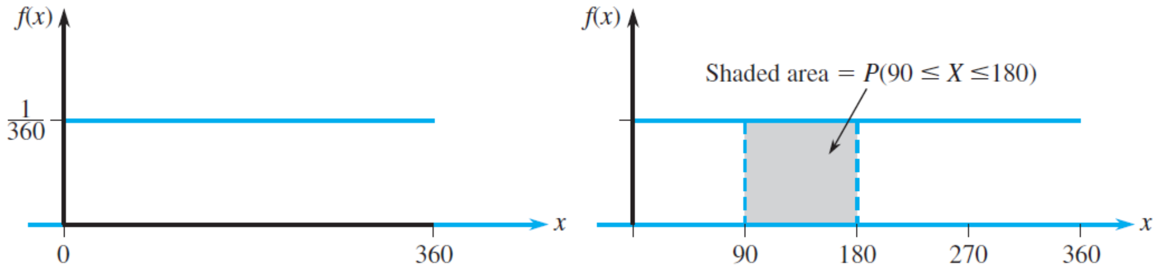
Lec 16

X has pdf f. $P(X \in A) = \int_A f(x)\,dx$



NOTE: Because whenever $0 \le a \le b \le 360$ in Example 51, $P(a \le X \le b)$ depends only on the width $b - a$ of the interval, X is said to have a uniform distribution.

Definition 17. A continuous random variable X is said to have a uniform distribution on the interval $[A, B]$ if the pdf of X is

$$f(x) = \begin{cases} \frac{1}{B-A} & \text{if } x \text{ is in } [A,B] \\ 0 & \text{o/w} \end{cases}$$

The graph of any uniform pdf looks like the graph in Figure above except that the interval of positive density is $[A, B]$ rather than $[0, 360]$.

Recall: When X is a discrete random variable, each possible value is assigned a 0 probability. This is not true of a continuous random variable because the area under a density curve that lies above any single value is zero:

$$P(X = a) = \int_a^a f(x)\,dx = 0$$

The fact that $P(X = c) = 0$ when X is continuous has an important practical consequence: The probability that X lies in some interval between a and b does not depend on whether the lower limit a or the upper limit b is included in the probability calculation:

$$P(a < X \le b) = P(a \le X \le b) = P(a \le X < b) = P(a < X < b)$$

Example 52. "Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars on a freeway during a period of heavy flow. The pdf of X is

$$f(x) = \begin{cases} 0.15e^{-0.15(x-5)} & x \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Check: $f \geq 0$ ✓

$$\int_{-\infty}^{\infty} f(x)\,dx$$

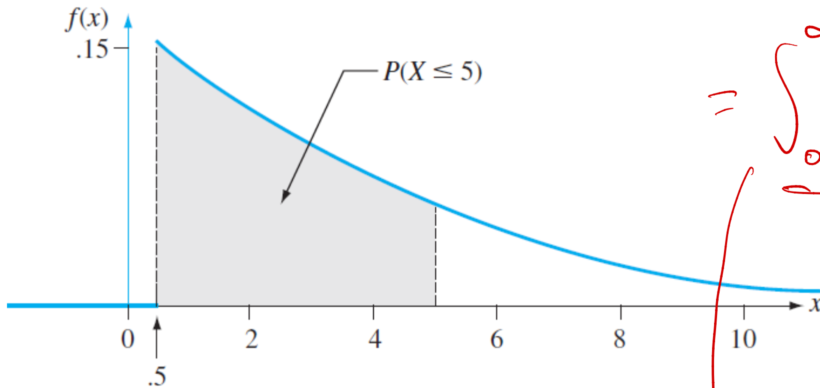
$$= \int_{0.5}^{\infty} 0.15\, e^{-0.15(x-.5)}\,dx$$

$$\left(e^{-0.15(x-.5)}\right)'$$

$$= -0.15\, e^{-0.15(x-.5)}$$

$$-e^{-0.15(x-0.5)}\Big|_{0.5}^{\infty}$$

$$= 0 + 1$$



The density curve for time headway

$$P(X \leq 5)$$

$$= \int_{-\infty}^{5} f(x)\,dx$$

$$= \int_{0.5}^{5} 0.15\, e^{-0.15(x-0.5)}\,dx$$

$$= -e^{-0.15(x-0.5)}\Big|_{0.5}^{5} = 1 - e^{-0.15 \times 4.5}$$

Example 53. (Exercise 1 on textbook page 146) The current in a certain circuit is a continuous random variable X with the following density function:

$$f(x) = \begin{cases} 0.075x + 0.2 & 3 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

a. Graph the pdf and verify that the total area under the density curve is indeed 1.

