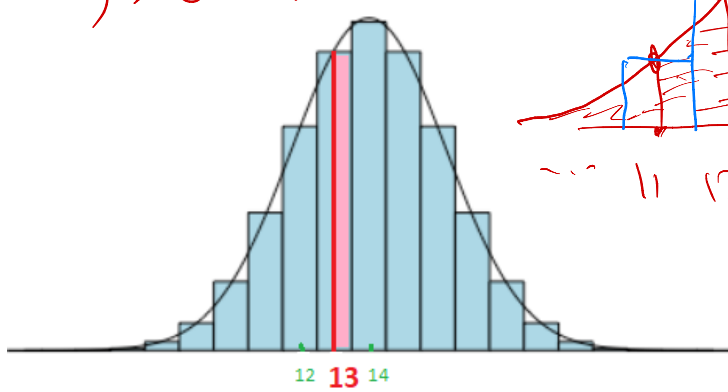
$$P(X \leq 13) \approx P(Y \leq 13) = P\left(\underset{N(0,1)}{Z} \leq \frac{13-15}{\sqrt{6}}\right) = P(Z \leq -0.82) \approx 0.206$$

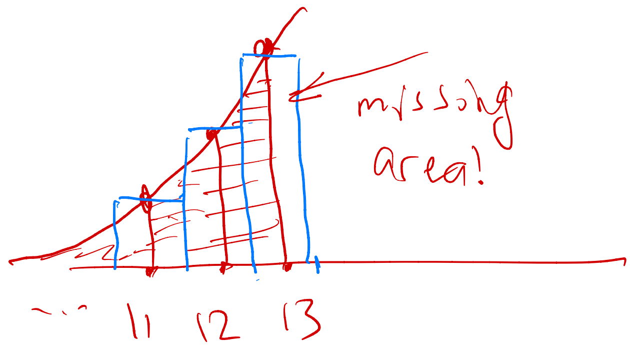
while the exact binomial calculation gives:

$$P(X \leq 13) = 0.267$$

The approximation is good! But still can be improved.

$P(Y \leq 13.5) \approx 0.271$





Normal approximation with the continuity correction.

Figure above shows that when we use $P(Y \leq 13)$ to approximate $P(X \leq 13)$, the normal approximation is _______________ than the exact binomial value. The area of the bars to the left of 13.5 gives $P(X \leq 13)$; the area under the curve to the left of 13 gives $P(Y \leq 13)$.

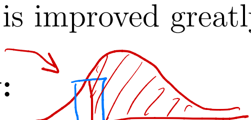
Correction:

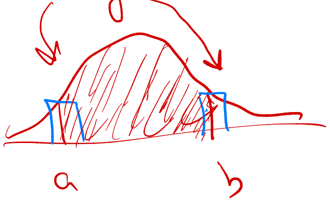
$$P(X \leq 13)$$

The result is improved greatly!

Summary:

$$P(X \leq x) \approx P(Y \leq x + 0.5)$$
$$P(X \geq x) \approx P(Y \geq x - 0.5)$$





e.g.

$P(X \leq 13) \approx P(Y \leq 13.5)$

$P(X \geq 13) \approx P(Y \geq 12.5)$

$P(5 \leq X \leq 18) \approx P(4.5 \leq Y \leq 18.5)$

$P(a \leq X \leq b) = P(a-0.5 \leq X \leq b+0.5)$

Example 67. (Exercise 55 on textbook page 169) Suppose only 75% of all drivers in a certain state regularly wear a seat belt. A random sample of 500 drivers is selected. What is the probability that

a. Between 360 and 400 (inclusive) of the drivers in the sample regularly wear a seat belt?

b. Fewer than 400 of those in the sample regularly wear a seat belt?

Solution. Let X = the number of drivers regularly wear a seat belt.

$p = 0.75 \quad n = 500 \qquad X \sim Bin(500, 0.75)$

$np = 500 \times 0.75 = 375 \geq 10$

$n(1-p) = 500 \times 0.25 = 125 \geq 10$

$Y \sim N(375, 93.75)$

$np(1-p) = 375 \times 0.25 = 93.75$

$P(360 \leq X \leq 400)$

$\approx P(359.5 \leq Y \leq 400.5)$

$= P\left(\frac{359.5 - 375}{\sqrt{93.75}} \leq Z \leq \frac{400.5 - 375}{\sqrt{93.75}}\right)$

$= P(-1.6 \leq Z \leq 2.63)$

$= P(Z \leq 2.63) - \underbrace{P(Z \leq -1.6)}$

$= 0.9957 - 0.0548$

$= 0.9409$

$= P(Z \geq 1.6)$

$= 1 - P(Z \leq 1.6)$

b) $P(X < 400) = P(X \leq 399)$

$\approx P(Y \leq 399.5) = P\left(Z \leq \frac{399.5 - 375}{\sqrt{93.75}}\right)$

$= 0.9943$