Lec12

$Var(X) = E[(X-E[X])^2] = E[X^2] - E(X)^2$

Variance of a Linear Function

> The variance of $aX + b$ is $Var(aX+b) = a^2 Var(X)$
>
> The standard deviation of $aX + b$ is $\sigma_{aX+b} = |a| \sigma_X$

Example 38. In Example 35,

$Var(X+b) = Var(X)$

$Var(aX) = a^2 Var(X)$

$\sigma_{aX} = \sqrt{Var(aX)} = \sqrt{a^2 Var(X)}$

$= |a| \sigma_X$

Example 39. An dealer sells three different models of freezers having 13.5, 15.9, and 19.1 cubic feet of storage space, respectively. Let X = the amount of storage space purchased by the next customer. Suppose that X has pmf

$13.5^2 \times 0.2 + 15.9^2 \times 0.5 + 19.1^2 \times 0.3 = B = 272.298$

$A = 13.5 \times 0.2 + 15.9 \times 0.5 + 19.1 \times 0.3 = 16.38$

x	13.5	15.9	19.1
$p(x)$	0.2	0.5	0.3

$B - A^2 = 3.9936$

a. Compute $E(X)$, $E(X^2)$, $\text{Var}(X) = \sigma^2$ and standard deviation σ.

$\sqrt{3.9936} \approx 2$

b. If the price of a freezer having capacity X cubic feet is $25X - 8.5$, what is the expected price paid by the next customer to buy a freezer?

$E[25X - 8.5] = 25E[X] - 8.5 = 401$

c. What is the variance and the standard deviation of the price $25X - 8.5$ paid by the next customer?

$Var(25X - 8.5) = 25^2 Var(X) = 2496$

$\sigma_{25X-8.5} = \sqrt{2496} \approx 50$

d. Suppose that although the rated capacity of a freezer is X, the actual capacity is $h(X) = X - 0.01X^2$. What is the expected actual capacity of the freezer purchased by the next customer?

$E[X - 0.01X^2]$

$= E[X] + E[-0.01X^2]$

$= E[X] - 0.01E[X^2]$

$= 16.38 - 0.01 \times 272.98 = 13.66$

We saw: Bernoulli(p) prob. of 1   Prob. of 0 $= 1-p$

3.4 The Binomial Probability Distribution

Note: $Bin(1,p) = Ber(p)$

\# of trials $Bin(n, p)$ → Prob. of success

Consider an experiment consisting of n trials, each of which has exactly two outcomes.

Example 40. Suppose 2% of all items produced from an assembly line are defective. We randomly sample 50 items and count how many are defective (and how many are not).

X

possible values $0, 1, 2, 3, \ldots, 50$

0.98^{50} 0.02^{50}

Definition 13. A **binomial experiment** has the following characteristics:

(1) Fixed # of identical & indep. experiments: n

(2) each results in success or failure

(3) Prob. of success: p

~~(4)~~

Note: The probability of failure is $1-p$.

More examples of binomial experiments:

- We toss a coin n times, let S = we observe a "head" and F = we observe a "tail".
- Each of the next n vehicles undergoing an emissions test, and let S denote a vehicle that passes the test and F denote one that fails to pass.
- Tossing a thumbtack n times, with S = point up and F = point down.
- The gender (S for female and F for male) is determined for each of the next n children born at a particular hospital.

The Binomial random Variable and distribution

In most binomial experiments, it is the # of successes, rather than knowledge of exactly which trials yielded S's, that is of interest.

Definition 14. The binomial random variable X associated with a binomial experiment consisting of n trials is defined as

$X =$ # of successes

Suppose, for example, that $n = 3$. Then there are eight possible outcomes for the experiment:

SSS 3   SSF 2   SFS 2   SFF 1   FSS 2   FSF 1   FFS 1   FFF 0

From the definition of X, we have that $X(SSF) =$, $X(SFF) =$, and so on.

p^3 $p(1-p)p = p^2(1-p)$ $(1-p)p^2$ $(1-p)^3$

n : $\underbrace{SSFFFS\cdots S}_{x\ S\text{'s}\quad n-x\ F\text{'s}} \rightarrow p^x(1-p)^{n-x}$

Possible values for X in an n-trial experiment are $x =$ <u>$0, 1, 2, \ldots, n$</u>. We will often write <u>$X \sim Bin(n,p)$</u> to indicate that X is a binomial random variable based on <u>n</u> trials with success probability <u>p</u>.

Because the pmf of a binomial random variable X depends on the two parameters n and p, we denote the pmf by <u>$b(x; n,p)$</u>, where n and p are known before the experiment starts.

parameters

To derive the expression of the pmf of a binomial random variable, consider the case $n = 3$ for which each outcome, its probability, and corresponding x value are displayed in the table below.

$$P(X=2) = P(SSF, SFS, FSS) = p^2(1-p) + p(1-p)p + (1-p)p^2$$

$$= 3p^2(1-p)$$

1 F

2 S's

ways to get exactly 2 S's

More generally:

$X \sim Bin(n,p) \qquad x = 0, 1, 2, \ldots, n$

$$b(x;n,p) = P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$n-x$ F's

x S's

x S's
$n-x$ F's

ways to get exactly x S's

$$= \frac{\overbrace{n(n-1)\cdots(n-x+1)}^{x \text{ terms}}}{x!} = \frac{n!}{x!(n-x)!}$$

n choose x

ways to choose x balls from a bag w/ n identical balls