b. Calculate $P(X \leq 4)$. How does this probability compare to $P(X < 4)$?

c. Calculate $P(3.5 \leq X \leq 4.5)$ and also $P(4.5 < X)$.

$P(X \leq 4)$

$P(3.5 \leq X \leq 4.5)$

$P(X > 4.5)$

4.2 Cumulative Distribution Functions and Expected Values

4.2.1 The Cumulative Distribution Function

Recall: The cumulative distribution function (cdf) $F(x)$ of a discrete random variable X with pmf $p(x)$ is defined for every number x in $\mathbb{R}$ by

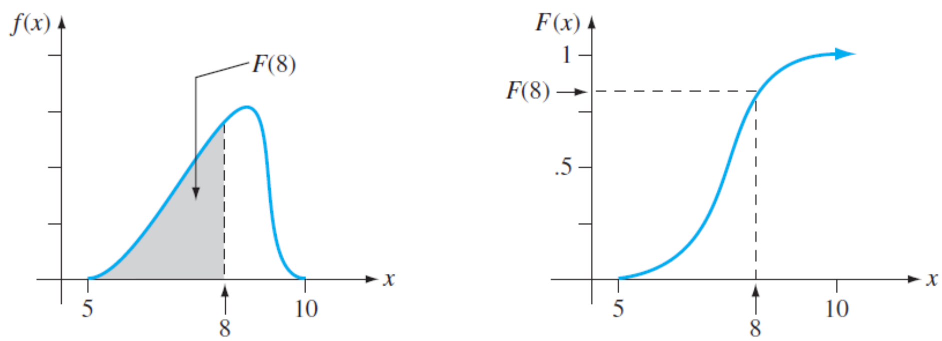
$$F(x) = P(X \leq x) = \sum_{y:\ y \leq x} p(y)$$

The cdf of a continuous random variable gives the same probabilities $P(X \leq x)$ and is obtained by replacing summation by integration.

The **cumulative distribution function** $F(x)$ for a continuous random variable X is defined for every number x by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(s)\,ds$$

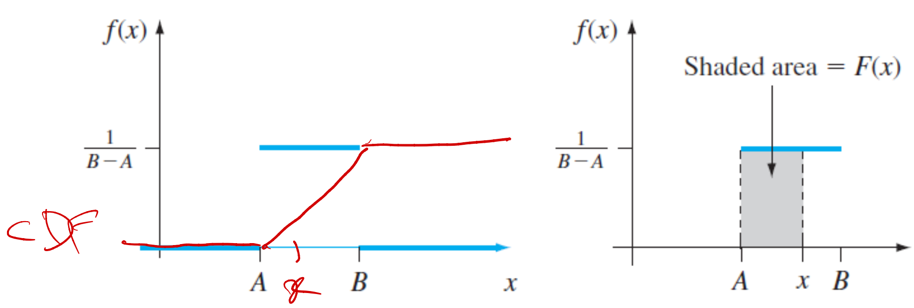
For each x, $F(x)$ is the area under the density curve to the left of x. This is illustrated in Figure below, where $F(x)$ increases smoothly as x increases.



A pdf and associated cdf

Example 54. Let X, the thickness of a certain metal sheet, have a uniform distribution on $[A, B]$. The density function is shown in Figure below.

$$\int_{-\infty}^{x} f(s)\,ds = \int_{A}^{x} \frac{1}{B-A}\,ds = \frac{1}{B-A}\, s \Big|_{A}^{x} = \frac{x-A}{B-A}$$



The pdf for a uniform distribution