$X \sim Poi(7)$ $\quad P(X \le 5) = 0.301$

Table A.2 Cumulative Poisson Probabilities *(cont.)*

$$F(x;\mu) = \sum_{y=0}^{x} \frac{e^{-\mu}\mu^{y}}{y!}$$

	μ										
	2.0	**3.0**	**4.0**	**5.0**	**6.0**	**7.0**	**8.0**	**9.0**	**10.0**	**15.0**	**20.0**
0	.135	.050	.018	.007	.002	.001	.000	.000	.000	.000	.000
1	.406	.199	.092	.040	.017	.007	.003	.001	.000	.000	.000
2	.677	.423	.238	.125	.062	.030	.014	.006	.003	.000	.000
3	.857	.647	.433	.265	.151	.082	.042	.021	.010	.000	.000
4	.947	.815	.629	.440	.285	.173	.100	.055	.029	.001	.000
5	.983	.916	.785	.616	.446	.301	.191	.116	.067	.003	.000
6	.995	.966	.889	.762	.606	.450	.313	.207	.130	.008	.000
7	.999	.988	.949	.867	.744	.599	.453	.324	.220	.018	.001
8	1.000	.996	.979	.932	.847	.729	.593	.456	.333	.037	.002
9		.999	.992	.968	.916	.830	.717	.587	.458	.070	.005
10		1.000	.997	.986	.957	.901	.816	.706	.583	.118	.011
11			.999	.995	.980	.947	.888	.803	.697	.185	.021
12			1.000	.998	.991	.973	.936	.876	.792	.268	.039
13				.999	.996	.987	.966	.926	.864	.363	.066
14				1.000	.999	.994	.983	.959	.917	.466	.105
15					.999	.998	.992	.978	.951	.568	.157

If X has a Poisson distribution with parameter μ, then

$$E(X) = \mu$$
$$\mathrm{Var}(X) = \mu$$

$\sigma_X = \sqrt{\mu}$

Lec15

Example 47. (Example 46 continued)

$E[X] = 2$

$\mathrm{Var}(X) = 2 \quad \sigma_X = \sqrt{2}$

$P(2-\sqrt{2} \le X \le 2+\sqrt{2})$
$= P(0.6 \le X \le 3.4)$
$= P(X = 1, 2, 3)$

$P(X \le 3) - P(X = 0)$
$= 0.857 - 0.135$

e^{-2}

Example 48. Suppose the number of accidents per month at an industrial plant has a Poisson distribution with mean 2.6. If we denote Y = the number of accidents per month,

then $Y \sim Poi(2.6)$

(a) Find the probability that there will be 4 accidents in the next month.

$P(Y=4) = e^{-\mu}\dfrac{\mu^4}{4!} = e^{-2.6}\dfrac{2.6^4}{4!}$

(b) Find the probability of two or more accidents in the

$$\sum_{k=2}^{\infty} e^{-2.6} \frac{2.6^k}{k!}$$

$$P(Y \geq 2)$$

$$= 1 - P(Y \leq 1) = 1 - \left(e^{-2.6}\frac{2.6^0}{0!} + e^{-2.6}\frac{2.6}{1!}\right)$$

(c) Find the probability of having between 3 and 6 accidents.

$$P(3 \leq Y \leq 6) = e^{-2.6}\left(\frac{2.6^3}{3!} + \frac{2.6^4}{4!} + \frac{2.6^5}{5!} + \frac{2.6^6}{6!}\right)$$

Example 49. Let X be the number of material anomalies occurring in a particular region of an aircraft disk. Some article proposes a Poisson distribution for X. Suppose that $\mu = 4$.

a. Compute both $P(X \leq 4)$ and $P(X < 4)$. $\leftarrow P(X \leq 3)$

b. Compute $P(4 \leq X \leq 8)$.

c. Compute $P(8 \leq X)$. $\rightarrow 1 - P(X \leq 7) = 1 - 0.949$

d. What is the probability that the number of anomalies exceeds its mean value by no more than one standard deviation?

Solution.

$$P(4 < X \leq 4 + \sqrt{4})$$

$$= P(5 \leq X \leq 6) = \cdots$$

4 Continuous Random Variables and Probability Distributions

Chapter 3 concentrated on the development of probability distributions for discrete random variables. In this chapter, we consider the second general type of random variable that arises in many applied problems: the continuous random variable

4.1 Probability Density Functions

Recall: A discrete random variable is one whose possible values either constitute a finite set or can be listed in an infinite sequence.

A random variable X is continuous if

(1) possible values comprise either a single interval on the number line (for some $a < b$, any number x between a and b is a possible value) or union of disjoint intervals, and

(2) $P(X=c)=0$ for any number c that is a possible value of X.

Examples:

- A chemical compound is randomly selected and its pH X is determined, then X is a continuous random variable because any pH value between 0 and 14 is possible.
- A location in the United States is selected and the height above sea level , Y, is observed. Then the set of all possible values of Y is the set of all numbers in the interval between 2282 and 14,494.
- The highest temperature of the day, Z, is observed and theoretically speaking, Z could be any numbers in $\mathbb{R}$.

Probability Distributions for Continuous Variables

Definition 16. Let X be a continuous random variable. Then the probability density function of X is a function $f(x)$ such that for any two numbers a and b with $a \leq b$,

pdf

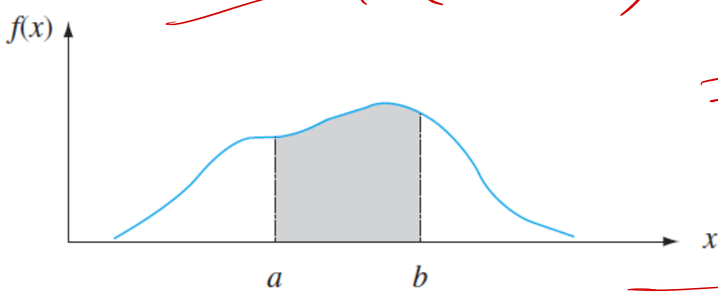
$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

That is, the probability that X takes on a value in the interval $[a, b]$ is the area above this interval and under the graph of the density function.

Note: $P(X = x) = P(x \leq X \leq x)$

$$= \int_x^x f(s)\,ds$$

$$= 0 \neq f(x)$$



$P(a \leq X \leq b)$ = the area under the density curve between a and b

For $f(x)$ to be a legitimate pdf, it must satisfy the following two conditions:

1. $f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Uniform(a, b)

pdf: $f(x) = \begin{cases} 0 & \text{if } x \notin [a,b] \\ \frac{1}{b-a} & \text{if } x \in [a,b] \end{cases}$

Example 51. Consider the reference line connecting the valve stem on a tire to the center point, and let X be the angle measured clockwise to the location of an imperfection. One possible pdf for X is

$$f(x) = \begin{cases} \dfrac{1}{360} & 0 \leq x \leq 360 \\ 0 & \text{otherwise} \end{cases}$$

Solution.

$a = 0$ $\quad$ $b = 360$