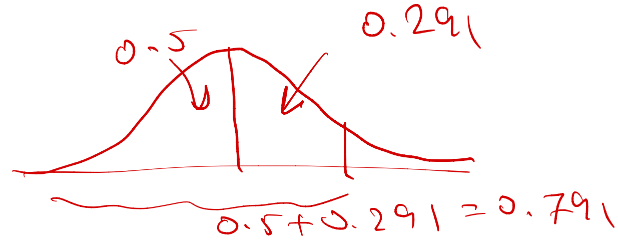
Lec 19

$\Phi(c) = P(Z \le c)$

Example 62. Determine the value of the constant c that makes the probability statement correct.

(a) $\Phi(c) = 0.9838$ $c = 2.14$



(b) $P(0 \le Z \le c) = 0.291$ 0.5

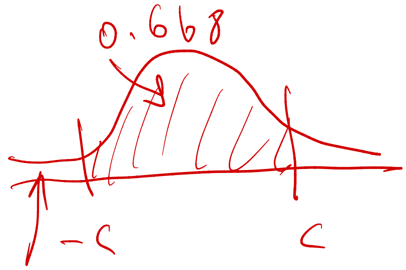
$P(Z \le c) = P(Z < 0) + P(0 \le Z \le c) = 0.791$

$c = 0.81$

(c) $P(c \le Z) = 0.121$ $P(Z \le c) = 1 - 0.121 = 0.879$

$1 - P(Z < c) = 1 - P(Z \le c)$ $c = 1.17$

(d) $P(-c \le Z \le c) = 0.668$



$0.668 + \frac{1-0.668}{2} = 0.834$

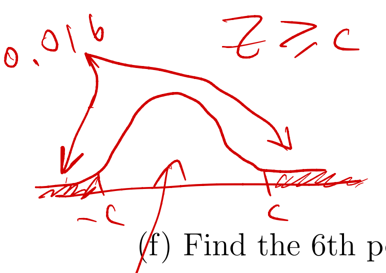
or

$0.5 + 0.334 = 0.834$

$c = 0.97$

(e) $P(c \le |Z|) = 0.016$

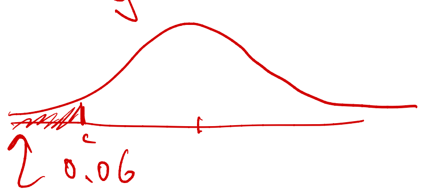
$Z \ge c$ or $-Z \ge c$ → $Z \le -c$



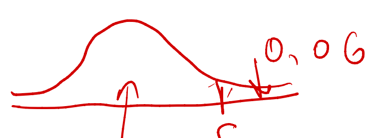
$1 - 0.016 + \frac{0.016}{2} = 0.992$

$c = 2.41$

(f) Find the 6th percentile for the standard normal curve



$c = -1.56$



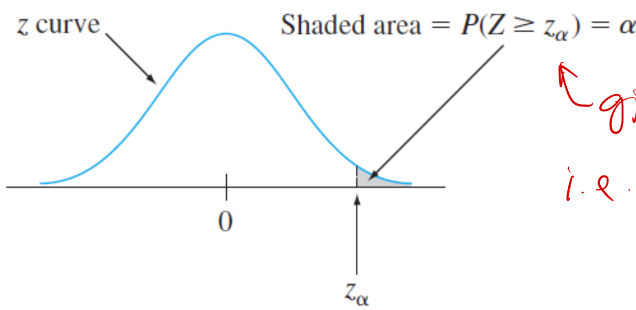
$1 - 0.06 = 0.94$

1.56

4.3.3 z_α Notation for z Critical Values

> **Notation:** z_α will denote the value on the z axis for which α of the area under the z curve lies to the right of z_α. (See Figure below.)

For example, $z_{.10}$ captures upper-tail area .10, and $z_{.01}$ captures upper-tail area .01.



z_α notation Illustrated

← given α, find z_α

i.e. find c:

$P(X \geq c) = \alpha$

so look up $1-\alpha$ in the table

Since α of the area under the z curve lies to the right of z_α, $1-\alpha$ of the area lies to its left. Thus z_α is the $100(1-\alpha)$th percentile of the standard normal distribution. By symmetry the area under the standard normal curve to the left of $-z_\alpha$ is also α. The z_α's are usually referred to as z critical values.

Example 63.

(a) Find $z_{0.0055}$

$1 - 0.0055$

$= 0.9945$

c

2.54

(b) Find $z_{0.6630}$

c

-0.42

0.6630

?

≈ 0.42

4.3.4 Nonstandard Normal Distributions

When $X \sim N(\mu, \sigma^2)$, probabilities involving X are computed by "standardizing."

If X has a normal distribution with mean μ and standard deviation σ, then

$$Z = \frac{X-\mu}{\sigma} \leftarrow \text{standard normal!} \quad (\text{standardization})$$

has a standard normal distribution. Thus

$$P(\underbrace{a \le X \le b}) = P\left(\frac{a-\mu}{\sigma} \le \underbrace{\frac{X-\mu}{\sigma}}_{Z} \le \frac{b-\mu}{\sigma}\right)$$

$$= P\left(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right)$$

$P(X \le b)$ & $P(X \ge a)$ similar

By standardizing, any probability involving X can be expressed as a probability involving a standard normal random variable Z, so that Appendix Table A.3 can be used.

$X \sim N(46.8, 1.75^2)$

Example 64. (Exercise 33 on textbook page 167) X = maximum speed of a car. A normal distribution with mean value 46.8 km/h and standard deviation 1.75 km/h is postulated. Consider randomly selecting a single such car.

$$P(X \le 50) = P\left(Z \le \underbrace{\frac{50-46.8}{1.75}}_{1.83}\right) \approx 0.9664$$

a. What is the probability that maximum speed is at most 50 km/h?

b. What is the probability that maximum speed is at least 48 km/h?

$$P(X \ge 48) = P\left(Z \ge \frac{48-46.8}{1.75}\right)$$

c. What is the probability that maximum speed differs from the mean value by at most 1.5 standard deviations?

d. Find the 75th percentile for the max speed.

Solution.

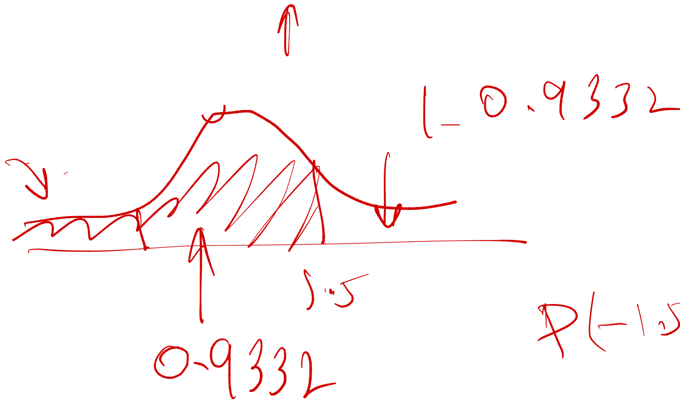
$$P(Z \ge 0.69)$$

$$= 1 - P(Z \le 0.69)$$

$$= 1 - 0.7549$$

$$P(46.8 - 1.5 \times 1.75 \le X \le 46.8 + 1.5 \times 1.75)$$

$$= P(-1.5 \le Z \le 1.5)$$



$$P(-1.5 \le Z \le 1.5)$$

$$= 0.9332 - (1 - 0.9332)$$